

SolutionOfSheet1

November 10, 2021

1 IPSA 2021 - Exercise 1

The goal of this exercise is to get used to practical image processing in *python / numpy / scipy*.

Do **NOT** use additional third party libraries such as *OpenCV* or *scikit-image* for the coding tasks in this course!

1.1 0 - Warm Up

```
In [18]: import imageio
import numpy as np
import scipy.ndimage as img
import matplotlib.pyplot as plt

import timeit, functools

In [19]: def imageRead(imgname, pilmode='L', arrtype=np.float):
        """
        Read an image file into a numpy array

        imgname: str
            name of image file to be read
        pilmode: str
            for luminance / intensity images use L
            for RGB color images use RGB

        arrtype: numpy dtype
            use np.float, np.uint8, ...
        """
        return imageio.imread(imgname, pilmode=pilmode).astype(arrtype)

def imageWrite(arrF, imgname, arrtype=np.uint8):
    """
    Write a numpy array as an image file
    the file type is inferred from the suffix of parameter imgname, e.g. .png
    arrF: array_like
        array to be written
    imgname: str
```

```
name of image file to be written
arrtype: numpy dtype
use np.uint8, ...
"""
imageio.imwrite(imgname, arrF.astype(arrtype))
```

To display an intensity image use:

```
In [20]: arrF = imageRead("portrait.png")
plt.imshow(arrF / 255, cmap='gray')
plt.xticks([]); plt.yticks([])
plt.show()
```



To display an (RGB) color image use:

```
In [24]: arrF = imageRead('asterixRGB.png', pilmode='RGB')
plt.imshow(arrF / 255)
plt.xticks([]); plt.yticks([])
plt.show()
```



1.2 1 - The emboss effect

In the Data folder for this exercise, you will find the intensity image `portrait.png`. Read it into a numpy array `arrF` and print the shape of this array to determine its number of rows and columns.

```
In [6]: filename = "portrait.png"

arrF = imageRead(filename)
print(f"The image '{filename}' has {arrF.shape[0]} rows and {arrF.shape[1]} columns.")
```

The image 'portrait.png' has 256 rows and 256 columns.

Different emboss implementations In the lecture, we discussed the idea of “embossing” an image such that the resulting image resembles a copper engraving. In fact, we discussed four different methods to accomplish this:

```
In [27]: def embossV1(arrF):
        """
        Naive Emboss: Iterate through all cells and explicitly compute the
        every gradient. Do so by computing the difference between the intensity
        values in the lower right and upper left cell.
        Add 255/2 = 128 to compensate for the induced value shift.
        Clip the results to the interval [0, 255].

        Slow implementation.
        """
```

```

M, N = arrF.shape
arrG = np.zeros((M,N))

for i in range(1,M-1):
    for j in range(1,N-1):
        arrG[i,j] = 128 + arrF[i+1,j+1] - arrF[i-1,j-1]
        arrG[i,j] = np.maximum(0, np.minimum(255, arrG[i,j]))

return arrG

```

```

In [28]: def embossV2(arrF):
        """
        Numpythonic Emboss: Use python-slicing to compute all gradients
        at once by subtracting the upper left (M-2 x N-2)-matrix
        from the lower right (M-2 x N-2)-matrix - to yield the gradients
        of the central (M-1 x N-1)-matrix.
        Add 255/2 = 128 to compensate for the induced value shift.
        Clip the results to the interval [0, 255].

        Fast implementation.
        """
        M, N = arrF.shape
        arrG = np.zeros((M,N))

        arrG[1:M-1,1:N-1] = 128 + arrF[2:,2:] - arrF[:-2,:-2]
        arrG = np.maximum(0, np.minimum(255, arrG))

        return arrG

```

```

In [29]: def embossV3(arrF):
        """
        Convolution Emboss: similar to CNNs, use a 3x3-mask which sweeps
        over all non-border-cells.
        Add 255/2 = 128 to compensate for the induced value shift.
        Clip the results to the interval [0, 255].

        Fast and practical (generalizable) implementation.
        """
        mask = np.array([[ -1,  0,  0],
                          [  0,  0,  0],
                          [  0,  0, +1]])

        arrG = 128 + img.correlate(arrF, mask, mode='reflect')
        arrG = np.maximum(0, np.minimum(255, arrG))

        return arrG

```

```

In [30]: def embossV4(arrF):
        """

```

Numpythonic Emboss: Use python-slicing to compute all gradients at once by subtracting the upper left (M-1 x N-1)-matrix from the lower right (M-1 x N-1)-matrix - to yield the gradients of the central (M-1 x N-1)-matrix. Notice, that only this matrix is stored, thus the dimensions are diminished in each direction by two and no black bordered will remain.

Add $255/2 = 128$ to compensate for the induced value shift.

Clip the results to the interval [0, 255], by using python's array boolean indexing. (Masks for the corner values.)

```
arrG = 128 + arrF[2:,2:] - arrF[:-2,:-2]
arrG[arrG < 0] = 0
arrG[arrG > 255] = 255
```

```
return arrG
```

Apply each of the above methods to arrF to produce a corresponding array arrG and write each of your results as a PNG image. Does the result you obtain from embossV4 differ from the results produced by the other methods? It should! Discuss the difference!

```
In [11]: methods = [embossV1, embossV2, embossV3, embossV4]
        arrGs = [arrF] * len(methods)

        fig, axs = plt.subplots(int(0.5*len(methods)), 2, figsize=(20,20))

        for i, mtd in enumerate(methods):
            arrGs[i] = mtd(arrF)

            ax = axs[int(i/2), i%2]
            ax.imshow(arrGs[i], cmap='gray')
            ax.set_title(f"{mtd.__name__}:", fontsize=40)
            ax.set_xticklabels([]); ax.set_yticklabels([])
            fig.tight_layout()
```

embossV1:



embossV2:



embossV3:



embossV4:



Notice that method embossV3 and embossV4 do not result in a black border.

A closer examination shows that embossV4 trims the image. Its results has dimension 254×254 . Only embossV3 computes non-zero values for the border.

```
In [12]: for i, arrG in enumerate(arrGs):
          print(f"Image dimensions when using method embossV{i+1}: {arrG.shape}")
          print("\tDoes %screate a black border.\n\n"%( "not " if arrG[0][100] != 0 else ""))
```

Image dimensions when using method embossV1: (256, 256)

Does create a black border.

Image dimensions when using method embossV2: (256, 256)

Does create a black border.

Image dimensions when using method embossV3: (256, 256)
Does not create a black border.

Image dimensions when using method embossV4: (254, 254)
Does not create a black border.

1.3 2 - Timing the emboss effect

In the lecture, we also performed experiments to determine the minimum average runtime of our different methods for the emboss effect. In this task you are supposed to conduct these experiments on your own machines.

1.3.1 2 a)

```
In [25]: def runtime_evaluation(data, methods, nRep, nRun):
          print("Mean runtimes for:")
          for method in methods:
              ts = timeit.Timer(func tools.partial(method, data)).repeat(nRep, nRun)
              print(f"{method.__name__}: \t{min(ts) / nRun} \t[sec]")

In [33]: methods = [embossV1, embossV2, embossV3, embossV4]
          nRep = 3
          nRun = 100

In [34]: filename0 = "portrait.png"
          arrF0 = imageRead(filename0)

          runtime_evaluation(arrF0, methods, nRep, nRun)
```

Mean runtimes for:

embossV1:	0.3334390802099915	[sec]
embossV2:	0.000724664239996855	[sec]
embossV3:	0.001031361499999548	[sec]
embossV4:	0.00036626205000175107	[sec]

1.3.2 2 b)

```
In [36]: filename1 = "asterix.png"
          arrF1 = imageRead(filename1)

          runtime_evaluation(arrF1, methods, nRep, nRun)
```

Mean runtimes for:

embossV1:	1.894169599070001	[sec]
embossV2:	0.005108834220000063	[sec]
embossV3:	0.0062503762200049095	[sec]
embossV4:	0.002401966990000801	[sec]

1.3.3 2 c) - Discuss your results. What do your experiments reveal? Is there any noteworthy difference between the runtimes for the two images? If so, what is the difference? What causes this difference? What does this tell you about image processing in general?

```
In [19]: n0, m0 = arrF0.shape
         n1, m1 = arrF1.shape
         print(f"The image '{filename0}' has dimensions {n0}x{m0} and thus {n0*m0} cells.")
         print(f"The image '{filename1}' has dimensions {n1}x{m1} and thus {n1*m1} cells.")
         print(f"That is almost {round(n1*m1/(n0*m0))}-times as much!")
```

The image 'portrait.png' has dimensions 256x256 and thus 65536 cells.
The image 'asterix.png' has dimensions 496x730 and thus 362080 cells.
That is almost 6-times as much!

These runtimes grow approximately linear wrt to the number of cells. That is as expected, since all implementations iterate over all pixels and thus have runtime complexity in $\mathcal{O}(mn)$.

1.4 3 - Working with RGB color images

```
In [ ]: arrF = imageRead('asterixRGB.png', pilmode='RGB')

         print(arrF.shape)
         plt.imshow(arrF / 255)
         plt.xticks([]); plt.yticks([])
         plt.show()
```

(496, 730, 3)



```
In [ ]: arrG = np.copy(arrF)
        arrG[:, :, 0] = 0

        print(arrG.shape)
        plt.imshow(arrG / 255)
        plt.xticks([]); plt.yticks([])
        plt.show()
```

(496, 730, 3)



We can see, that all values of the third RGB-channel were set to zero. Thus the **image does no longer contain the color red**. Red is replaced by black.

Notice that asterix's trousers are black. And the white background is only composed out of the colors green and blue.

Explanation of the code `arrG[:, :, 0] = 0` operates on all cells (`[:, :]`) and the first (zeroth) of the color channels (which are stored in the third position). Thus the red-value for every pixel is set to zero/black.

1.5 4 - Getting used to slicing (part 1)

Read image `asterix.png` into an array `arrF` and image `portrait.png` into an array `arrG`.

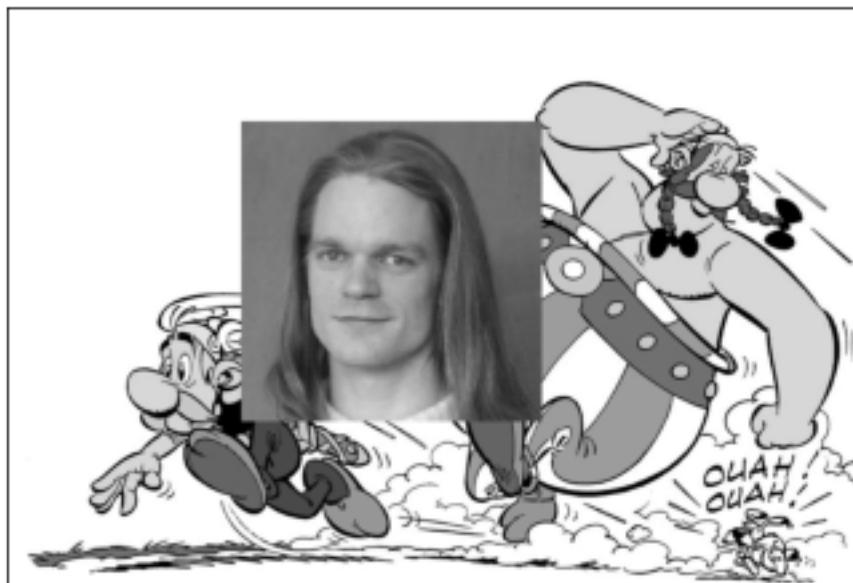
Now, create a copy `arrH` of `arrF` and then - without using for loops - paste `arrG` into `arrH` such that the upper left corner of `arrG` is at array coordinate `[i, j] = [100, 200]` in `arrH`. Write your result as a PNG image.

```
In [24]: arrF = imageRead('asterix.png')
         arrG = imageRead('portrait.png')

         arrH = np.copy(arrF)
         m, n = arrG.shape

         i, j = 100, 200
         arrH[i:i+m, j:j+n] = arrG

In [25]: plt.imshow(arrH / 255, cmap='gray')
         plt.xticks([]); plt.yticks([])
         plt.show()
```



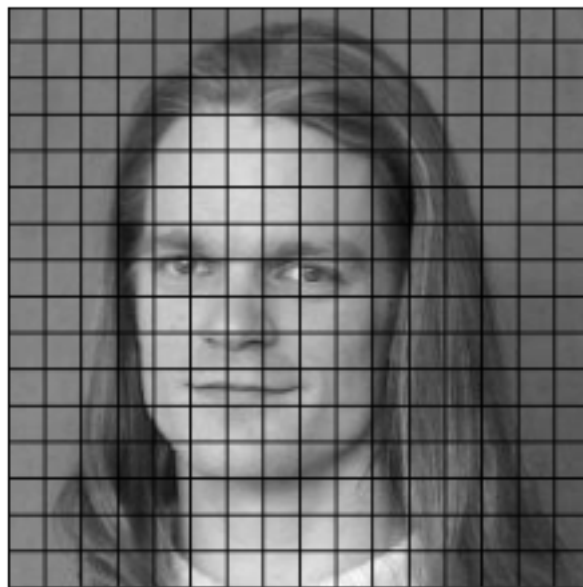
1.6 5 - Getting used to slicing (part 2)

Read image `portrait.png` into an array `arrF`. Then - without using for loops - set all the intensities of the pixels in every 16th column and every 16th row of `arrF` to zero. Write your result as a PNG image.

```
In [27]: arrF = imageRead('portrait.png')
```

```
d = 16
arrF[:,::d] = 0
arrF[:,d:,] = 0
```

```
In [28]: plt.imshow(arrF / 255, cmap='gray')
plt.xticks([]); plt.yticks([])
plt.show()
```



1.7 6 - Getting used to meshgrids (part 1)

Read image `portrait.png` into an array `arrF`. Then - without using for loops - set the intensities of all its pixels situated within an ellipse of *width* $2 * 50$ and *height* $2 * 85$ which is centered at array coordinates $[c_i, c_j] = [128, 110]$ to 255. Write your result as a PNG image.

```
In [75]: arrF = imageRead('portrait.png')
m, n = arrF.shape
```

```
width_offset = 50
height_offset = 85
ci, cj = 128, 110
```

```
In [76]: # Use meshgrid to create an index mask.
rs, cs = np.meshgrid(np.arange(m), np.arange(n), indexing='ij')

# For the ellipse use a similar formula like the unit circle
# Simply offset the origin by the defined center, and
# scale the height (y-axis) and width (x-axis) differently.
mask = ((rs-ci)**2 / height_offset**2 + (cs-cj)**2 / width_offset**2 <= 1)

# Apply the change to the image - in the area of the mask.
arrF[mask] = 255

In [77]: plt.imshow(arrF / 255, cmap='gray')
plt.xticks([]); plt.yticks([])
plt.show()
```



1.8 7 - Getting used to meshgrids (part 2)

```
In [35]: arrF = imageRead('portrait.png')
radii = [32, 16, 64]

In [36]: def circle_cutouts(arrF, r=32):
m, n = arrF.shape
```

```

# Create a meshgrid for only one circle (with diameter 2*r).
rs, cs = np.meshgrid(np.arange(2*r), np.arange(2*r), indexing='ij')

# Define the circle cutout.
# Notice that the OUTSIDE of the circle is masked ('>').
mask = ((rs-r)**2 + (cs-r)**2 > r**2)

# Tile the circle as often as it fits in the dimensions
mask = np.tile(mask, (m//(2*r), n//(2*r)))

# Apply the mask
arrG = np.copy(arrF)
arrG[mask] = 255

return arrG

```

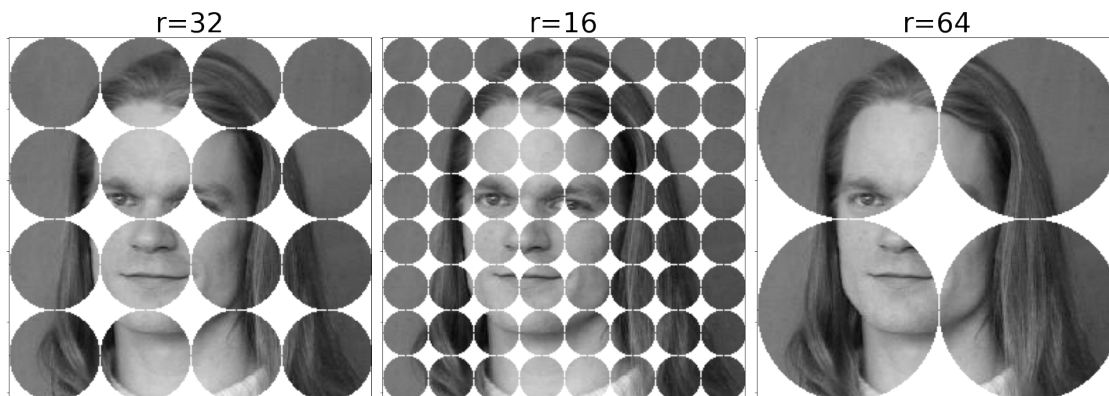
```
In [38]: fig, axs = plt.subplots(1, 3, figsize=(20,20))
```

```

for i, radius in enumerate(radii):
    # Generate the image with the circle cutouts
    arrG = circle_cutouts(arrF, r=radius)

    # Arrange the image(s) in one figure
    ax = axs[i]
    ax.imshow(arrG, cmap='gray')
    ax.set_title(f"r={radius}", fontsize=35)
    ax.set_xticklabels([]); ax.set_yticklabels([])
    fig.tight_layout()

```



Runtime measurements

```
In [ ]: nRep = 3
        nRun = 100
```

```
ts = timeit.Timer(func tools.partial(circle_cutouts, arrF)).repeat(nRep, nRun)
print(f"{min(ts) / nRun} [sec]")
```

0.00025478838999788423 [sec]

1.9 8 - Getting used to meshgrids (part 3)

Again - without using for loops - create an image of size $M \times N$ where $M = N = 256$ that displays the following function

$$f[x,y] = \frac{1}{2} \left(\sin \left(2\pi \nu \frac{x}{N-1} + \frac{\pi}{2} (N-1) \right) + 1 \right) * 255$$

```
In [43]: def sin_func(x, y, nu, n):
        sinus_term = np.sin(2 * np.pi * nu * x / (n-1) + np.pi / 2 * (n-1))
        return 0.5 * (sinus_term + 1) * 255
```

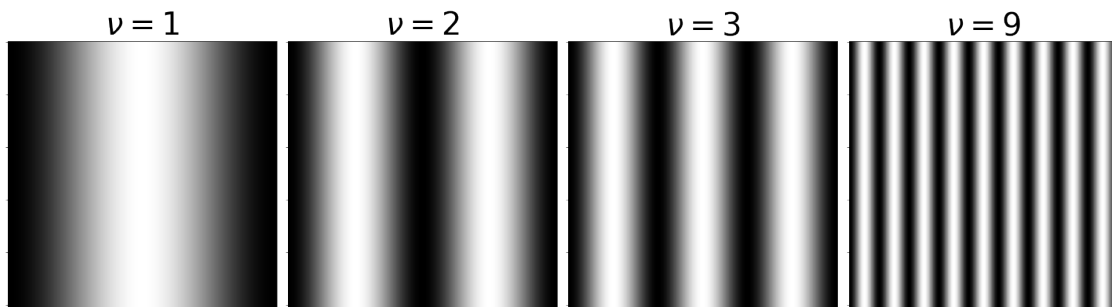
```
In [46]: # Generate a meshgrid for all plots
        xs, ys = np.meshgrid(np.arange(m), np.arange(n))

        nu_list = [1., 2., 3., 9.]
        m = n = 256
```

```
In [47]: fig, axs = plt.subplots(1, len(nu_list), figsize=(20,20))

        for i, nu in enumerate(nu_list):
            # Generate the image with the since function
            arrF = sin_func(xs, ys, nu, n)

            # Arrange the image(s) in one figure
            ax = axs[i]
            ax.imshow(arrF, cmap='gray')
            ax.set_title(r"$\nu$=%i"%nu, fontsize=40)
            ax.set_xticklabels([]); ax.set_yticklabels([])
            fig.tight_layout()
```



1.10 9 - Getting used to meshgrids (part 4)

Again - without using for loops - create an image of size $M \times N$ where $M = N = 256$ that displays the following function

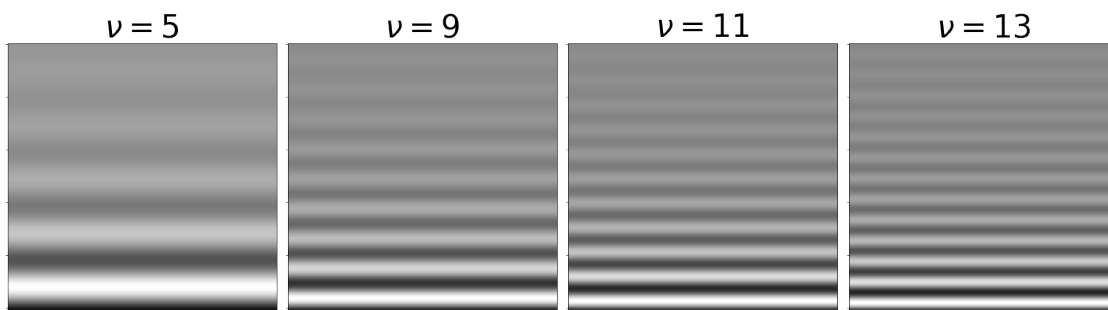
$$f[x, y] = \frac{1}{2} \left(e^{-\frac{4y}{M}} \sin \left(2\pi \nu \frac{y}{M-1} + \frac{\pi}{2}(M-1) \right) + 1 \right) * 255$$

```
In [50]: def dampedVerticalSineWave(M, N, nu):  
         xs, ys = np.meshgrid(np.arange(M), M-1-np.arange(N))  
         return 0.5 * (np.exp(-4 / M * ys) * np.sin(2 * np.pi * nu * ys / (M-1) + np.pi / 2))
```

```
In [51]: nu_list = [5., 9., 11., 13.]
```

```
In [52]: fig, axs = plt.subplots(1, len(nu_list), figsize=(20,20))
```

```
for i, nu in enumerate(nu_list):  
    arrF = dampedVerticalSineWave(256, 256, nu)  
  
    ax = axs[i]  
    ax.imshow(arrF, cmap='gray')  
    ax.set_title(r"$\nu$=%i"%nu, fontsize=40)  
    ax.set_xticklabels([]); ax.set_yticklabels([])  
    fig.tight_layout()
```



1.11 10 - Getting used to universal functions

Again - without using for loops - implement a method that turns a given intensity image function $f[x, y]$ into another function $g[x, y]$ where

$$g[x, y] = \cos \left(f[x, y] \nu \frac{2\pi}{255} \right) 127.5 + 127.5$$

```
In [12]: def solarize(arrF, nu):  
         return np.cos(arrF * nu * 2 * np.pi / 255.) * 127.5 + 127.5
```

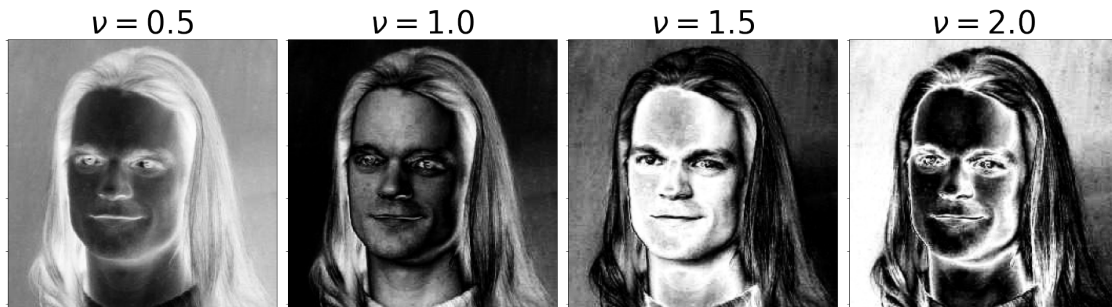
```
In [13]: nu_list = [0.5, 1.0, 1.5, 2.0]  
         arrF = imageRead('portrait.png')
```

```
In [14]: fig, axs = plt.subplots(1, len(nu_list), figsize=(20,20))

for i, nu in enumerate(nu_list):
    arrG = solarize(arrF, nu)

    ax = axs[i]
    ax.imshow(arrG, cmap='gray')

    ax.set_title(r"$\nu$="+ f"{nu:.1f}", fontsize=40)
    ax.set_xticklabels([]); ax.set_yticklabels([])
    fig.tight_layout()
```



```
In [ ]:
```