Metodo de diferencias finitas aplicado a:

$$w = \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial x^2} \; ; \; \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = \frac{\partial u}{\partial y}\frac{\partial w}{\partial x} - \frac{\partial u}{\partial x}\frac{\partial w}{\partial y}$$

Comenzamos con u

$$\frac{\partial^2 u}{\partial y^2} = \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{h^2}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

de modo que:

① $$w_{i,j} = -\frac{4u_{i,j}}{h^2} + \frac{u_{i,j-1} + u_{i-1,j} + u_{i,j+1} + u_{i+1,j}}{h^2}$$

Para el segundo termino:

$$\frac{\partial^2 w}{\partial x^2} = \frac{w_{i+1,j} - 2w_{i,j} + w_{i-1,j}}{h^2}$$

$$\frac{\partial^2 w}{\partial y^2} = \frac{w_{i,j+1} - 2w_{i,j} + w_{i,j-1}}{h^2}$$

$$\frac{\partial u}{\partial y}\frac{\partial w}{\partial x} = \left[\frac{u_{i,j+1} - u_{i,j-1}}{2h}\right]\left[\frac{w_{i+1,j} - w_{i-1,j}}{2h}\right]$$

$$\frac{\partial u}{\partial x}\frac{\partial w}{\partial y} = \left[\frac{u_{i+1,j} - u_{i-1,j}}{2h}\right]\left[\frac{w_{i,j+1} - w_{i,j-1}}{2h}\right]$$

② $$\frac{\nu}{h^2}\left(-4w_{i,j} + w_{i+1,j} + w_{i-1,j} + w_{i,j+1} + w_{i,j-1}\right) = \left[\frac{(u_{i,j+1} - u_{i,j-1})(w_{i+1,j} - w_{i-1,j})}{4h^2}\right] - \left[\frac{(u_{i+1,j} - u_{i-1,j})(w_{i,j+1} - w_{i,j-1})}{4h^2}\right]$$

Despejando en la Primera

$$+4u_{i,j} = u_{i,j-1} + u_{i-1,j} + u_{i+1,j} + u_{i,j+1} - u_{i,j}h^2$$

$$\rightarrow u_{i,j} = \frac{1}{4}\left[u_{i,j-1} + u_{i-1,j} + u_{i+1,j} + u_{i,j+1} - u_{i,j}h^2\right]$$

Despejando Para la segunda

$$u_{i,j} = \frac{R}{4}\left[(w_{i+1,j} + w_{i-1,j} + w_{i,j+1} + w_{i,j-1}) - \frac{(u_{i,j+1} - u_{i,j-1})(w_{i+1,j} - w_{i-1,j})}{4} + \frac{(u_{i+1,j} - u_{i-1,j})(w_{i,j+1} - w_{i,j-1})}{4}\right]$$

mostrando así las expresiones iniciales