

# **MEMS Accelerometer developed in COMSOL, SIMULINK and VHDL-AMS.**



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# Parallel Plate Capacitor Accelerometer

## *Introduction*

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Nowadays researching and designing are based on automatic and computerized tools. Paper is useful to reach a basic idea of the project allowing the designer to implement it on a computer. Micro system designing techniques need powerful tools because of complexity of MEMS. Our aim is to realize a simple mono-axial parallel plate capacitor accelerometer. In particular, our attention is on the designing and simulation of an accelerometer for car airbag application with COMSOL Multiphysics comparing the results with Matlab Simulink and VHDL-AMS modeling. Realized accelerometer is based on a mass-dashpot-spring system interfaced with a differential capacitor that allows to detect both positive and negative accelerations along the x-axis. Finally it will be realized a complete model including the accelerometer and a possible interface circuit using VHDL-AMS.

## Model Specifications

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Accelerometers for airbag applications need of high speed and accurate response allowing car control unit to make the right choice saving the driver life. For this reason it is important to study a car crash in order to define the specifications of our project. Our aim is the creation of mono axial accelerometer that can be used for both front and lateral car crashes. For our application we consider a car crash into a wall because it is the simplest example to define the most important parameters for the device. Another reason is that a lateral crash has a threshold lower than a front car crash so to define the input acceleration range we refer to a front one. Our starting point is to consider a car that travels at constant velocity  $V_0$  equal to 60 km/h. Considering a stop time  $t_s$  of 1/10 second and a stop space  $S_s$  of 1 meter we have:

$$S = t \cdot V_0 - \frac{1}{2} \cdot a \cdot t^2$$

Thus:

$$S_s = t_s \cdot V_0 - \frac{1}{2} \cdot a \cdot t_s^2$$

We get an average acceleration of:

$$a = 2 \cdot \frac{S_s - t_s \cdot V_0}{t_s^2}$$

It corresponds a number of g equal to:

$$n_g = 2 \cdot \frac{S_s - t_s \cdot V_0}{t_s^2} \cdot \frac{1}{g_0}$$

Substituting we get:

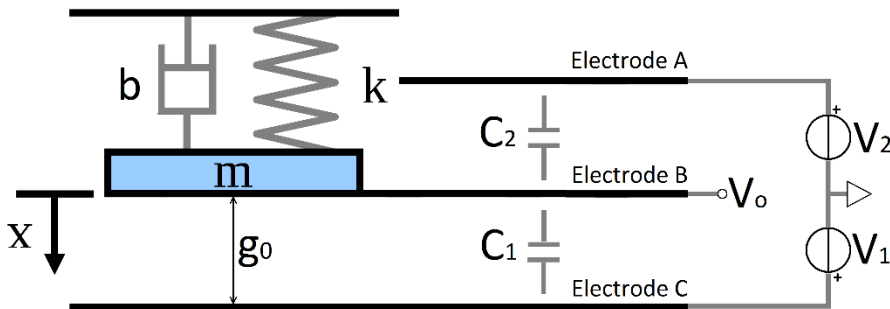
$$n_g = 14 \text{ g}$$

As we can see the acceleration is very high so our MEMS accelerometer needs a big range of sensing. In particular, from [Reference 5](#) and [6](#), we found that the typical airbag acceleration threshold is around 8 g and the time between crash and airbag opening is approximately 100 ms. Time is divided into 20 ms taken from the car control unit to make the right decision and 80 ms to the mechanical opening of the airbag. Starting from this point, we assume that accelerometer would send a correct value within about 15 ms allowing a correct sampling of signal in the remaining time. In reality a crash is more complex because depends on the type of vehicle, on its materials and on the crash dynamic. To simplify our project we consider the following specifications:

- Acceleration threshold value: 8 g.
- Acceleration input range higher than 8 g.
- Rise Time < 10 ms.
- Output Signal compatibility for standard electronic voltages (1 V, 2.5 V, 3.3 V, 5 V typical values).

## Model Definition

Accelerometers are useful sensors that can detect an acceleration along one axis or more, like in 2D or 3D systems. Capacitive accelerometers are based on mass-dashpot-spring system and they convert the displacement between two or more plates into a capacitance variation. The mass is realized with a conductive material and it corresponds to the movable plate. They can be based on a single capacitor or on a differential capacitor structure. First one is used to measure only positive acceleration while the second one is used for both positive and negative. [Figure 1](#) shows the device model.



*Figure 1: Device model: mass, dashpot, spring, movable plate and two electrodes that create two capacitors.*

From the device model we can define model equations, output voltage function and the frequency domain parameters. All of these equations allow to design our accelerometer in order to obtain the given specifications.

## Model Equations

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From [Figure 1](#) we can extract the basic mechanical equation:

$$m \cdot g'' = m \cdot a_{ext} - k \cdot g - b \cdot g' \quad (1)$$

Where  $a_{ext}$  is the external acceleration,  $m$  is the mass value,  $k$  is the spring constant,  $g$  is the gap variation and  $b$  the damping coefficient.

### Electrostatic Forces

The system includes two DC bias voltage generators,  $V_1$  and  $V_2$  connected to two electrodes. The electrode B is the movable plate connected to the mechanical system and it corresponds to our electrical output signal. The two voltage generators in fact, are useful to observe the capacitance variations but we have to take into account their related electrostatic forces. The relation (1) becomes:

$$m \cdot g'' = m \cdot a_{ext} - k \cdot g - b \cdot g' + F_{el} \quad (2)$$

Where:

$$F_{el} = F_{el1} - F_{el2}$$

$$F_{el1} = \frac{\epsilon_0 \cdot A \cdot V_1^2}{2 \cdot (g_0 - g)^2}$$

$$F_{el2} = \frac{\epsilon_0 \cdot A \cdot V_2^2}{2 \cdot (g_0 + g)^2}$$



### *Damping*

Damping coefficient is related to the interactions between the moving mechanical components and the air surrounding them. In this model we describe the behavior of the gas in a differential capacitive accelerometer. [Reference 1](#) tells us that for a system with a single degree of freedom (mass-dashpot-spring system) it is possible to model the viscous damping with the Rayleigh damping equation:

$$b = \alpha \cdot m + \beta \cdot k \quad (3)$$

Where  $\alpha$  and  $\beta$  depends on the system and can be calculated using the frequency analysis. In particular:

$$\alpha = \frac{1}{6 \cdot \pi \cdot f_0 \cdot Q}$$

$$\beta = \frac{4 \cdot \pi \cdot f_0}{3 \cdot Q}$$

On the other hand [Reference 2](#) tells us that for large accelerations and big displacements the damping coefficient is a function of the deflection. Its mathematical expression can be written as:

$$b = \frac{1}{2} \cdot u \cdot A^2 \cdot \left( \frac{1}{(g_0 - g)^3} + \frac{1}{(g_0 + g)^3} \right) \quad (4)$$

Where  $u$  is the air viscosity equal to  $1.81 \cdot 10^{-5} \frac{kg}{m \cdot s}$ ,  $A$  is the area between the plates and  $g_0$  is the starting value of the displacement

g. Since the combination of the equations (3) and (4) give us a complex model, our aim is to minimize the damping coefficient variation in order to combine the following simplified equations:

$$b_0 = u \cdot \frac{A^2}{g_0^3} \quad (5)$$

$$b_0 = \alpha \cdot m + \beta \cdot k \quad (6)$$

### *Maximum Acceleration*

The maximum acceleration is important to define the input range of our system. We consider that the  $g$  variation is between  $-g_0 + g_{min}$  and  $g_0 - g_{min}$ . These limits strongly depend on the mechanical realization of the accelerometer. Neglecting the effects of electrostatic forces and damper coefficient variation we find a theoretical value of the maximum acceleration. Imposing the value of  $g$  equal to  $g_0 - g_{min}$  we find the maximum acceleration that we can apply towards the mobile plate B. Recalling the basic equation (1) and imposing that for  $g = g_0 - g_{min}$  the resulting force becomes zero, we have:

$$0 = m \cdot a_{ext+} - k \cdot (g_0 - g_{min})$$

So the maximum positive acceleration is:

$$a_{ext+} = \frac{k \cdot (g_0 - g_{min})}{m}$$

Imposing that for  $g = -g_0 + g_{min}$  the resulting force becomes zero:

$$0 = m \cdot a_{ext-} - k \cdot (-g_0 + g_{min})$$

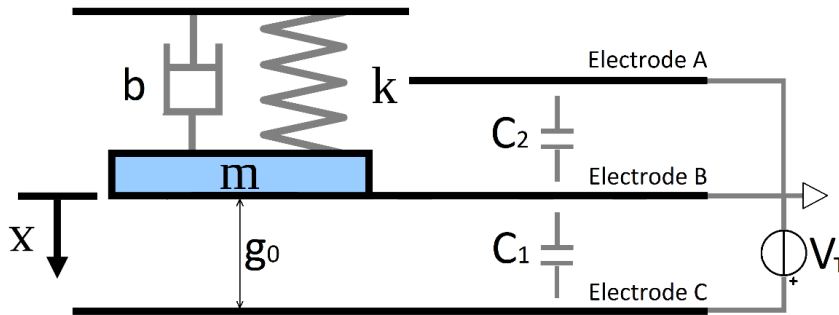
So the maximum negative acceleration is:

$$a_{ext-} = \frac{k \cdot (-g_0 + g_{min})}{m}$$

In reality these maximum values are only theoretical. The real maximum acceleration is a bit smaller for both negative and positive maximum displacements because of the electrostatic force and the damper coefficient variation. For this reason it is important to minimize the variation of damper coefficient and the maximum value of the resulting electrostatic force when the maximum acceleration is applied.

#### *Pull-In Voltage*

The definition of the pull-in voltage of our accelerometer is done considering no external forces and the only action of  $V_T$ . In this way electrodes A and B are at zero potential. The circuit for the pull-in voltage is showed in [Figure 2](#):



*Figure 2: Pull-In evaluation circuit.*

The resulting force becomes:

$$F_{Tot} = -k \cdot g + \frac{\varepsilon_0 \cdot A \cdot V_T^2}{2} \cdot \left[ \frac{1}{(g_0 - g)^2} \right]$$

Now we consider the first derivative of  $F_{Tot}$ .

$$\frac{dF_{Tot}}{dg} = -k + \frac{\varepsilon_0 \cdot A \cdot V_T^2}{1} \cdot \left[ \frac{1}{(g_0 - g)^3} \right]$$

Since we want that for a positive  $dg$  the resulting  $dF_{Tot}$  becomes negative to have a negative feedback we impose that:

$$\frac{dF_{Tot}}{dg} = -k + \frac{\varepsilon_0 \cdot A \cdot V_T^2}{1} \cdot \left[ \frac{1}{(g_0 - g)^3} \right] < 0$$

Considering the upper limit we have:

$$\frac{dF_{Tot}}{dg} = -k + \frac{\varepsilon_0 \cdot A \cdot V_T^2}{1} \cdot \left[ \frac{1}{(g_0 - g)^3} \right] = 0$$

So we obtain  $k_{PI}$ :

$$k_{PI} = \frac{\varepsilon \cdot A \cdot V_T^2}{(g_0 - g)^3}$$

Now substituting  $k_{PI}$  in  $F_{Tot}=0$  we found:

$$F_{Tot} = -k_{PI} \cdot g + \frac{\varepsilon_0 \cdot A \cdot V_T^2}{2} \cdot \left[ \frac{1}{(g_0 - g)^2} \right] = 0$$

Thus:

$$F_{Tot} = -\frac{\varepsilon_0 \cdot A \cdot V_T^2}{(g_0 - g)^3} \cdot g + \frac{\varepsilon_0 \cdot A \cdot V_T^2}{2} \cdot \left[ \frac{1}{(g_0 - g)^2} \right] = 0$$

$$-g + \frac{(g_0 - g)}{2} = 0$$

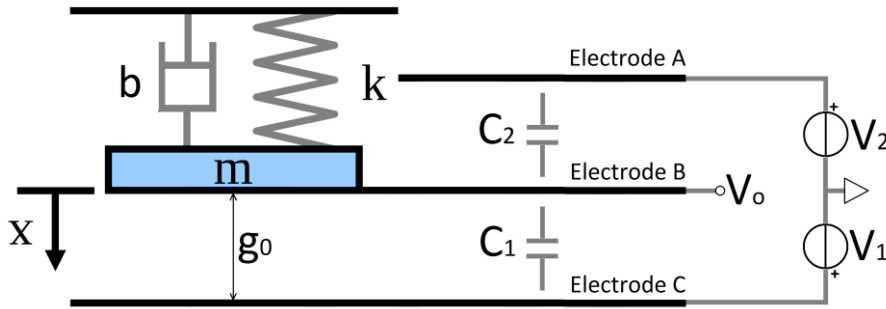
$$g_{PI} = \frac{g_0}{3}$$

Now from  $k_{PI}$  we can obtain the pull-in voltage:

$$k_{PI} = \frac{\varepsilon_0 \cdot A \cdot V_{PI}^2}{(g_0 - g)^3} = \frac{\varepsilon_0 \cdot A \cdot V_{PI}^2}{\left(g_0 - \frac{g_0}{3}\right)^3}$$

$$V_{PI} = \sqrt{\frac{8 \cdot k_{PI} \cdot g_0^3}{27 \cdot \varepsilon_0 \cdot A}}$$

The same result can be obtained if we consider  $V_T$  applied between A and B electrodes. Normally  $C_1$  is equal to  $C_2$  at the equilibrium position. Remembering the system showed [Figure 3](#):

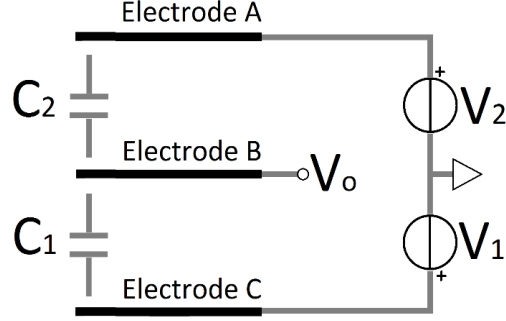


*Figure 3: Device model.*

If we impose  $V_1 = -V_2$  we have that with no external forces, for any value of  $V_1$  and  $V_2$ , the resultant electrostatic force is zero so we do not have to consider the Pull-In effect.

### Output Voltage Function

The output function represents the relationship between the variation of the capacitances  $\Delta C$  and the voltage  $V_o$ . Applying the superimposition principle on the schematic in [Figure 4](#) we can calculate the output function as follow.



*Figure 4: Output circuit.*

Considering  $V_2=0$ , we have:

$$V'_{out}(s) = \frac{\frac{1}{s \cdot C_2}}{\frac{1}{s \cdot C_2} + \frac{1}{s \cdot C_1}} \cdot V_1 = \frac{s \cdot C_1}{s \cdot C_1 + s \cdot C_2} \cdot V_1$$

Considering  $V_1=0$ , we have:

$$V''_{out}(s) = \frac{\frac{1}{s \cdot C_1}}{\frac{1}{s \cdot C_2} + \frac{1}{s \cdot C_1}} \cdot V_2 = \frac{s \cdot C_2}{s \cdot C_1 + s \cdot C_2} \cdot V_2$$

Thus:

$$V_{out}(s) = V'_{out}(s) + V''_{out}(s) = \frac{(V_1 \cdot C_1 + V_2 \cdot C_2)}{(C_1 + C_2)}$$

Calling  $C_0$  the value of the two capacitors when the  $g$  variation is zero (zero external acceleration) we can write that  $C_1 = C_0 + \Delta C$  and  $C_2 = C_0 - \Delta C$ . Since  $V_1 = -V_2$  we obtain:

$$V_{out}(s) = \frac{\Delta C \cdot V_1(s)}{C_0}$$

The resulting gain is:

$$\frac{V_{out}(s)}{\Delta C \cdot V_1(s)} = \frac{1}{C_0}$$

The last relation shows the importance of the starting value of the capacitors because it is the gain of our system. The capacitors can be calculated as a function of the displacement  $g$ :

$$C_1 = \frac{\varepsilon_0 \cdot A}{(g_0 - g)}$$

And:

$$C_2 = \frac{\varepsilon_0 \cdot A}{(g_0 + g)}$$

At zero  $g$  displacement  $C_1$  and  $C_2$  are equal to:

$$C_0 = \frac{\varepsilon_0 \cdot A}{g_0}$$

From these relations we can write that:

$$C_1 - C_2 = 2 \cdot \Delta C = \frac{\varepsilon_0 \cdot A}{(g_0 - g)} - \frac{\varepsilon_0 \cdot A}{(g_0 + g)} = \frac{\varepsilon_0 \cdot A \cdot g \cdot 2}{g_0^2 - g^2}$$

We finally obtain:

$$\Delta C = \frac{\varepsilon_0 \cdot A \cdot g}{g_0^2 - g^2}$$

### Final State Equations

From the model equations showed before we can define the final state equations choosing  $g$  as state variable. The following equations will be used in Simulink to create the schematic block of our accelerometer.

$$\begin{cases} X_{1,0} = g_0 \\ X_1 = g \\ X_2 = g' \end{cases}$$

$$\begin{cases} X_1' = X_2 \\ X_2' = \frac{1}{m} \cdot \left( m \cdot a_{ext} - k \cdot X_1 - u \cdot \frac{A^2}{X_{1,0}^3} \cdot X_2 + \frac{\varepsilon \cdot A \cdot V_1^2}{2} \cdot \left[ \frac{1}{(X_{1,0} - X_1)^2} - \frac{1}{(X_{1,0} + X_1)^2} \right] \right) \end{cases}$$

And:

$$V_{out}(s) = \frac{\Delta C \cdot V_1(s)}{C_0}$$

Where:

$$\Delta C = \frac{\varepsilon_0 \cdot A \cdot g}{g_0^2 - g^2}$$

$$C_0 = \frac{\varepsilon_0 \cdot A}{g_0}$$



### *Frequency Domain Analysis*

Starting from the simplest model equation (1):

$$m \cdot g'' = m \cdot a_{ext} - k \cdot g - b_0 \cdot g'$$

We can write the Laplace's transform:

$$m \cdot s^2 \cdot X(s) = U(s) - k \cdot X(s) - b_0 \cdot s \cdot X(s)$$

Where  $X(s)$  is  $g$  and  $U(s)$  is  $m \cdot a_{ext}$ .

The transfer function can be written as:

$$F(s) = \frac{X(s)}{U(s)} = \frac{1/m}{s^2 + \frac{b_0}{m} \cdot s + \frac{k}{m}}$$

Now we can define the main parameters of a second order system:

1) Quality Coefficient  $Q$ :

$$Q = \frac{\omega_n \cdot m}{b_0}$$

2) Damping Ratio:

$$\zeta = \frac{1}{2 \cdot Q} = \frac{b_0}{2 \cdot \sqrt{m \cdot k}}$$

3) Resonance Frequency:

$$\omega_n = \sqrt{\frac{k}{m}}$$

## Design

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### Starting Point

In the [Reference 7](#) is showed our previous accelerometer project based on parallel plate capacitor principle. Our starting point is an accelerometer with the following parameters:

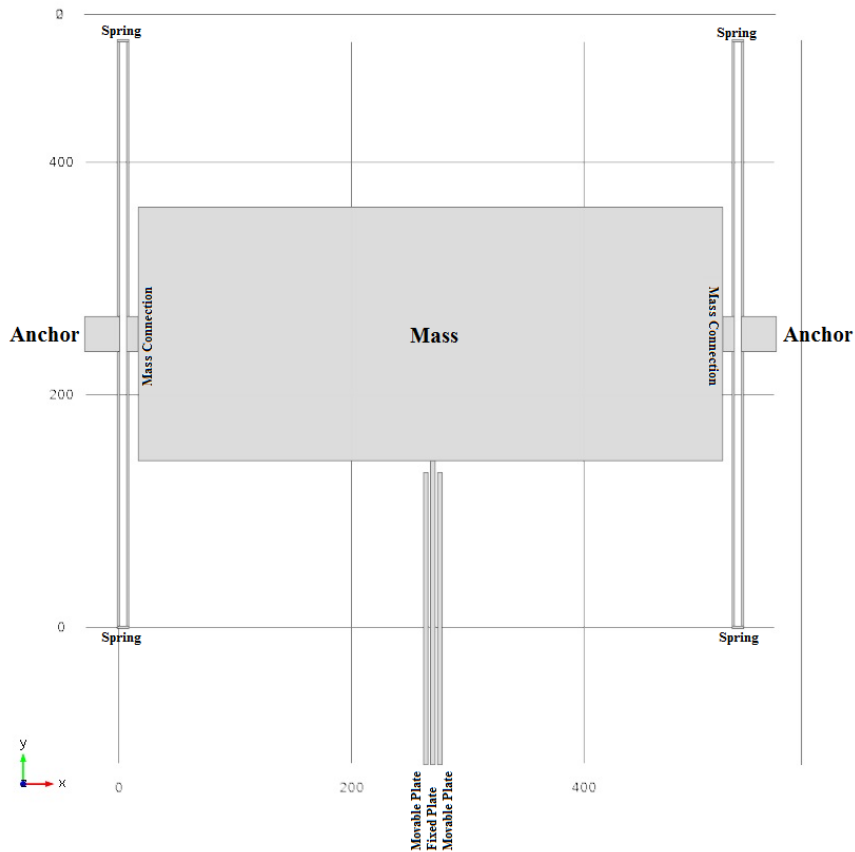
- Spring Constant  $k = 3 \text{ N/m}$ .
- Mass  $m = 2.79 \cdot 10^{-7} \text{ kg}$ .
- Damping Coefficient  $b_0 = 0.0017 \text{ kg/s}$ .
- Resonance Frequency  $f_n = 522 \text{ Hz}$ .
- Rise Time  $t_r = 0.002 \text{ s}$ .
- Quality Factor  $Q = 0.55$ .
- Capacitance Plates Distance  $g_0 = 100 \text{ }\mu\text{m}$ .
- Maximum Acceleration = 11 g.
- Bias Voltage  $V_1 = 5 \text{ V}$ .
- Maximum Output Voltage = 0.5 V.

These values are obtained by using a mathematical design approach. In this project instead we are interested to a more physical one so we need to wonder ourselves if these values are effectively usable. Considering the mass value we observe that from a physical point of view it is too big to be realized, since it requires a dimensions of the order of mm. An accelerometer MEMS has typical dimensions of  $\mu\text{m}$  as we can observe in the [Reference 8](#). Since the frequency response depends on the  $k$ ,  $m$  and  $b_0$  we need to re-design our accelerometer in order to obtain a realizable device. The mass will be decreased and will expect an increase of the resonance frequency and of the band of our device. The Rise Time will be decreased giving better time response. Another consideration is related to the capacitance plates

distance. Its value is too big because the capacitor realized inside the MEMS use the air as dielectric. Since the air is all around the plates and not only between them, the parasitic capacitances can influence the nominal one. For this reason we need to consider a lower starting distance reducing also the maximum displacement of our device. In the following paragraphs will be showed a new design approach. The previous project started from a mathematical model in the time domain using the Rise Time and the Overshoot to obtain the final values of the mass, spring constant and damping coefficient. The newer approach instead, starts from a physical point of view evaluating the time and frequency response at the end, in order to verify if they met our starting specifications.

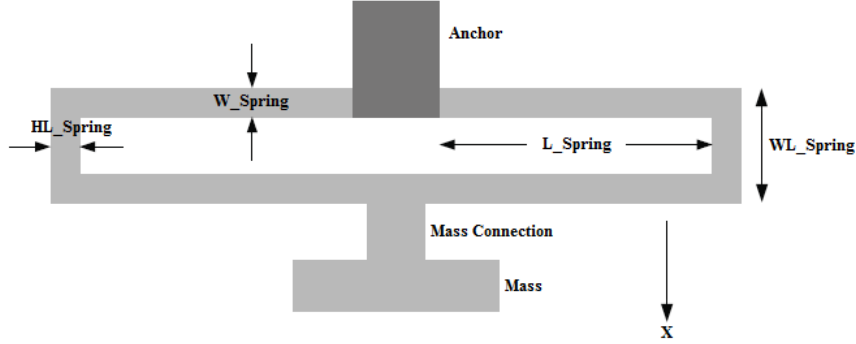
### Project

Parallel plate capacitor accelerometers are based on different mechanical structures with various shapes and mechanical characteristics. Our choice is to use a simple structure based on folded springs and two variable capacitors. It is showed in [Figure 5](#).



*Figure 5: Parallel plate capacitor accelerometer based on folded springs.*

In [Reference 9](#) we observe that the typical material is polycrystalline silicon. Analyzing the single suspension structure showed in [Figure 6](#) we can determinate the spring constant along the x-axis.



*Figure 6: Suspension structure.*

From [Reference 3](#) we have that for each beam the spring constant can be written as:

$$k_s = \frac{1}{2} \frac{W_{Spring}^3 \cdot Thickness}{L_{Spring}^3} \cdot E_R$$

Where  $W_{Spring}$  is the beam width,  $L_{Spring}$  is the beam length, Thickness is the thickness of the entire MEMS structure and  $E_R$  is the Young's modulus of the material. For the polycrystalline silicon  $E_R$  is equal to 160 GPa. Since there are four folded beams they behave as four springs connected in parallel. So the total spring constant along the x direction is:

$$k = 4 \cdot k_s = 2 \frac{W_{Spring}^3 \cdot Thickness}{L_{Spring}^3} \cdot E_R$$

Where  $k$  is the constant of spring for one folded beam,  $L_{\text{Spring}}$  is the beam length,  $W_{\text{Spring}}$  is the spring width, Thickness is the structure thickness,  $E_R$  is the Young's modulus of the structural material. [Reference 3](#) tells us that the lowest is the spring constant the higher is the sensitivity of the accelerometer. Typical  $k$  spring constants are from 0.01 N/m to 100 N/m ([Reference 4](#)). Choosing:

$k = 20$  N/m,  $W_{\text{Spring}} = 2 \mu\text{m}$  and  $L_{\text{Spring}} = 235 \mu\text{m}$  we obtain the device thickness:

$$\text{Thickness} = k \cdot \frac{L_{\text{Spring}}^3}{W_{\text{Spring}}^3 \cdot 2 \cdot E_R} = 101 \mu\text{m}$$

The others dimensions  $WL_{\text{Spring}}$  and  $HL_{\text{Spring}}$  are chosen equal to  $10 \mu\text{m}$  and  $2 \mu\text{m}$  respectively. The Anchor instead has the follow dimensions:  $L_{\text{Anchor}} = 30 \mu\text{m}$ ,  $\text{Height} = \text{Thickness} + Z_{\text{Space}}$  where  $Z_{\text{Space}} = 2 \mu\text{m}$ . The  $Z_{\text{Space}}$  is the  $z$  empty space between the suspensions structure composed by Spring + Mass + Movable Plate and the Anchor plane. In this way the movable structure is free to move.

From the device thickness we can define the starting value of the two capacitors. Remembering that:

$$C = \frac{\varepsilon_0 \cdot A}{g_0}$$

Where  $A$  is the area of the plates,  $g$  is the initial distance between two plates and  $\varepsilon_0$  is the vacuum permittivity. The area depends on the length and thickness of the fixed plates. Since the thickness is decided by the spring we have to impose the initial distance  $g_0$  and the fixed

plates length. Choosing  $L\_FixPlate = 250 \mu m$  and  $g_0 = 2 \mu m$  we obtain the initial  $C_0$ :

$$C_0 = \frac{\varepsilon_0 \cdot A}{g_0} = \frac{\varepsilon_0 \cdot Width \cdot L\_FixPlate}{g_0} = 112 fF$$

The width of the movable and fixed plates are chosen equal to  $4 \mu m$ .

To define the value of the mass we impose the gap variation at the threshold acceleration for the airbag application equal to  $8 g$ . From the relation (1) neglecting for simplicity the electrostatic force we obtain the value of the mass. Imposing  $g = 0.1 \mu m$  we have:

$$m = k \cdot \frac{g}{a_{ext}} = 2.55 \cdot 10^{-8} kg$$

From the mass definition we can design its physical dimensions neglecting the movable plate, the mass connections and the springs contributes. The mass connections are useful to join the mass and the two folded springs. Since the mass is a simple parallelepiped we can define its volume as:

$$V = Thickness \cdot L\_Mass \cdot W\_Mass$$

Knowing the density value of the polycrystalline silicon  $2320 kg/m^3$  and imposing  $L\_Mass = 500 \mu m$  we obtain:

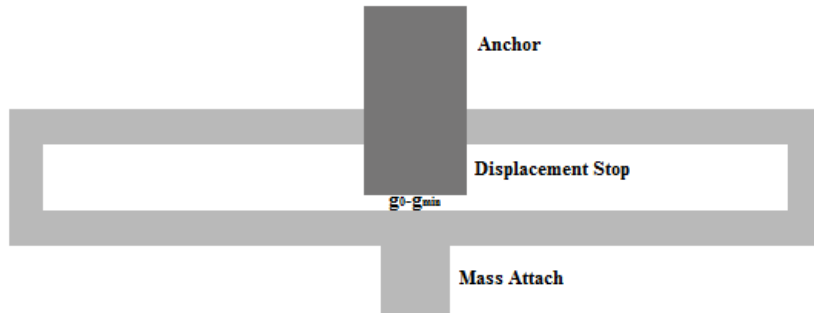
$$W\_Mass = \frac{V}{L\_Mass \cdot Thickness} = 217 \mu m$$

From the damping formula (5) we can define the value of the damping coefficient related to  $g$  equal to 0:

$$b_0 = \frac{u \cdot A^2}{g_0^3} = 0.00145 \frac{kg}{s}$$

### *Displacement Mechanical Stops*

Since the movable plate cannot collide into a fixed plate the displacement variation must be considered limited. In our case these limits are mechanically realized using the Anchors. Displacement stop is showed in [Figure 7](#).



*Figure 7: Displacement stops.*

The distance between the Anchor and the Mass Attach is  $g_0 - g_{min}$ . As soon as the Anchor touches the mass attach (due to high input acceleration) we have the displacement stopping.



Imposing  $g_{min}$  equal to  $1.8 \mu\text{m}$  we have:

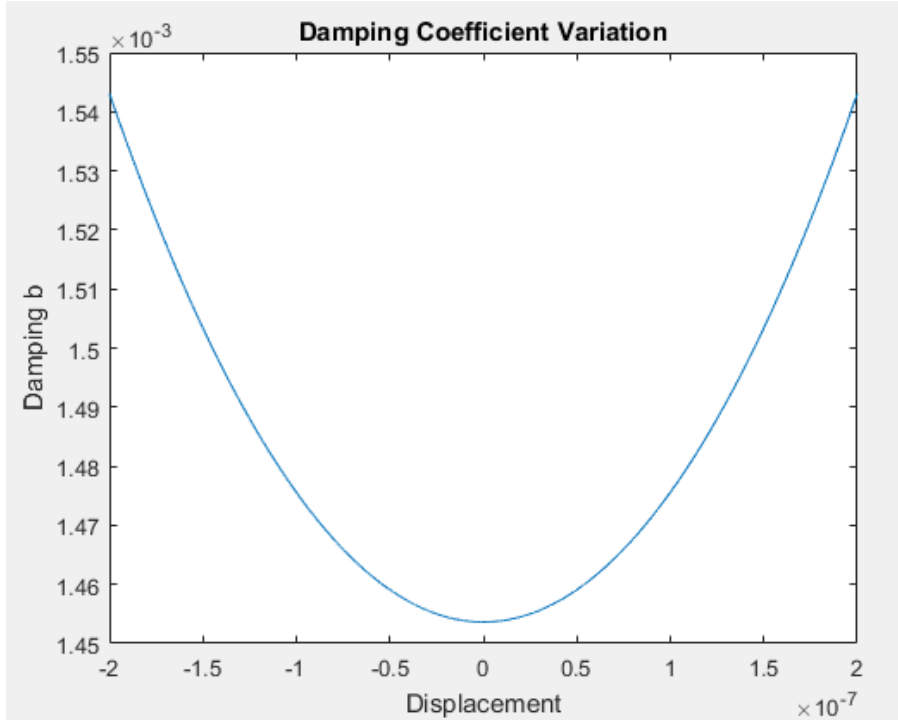
$$a_{ext+} = \frac{k \cdot (g_0 - g_{min})}{m} = 157 \frac{m}{s^2} \rightarrow 16 g$$

$$a_{ext-} = \frac{k \cdot (g_{min} - g_0)}{m} = -157 \frac{m}{s^2} \rightarrow -16 g$$

These values are higher than the airbag threshold acceleration equal to  $8 g$  so the input range is respected. Displacements stops are not realized inside the COMSOL model since they imply too big simulation effort and time.

### *Damping Coefficient Effect*

In order to validate our approximation on the damping coefficient we need to plot the damping coefficient variation using the formula (4). Considering a variation of  $g$  between  $g_0 - g_{min}$  and  $-g_0 + g_{min}$  we obtain the graph showed in [Figure 8](#).



*Figure 8: Damping coefficient variation plot.*

As we can see its variation is limited in fact the percentage of the variation is equal to:

$$b\% = 100 \cdot \left(1 - \frac{b_0}{b_{max}}\right) = 100 \cdot \left(1 - \frac{0.00145}{0.00154}\right) = 5.8 \%$$

### *Pull-In Voltage Evaluation*

Using the designed value we can evaluate the Pull-In voltage:

$$V_{PI} = \sqrt{\frac{8 \cdot k \cdot g_0^3}{27 \cdot \epsilon_0 \cdot A}} = 14.5 \text{ V}$$

Inside COMSOL model will not simulate the pull-in voltage since from the model definition we will not be able to realize the Pull-In circuit showed in [Figure 2](#).

### *Electrostatic Forces Evaluation*

The electrostatic forces effect depends on the displacement  $g$ :

$$F_{el} = F_{el1} - F_{el2} = \frac{\epsilon_0 \cdot A \cdot V_P^2}{2} \left[ \frac{1}{(g_0 - g)^2} - \frac{1}{(g_0 + g)^2} \right]$$

We can evaluate the maximum values of  $F_{el}$  at  $g = g_0 - g_{min}$  and  $g = -g_0 + g_{min}$ . Choosing  $V_1 = 1 \text{ V}$  and  $V_2 = -1 \text{ V}$  we calculate the equivalent accelerations of electrostatic forces:

$$A_{el+} = \frac{\epsilon_0 \cdot A \cdot V_P^2}{2 \cdot m} \left[ \frac{1}{(g_0 - g_0 + g_{min})^2} - \frac{1}{(g_0 + g_0 - g_{min})^2} \right] = 0.44 \frac{m}{s^2} \rightarrow 0.04g$$

$$A_{el-} = - \frac{\epsilon_0 \cdot A \cdot V_P^2}{2 \cdot m} \left[ \frac{1}{(g_0 + g_0 - g_{min})^2} - \frac{1}{(g_0 - g_0 + g_{min})^2} \right] = -0.44 \frac{m}{s^2} \rightarrow -0.04g$$

These accelerations are coherent with respect to the external one. Their values are smaller than the maximum acceleration equal to 16 g and so the introduced error is limited. From this analysis we can consider a good choice the designed values for the accelerometer.

### Output Voltage Range

The calculation of the output voltage range is dependent from the maximum  $\Delta C$  variations:

$$\Delta C_{max+} = \frac{\varepsilon_0 \cdot A \cdot g}{g_0^2 - g^2} \Big|_{g=g_0-g_{min}} = \frac{\varepsilon_0 \cdot A \cdot (g_0 - g_{min})}{g_0^2 - (g_0 - g_{min})^2} = 11.3 \text{ fF}$$

So the maximum output signal  $V_0$  is:

$$V_{Out}(s)_{max+} = \frac{\Delta C_{max+} \cdot V_1(s)_{max}}{C_0} = 0.1 \text{ V}$$

This value corresponds to an external acceleration equal to 16 g. For an external acceleration of -16 g we have:

$$\Delta C_{max-} = \frac{\varepsilon_0 \cdot A \cdot g}{g_0^2 - g^2} \Big|_{g=g_{min}-g_0} = \frac{\varepsilon_0 \cdot A \cdot (g_{min} - g_0)}{g_0^2 - (g_{min} - g_0)^2} = -11.3 \text{ fF}$$

And:

$$V_{Out}(s)_{max-} = \frac{\Delta C_{max-} \cdot V_1(s)_{max}}{C_0} = -0.1 \text{ V}$$

Considering an external acceleration equal to 8 g we have:

$$\Delta C_{8g} = \frac{\varepsilon_0 \cdot A \cdot g}{g_0^2 - g^2} \Big|_{g_{8g}=0.1 \mu m} = \frac{\varepsilon_0 \cdot A \cdot g_{8g}}{g_0^2 - g_{8g}^2} = 5.6 \text{ fF}$$

That corresponds to:

$$V_{Out}(s)_{max} = \frac{\Delta C_{8g} \cdot V_1(s)_{max}}{C_0} = 0.05 \text{ V}$$

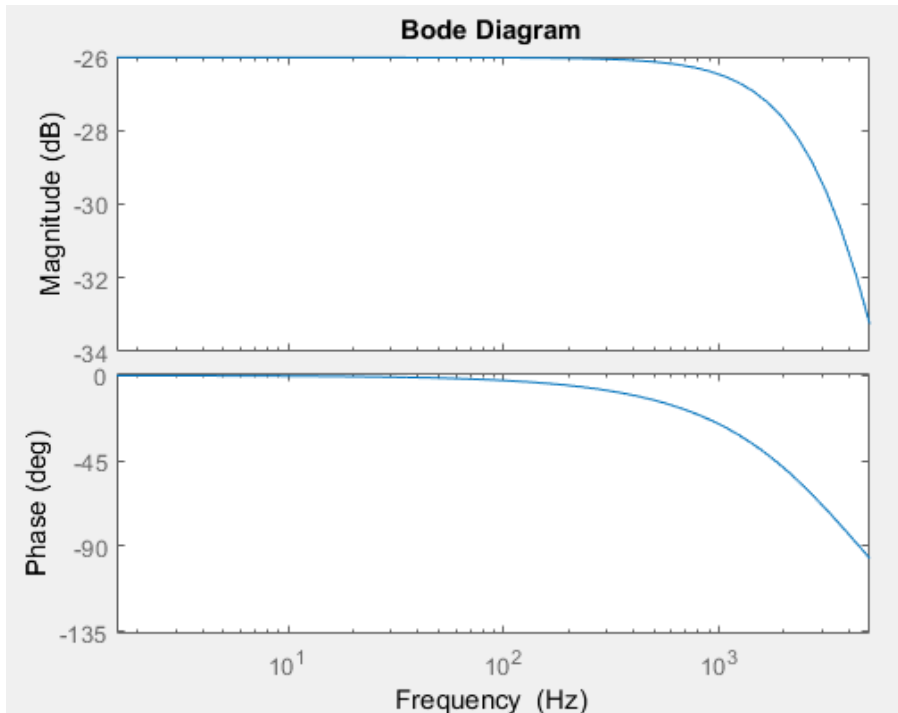
The same results (with an opposite sign) can be obtained with an acceleration of -8 g.

### Frequency & Time Response

Knowing  $k$ ,  $b_0$  and  $m$  we can evaluate the quality factor  $Q$ , the damping ratio and the resonance frequency. We obtain:

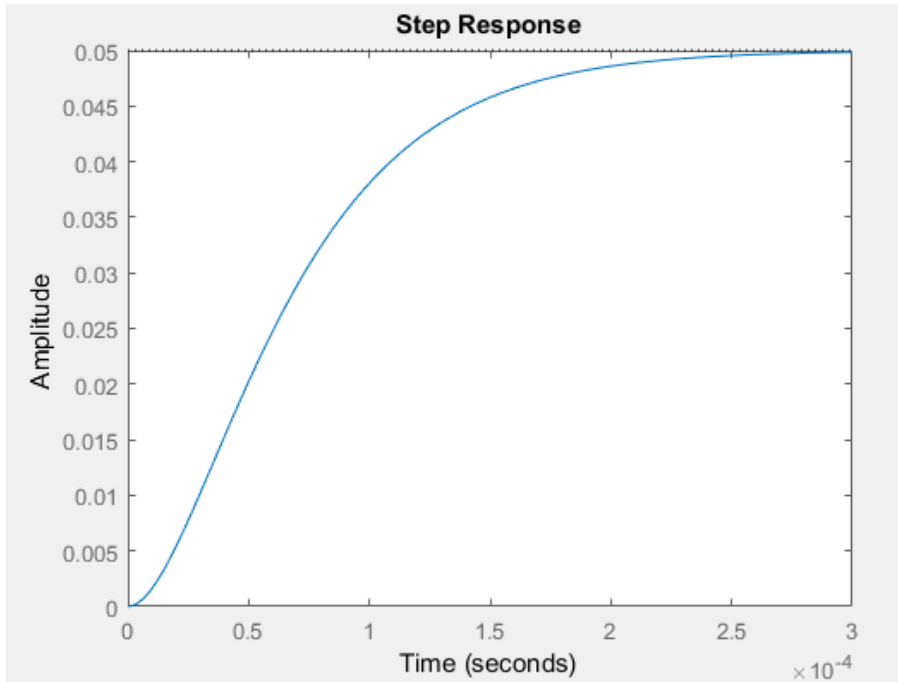
$$Q = \frac{\omega_n \cdot m}{b_0} = 0.49, \quad \zeta = \frac{1}{2 \cdot Q} = \frac{b_0}{2 \cdot \sqrt{m \cdot k}} = 1.01, \quad f_0 = \sqrt{\frac{k}{m}} \cdot \frac{1}{2\pi} = 4459 \text{ Hz}$$

Moreover we can plot the Bode diagram and the Step Response of our mechanical system neglecting for simplicity the electrostatic forces and the damping coefficient variation. [Figure 9](#) shows the Bode Diagram obtained with Matlab:



*Figure 9: Bode Diagram.*

Figure 10 shows the Step Response obtained with Matlab:

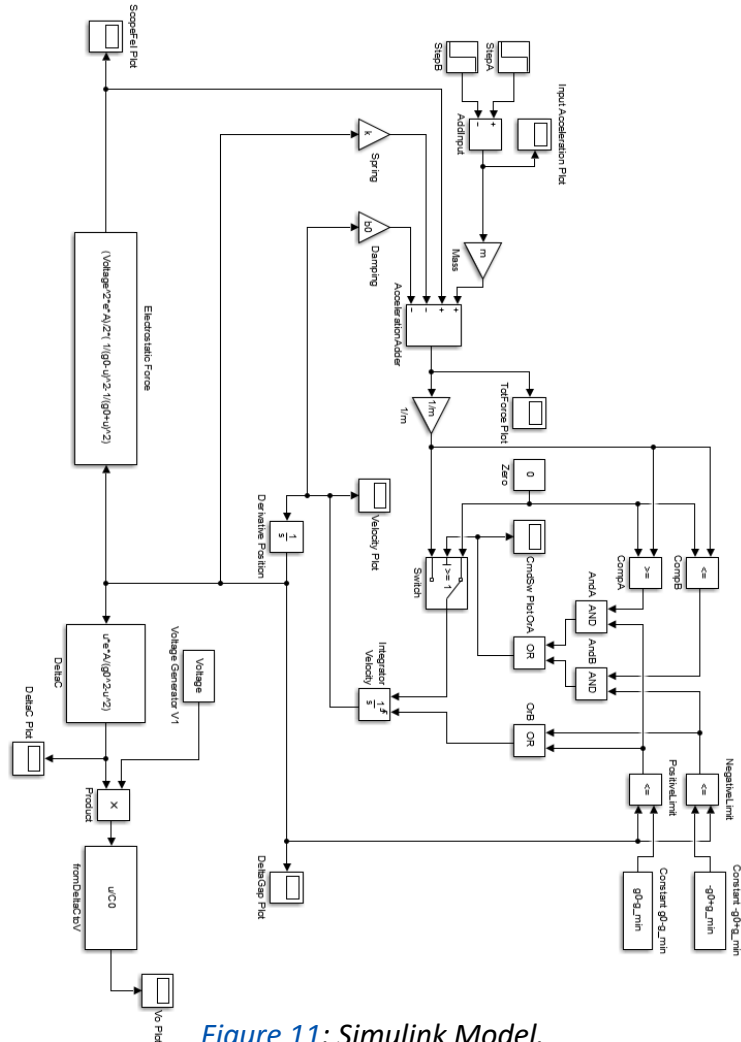


*Figure 10: Step Response Diagram.*

As we can see the Rise Time is lower than 2 ms and the overshoot is zero so the specifications are met. In particular the time in order to reach the 100 % of the final value is equal to about 300  $\mu$ s.

### *SIMULINK Model and Results*

The realization of the accelerometer model in Simulink was done through a graphical interface using the Final State Equations obtained before. In [Figure 11](#) is showed the final model.



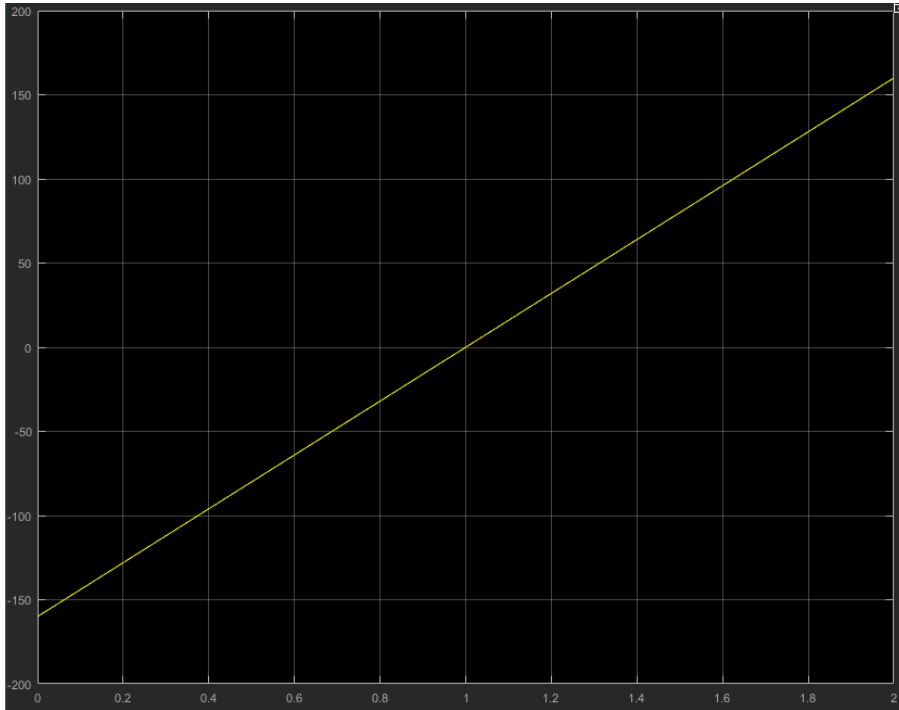
*Figure 11: Simulink Model.*

On the left we have the input step. The input acceleration is multiplied by the mass  $m$  through the **Mass** block. The next block is **AccelerationAdder**. This adder sums the input acceleration, the spring force, the dashpot force and resultant electrostatic force. The resultant force is applied to our movable plate divided by the mass and integrated by **Integrator Velocity** obtaining the velocity. Finally we obtain the position variation  $g$  from **Derivative Position** block. The displacement  $g$  is used by **Damping**, **Electrostatic Force** and **Spring** blocks. As we can see the resultant acceleration passes through the **Switch** block. It allows with some other blocks the saturation of displacement  $g$  to  $g_0 - g_{min}$  for a positive resultant acceleration and to  $-g_0 + g_{min}$  for a negative one. The comparators **PositiveLimit** and **NegativeLimit** give us an output equal to '1' logic when  $g$  exceeds one of the two limits. In these cases the **OrB** resets the **IntegratorVelocity** block in order to force the velocity to a zero value. When we have the positive saturation  $g = g_0 - g_{min}$  and the resultant acceleration continues to be positive we have to force it to zero so the **CompA** in AND with the **PositiveLimit** block, forces the resultant acceleration to zero through the **Switch** block. Otherwise if the resultant acceleration changes its sign the system works normally. The same thing happens when  $g$  is equal to the negative limit. In fact **CompB** in AND with **NegativeLimit** block forces the acceleration to zero and if it continues to be negative. Finally we use the displacement  $g$  to calculate  $\Delta C$  through **DeltaC** block. The output is multiplied by the voltage generator **VoltageGenerator V1** that represents the generators  $V_1$  and  $V_2$ . The resultant output is applied to **fromDeltaCtoV** block in order to obtain the final  $V_0$  signal.



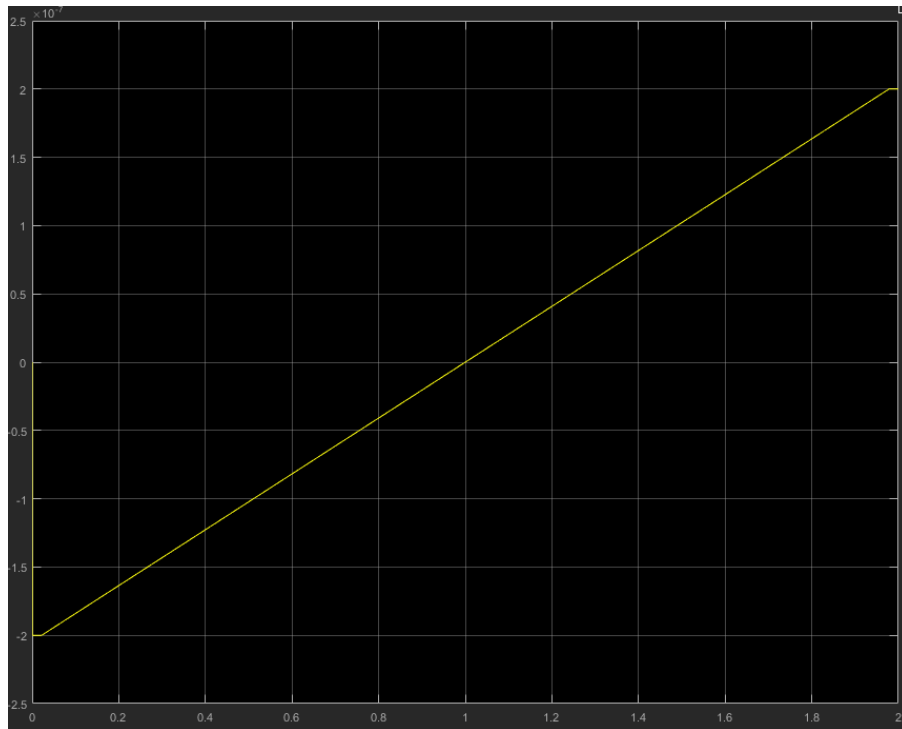
*SIMULINK: Acceleration sweep from -16 g to 16 g*

In [Figure 12](#) is showed the input acceleration that sweeps from -16 g to 16 g. The maximum values are little higher than the input range in order to evaluate also the displacement stops modelling.



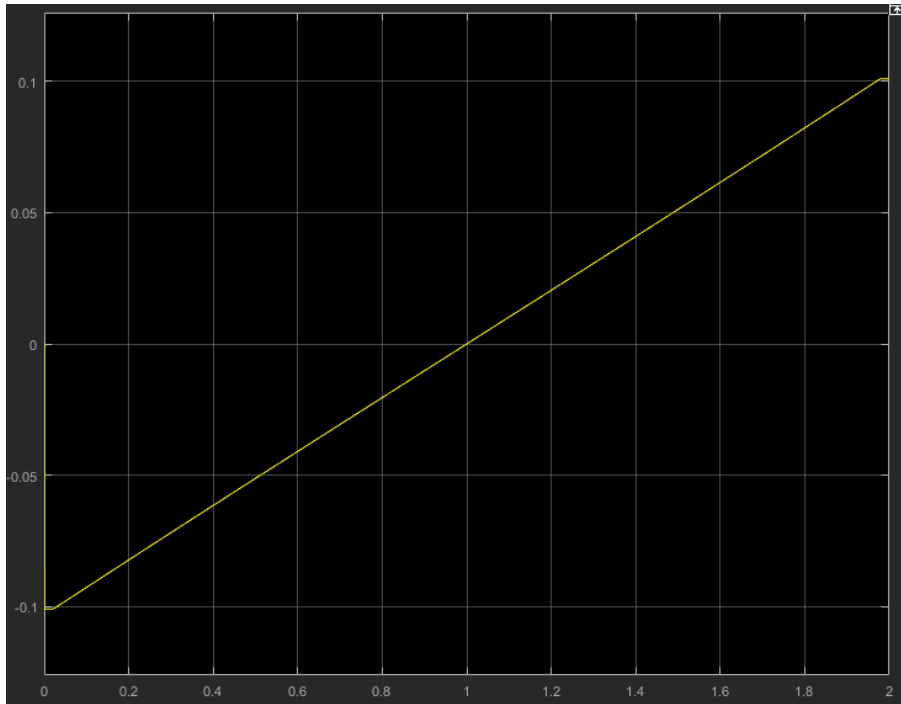
*Figure 12: Input Sweep Acceleration [m/s²].*

The related displacement is showed in [Figure 13](#).



*Figure 13: Displacement behavior [ $\mu\text{m}$ ].*

As we can see the displacement saturates as soon as are reached -16 g or 16 g input accelerations. Finally we observe the output voltage in [Figure 14](#).



*Figure 14: Output Voltage [Volt].*

All results respects the expected values so we can affirm the model works correctly.

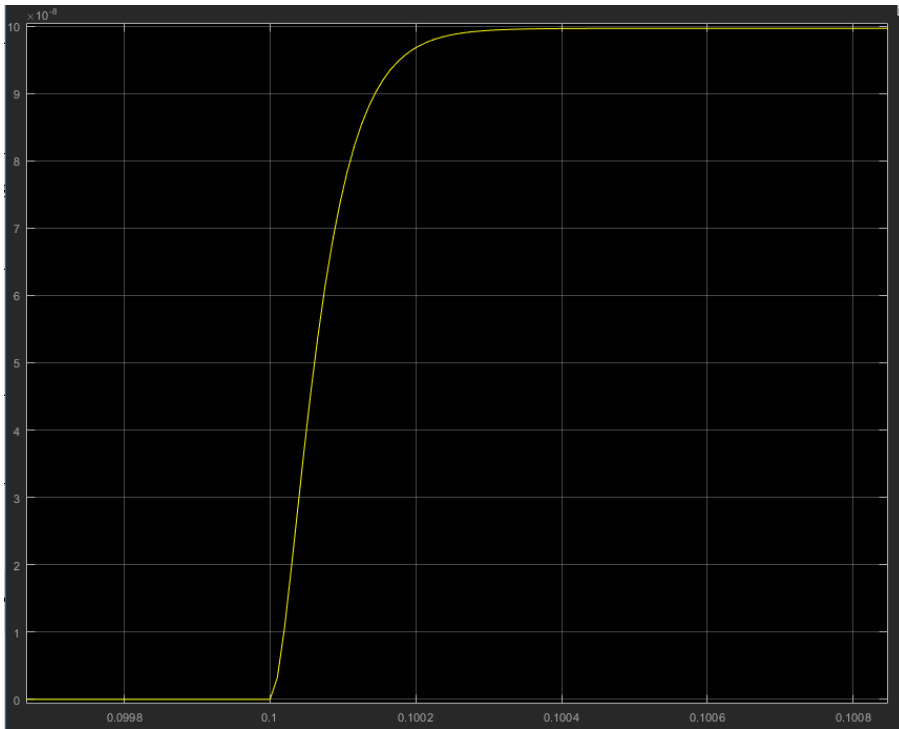
*SIMULINK: Step Response at 8 g*

In [Figure 15](#) is showed the input step acceleration from 0 to 8 g.



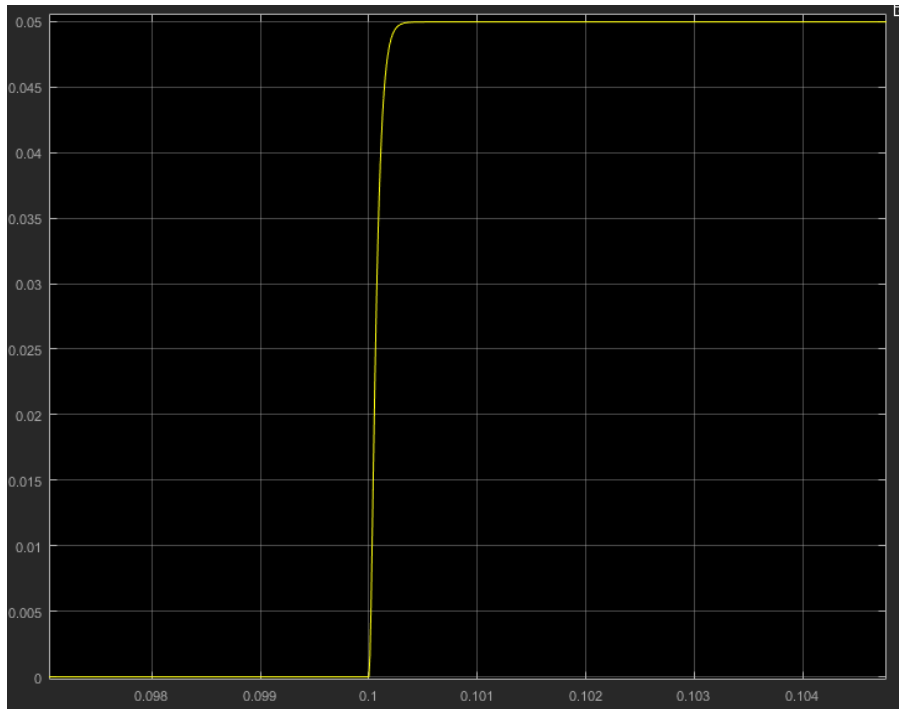
*Figure 15: Input Step Acceleration [m/s²].*

The related displacement is showed in [Figure 16](#).



*[Figure 16](#): Displacement behavior [ $\mu\text{m}$ ].*

As we can see it results equal to 0.1  $\mu\text{m}$  like the design specification. The Rise Time in order to reach the 100 % of the final value is equal to about 300  $\mu\text{s}$ . In [Figure 17](#) we have the output voltage behavior.



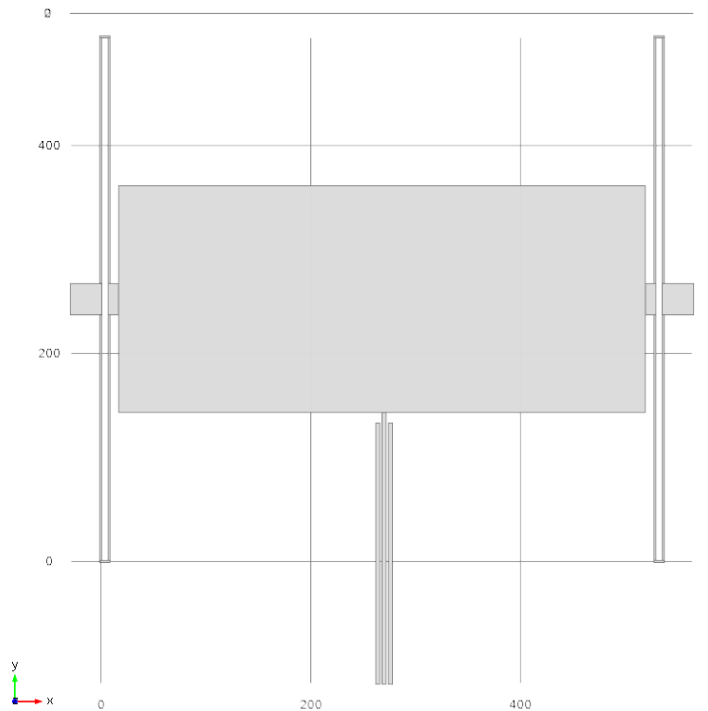
*Figure 17: Output Voltage [Volt].*

The output voltage is 0.05 V exactly as from specifications. All results respects the expected values so we can affirm the model works correctly.

## COMSOL Model and Results

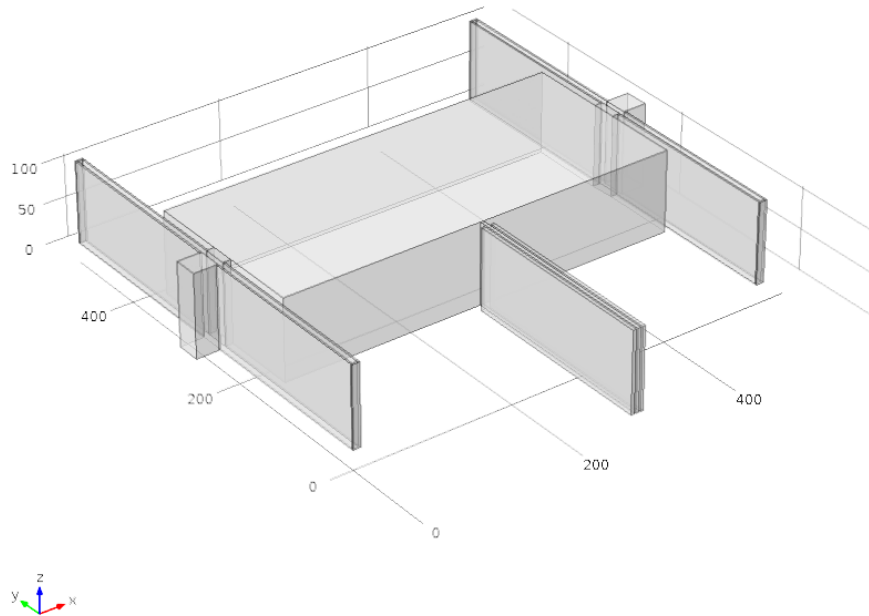
---

Using COMSOL the accelerometer MEMS is realized by using a 2D approach with a final extrusion of each blocks. The 2D view is showed in [Figure 18](#).



*Figure 18: 2D MEMS Accelerometer.*

The 3D view instead is showed in [Figure 19](#).

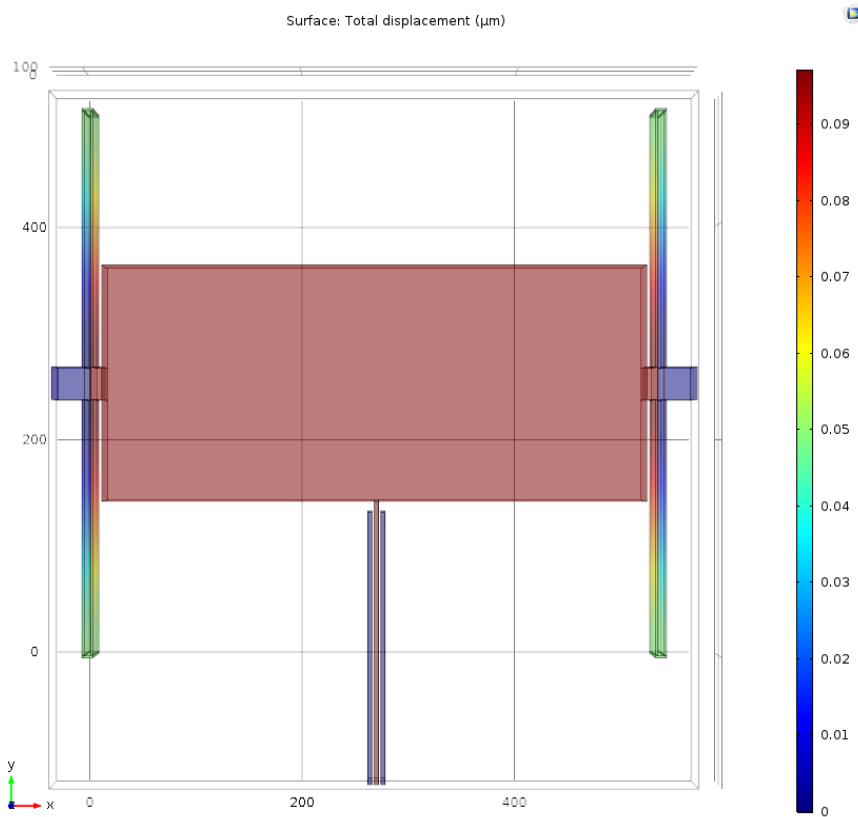


*Figure 19: 3D MEMS Accelerometer.*



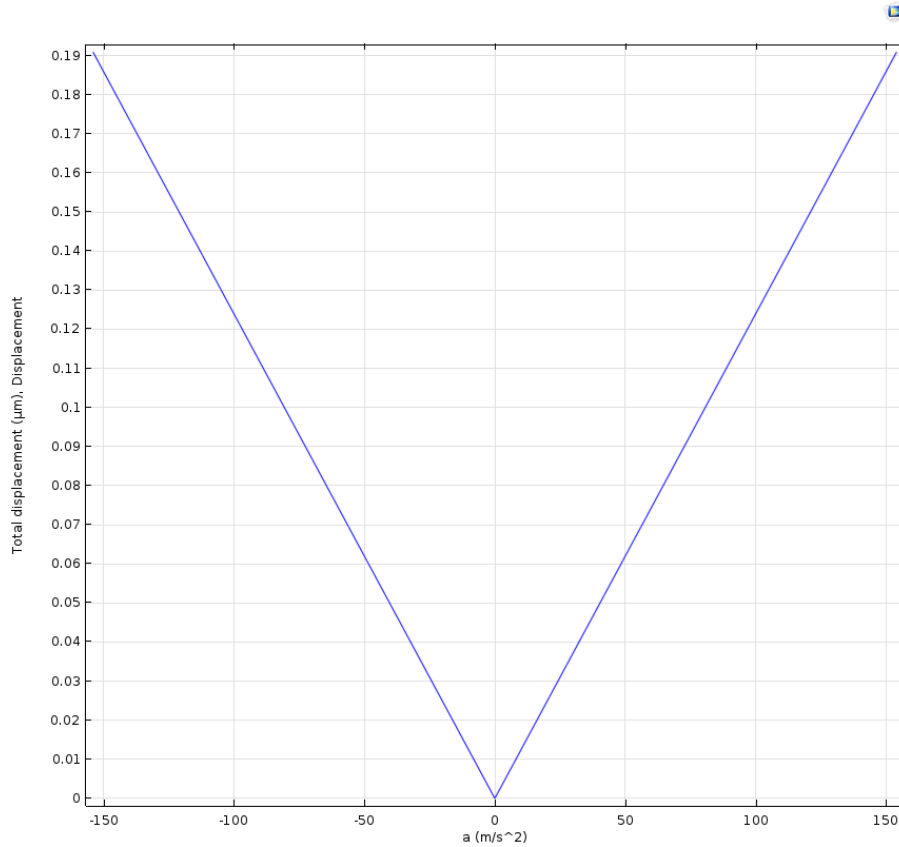
*COMSOL: Acceleration sweep from -16 g to 16 g*

Computing the Study 1 the input acceleration sweeps from -16 g to 16 g. In Figure 20 is showed 3D displacement obtained at 8 g. It is about 0.1  $\mu\text{m}$  and so the device works properly.



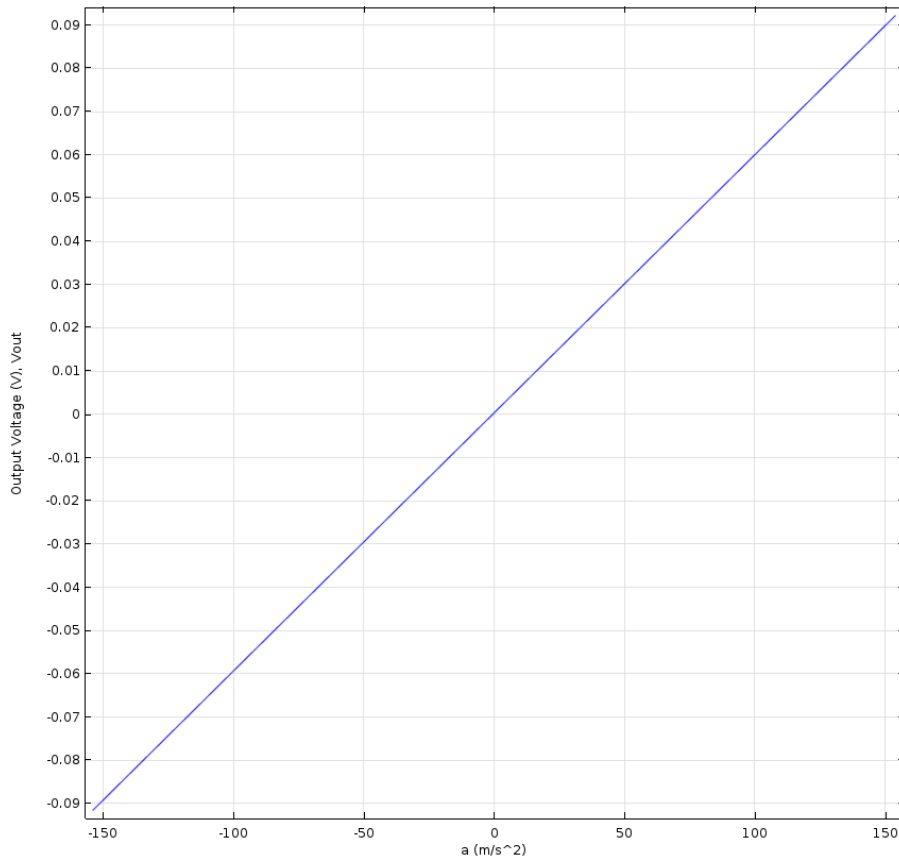
*Figure 20: 3D displacement at 8 g.*

In [Figure 21](#) is showed the absolute value of the Displacement vs Acceleration Graph.



*Figure 21: Displacement vs Acceleration Graph.*

In [Figure 22](#) is showed the Output Voltage vs Acceleration Graph.

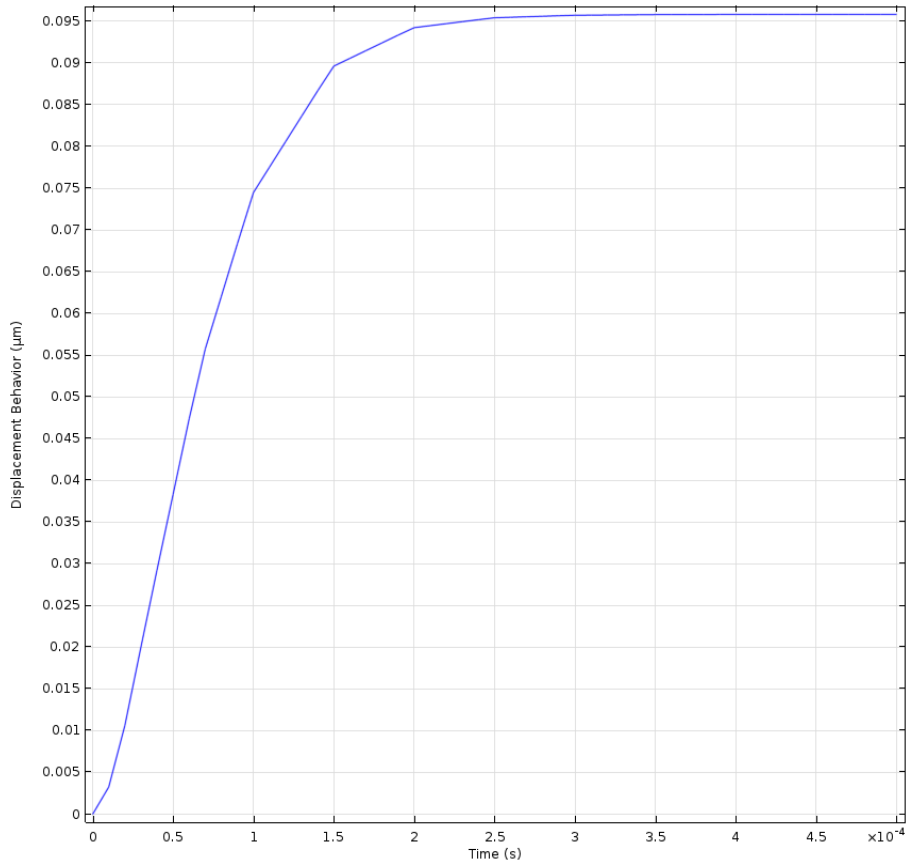


*Figure 22: Output Voltage s Acceleration Graph.*

The output voltage behaves exactly as from specifications. All results respects the expected values so we can affirm the model works correctly.

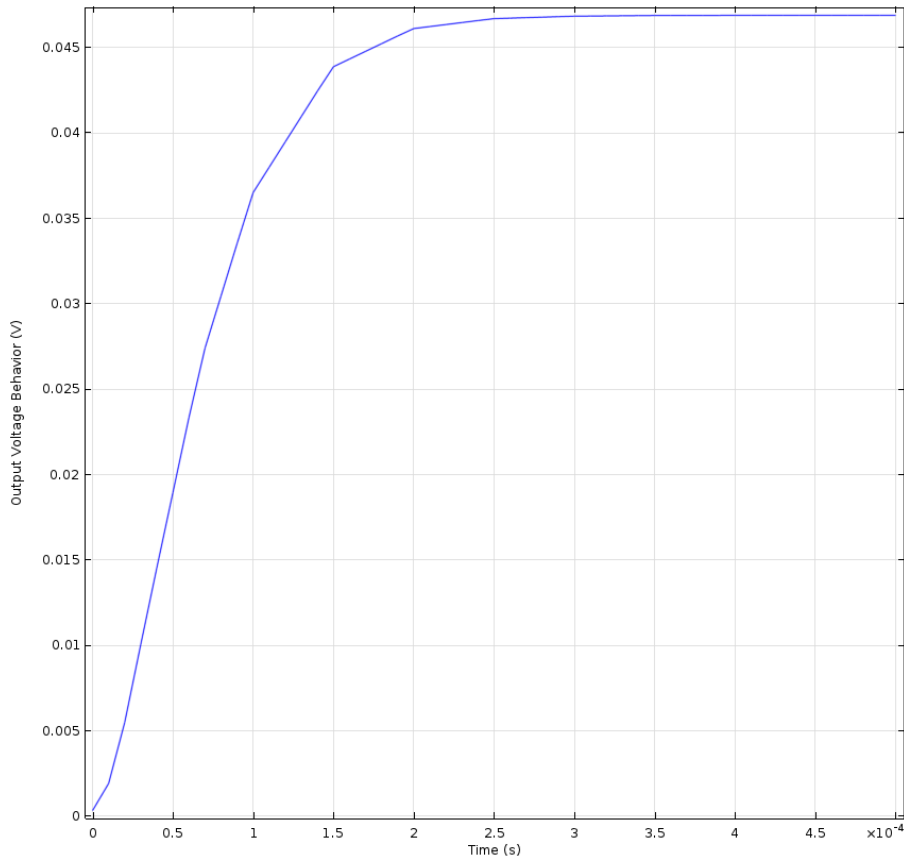
### *COMSOL: Step Response at 8 g*

Computing the Study 2 we apply an input step acceleration equal to 8 g. In [Figure 23](#) is showed the Displacement behavior.



*Figure 23: Step Response.*

In [Figure 24](#) is showed the output voltage behavior.

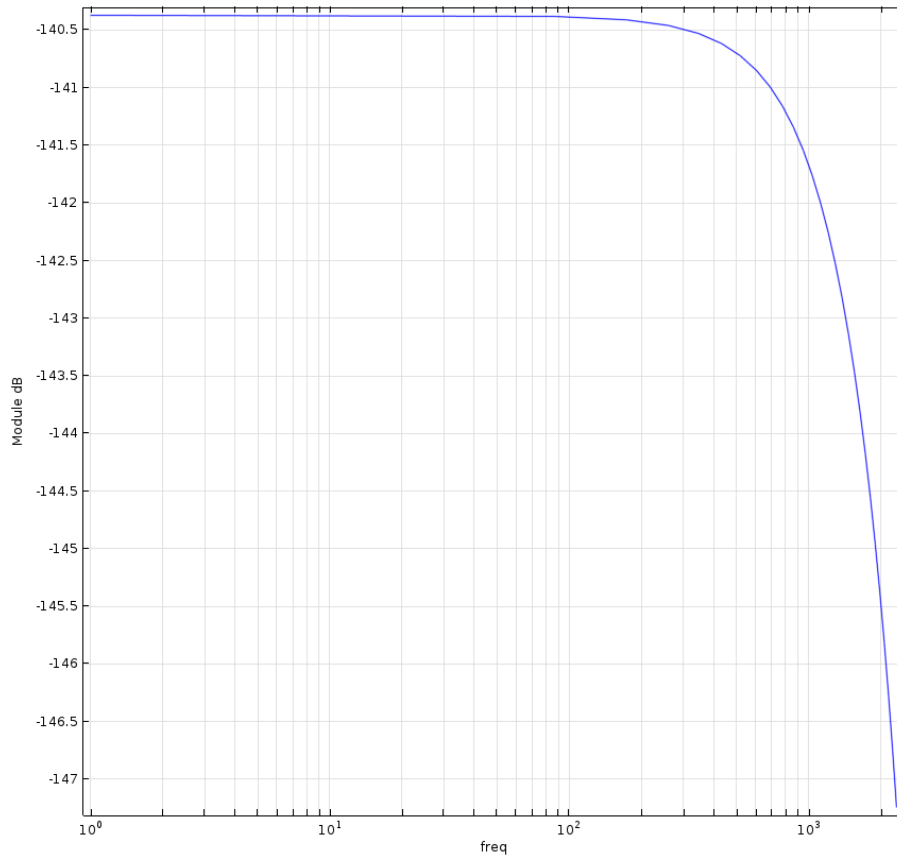


*Figure 24: Output Voltage Response.*

As we can see the Rise Time is around 300  $\mu$ s as from the specifications. The device works properly.

### *COMSOL: Bode Diagram and Frequency Analysis*

Computing the Study 3 we obtain the Bode Diagram. In [Figure 25](#) is showed the module.



*Figure 25: Module Bode Diagram.*

As we can see the device works properly in fact the graph is similar to the theory.

## Proposed Circuit for MEMS Interface

The proposed output circuit is a block that generates a digital output if the acceleration threshold of 8 g is exceeded for both positive and negative ones. Since the  $V_0$  at 8 g is 0.05 V we can use an analog ComparatorA with a threshold  $V_{th+}$  around 0.05 V. The same can be done for the negative acceleration of -8 g using the ComparatorB. In this case we have to use a negative threshold  $V_{th-}$  around -0.05 V. The  $V_{COMPA}$  and  $V_{COMPB}$  will be 1 logic when the thresholds will be exceeded and so the corresponding D Flip-Flop will be set. We have chosen to use two comparators because in this way the accelerometer can be mounted on the final target in two possible direction. In a more general analysis the two threshold values can be different. The circuit is the following:

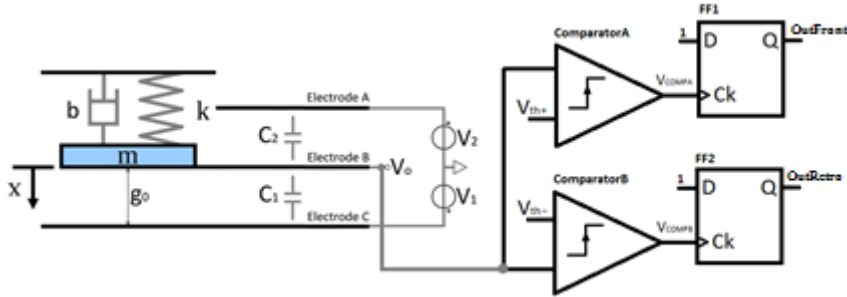


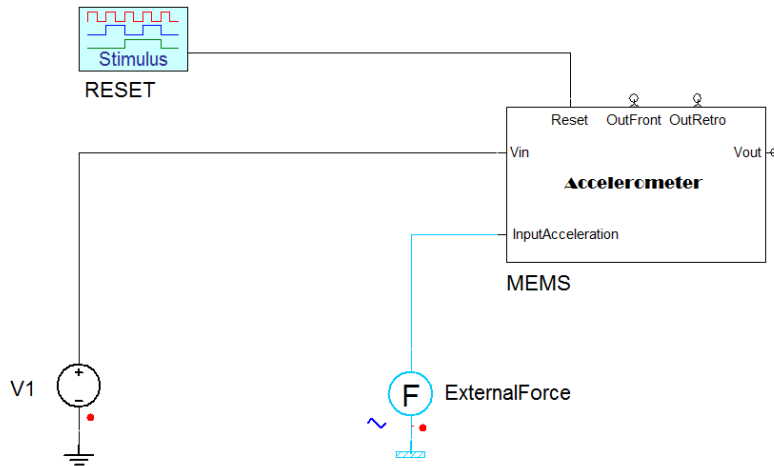
Figure 26: Output Circuit.

In this way as soon as one of the acceleration thresholds are exceeded the output of the corresponding comparator will be at '1' logic. A processor can use it as an interrupt or a digital input.

## VHDL-AMS Model

---

In [Figure 27](#) is showed the schematic used to test our final device. It includes both MEMS and Output Circuit.



*Figure 27: Final Device Model.*

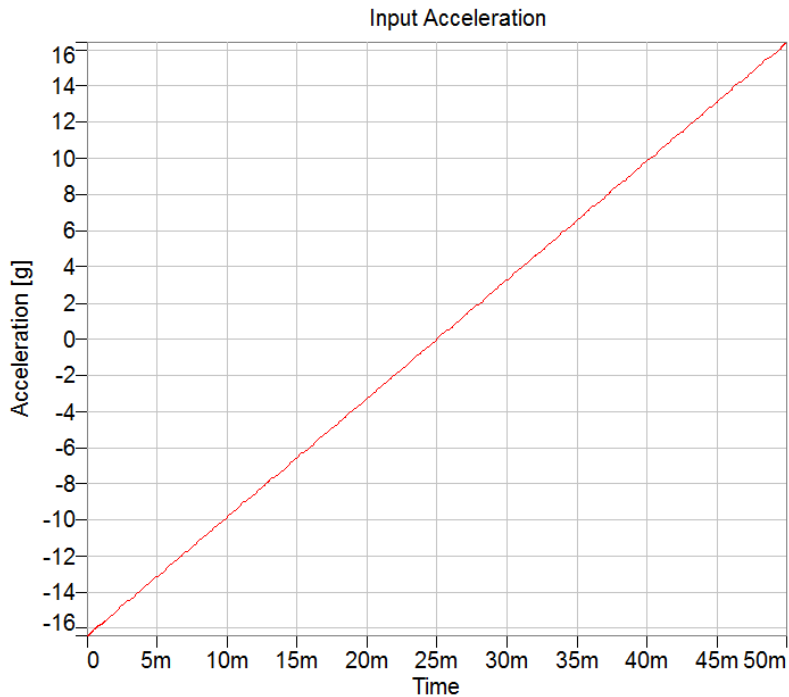
The accelerometer symbol was created through Simplorer thanks to the Symbol Editor tool included internally. It allowed us to define the shape and the pin disposition. The quality of simulation is more limited than Simulink and COMSOL because the graphical user interface is simpler with less functionalities. This limits the evaluation of the results. The simulation use a triangular input acceleration in order to evaluate the dynamic response when  $g$  varies from  $-0.2 \mu\text{m}$



to  $0.2 \mu\text{m}$ . The Reset input is used to reset the two Flip-Flop created internally inside the MEMS.

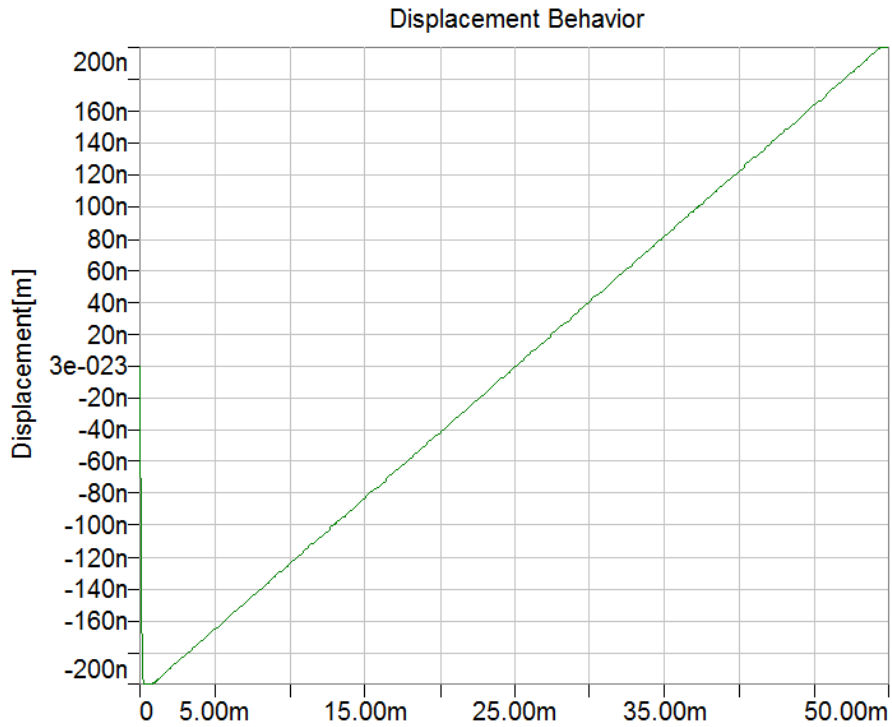
*VHDL-AMS: Acceleration sweep from -16 g to 16 g*

In [Figure 28](#) is showed the input acceleration that sweeps from -16 g to 16 g. The maximum values are little higher than the input range in order to evaluate also the displacement stops modelling.



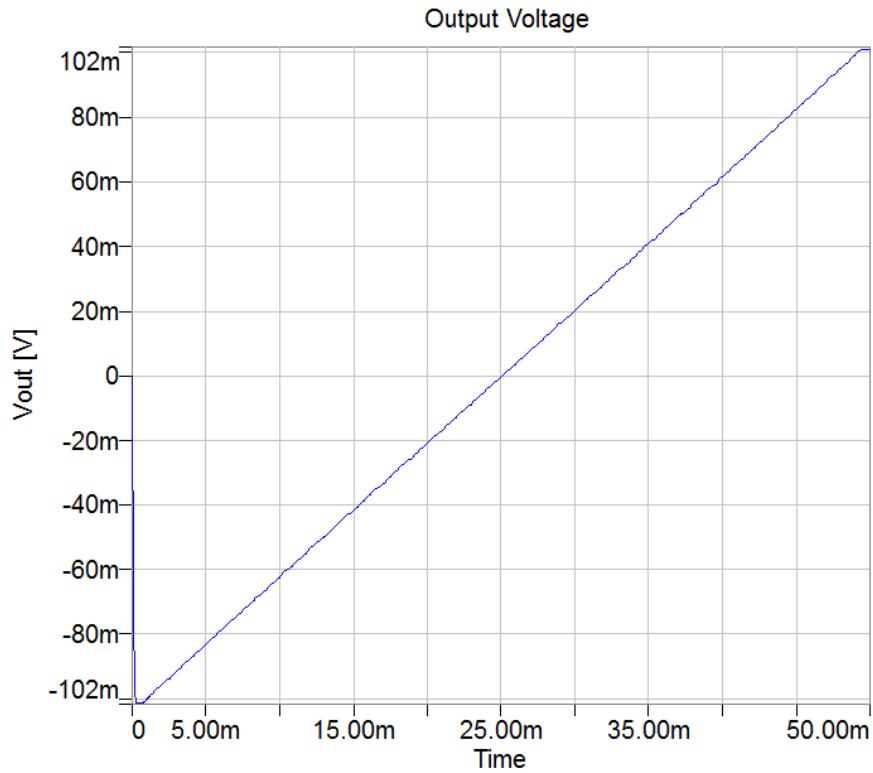
*Figure 28: Input Acceleration [g].*

The related displacement is showed in [Figure 29](#).



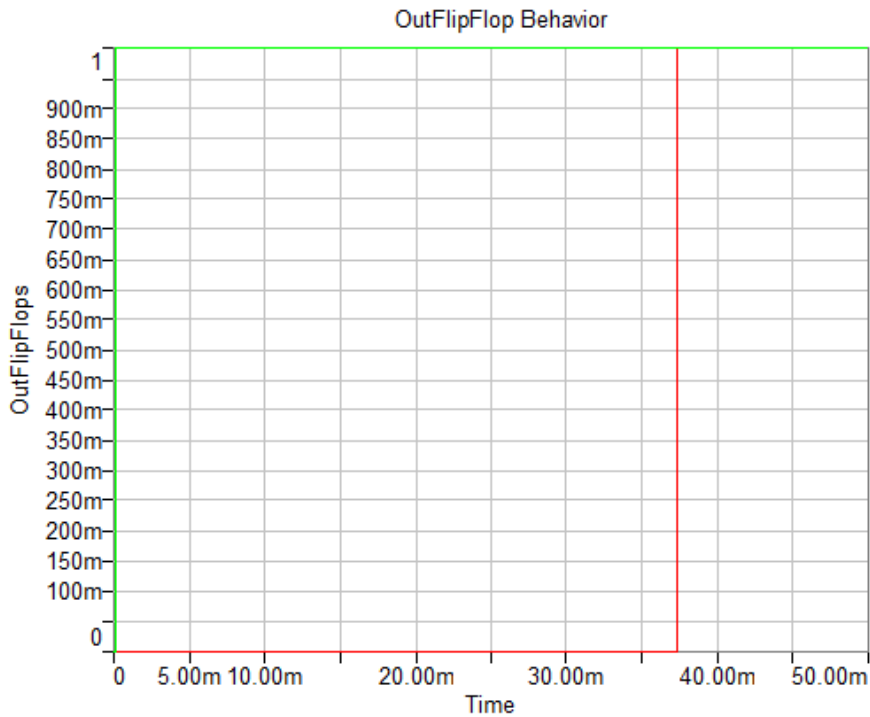
*Figure 29: Displacement behavior [m].*

As we can see the displacement saturates as soon as are reached -16 g or 16 g input acceleration. Finally we observe the output voltage in [Figure 30](#).



*Figure 30: Output Voltage.*

The outputs of the two Flip-Flop are set as soon as the acceleration becomes higher than 8 g or lower than -8 g as we can see in the [Figure 31](#).



*Figure 31: Flip Flop Outputs.*

The green line is related to the output OutRetro and becomes '1' as soon as the simulation starts since the first values of the acceleration are lower than -8 g. As soon as the acceleration reach the positive threshold acceleration of 8 g the OutFront becomes '1'. This two outputs will remain active until the Reset will not be activated. All results respects the expected values so we can affirm the model works correctly.

## Comparison

---

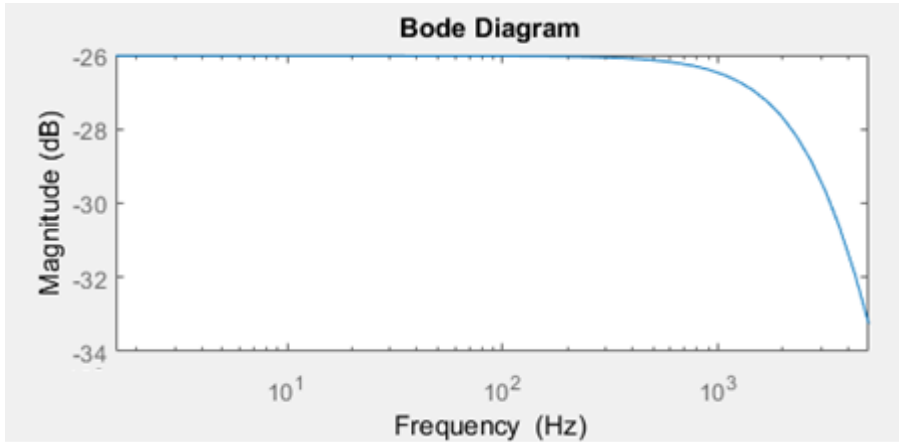
### *Developing tools*

Nowadays the modeling and the developing tool choice are important for the maximization of the efficiency during a project. In fact, changing the tool we can have different approaches to the problem. In our project, we have used COMSOL, SIMULINK and VHDL-AMS. For each of them we can analyze their advantages and drawbacks. If the implemented model needs of a more physical approach, the best choice is COMSOL. It offers a lower mathematical approach since we do not need to insert equations inside the model but we only need to know which are the physic of the model and the main physical parameters that are included inside it. Therefore, the description is simply done by the insertion of parameters. COMSOL offers a graphical environment that is oriented to the physics and manufacturing of the device. Moreover, it shows the real behavior of the device including a multi-physics analysis. On the other hand, if we need to develop our device quickly we can use VHDL-AMS. It is a device description language that allows to describe a physical object simply writing a code row for each physical equation. In this case, if we need many equations we can simply write few code rows. VHDL-AMS offers also the possibility to create an interface between analog and digital worlds. For example, we can describe a MEMS and also the digital system to use it. Since it is a language, it is more difficult to debug the system. A more graphical approach like COMSOL can offer a better analysis for solving problems. VHDL-AMS offers higher modeling level with respect COMSOL. The highest level approach is offered by SIMULINK. In this case each equation is implemented by some blocks and for this reason we cannot describe too complex

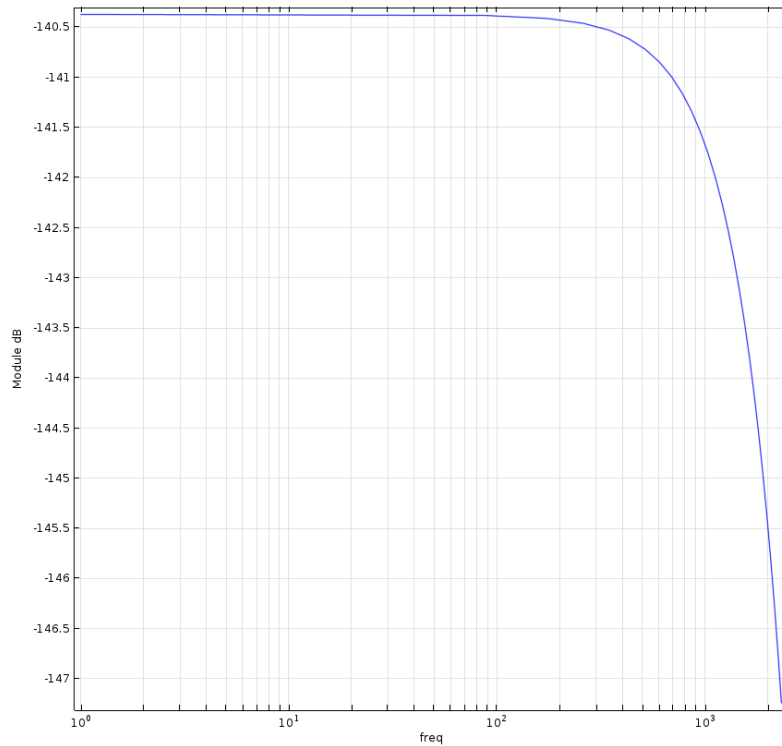
model. SIMULINK offers a graphical environment oriented to a mathematical approach and it allows the simplest debugging since the system is graphically implemented and each block has simple function that can be analyzed to find the problem. Since SIMULINK is integrated in MATLAB environment, it offers many functionalities including the possibility to use VHDL-AMS to describe block inside model implementation.

### *Results*

As we have seen before each tools give us good results that are comparable to the theoretical ones but there are some little differences between results. The output voltage at 8 g has little variations between COMSOL, SIMULINK and VHDL-AMS but they are limited at few millivolts. The Step Response instead is comparable and the time to reach the 100 % of the final value is exactly 300  $\mu$ s. The most important difference that we have found is related to the Bode diagram obtained by SIMULINK and COMSOL. In the first case, since the considered model was of the second order, bode diagram shows a slope of -40 dB/Dec after the flat band response in low frequency. In COMSOL instead, since the tool is really powerful, the Bode diagram has a different behavior at high frequency. [Figure 32](#) and [Figure 33](#) show the two Bode diagrams.



*Figure 32: SIMULINK Bode Diagram.*



*Figure 33: COMSOL Bode Diagram.*

The difference between the magnitudes of the two graphs is due to the fact the Simulink consider a unitary input acceleration as input for the Bode diagram evaluation. COMSOL instead uses 8 g input acceleration that is multiplied by the mass value so there are a difference in terms of dB but the shape in low frequency is the same. At higher frequency, COMSOL takes into account higher order of the system and so the behavior is different, with higher slope.



## Script Matlab

---

```

close all;
clear all;
%-----Constant Spring-----
k=20;
Eg=160E9;
W_Spring=2e-6;
L_Spring=235E-6;
Thickness=k/2/(W_Spring/L_Spring)^3/Eg
%-----Capacitance-----
g0=2e-6;
L_Plate=250e-6;
A=Thickness*L_Plate;
e=8.854187e-12;
C0=A*e/g0;
%-----Displacement-----
g=1e-07;
Threshold_Acceleration=9.81*8;
m= k*g/Threshold_Acceleration;
%-----W Mass Calculation-----
Density=2320;
Volume=m/Density;
L_Mass=500e-6;
W_Mass=Volume/L_Mass/Thickness;
%-----Damping-----
u0=1.81e-5;
b0=u0*A^2/g0^3
g_min=1.8e-6;
figure
x=linspace(-g0+g_min,g0-g_min,10000);
plot(x,1/2*u0*A^2.*( 1./(g0-x).^3+1./(g0+x).^3))
title('Damping Coefficient Variation')
xlabel('Displacement')
ylabel('Damping b')
bmax=1/2*u0*A^2*( 1/(g0-(g0-g_min))^3+1/(g0+(g0-g_min))^3);
deltab=(1-b0/bmax)*100;
%-----A Max-----
a_max=k*(g0-g_min)/m/9.81;
a_min=k*(-g0+g_min)/m/9.81;
%-----delta C-----
x=0.1e-6;
deltac=x*e*A/(g0^2-x^2);
Voltage=1;
Vout=deltac*Voltage/C0;
%-----Electrostatic Forces-----
a_elMax=(e*A*(Voltage^2)/2/m)*(1/(g_min^2)-1/(2*g0-g_min)^2);
a_elMin=-(e*A*Voltage^2/2/m)*(-1/g_min^2+1/(2*g0-g_min)^2);
%-----Bode Diagram-----
s=tf('s');
f=(1/m)/(s^2+b0/m*s+k/m);
[num,den]=tfdata(f,'v');
step(f)
figure
bode(f,{10, 5000*2*pi})
fn=sqrt(k/m)/2/pi
Q=fn*pi^2*m/b0
alpha=4*pi*fn/3/Q
beta=1/6/pi/fn/Q
c= alpha*m+beta*k;
%-----Evaluation of V Pull-In -----
VPull=sqrt(8*k*g0^3/27/e/A)
%-----Input Acceleration Simulink -----
a=78

```

## *Modeling Instructions*

---

From the **File** menu, choose **New**.

### **NEW**

- 1 In the **New** windows, click **Model Wizard**.

### **MODEL WIZARD**

- 2 In the **Model Wizard** window, click **3D**.
- 3 In the **Select physics** tree, select **Structural Mechanics > Electromechanics(emi)**.
- 4 Click **Add**.
- 5 Click **Study**.
- 6 In the **Select study** tree, select **Preset Studies for Selected Physics Interfaces>Stationary**.
- 7 Click **Done**.
- 8 Finally save your project.

### **GLOBAL**

On the **Home** toolbar, click **Parameters**.

### **DEFINITIONS**

#### **Parameters**

- 1 In the **Settings** window for Parameters, locate the **Parameters** section.
- 2 In the table, enter the following settings:

Name	Expression	Value	Description
a	$78[\text{m/s}^2]$	78 m/s <sup>2</sup>	Input Acceleration
alpha	$4\pi f_0/(3Q)$	38040 Hz	Alpha Rayleigh
beta	$1/(6\pi f_0 Q)$	2.4231-5 s	Beta Rayleigh
d	$2[\mu\text{m}]$	2E-6 m	Initial Distance Between Plates
f0	4458[Hz]	4458 Hz	Theoretical Resonance Frequency
HL_Spring	$2[\mu\text{m}]$	2E-6 m	Vertical Width of Spring
L_Anchor	$30[\mu\text{m}]$	3E-5 m	Anchor Length
L_Attach	$10[\mu\text{m}]$	1E-5 m	Attach Length
L_FixPlate	$250[\mu\text{m}]$	2.5E-4 m	Fixed Plate Length
L_Mass	$500[\mu\text{m}]$	5E-4 m	Mass Length
L_MovPlate	$L_{\text{FixPlate}} + 10[\mu\text{m}]$	2.6E-4	Movable Plate Length
L_Spring	$235[\mu\text{m}]$	2.35E-4 m	Spring Length
Q	0.49	0.49	Theoretical Quality Factor
Thickness	$101[\mu\text{m}]$	1.014E-4 m	Thickness of Accelerometer
V1	2.5[V]	2.5 V	Amplitude Voltage Generator
V2	-2.5[V]	-2.5 V	Amplitude Voltage Generator
W_FixPlate	$4[\mu\text{m}]$	4E-6 m	Fixed Plate Width
W_MovPlate	$4[\mu\text{m}]$	4E-6 m	Movable Plate Width
W_Spring	$2[\mu\text{m}]$	2E-6 m	Spring Width
WL_Spring	$10[\mu\text{m}]$	1E-5 m	Horizontal Length of Spring
W_Mass	$217[\mu\text{m}]$	2.17E-4 m	Mass Width
Z_Space	$2[\mu\text{m}]$	2E-6 m	Z_Space from 0 for Movable Structure

### *GEOMETRY 1*

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.
- 2 In the **Settings** window for Geometry, locate the **Units** section.
- 3 From the **Length unit** list, choose  $\mu\text{m}$ .

### **Anchor Plane (wp1)**

- 1 On the **Geometry** toolbar, click **Work Plane**.
- 2 In the **Settings** window for Work Plane 1, locate the **Label** text field and type Anchor Plane.

### **Anchor1 (r1)**

- 1 Under **Anchor Plane** click **Plane Geometry**.
- 2 On the **Work Plane** toolbar locate **Primitive** choose **Rectangle**.
- 3 In the **Settings** window for Rectangle, locate the **Label** text field and type Anchor1.
- 4 In the **Settings** window for Anchor1, locate the **Size and Shape** section.
- 5 In the **Width** text field, type L\_Anchor.
- 6 In the **Height** text field, type L\_Anchor.
- 7 In the **Settings** window for Anchor1, locate the **Position** section.
- 8 In the **xw** text field, type -L\_Anchor+W\_Spring.
- 9 In the **yw** text field, type HL\_Spring+L\_Spring.

## **Anchor2 (r2)**

- 1 Under **Anchor Plane** click **Plane Geometry**.
- 2 On the **Work Plane** toolbar locate **Primitive** choose **Rectangle**.
- 3 In the **Settings** window for Rectangle, locate the **Label** text field and type Anchor2.
- 4 In the **Settings** window for Anchor2, locate the **Size and Shape** section.
- 5 In the **Width** text field, type  $L\_Anchor$ .
- 6 In the **Height** text field, type  $L\_Anchor$ .
- 7 In the **Settings** window for Anchor1, locate the **Position** section.
- 8 In the **xw** text field, type  $WL\_Spring - W\_Spring + L\_Mass - W\_Spring + WL\_Spring - W\_Spring + 2 * L\_Attach$ .
- 9 In the **yw** text field, type  $HL\_Spring + L\_Spring$ .

## **Structure Plane (wp2)**

- 1 On the **Geometry** toolbar, click **Work Plane**.
- 2 In the **Settings** window for Work Plane 2, locate the **Label** text field and type Structure Plane.
- 3 In the **Settings** window for Structure Plane, locate the **z-coordinate** text field and type  $Z\_Space$ .

### **Mass (r1)**

- 1 Under **Structure Plane** click **Plane Geometry**.
- 2 On the **Work Plane** toolbar locate **Primitive** choose **Rectangle**.
- 3 In the **Settings** window for Rectangle, locate the **Label** text field and type Mass.
- 4 In the **Settings** window for Mass, locate the **Size and Shape** section.
- 5 In the **Width** text field, type  $L_{\text{Mass}}$ .
- 6 In the **Height** text field, type  $W_{\text{Mass}}$ .
- 7 In the **Settings** window for Mass, locate the **Position** section.
- 8 In the **xw** text field, type  $WL_{\text{Spring}} - W_{\text{Spring}} + L_{\text{Attach}}$ .
- 9 In the **yw** text field, type  $HL_{\text{Spring}} + L_{\text{Spring}} - (W_{\text{Mass}} - L_{\text{Anchor}})/2$ .

### **Movable Plate (r2)**

- 1 Under **Structure Plane** click **Plane Geometry**.
- 2 On the **Work Plane** toolbar locate **Primitive** choose **Rectangle**.
- 3 In the **Settings** window for Rectangle, locate the **Label** text field and type Movable Plate.
- 4 In the **Settings** window for Movable Plate, locate the **Size and Shape** section.
- 5 In the **Width** text field, type  $W_{\text{MovPlate}}$ .
- 6 In the **Height** text field, type  $L_{\text{MovPlate}}$ .

- 7 In the **Settings** window for Movable Plate, locate the **Position** section.
- 8 In the **xw** text field, type  $WL\_Spring - W\_Spring + L\_Attach + L\_Mass/2$ .
- 9 In the **yw** text field, type  $HL\_Spring + L\_Spring - (W\_Mass - L\_Anchor)/2 - L\_MovPlate$ .

### **FixedPlate1 (r3)**

- 1 Under **Structure Plane** click **Plane Geometry**.
- 2 On the **Work Plane** toolbar locate **Primitive** choose **Rectangle**.
- 3 In the **Settings** window for Rectangle, locate the **Label** text field and type FixedPlate1.
- 4 In the **Settings** window for FixedPlate1, locate the **Size and Shape** section.
- 5 In the **Width** text field, type  $W\_FixPlate$ .
- 6 In the **Height** text field, type  $L\_FixPlate$ .
- 7 In the **Settings** window for FixedPlate1, locate the **Position** section.
- 8 In the **xw** text field, type  $WL\_Spring - W\_Spring + L\_Attach + L\_Mass/2 + d + W\_MovPlate$ .
- 9 In the **yw** text field, type  $HL\_Spring + L\_Spring - (W\_Mass - L\_Anchor)/2 - L\_MovPlate$ .

### **FixedPlate2 (r4)**

- 1 Under **Structure Plane** click **Plane Geometry**.
- 2 On the **Work Plane** toolbar locate **Primitive** choose **Rectangle**.
- 3 In the **Settings** window for Rectangle, locate the **Label** text field and type FixedPlate2.
- 4 In the **Settings** window for FixedPlate2, locate the **Size and Shape** section.
- 5 In the **Width** text field, type  $W\_FixPlate$ .
- 6 In the **Height** text field, type  $L\_FixPlate$ .
- 7 In the **Settings** window for FixedPlate2, locate the **Position** section.
- 8 In the **xw** text field, type  $WL\_Spring - W\_Spring + L\_Attach + L\_Mass/2 - d - W\_FixPlate$ .
- 9 In the **yw** text field, type  $HL\_Spring + L\_Spring - (W\_Mass - L\_Anchor)/2 - L\_MovPlate$ .

### **Attach1 (r5)**

- 1 Under **Structure Plane** click **Plane Geometry**.
- 2 On the **Work Plane** toolbar locate **Primitive** choose **Rectangle**.
- 3 In the **Settings** window for Rectangle, locate the **Label** text field and type Attach1.
- 4 In the **Settings** window for Attach1, locate the **Size and Shape** section.
- 5 In the **Width** text field, type  $L\_Attach$ .
- 6 In the **Height** text field, type  $L\_Anchor$ .



- 7 In the **Settings** window for Attach1, locate the **Position** section.
- 8 In the **xw** text field, type WL\_Spring-W\_Spring.
- 9 In the **yw** text field, type HL\_Spring+L\_Spring.

### **Attach2 (r6)**

- 1 Under **Structure Plane** click **Plane Geometry**.
- 2 On the **Work Plane** toolbar locate **Primitive** choose **Rectangle**.
- 3 In the **Settings** window for Rectangle, locate the **Label** text field and type Attach2.
- 4 In the **Settings** window for Attach2, locate the **Size and Shape** section.
- 5 In the **Width** text field, type L\_Attach.
- 6 In the **Height** text field, type L\_Anchor.
- 7 In the **Settings** window for Attach2, locate the **Position** section.
- 8 In the **xw** text field, type WL\_Spring-W\_Spring+L\_Mass+L\_Attach.
- 9 In the **yw** text field, type HL\_Spring+L\_Spring.

### **Spring Plane (wp3)**

- 1 On the **Geometry** toolbar, click **Work Plane**.
- 2 In the **Settings** window for Work Plane 3, locate the **Label** text field and type Spring Plane.
- 3 In the **Settings** window for Spring Plane, locate the **z-coordinate** text field and type Z\_Space.

### **Spring1.1 (r1)**

- 1 Under **Spring Plane** click **Plane Geometry**.
- 2 On the **Work Plane** toolbar locate **Primitive** choose **Rectangle**.
- 3 In the **Settings** window for Rectangle, locate the **Label** text field and type Spring1.1.
- 4 In the **Settings** window for Spring1.1, locate the **Size and Shape** section.
- 5 In the **Width** text field, type W\_Spring.
- 6 In the **Height** text field, type L\_Spring.
- 7 In the **Settings** window for Spring1.1, locate the **Position** section.
- 8 In the **xw** text field, type 0.
- 9 In the **yw** text field, type HL\_Spring.

### **Spring1.2 (r2)**

- 1 Under **Spring Plane** click **Plane Geometry**.
- 2 On the **Work Plane** toolbar locate **Primitive** choose **Rectangle**.
- 3 In the **Settings** window for Rectangle, locate the **Label** text field and type Spring1.2.
- 4 In the **Settings** window for Spring1.2, locate the **Size and Shape** section.
- 5 In the **Width** text field, type W\_Spring.
- 6 In the **Height** text field, type L\_Spring.
- 7 In the **Settings** window for Spring1.2, locate the **Position** section.

- 8 In the **xw** text field, type WL\_Spring-W\_Spring.
- 9 In the **yw** text field, type HL\_Spring.

### **SpringL1 (r3)**

- 1 Under **Spring Plane** click **Plane Geometry**.
- 2 On the **Work Plane** toolbar locate **Primitive** choose **Rectangle**.
- 3 In the **Settings** window for Rectangle, locate the **Label** text field and type SpringL1.
- 4 In the **Settings** window for SpringL1, locate the **Size and Shape** section.
- 5 In the **Width** text field, type WL\_Spring.
- 6 In the **Height** text field, type HL\_Spring.
- 7 In the **Settings** window for SpringL1, locate the **Position** section.
- 8 In the **xw** text field, type 0.
- 9 In the **yw** text field, type 0.

### **Spring2.1 (r4)**

- 1 Under **Spring Plane** click **Plane Geometry**.
- 2 On the **Work Plane** toolbar locate **Primitive** choose **Rectangle**.
- 3 In the **Settings** window for Rectangle, locate the **Label** text field and type Spring2.1.
- 4 In the **Settings** window for Spring2.1, locate the **Size and Shape** section.

- 5 In the **Width** text field, type  $W_{\text{Spring}}$ .
- 6 In the **Height** text field, type  $L_{\text{Spring}}$ .
- 7 In the **Settings** window for Spring2.1, locate the **Position** section.
- 8 In the **xw** text field, type 0.
- 9 In the **yw** text field, type  $HL_{\text{Spring}} + L_{\text{Spring}} + L_{\text{Anchor}}$ .

### **Spring2.2 (r5)**

- 1 Under **Spring Plane** click **Plane Geometry**.
- 2 On the **Work Plane** toolbar locate **Primitive** choose **Rectangle**.
- 3 In the **Settings** window for Rectangle, locate the **Label** text field and type Spring2.2.
- 4 In the **Settings** window for Spring2.2, locate the **Size and Shape** section.
- 5 In the **Width** text field, type  $W_{\text{Spring}}$ .
- 6 In the **Height** text field, type  $L_{\text{Spring}}$ .
- 7 In the **Settings** window for Spring2.2, locate the **Position** section.
- 8 In the **xw** text field, type  $WL_{\text{Spring}} - W_{\text{Spring}}$ .
- 9 In the **yw** text field, type  $HL_{\text{Spring}} + L_{\text{Spring}} + L_{\text{Anchor}}$ .

### **SpringL2 (r6)**

- 1 Under **Spring Plane** click **Plane Geometry**.
- 2 On the **Work Plane** toolbar locate **Primitive** choose **Rectangle**.

- 3 In the **Settings** window for Rectangle, locate the **Label** text field and type SpringL2.
- 4 In the **Settings** window for SpringL2, locate the **Size and Shape** section.
- 5 In the **Width** text field, type WL\_Spring.
- 6 In the **Height** text field, type HL\_Spring.
- 7 In the **Settings** window for SpringL2, locate the **Position** section.
- 8 In the **xw** text field, type 0.
- 9 In the **yw** text field, type  $HL\_Spring + 2 * L\_Spring + L\_Anchor$ .

### Spring3.1 (r7)

- 1 Under **Spring Plane** click **Plane Geometry**.
- 2 On the **Work Plane** toolbar locate **Primitive** choose **Rectangle**.
- 3 In the **Settings** window for Rectangle, locate the **Label** text field and type Spring3.1.
- 4 In the **Settings** window for Spring3.1, locate the **Size and Shape** section.
- 5 In the **Width** text field, type W\_Spring.
- 6 In the **Height** text field, type L\_Spring.
- 7 In the **Settings** window for Spring3.1, locate the **Position** section.
- 8 In the **xw** text field, type  $WL\_Spring + 2 * L\_Attach + L\_Mass - 2 * W\_Spring$ .
- 9 In the **yw** text field, type HL\_Spring.

### Spring3.2 (r8)

- 1 Under **Spring Plane** click **Plane Geometry**.
- 2 On the **Work Plane** toolbar locate **Primitive** choose **Rectangle**.
- 3 In the **Settings** window for Rectangle, locate the **Label** text field and type Spring3.2.
- 4 In the **Settings** window for Spring3.2, locate the **Size and Shape** section.
- 5 In the **Width** text field, type  $W\_Spring$ .
- 6 In the **Height** text field, type  $L\_Spring$ .
- 7 In the **Settings** window for Spring3.2, locate the **Position** section.
- 8 In the **xw** text field, type  $WL\_Spring + L\_Mass - 3 * W\_Spring + WL\_Spring + 2 * L\_Attach$ .
- 9 In the **yw** text field, type  $HL\_Spring$ .

### SpringL3 (r9)

- 1 Under **Spring Plane** click **Plane Geometry**.
- 2 On the **Work Plane** toolbar locate **Primitive** choose **Rectangle**.
- 3 In the **Settings** window for Rectangle, locate the **Label** text field and type SpringL3.
- 4 In the **Settings** window for SpringL3, locate the **Size and Shape** section.
- 5 In the **Width** text field, type  $WL\_Spring$ .
- 6 In the **Height** text field, type  $HL\_Spring$ .

- 7 In the **Settings** window for SpringL3, locate the **Position** section.
- 8 In the **xw** text field, type  $WL\_Spring + L\_Mass - 2*W\_Spring + 2*L\_Attach$ .
- 9 In the **yw** text field, type 0.

### **Spring4.1 (r10)**

- 1 Under **Spring Plane** click **Plane Geometry**.
- 2 On the **Work Plane** toolbar locate **Primitive** choose **Rectangle**.
- 3 In the **Settings** window for Rectangle, locate the **Label** text field and type Spring4.1.
- 4 In the **Settings** window for Spring4.1, locate the **Size and Shape** section.
- 5 In the **Width** text field, type  $W\_Spring$ .
- 6 In the **Height** text field, type  $L\_Spring$ .
- 7 In the **Settings** window for Spring4.1, locate the **Position** section.
- 8 In the **xw** text field, type  $WL\_Spring + L\_Mass - 2*W\_Spring + 2*L\_Attach$ .
- 9 In the **yw** text field, type  $HL\_Spring + L\_Spring + L\_Anchor$ .

### **Spring4.2 (r11)**

- 1 Under **Spring Plane** click **Plane Geometry**.
- 2 On the **Work Plane** toolbar locate **Primitive** choose **Rectangle**.

- 3 In the **Settings** window for Rectangle, locate the **Label** text field and type Spring4.2.
- 4 In the **Settings** window for Spring4.2, locate the **Size and Shape** section.
- 5 In the **Width** text field, type  $W\_Spring$ .
- 6 In the **Height** text field, type  $L\_Spring$ .
- 7 In the **Settings** window for Spring4.2, locate the **Position** section.
- 8 In the **xw** text field, type  $WL\_Spring + L\_Mass - 3*W\_Spring + WL\_Spring + 2*L\_Attach$ .
- 9 In the **yw** text field, type  $HL\_Spring + L\_Spring + L\_Anchor$ .

### **SpringL4 (r12)**

- 1 Under **Spring Plane** click **Plane Geometry**.
- 2 On the **Work Plane** toolbar locate **Primitive** choose **Rectangle**.
- 3 In the **Settings** window for Rectangle, locate the **Label** text field and type SpringL4.
- 4 In the **Settings** window for SpringL4, locate the **Size and Shape** section.
- 5 In the **Width** text field, type  $WL\_Spring$ .
- 6 In the **Height** text field, type  $HL\_Spring$ .
- 7 In the **Settings** window for SpringL4, locate the **Position** section.
- 8 In the **xw** text field, type  $WL\_Spring + L\_Mass - 2*W\_Spring + 2*L\_Attach$ .
- 9 In the **yw** text field, type  $HL\_Spring + 2*L\_Spring + L\_Anchor$ .



### **Anchor Extrude (ext1)**

- 1 On the **Geometry** toolbar, click **Extrude**.
- 2 In the **Settings** window for Extrude 1, locate the **Label** text field and type Anchor Extrude.
- 3 In the **Settings** window for Anchor Extrude, locate the **General** section.
- 4 Select Anchor Plane(wp1) in the Work plane field and click on the wp1 Plane in the **Graphics** view.
- 5 In the **Settings** window for Anchor Extrude, locate the **Distance from Plane** section.
- 6 Type Thickness + Z\_Space in the Distances( $\mu\text{m}$ ) field.

### **Structure Extrude (ext2)**

- 1 On the **Geometry** toolbar, click **Extrude**.
- 2 In the **Settings** window for Extrude 2, locate the **Label** text field and type Structure Extrude.
- 3 In the **Settings** window for Structure Extrude, locate the **General** section.
- 4 Select Structure Plane(wp2) in the Work plane field and click on the wp2 Plane in the **Graphics** view.
- 5 In the **Settings** window for Structure Extrude, locate the **Distance from Plane** section.
- 6 Type Thickness in the Distances( $\mu\text{m}$ ) field.

### Spring Extrude (ext3)

- 1 On the **Geometry** toolbar, click **Extrude**.
- 2 In the **Settings** window for Extrude 3, locate the **Label** text field and type Spring Extrude.
- 3 In the **Settings** window for Structure Extrude, locate the **General** section.
- 7 Select Spring Plane(wp3) in the Work plane field and click on the wp3 Plane in the **Graphics** view.
- 4 In the **Settings** window for Structure Extrude, locate the **Distance from Plane** section.
- 5 Type Thickness in the Distances( $\mu\text{m}$ ) field.

### Box1 (blk1)

- 1 On the **Geometry** toolbar, click **Block**.
- 2 In the **Settings** window for Block 1, locate the **Label** text field and type Box1.
- 3 In the **Settings** window for Box1, locate the **Size and Shape** section.
- 4 In the **Width** text field, type 595.
- 5 In the **Depth** text field, type 640.
- 6 In the **Height** text field, type Thickness+8.
- 7 In the **Settings** window for Box1, locate the **Position** section.
- 8 In the **x** text field, type -30.
- 9 In the **y** text field, type -120.
- 10 In the **z** text field, type -2.

11 On the **Geometry** toolbar, click **Build All**.

## MATERIALS

On the **Home** toolbar, click **Windows** and choose **Add Material**.

### *ADD MATERIAL*

- 1 Go to the **Add Material** window.
- 2 In the tree, select **Liquids and Gases>Gases>Air**.
- 3 Click **Add to Component** in the window toolbar.

### *ADD MATERIAL*

- 1 Go to the **Add Material** window.
- 2 In the tree, select **MEMS>Semiconductors>Si – Polycrystalline Silicon**.
- 3 Click **Add to Component** in the window toolbar.
- 4 On the **Home** toolbar, click **Add Material** to close the **Add Material** window.

### *MATERIALS*

#### **Air (mat1)**

- 1 In the **Model Builder** window, under **Component 1 (comp1)>Materials** click **Air (mat1)**.
- 2 In the **Settings** window for Box1, locate the **Material Contents** section.

- 3 Locate **Relative permittivity** in the table and type 1 in the value field.
- 4 Select Domains 1 only.

### **Si – Polycrystalline Silicon (mat2)**

- 1 In the **Model Builder** window, under **Component 1 (comp1)>Materials** click **Si – Polycrystalline Silicon (mat2)** .
- 2 Select all Domains except 1 .

### *ELECTROMECHANICS (EMI)*

- 1 On the **Physics** toolbar, click **Domains** and choose **Linear Elastic Material**.
- 2 Select all Domains except 1.

### **Damping 1**

- 1 In the **Model Builder** window under **Component 1 (comp1)>Electromechanics (emi)** click **Linear Elastic Material 1**.
- 2 On the **Physics** toolbar, click **Attributes** and choose **Damping**.
- 3 In the **Settings** window for Damping, locate the **Damping Settings** section.
- 4 In the  $\alpha_{dM}$  text field, type alpha.
- 5 In the  $\beta_{dK}$  text field, type beta.

### **Fixed Constraint 1**

- 1 On the **Physics** toolbar, click **Domains** and choose **Fixed Constraint**.
- 2 Select Domains 2,11,13 and 19 only.

### **Gravity 1**

- 1 On the **Physics** toolbar, click **Domains** and choose **Gravity**.
- 2 Select all Domains except 1.
- 3 In the **Settings** window for Gravity 1, locate the **Gravity** section.
- 4 In the x component field, type a.

### **Ground 1**

- 1 On the **Physics** toolbar, click **Boundaries** and choose **Ground**.
- 2 Select Boundaries 3.

### **Terminal 1**

- 3 On the **Physics** toolbar, click **Boundaries** and choose **Terminal**.
- 4 Select Boundaries 73,74,75,76,77 and 78 only.
- 5 In the **Settings** window for Terminal 1, locate the **Terminal** section.
- 6 In the **Terminal Type** select Voltage.
- 7 In the **Electric Potential** field type V1.

## Terminal 2

- 1 On the **Physics** toolbar, click **Boundaries** and choose **Terminal**.
- 2 Select Boundaries 60,61,62,63,64 and 65 only.
- 3 In the **Settings** window for Terminal 2, locate the **Terminal** section.
- 4 In the **Terminal Type** select Voltage.
- 5 In the **Electric Potential** field type V2.

## Floating Potential 1

- 8 On the **Physics** toolbar, click **Boundaries** and choose **Floating Potential**.
- 9 Select Boundaries 66,67,68,69 and 71 only.

## MESH

- 1 In the **Model Builder** window click on **Mesh 1** under **Component 1 (Comp1)**.
- 2 In the **Settings** window for Mesh, locate the **Element Size** field.
  - 3 Select Coarse.

## DEFINITIONS

### Displacement (dom1)

- 4 On the **Home** toolbar, click **Definitions>Probes** and add **Domain Probe**.
- 5 In the **Settings** window for Domain Probe 1, locate the **Label** field.

- 6 Type Displacement.
- 7 Select Domains 10 and 12 in the **Source Selection** section.
- 8 Expand **Expression** section.
- 9 Type emi.disp in **Expression** field.

### **Vout (dom2)**

- 1 On the **Home** toolbar, click **Definitions>Probes** and add **Domain Probe**.
- 2 In the **Settings** window for Domain Probe 2, locate the **Label** field.
- 3 Type Vout.
- 4 Select Domains 12 in the **Source Selection** section.
- 5 Expand **Expression** section.
- 6 Type emi.V0\_fp1 in **Expression** field.
- 7 Activate the **Description** check box and type Output Voltage.

### **ADD STUDY**

- 1 On the **Home** toolbar, click **Add Study** to open the **Add Study** window.
- 2 Go to the **Add Study** window.
- 3 Find the **Studies** subsection. In the **Select study** tree, select **Custom Studies> Frequency Domain**.
- 4 Click **Add Study** in the window toolbar.

## ADD STUDY

- 1 On the **Home** toolbar, click **Add Study** to open the **Add Study** window.
- 2 Go to the **Add Study** window.
- 3 Find the **Studies** subsection. In the **Select study** tree, select **Preset Studies> Time Dependent**.
- 4 Click **Add Study** in the window toolbar.
- 5 On the **Home** toolbar, click **Add Study** to close the **Add Study** window.

## STUDY 1

- 1 Under **Study 1>Step 1: Stationary** click on the **Settings** window.
- 2 Locate the **Study Extensions** section.
- 3 Activate **Auxiliary sweep** clicking inside the check box.
- 4 Click on the plus symbol to add a new parameter.
- 5 Inside the **Parameter name** select a and type range(-154,7,154) in the **Parameter value list** field.
- 6 Type  $m/s^2$  in **Parameter unit** field.
- 7 On the **Home** toolbar, click **Compute**.

## STUDY 2

- 1 Under **Study 2>Step 1: Frequency Domain** click on the **Settings** window.
- 2 Locate the **Study Settings** section.
- 3 Type range(10,500,6000) in the **Frequencies** field.



- 4 On the **Home** toolbar, click **Compute**.

### STUDY 3

- 1 Under **Study 3>Step 1: Time Dependent** click on the **Settings** window.
- 2 Locate the **Study Settings** section.
- 3 Type range(0,1.0e-5,0.0015) in the **Times** field.
- 4 On the **Home** toolbar, click **Compute**.

### RESULTS

#### 1D Plot Displacement vs Acceleration

- 1 On the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.
- 2 In the **Settings** window for 1D Plot Group created, locate the **Label** field.
- 3 Type 1D Plot 1.
- 4 Locate and click on **1D Plot 1** in the **Model Builder** window.
- 5 On the **1D Plot 1** Toolbar click on **Table Graph**.
- 6 In the **Settings** window for Table Graph 1, locate the **Label** field.
- 7 Type Displacement vs Acceleration.
- 8 In the **Settings** window for Table Graph 1, locate the **Data** section.
- 9 Select Probe Table 1 in **Table** field.

- 10 Select  $a(m/s^2)$  in **x-axis data** field.
- 11 Select Manual in **Plot columns** field.
- 12 Select **Total displacement ( $\mu m$ ), Displacement**.
- 13 Click on **Plot** in the **1D Plot 1** toolbar.

### **1D Plot Vout vs Acceleration**

- 1 On the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.
- 2 In the **Settings** window for 1D Plot created, locate the **Label** field.
- 3 Type 1D Plot 2.
- 4 Locate and click on **1D Plot 2** in the **Model Builder** window.
- 5 On the **1D Plot 2** Toolbar click on **Table Graph**.
- 6 In the **Settings** window for Table Graph 2, locate the **Label** field.
- 7 Type Vout vs Acceleration.
- 8 In the **Settings** window for Table Graph 2, locate the **Data** section.
- 9 Select Probe Table 1 in **Table** field.
- 10 Select  $a(m/s^2)$  in **x-axis data** field.
- 11 Select Manual in **Plot columns** field.
- 12 Select **Output Voltage**.
- 13 Click on **Plot** in the **1D Plot 2** toolbar.

## 1D Plot Module Bode Diagram

- 1 On the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.
- 2 In the **Settings** window for 1D Plot created, locate the **Label** field.
- 3 Type 1D Plot 3.
- 4 In the **Settings** window for 1D Plot 3, locate the **Axis** section.
- 5 Activate the check box **x-axis log scale**.
- 6 Locate and click on **1D Plot 3** in the **Model Builder** window.
- 7 On the **1D Plot 3** Toolbar click on **Global**.
- 8 In the **Settings** window for Global 1, locate the **Label** field.
- 9 Type Module Bode Diagram.
- 10 In the **Settings** window for Module Bode Diagram, locate the **Data** section.
- 11 Select Study 2/Solution 2 (sol2).
- 12 In the **Settings** window for Module Bode Diagram, locate the **y - Data** section.
- 13 Type  $20 \cdot \log_{10}(\text{abs}(\text{dom1}))$  in the **Expression** field.
- 14 Type Module dB in the **Description** field.
- 15 Click on **Plot** in the **1D Plot 3** toolbar.

## 1D Plot Displacement Time Response

- 1 On the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.
- 2 In the **Settings** window for 1D Plot created, locate the **Label** field.

- 3 Type 1D Plot 4.
- 4 Locate and click on **1D Plot 4** in the **Model Builder** window.
- 5 On the **1D Plot 4** Toolbar click on **Global**.
- 6 In the **Settings** window for Global 2, locate the **Label** field.
- 7 Type Displacement Time Response.
- 8 In the **Settings** window for Time Response, locate the **Data** section.
- 9 Select Study 3/Solution 3 (sol3).
- 10 In the **Settings** window for Module Bode Diagram, locate the **y - Data** section.
- 11 Type dom1 in the **Expression** field.
- 12 Type Displacement Behavior in the **Description** field.
- 13 Click on **Plot** in the **1D Plot 4** toolbar.

### **1D Plot Output Voltage Time Response**

- 14 On the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.
- 15 In the **Settings** window for 1D Plot created, locate the **Label** field.
- 16 Type 1D Plot 5.
- 17 Locate and click on **1D Plot 5** in the **Model Builder** window.
- 18 On the **1D Plot 5** Toolbar click on **Global**.
- 19 In the **Settings** window for Global 3, locate the **Label** field.
- 20 Type Vout Time Response.
- 21 In the **Settings** window for Time Response, locate the **Data** section.

- 22 Select Study 3/Solution 3 (sol3).
- 23 In the **Settings** window for Module Bode Diagram, locate the **y - Data** section.
- 24 Type dom2 in the **Expression** field.
- 25 Type Output Voltage Behavior in the **Description** field.
- 26 Click on **Plot** in the **1D Plot 5** toolbar.

## VHDL-AMS Model

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```

----- VHDLAMS MODEL accelerometer -----
LIBRARY ieee;
use ieee.fluidic_systems.all;
use ieee.electrical_systems.all;
use ieee.mechanical_systems.all;
use ieee.thermal_systems.all;
use ieee.math_real.all;
use ieee.std_logic_1164.all;
----- ENTITY DECLARATION accelerometer -----
ENTITY accelerometer IS
  GENERIC(m : MASS := 2.548e-08;
    b0: DAMPING := 0.00145;
    C0: CAPACITANCE:=1.122e-13;
    u0: VISCOSITY:= 1.81e-5;
    Ampiezza: VOLTAGE :=1.0;
    k : STIFFNESS := 20.0;
    A : REAL := 2.535e-08;
    g0 : DISPLACEMENT := 2.0e-6;
    g_min: DISPLACEMENT := 1.8e-6;
    e0: REAL := 8.854e-12;
    ThreShold: REAL :=0.05);
  PORT(
    Reset: In bit ;
    OutFront: OUT bit:= '0';
    OutRetro: OUT bit:= '0';
    TERMINAL InputAcceleration: TRANSLATIONAL;
    TERMINAL IN1 : ELECTRICAL;
    TERMINAL OUT1 : ELECTRICAL);
END ENTITY accelerometer;
----- ARCHITECTURE DECLARATION simple -----
ARCHITECTURE simple OF accelerometer IS
  QUANTITY Vin ACROSS IN1 TO Electrical_Ref;
  QUANTITY Vout ACROSS Iout THROUGH OUT1 to electrical_ref;
  QUANTITY g ACROSS Fext THROUGH InputAcceleration;
  QUANTITY Fel: FORCE:=0.0;
  QUANTITY deltaC: CAPACITANCE:=0.0;
  SIGNAL clockP,clockN: std_logic:= '0';
  BEGIN
    if g>(g0-g_min) USE
      g:=(g0-g_min);

```

```

elseif g<(-g0+g_min) USE
g=(-g0+g_min);
else
g'dot'dot== Fext/m - k*g/m - b0*g'dot/m+Fel/m;
end use;
Fel==(Ampiezza**2.0*e0*A)/2.0*( 1.0/(g0-g)**2.0-1.0/(g0+g)**2.0);
deltaC==e0*A*g/(g0**2.0-g**2.0);
Vout==Vin*deltaC/C0;

process (Vout'ABOVE(Threshold)) is
BEGIN
if Vout'ABOVE(Threshold) then
clockP<='1';
else
clockP<='0';
end if;
end process;

process (Vout'ABOVE(-Threshold)) is
BEGIN
if NOT Vout'ABOVE(-Threshold) then
clockN<='1';
else
clockN<='0';
end if;
end process;

Process(clockP,Reset)is
BEGIN
if(Reset='0')then
OutFront<='0';
elseif(clockP'event AND ClockP='1')then
OutFront<='1';
end if;
end process;

Process(clockN,Reset)is
BEGIN
if(Reset='0')then
OutRetro<='0';
elseif(clockN'event AND ClockN='1')then
OutRetro<='1';
end if;
end process;
END ARCHITECTURE simple;

```

Observing the model, we note that there are two sections: Entity and Architecture. The Entity includes the GENERIC statement for the definition of all parameters used in our system. Moreover it includes the PORT section that describes both digital ports and terminals:

- Reset: digital input for the reset of two D Flip-Flop;
- OutFront: digital output of the positive threshold D Flip-Flop;
- OutRetro: digital output of the negative threshold D Flip-Flop;
- InputAcceleration: translational input for the external acceleration;
- IN1: electrical input for the bias generator  $V_1$ ;
- OUT1: electrical output for the output signal  $V_O$ .

Inside the Architecture section we found the definition of all quantities in order to assign them to the corresponding terminal. We have also defined internal quantities and digital signals. After the BEGIN statement we found both continuous and discrete descriptions.

### ***Continuous-Time Section***

The description is the following:

```

if g>(g0-g_min) USE
g==(g0-g_min);
elseif g<(-g0+g_min) USE
g==(-g0+g_min);
else
g'dot'dot== Fext/m - k*g/m -b0*g'dot/m+Fel/m;
end use;

Fel==(Ampiezza**2.0*e0*A)/2.0*( 1.0/(g0-g)**2.0-1.0/(g0+g)**2.0);
deltaC==e0*A*g/(g0**2.0-g**2.0);
Vout==Vin*deltaC/C0;

```

As we can see there is the “If-Elsif-Else” statement for the saturation modeling for both negative and positive limit. If no saturations occur,



the system uses the complete differential equation in order to calculate g variation:

$$m \cdot g'' = m \cdot a_{ext} - k \cdot g - b_0 \cdot g' + \frac{\varepsilon_0 \cdot A \cdot V_P^2}{2} \left[ \frac{1}{(g_0 - g)^2} - \frac{1}{(g_0 + g)^2} \right]$$

Here the saturation description is simpler than Simulink. Since we use a differential equation and we do not have to set to zero the acceleration or the velocity because this happens automatically inside the equation. After the “If-Else-Else” statement, we find the  $\Delta C$  and  $V_O$  formulas.

### ***Discrete-Time Section***

The description is showed below:

```
FFA:Process(clockP,Reset)is
BEGIN
if(Reset='0')then
OutFront<='0';
elseif(clockP'event AND ClockP='1')then
OutFront<='1';
end if;
end process;
```

```
FFB:Process(clockN,Reset)is
BEGIN
if(Reset='0')then
OutRetro<='0';
elseif(clockN'event AND ClockN='1')then
OutRetro<='1';
end if;
end process;
```

It includes two D Flip-Flop with two different clocks: ClockN and ClockP. These are the two outputs of the comparators. The first one is related to RetroComparator and the second one to FrontComparator. They are used in the same way described inside

the Simulink model.

### ***Continuous-Discrete Interface***

The description is showed below:

```
Process (Vout'ABOVE(Threshold)) is
BEGIN
if Vout'ABOVE(Threshold) then
clockP<='1';
else
clockP<='0';
end if;
end process;
Process (Vout'ABOVE(-Threshold)) is
BEGIN
if NOT Vout'ABOVE(-Threshold) then
clockN<='1';
else
clockN<='0';
end if;
end process;
```

Two comparators are the connection between the continuous and discrete sections. The first one is the FrontComparator and it asserts the clockP signal. The second one is the RetroComparator and it asserts the clockN signal.

## *References*

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1. COMSOL Tutorial Frequency Response of a Biased Resonator—3D
2. SENSING AND CONTROL OF MEMS ACCELEROMETERS  
USING KALMAN FILTER by KAI ZHANG.
3. Optimization of MEMS capacitive accelerometer by  
Mekkakia Maaza Nasreddine and Mourad Benmessaoud.
4. From MEMS to Bio-MEMS and Bio-NEMS: Manufacturing  
Techniques and Applications by Marc J. Madou.
5. [www.aci.it](http://www.aci.it)
6. [www.quattroruote.it](http://www.quattroruote.it)
7. Project: VHDL-AMS & MEMS Accelerometer by Fabio Grassi,  
Alessandro Nadal and Stefano Iuliano.
8. Surface Micromachined Accelerometer tutorial by COMSOL team.
9. Design Optimization of MEMS Comb Accelerometer  
Kanchan Sharma, Isaac G. Macwan, Linfeng Zhang, Lawrence  
Hmurcik, Xingguo Xiong.