Punto 2

Encuentre la Serie de fourier de la función $f(t) = t$ Para el intervalo $(-\pi, \pi)$ y $f(t+2\pi) = f(t)$; animar los primeros 50 armonicos.

$$f(t) = 2\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \operatorname{sen}(nt)$$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \operatorname{Cos}(n\omega_0 t) + b_n \operatorname{Sen}(n\omega_0 t)\right) = t$$

note ahora que:

$$a_0 = \frac{2}{T}\int_{-T/2}^{T/2} f(t)dt = \frac{2}{2\pi}\int_{-\pi}^{\pi} t\,dt = \frac{1}{\pi}\left.\frac{t^2}{2}\right|_{-\pi}^{\pi} = \frac{1}{\pi}\left(\frac{\pi^2}{2} - \frac{(-\pi)^2}{2}\right) = 0$$

note que Tambien se tiene que:

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t) \operatorname{Cos}(n\omega_0 t)\,dt \quad , \quad b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t) \operatorname{Sen}(n\omega_0 t)\,dt$$

Por ende

$$a_n = \frac{2}{2\pi}\int_{-\pi}^{\pi} t \operatorname{Cos}(nt)\,dt = \frac{t\operatorname{Sen}(nt)}{n} - \int \frac{\operatorname{Sen}(nt)}{n}\,dt = \left.\frac{t\operatorname{Sen}(nt)}{n} - \frac{\operatorname{Cos}(nt)}{n^2}\right|_{-\pi}^{\pi}$$

$u = t \quad dV = \operatorname{Cos}(nt)$
$du = dt \quad V = \operatorname{Sen}(nt)/n$

$$a_n = \frac{1}{\pi}\left(\frac{\pi \operatorname{Sen}(n\pi)}{n} - \frac{\pi\operatorname{Sen}(-n\pi)}{n} - \frac{\operatorname{Cos}(n\pi)}{n^2} + \frac{\operatorname{Cos}(-n\pi)}{n^2}\right) \rightarrow \begin{array}{l}\operatorname{Cos}(-x) = \operatorname{Cos}(x) \\ \operatorname{Sen}(-x) = -\operatorname{Sen}(x)\end{array}$$

$a_n = \frac{2}{n}\operatorname{Sen}(n\pi)$ Pero note $\operatorname{Sen}(n\pi)$ siempre es 0 si $n \in \mathbb{Z}$

ahora $b_n = \frac{2}{2\pi}\int_{-\pi}^{\pi} t\,Sen(nt)\,dt$ $\quad u = t \quad dV = Sen(nt)\,dt$ $\quad du = dt \quad V = -\frac{Cos(nt)}{n}$

$$b_n = \frac{1}{\pi}\left(\frac{-t}{n}Cos(nt) + \int_{-\pi}^{\pi}\frac{Cos(nt)}{n}dt\right) = \frac{1}{\pi}\left(-\frac{t}{n}Cos(nt) + \frac{Sen(nt)}{n^2}\right)\Big|_{-\pi}^{\pi}$$

$$b_n = \frac{1}{\pi}\left(-\frac{\pi}{n}Cos(n\pi) - \left(-\frac{(-\pi)}{n}Cos(n\pi)\right) + \frac{Sen(n\pi)}{n^2} - \frac{(Sen(-n\pi))}{n^2}\right)$$

$$b_n = \frac{2}{\pi n^2}Sen(n\pi) - \frac{2}{n}Cos(n\pi) \quad \text{Como } n \in \mathbb{Z}$$

$b_n = \frac{2}{n}Cos(n\pi)$ note que Para n Par $Cos(\pi n) = 1$
caso contrario $cos(\pi n) = -1$

de modo que $b_n = (-1)^n \frac{2}{n}$ dando así:

la serie de fourier de t sería:

$\hat{f}(t) = 2\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}sen(nt)$ → mostrando así la expresión inicial