Getting Started

At Kennedy Middle School, the probability that a student takes Technology and Spanish is 0.087. The probability that a student takes Technology is 0.68. What is the probability that a student takes Spanish given that the student is taking Technology?

Probability Functions

Data Science Immersive

FLATIRON SCHOOL

Agenda

- Introduce probability functions - how to formulaically represent probabilities?
- Test our understanding with examples and practices
- Introduce concept and application of binomial distribution

After today, you will be able to...

- Distinguish between discrete and continuous random variables.
- Compute probabilities, cumulative probabilities, means and variances for discrete random variables.
- Identify binomial random variables and their characteristics.
- Calculate probabilities of binomial random variables.

Terminology

Random Variable

Is a variable whose value results from the process of a random experiment

Types of Random Variables:

Qualitative Random Variables

The possible values vary in kind but not in numerical degree. They are also called categorical variables. (Favorite TV show, Relationship status, College Major)

Quantitative Random Variables

There are two types of quantitative random variables.

- Discrete Random Variable: When the random variable can assume only a countable, sometimes infinite, number of values. (# of cousins, years in school)
- Continuous Random Variable: When the random variable can assume an uncountable number of values in a line interval. (Age, Height, Weight)

Probability Functions

Probability Function

A probability function is a mathematical function that provides probabilities for the possible outcomes of the random variable, X. It is typically denoted as $f(x)$.

Probability Mass Function (PMF)

If the random variable is a discrete random variable, the probability function is usually called the probability mass function (PMF). If X is discrete, then $f(x)=P(X=x)$. In other words, the PMF for a constant, x, if the probability that the random variable X is equal to x.

Probability Density Function (PDF)

If the random variable is a continuous random variable, the probability function is usually called the probability density function (PDF). Contrary to the discrete case, $f(x)\neq P(X=x)$.

The probability of any given point in a PDF is 0!

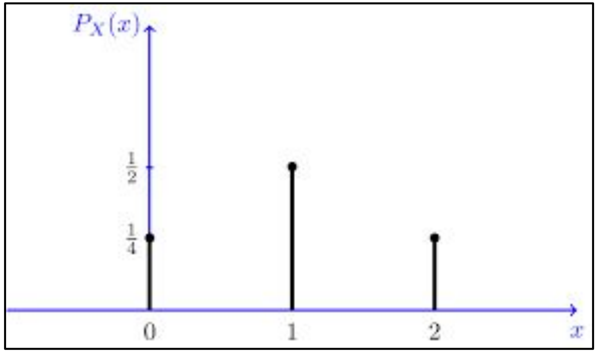
Probability Mass Function

- Probability Mass Function is a function that maps the frequency of a **discrete** set of data to distribution

$$fX(x) = P(X = x)$$

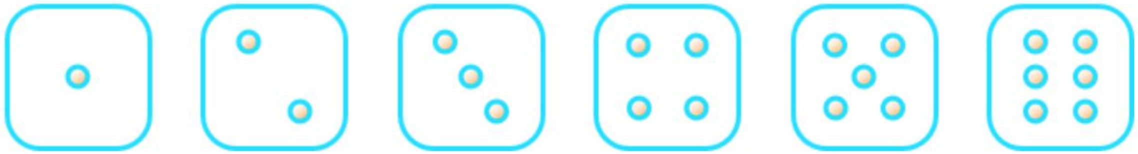
- The values of pmf must sum to 1
- f(x) can only take on values [0,1]

$$f_X(x) = \Pr(X = x) = P(\{s \in S : X(s) = x\})$$

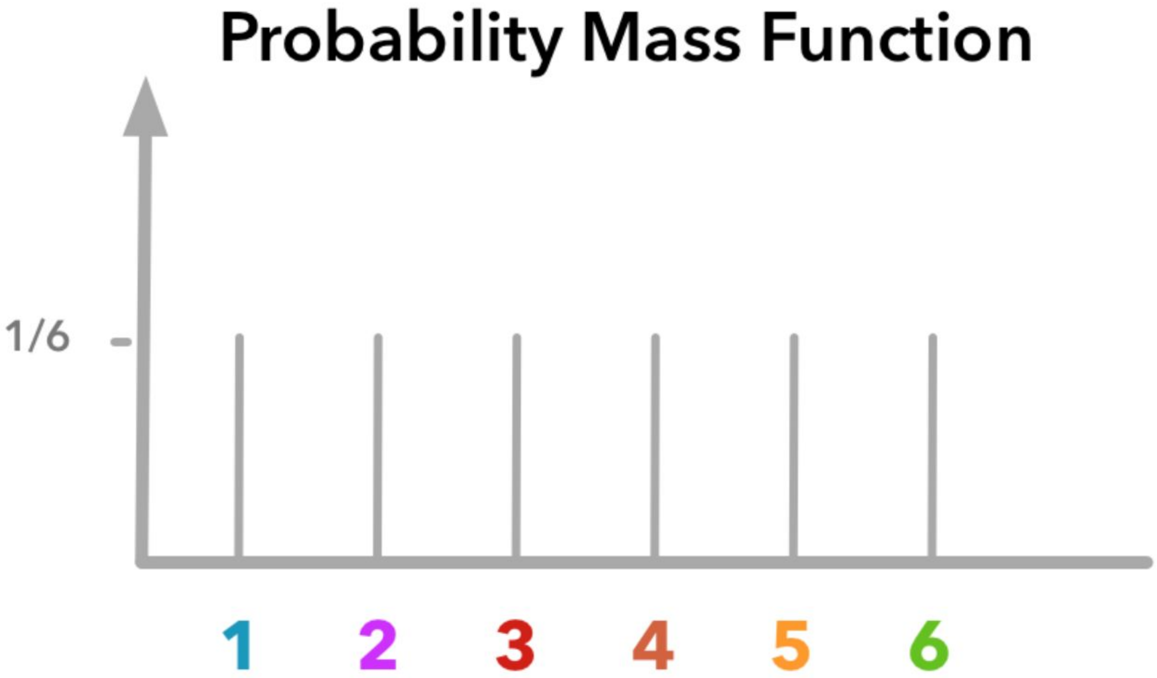


Probability Mass Function - Example

- Draw the probability distribution of a fair, standard dice roll
  - What are the values the random variable could possibly take on?
  - What is the probability laying at each value the rv takes on?



Probability Mass Function - Example



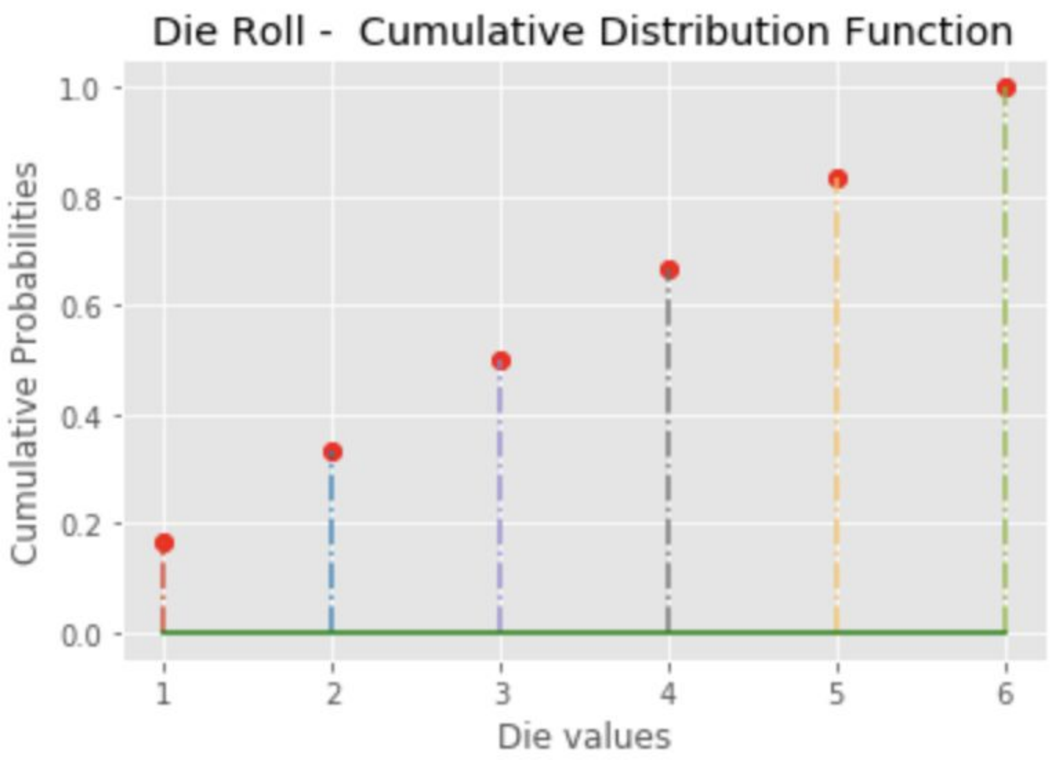
Cumulative Mass Function

Cumulative frequency sums the frequencies <= a particular value.

$$F(x) = P(X \leq x)$$

Outcome	Frequency	Cumulative Frequency	Cumulative frequency probability
1	1	1	.167
2	1	2	.33
3	1	3	.5
4	1	4	.67
5	1	5	.833
6	1	6	1

Cumulative Mass Function



Cumulative Mass Function

The following is a table showing how 10 students rated the movie “US” on a scale of 1-5. Complete the table and then draw the CMF.

Outcome	Frequency	Cumulative Frequency	Cumulative frequency probability
1	1	1	
2	2		
3	3		
4	3		
5	2		

Discrete Random Variable

Expected Value (or mean) of a Discrete Random Variable:

$$\mu = E(X) = \sum x_i f(x_i)$$

The formula means that we multiply each value, *x*, in the support by its respective probability, *f(x)*, and then add them all together. It can be seen as an average value but weighted by the likelihood of the value.

Deal or No Deal

You have 4 suitcases in front of you that have a value of $1, $1,000, $10,000 and $100,000. You can choose 1 suitcase and win that amount of money or take a offer of $25,000.

If you were to base your decision solely on the expected value of a suitcase, what would you do?

Deal or No Deal

Variance of a Discrete Random Variable
The variance of a discrete random variable is given by:

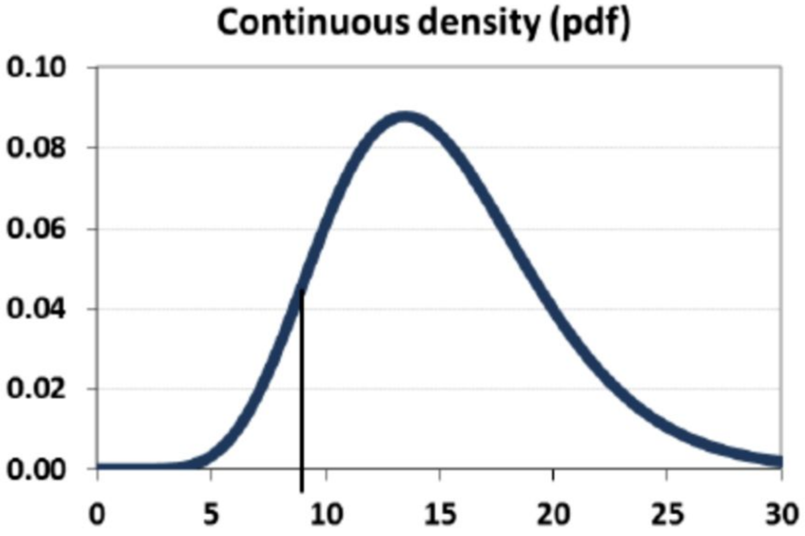
$$\sigma = \mathrm{SD}(X) = \sqrt{\mathrm{Var}(X)} = \sqrt{\sigma^2}$$

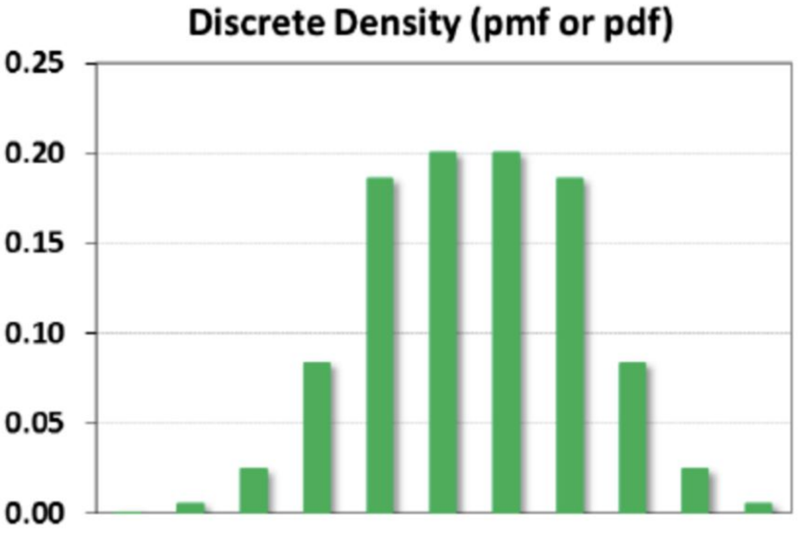
Standard Deviation of a Discrete Random Variable
The standard deviation of a random variable, *X*, is the square root of the variance.

$$\sigma^2 = \mathrm{Var}(X) = \sum (x_i - \mu)^2 f(x_i)$$

Continuous Probability Functions

If the data is continuous the distribution is modeled using a probability density function (PDF).

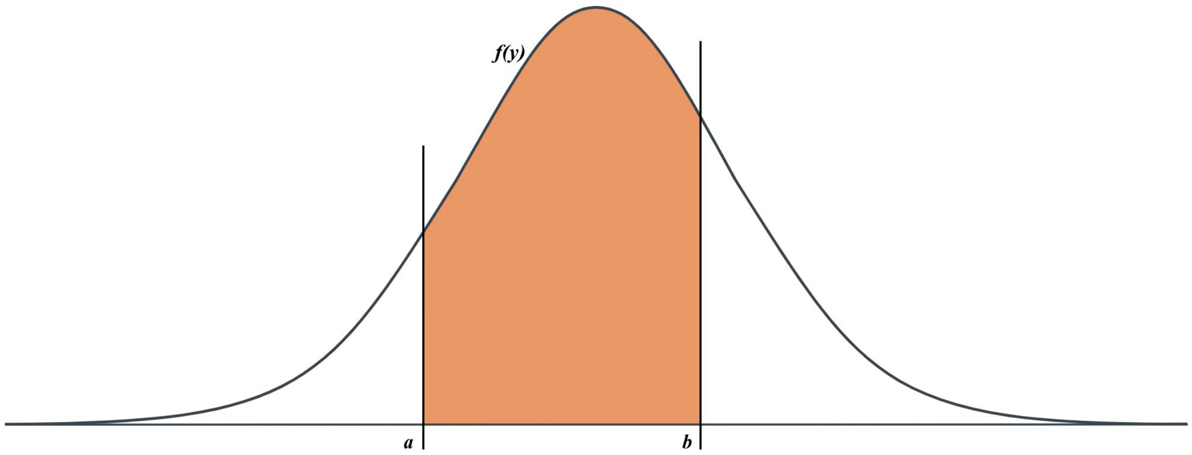




The probability of any given point in a PDF is 0!

Continuous Probability Functions

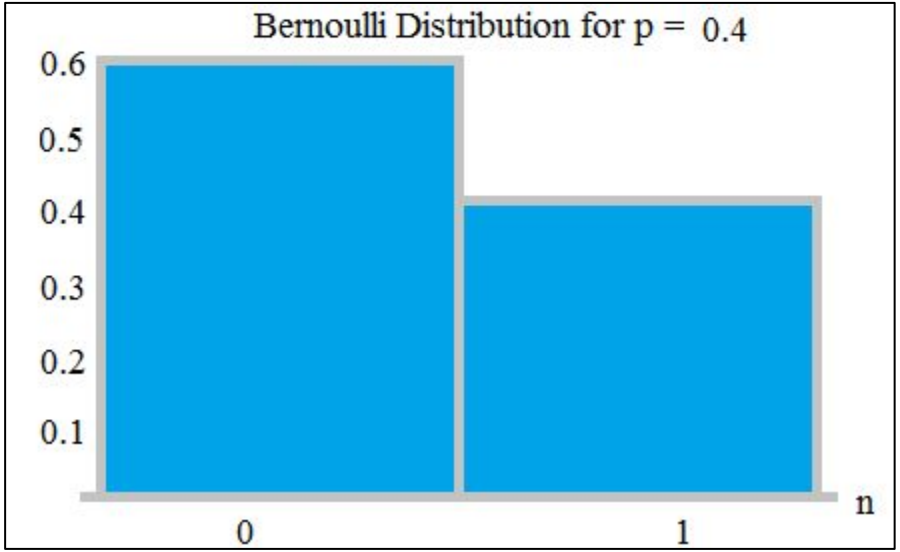
We define the probability distribution function (PDF) of *Y* as *f(y)* where: *P(a<Y<b)* is the area under *f(y)* over the interval from *a* to *b*.



The Bernoulli Trial

Bernoulli Trials

Bernoulli trials are experiments with two outcomes, such as coin flip, win/lose etc



How can a dice roll experiment be thought of as Bernoulli Trials?

The Binomial Distribution

Binomial distribution

The binomial distribution is a special discrete distribution where there are two distinct complementary outcomes, a “success” and a “failure”.

A binomial random variable is random variable that represents the number of successes in n successive independent trials of a Bernoulli experiment.

Some example uses include the number of heads in n coin flips, the number of disk drives that crashed in a cluster of 1000 computers, and the number of advertisements that are clicked when 40,000 are served.

The Binomial Distribution

The Binomial Distribution

We have a binomial experiment if ALL of the following four conditions are satisfied:

1. The experiment consists of n identical trials.
2. Each trial results in one of the two outcomes, called success and failure.
3. The probability of success, denoted p, remains the same from trial to trial.
4. The n trials are independent. That is, the outcome of any trial does not affect the outcome of the others.

If the four conditions are satisfied, then the random variable *X*=number of successes in *n* trials, is a binomial random variable with

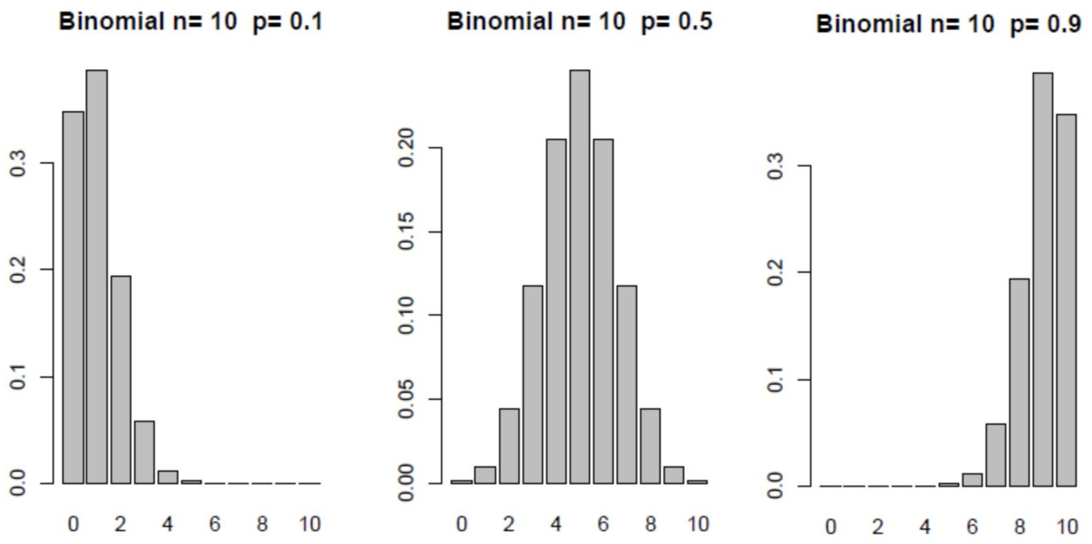
$\mu = E(X) = np$ (Mean)

$\mathrm{Var}(X) = np(1-p)$ (Variance)

$\mathrm{SD}(X) = \sqrt{np(1-p)}$, where p is the probability of the "success." (Standard Deviation)

Binomial Distributions

The formula defined above is the probability mass function, pmf, for the Binomial. We can graph the probabilities for any given *n* and *p*. The following distributions show how the graphs change with a given n and varying probabilities.



The Binomial Distribution

An FBI survey shows that about 80% of all property crimes go unsolved. Suppose that in your town 3 such crimes are committed and they are each deemed independent of each other. What is the probability that 1 of 3 of these crimes will be solved?

The example above and its formula illustrates the motivation behind the binomial formula for finding exact probabilities.

The Binomial Distribution

For the FBI Crime Survey example, what is the probability that at least one of the crimes will be solved?

The Binomial Formula

For a binomial random variable with probability of success, p, and n trials...

$$f(x) = P(X = x) = \frac{n!}{x!(n-x)!}p^x(1-p)^{n-x} \text{ for } x = 0, 1, 2, \ldots, n$$

The Binomial Distribution

Two fair coins being flipped should land the same way 50%. Two coins are flipped at the same time, what is the probability that they will land the same way 8 or more times?

The Binomial Formula

For a binomial random variable with probability of success, p, and n trials...

$$f(x) = P(X = x) = \frac{n!}{x!(n-x)!}p^x(1-p)^{n-x} \text{ for } x = 0, 1, 2, \ldots, n$$