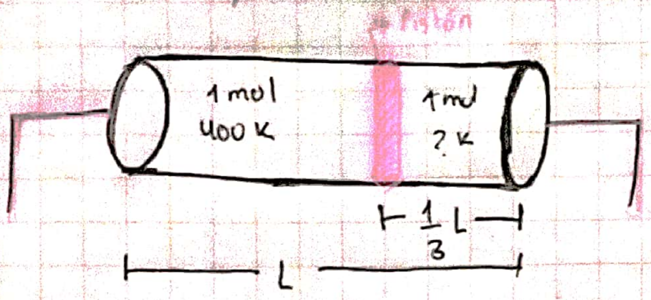

$A = 0.01 m^2$ y alambre de
$\ell = 0.30$ m cobre.

a) Encuentre la temperatura de equilibrio de la sección derecha antes de conectar el alambre de cobre ($T_0^2 = 200K$)

Aplicamos ec de gases ideales:

$$P_1 V_1 = nRT_1$$
$$P_2 V_2 = nRT_2$$

En equilibrio: $P_1 = P_2$

$$\frac{\cancel{nR}T_1}{V_1} = \frac{\cancel{nR}T_2}{V_2}$$

$$T_2 = T_1 \frac{V_2}{V_1} \quad \rightarrow \quad \begin{aligned} V_1 &= \pi r^2 (L - L/3) = \pi r^2 2L/3 \\ V_2 &= \pi r^2 (L/3) = \pi r^2 L/3 \end{aligned}$$

$$T_2 = 400K \frac{\cancel{\pi r^2 L/3}}{\cancel{\pi r^2} 2\cancel{L/3}}$$

$$T_2 = \frac{400K}{2} = 200K$$

b) No se realiza trabajo y el proceso es lo suficientemente lento. Use la primera ley de la termodinámica y la ley de transferencia de Fourier para encontrar:

$$n C_v \frac{dT_1}{dt} = -\frac{kA}{\ell}(T_1 - T_2)$$

$$n C_v \frac{dT_2}{dt} = \frac{kA}{\ell}(T_1 - T_2)$$

$C = \frac{kA}{nC_v \ell}$ y $C_v = 3/2 R$.

Las condiciones iniciales de las derivadas están bien definidas.

$$\left.\frac{dT_1}{dt}\right|_{t=0} = -C(T_1^0 - T_2^0)$$

$$\left.\frac{dT_2}{dt}\right|_{t=0} = C(T_1^0 - T_2^0)$$

Primera ley:

$$\Delta U = n c_v \Delta T = \Delta Q - \Delta W$$

Como no se realiza trabajo: $\Delta W = 0$

$$\Delta U = n c_v \Delta T = \Delta Q$$

$$\rightarrow dU = n c_v dT = dQ$$

Ley de transferencia de Fourier:

$$\frac{dQ}{dt} = \frac{-KA(T_2 - T_1)}{l}$$

$$\rightarrow \frac{dQ_1}{dt} = \frac{-KA(T_1 - T_2)}{l}$$

$$n c_v \frac{dT_1}{dt} = \frac{-KA(T_1 - T_2)}{l}$$

$$\rightarrow \frac{dQ_2}{dt} = \frac{-KA(T_2 - T_1)}{l}$$

$$n c_v \frac{dT_2}{dt} = \frac{KA(T_1 - T_2)}{l}$$

obteniendo:

$$\frac{dT_1}{dt} = \frac{-KA(T_1 - T_2)}{n c_v l} = -C(T_1 - T_2)$$

$$C = \frac{KA}{n c_v l}$$

$$\frac{dT_2}{dt} = \frac{KA(T_1 - T_2)}{n c_v l} = C(T_1 - T_2)$$

$$\left.\frac{dT_1}{dt}\right|_{t=0} = -C(200K)$$

$$\left.\frac{dT_2}{dt}\right|_{t=0} = C(200K)$$

Encuentre analíticamente la solución del sistema de ecuaciones

$T_1^0 = 400\,K$
$T_2^0 = 200\,K$

$$\frac{d}{dt}\begin{pmatrix}T_1\\T_2\end{pmatrix} = -C\underbrace{\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}}_{A}\begin{pmatrix}T_1\\T_2\end{pmatrix}$$

$$P(\lambda) = \det(A - \lambda I)$$

$$P(\lambda) = \det\begin{pmatrix}-C-\lambda & C\\ C & -C-\lambda\end{pmatrix}$$

$$P(\lambda) = (-C-\lambda)^2 - C^2$$

$$= \cancel{-C^2} + 2C\lambda + \lambda^2 \cancel{-C^2} = 0$$

$$\lambda^2 + 2C\lambda = 0$$

$\lambda = 0$, $\lambda = -2C$

$\lambda = 0$:

$$\begin{bmatrix}-C & C\\ C & -C\end{bmatrix}\begin{pmatrix}a_{11}\\ a_{12}\end{pmatrix} = 0$$

$-Ca_{11} + Ca_{12} = 0$
$Ca_{11} - Ca_{12} = 0$

$-a_{11} + a_{12} = 0 \quad \rightarrow \quad v_1 = \begin{pmatrix}1\\1\end{pmatrix} = a^{(1)}$
$[a_{11} = a_{12}]$

$\lambda = -2C$

$$\begin{bmatrix}C & C\\ C & C\end{bmatrix}\begin{pmatrix}a_{21}\\ a_{22}\end{pmatrix} = 0$$

$Ca_{21} + Ca_{22} = 0$

$a_{21} = -a_{22} \quad \rightarrow \quad v_2 = \begin{pmatrix}1\\-1\end{pmatrix} = a^{(2)}$

$$\begin{pmatrix}T_1\\T_2\end{pmatrix} = c_1\begin{pmatrix}1\\1\end{pmatrix}e^{\lambda t} + c_2\begin{pmatrix}1\\-1\end{pmatrix}e^{\lambda t}$$

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$$\begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} \cancel{e^{0t}}^{\to 1} + C_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-2ct}$$

$$T_1 = C_1 + C_2 e^{-2ct}$$

$$T_2 = C_1 - C_2 e^{-2ct}$$

$\rightarrow$ $T_1(0) = C_1 + C_2$

$400 = C_1 + C_2 \rightarrow C_1 = 400 - C_2$

$T_2(0) = C_1 - C_2$

$200 = C_1 - C_2$

$200 = 400 - C_2 - C_2$

$-200 = -2C_2$

$C_2 = 100$

$\rightarrow$ $C_1 = 400 - 100$

$C_1 = 300.$

Por lo tanto:

$$T_1 = 300 + 100e^{-2ct}$$

$$T_2 = 300 - 100e^{-2ct}$$