Starting Off:

1. What are the chances that you will be on time to class in the morning?
2. What are the chances if it is a Friday?
3. What are the chances if it is Friday and it is raining really hard?
4. How did you go about coming up with your answer?

Foundation of Probabilities

Data Science Immersive

August 6, 2019

FLATIRON SCHOOL

Probability Background

- The goal of this unit is to make inferences about the population based on the sample.
    - If you want to predict the chance that a house will sell for more than a million dollars, you need to be able to calculate the probability, then update that probability based on some conditions.
- In order to learn about inference, we need to learn a few more things first. Inference requires an understanding of probability , random variables, and probability distributions . This Lesson will focus on the first step, probability.
- Allows us to make informed decision
    - Should you make a living by purchasing lottery?
    - The Monte Carlo Fallacy
- Allows us to speak about uncertainty in an informed way
- **INTERVIEWS!**

Foundations of Probabilities

- Agenda today
  - Set & Set Theory
  - Independent Probabilities
  - Permutations & Combinations

After today, you'll be able to...

- Gain an intuitive understanding of why we need to learn probabilities
- Calculate and compute examples of independent probabilities
- Apply permutations and combinations to real life examples

Probability Notation

Probability is the likelihood of an outcome. Before we can properly define probability, we must first define 'events.' It is helpful and convenient to denote the collection of events as a single letter rather than list all possible outcomes.

Event
a collection of outcomes, typically denoted by capital letters such as A, B, C, etc...

- Suppose we ask 30 students to record their eye color. We can define an event B to be blue eye color. In other words, let $B=\{blue\ eyes\}$.

Probability Notation

Converting to Probability Notation

1. Identifying the outcome event of interest: {Getting a Tail when we toss a fair coin}.
2. Use a single letter or word to represent this outcome of interest: T={Getting a Tail when we toss a fair coin}, for instance.
3. State your interest in the probability of this outcome: P(T) which is read, "Probability of getting a Tail when we toss a fair coin."

Now let's complicate things. Consider again tossing a fair coin. We stated P(T) was the probability we get a tail when we toss the coin. How would one write the probability statement if the outcome was getting two tails when the coin was tossed twice?

Probability Notation

Now let's complicate things. Consider again tossing a fair coin. We stated P(T) was the probability we get a tail when we toss the coin. How would one write the probability statement if the outcome was getting two tails when the coin was tossed twice?

Applying the steps we get...

1. Identify the outcome of the event: *Getting a tail on both tosses of a fair coin.*
2. Use a single letter or word to represent this outcome of interest: *We can write this as T,T, where the first T represents the outcome of the first toss and the second T as the outcome of the second toss.*
3. State your interest in the probability of this outcome: *P(T,T) which is read, "Probability of getting a Tail on the first and second toss."*

*Note: Often the comma is eliminated and P(T,T) is written as P(TT).

Set Theory

Outcome Space

- The outcome space of a scenario is all the possible outcomes that can occur and is often denoted S. The outcome space may also be referred to as the sample space

- Consider the experiment where two fair six-sided dice are rolled and their face values recorded. Write down the outcome space.

- In probability theory, a set is denoted as a well-defined collection of **distinct** objects.
- Mathematically, you can define a set by S. If an element x belongs to a set S, then you'd write $x \in S$. On the other hand, if x does not belong to a set S, then you'd write $x \notin S$.

Set Theory

- Consider the experiment where two fair six-sided dice are rolled and their face values recorded.

- **Directions:** Write out the event in probability notation and then identify the outcome space.

Subset & Set Operations

Subset

Set T is a subset of set S if every element in set T is also in set S. The mathematical notation for a subset is $T \subseteq S$.

- $T = \{2,3,4\}$
- $S = \{1,2,3,4,5\}$
- $T \subseteq S$

Set Operations

Now that we know how to denote events, the next step is to use the set notation to represent set operations. Each operation will also be presented in a Venn Diagram. Set operations are important because they allow us to create a new event by manipulation of other events

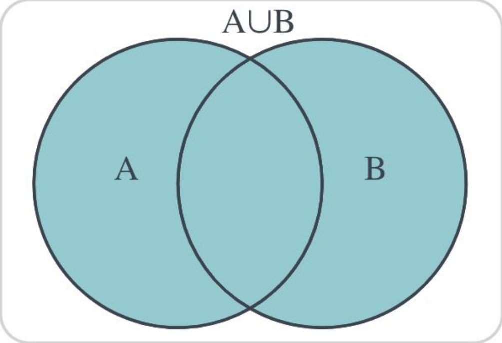
Set Operations

Union

The union of two events, A and B, contains all of the outcomes that are in A, B or both. In statistics, ‘or’ means at least one event occurs and therefore includes the event where both occur,

The union of A and B is denoted:

$$A \cup B$$



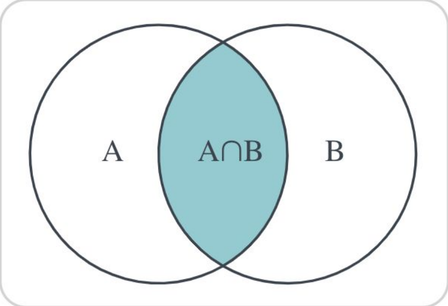
{outcomes in both A or B or both}

Intersection

The intersection of two events, A and B, contains all of the outcomes that are in both A and B.

The intersection is denoted by:

$$A \cap B$$



{outcomes in both A and B}

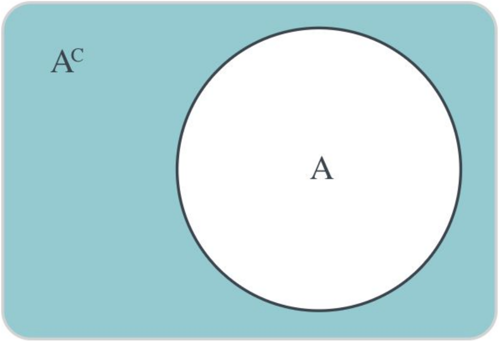
Set Operations

Complement

The complement of an event, A, contains all of the outcomes that are not in A.

The complement can be denoted as:

$$A^c, \bar{A}, \text{ or } A'$$



{outcomes not in *A*}

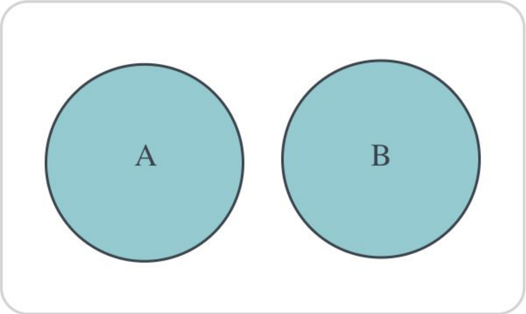
Mutually Exclusive

A and *B* are called mutually exclusive (or disjoint) if the occurrence of outcomes in *A* excludes the occurrence of outcomes in *B*.

There are no elements in $A \cap B$ and thus:

$$A \cap B = \varnothing$$

The empty set, denoted as $\varnothing$, is an event that contains no outcomes.



$A \cap B = \varnothing$

Set Operations

Suppose events A, B, and C are events of a particular scenario. Write the following using event notation.

1. At least one event occurs.
2. None of the events occur.
3. Only A occurs.

Set Operations

Let's go back to the example where we roll two fair six-sided die. Given the following events:

$$A = (3, 5)$$

$$B = \text{a 4 is rolled on the first die}$$

$$C = \text{a 5 is rolled on the second die}$$

$$D = \text{the sum of the dice is 7}$$

$$E = (7, 4)$$

Find $B \cap D$ and $B \cup D$

Practice:

- $B \cup C$
- $B \cap C$

Independent Probabilities

- What is probabilities?

  Probability theory is the study on the **frequency** of a given event occurring in some context.

Probability Properties

Probability theory is the study on the frequency of a given event occurring in some context.

Probability of an event

Probabilities will always be between (and including) 0 and 1. A probability of 0 means that the event is impossible. A probability of 1 means an event is guaranteed to happen.We denote the probability of event A as P(A).

$$0 \leq P(A) \leq 1$$

Probability of a complement

If A is an event, then the probability of A is equal to 1 minus the probability of the complement of A, A'.

$$P(A) = 1 - P(A') \text{ or } 1 = P(A) + P(A')$$

Probability Properties

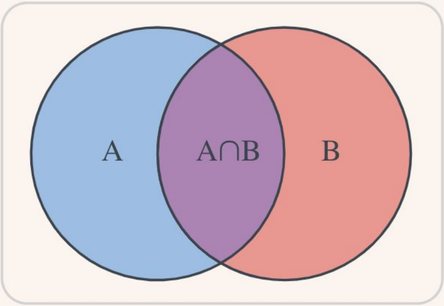
Probability of the empty set

If A and B are mutually exclusive, then A∩B=∅.
Therefore, P(A∩B)=0. This is important when we consider mutually exclusive (or disjoint) events.

Probability of the union of two events

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If A and B are mutually exclusive, then $P(A) + P(B)$.



Probability Properties

Directions: Use the information given below to work out your answer to the following questions.

Given $P(A)=0.6$, $P(B)=0.5$, and $P(A \cap B)=0.2$.

1. Find $P(A')$.
2. Find $P(A \cap B')$.
3. Find $P(B \cap A')$.
4. Find $P(A \cup B)$.

•

Independent Probabilities

What is the probability of event *A* occurring?

- Event *A* is known as the event space, and all possible events are known as the sample space
- P(A) = A / (all possible events)
- For example, what is the probability of drawing an Ace in a deck of cards?

Independent Events

Two events, A and B, are considered independent events if the probability of A occurring is not changed based on any knowledge of the outcome of B.

Dependent Events

Two events are not independent, or dependent if the knowledge of the outcome of B changes the probability of A.

Independent Probabilities

We can check for independence of two events by showing that any ONE of the following is true. For any given probabilities for events A and B, the events are independent if:

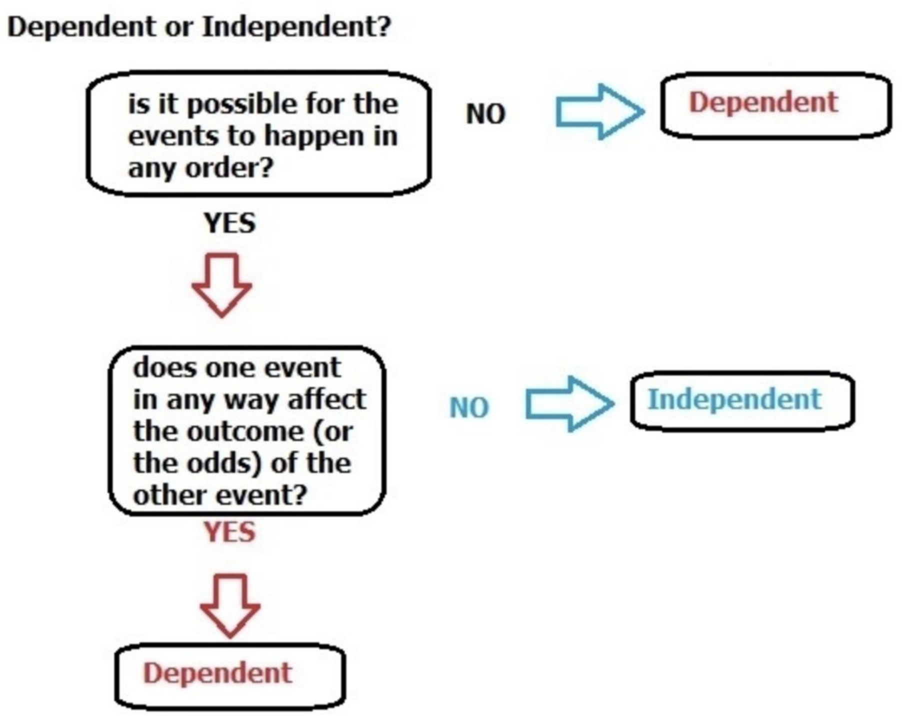
1. $P(A \cap B) = P(A) \cdot P(B)$
2. $P(A|B) = P(A)$

   or,

3. $P(B|A) = P(B)$

Independent Probabilities

- What are some examples of independent events?
- What are some examples of dependent events?



Independent Probabilities

You apply to grad schools at Harvard (H) and PSU (P). Probability accepted into Harvard is 0.3. Probability accepted into PSU is 0.6. The probability of being accepted to both is 0.25. Noting the probabilities:

$P(H)=0.3$,

$P(P)=0.6$, and

$P(H \cap P)=.25$

Are events getting accepted into Harvard and into Penn State independent events?

Independent Probabilities

Independent vs. Mutually Exclusive

Students often confuse independent events and mutually exclusive events. The two terms mean very different things. Recall that if two events are mutually exclusive, they have no elements in common and thus cannot both happen at the same time. In fact, mutually exclusive events are dependent. If A and B are mutually exclusive events, then...

$P(A \cap B) = 0 \neq P(A)P(B)$

Combinatorics - Example

- What is the probability of getting exactly 4 heads out of 6 tosses?

Combinatorics

- We throw around combinatorics language in our daily life, sometimes incorrectly. For example, we ask what is our safe combination, even though in reality we are really asking what is our safe permutations.
- So what is the difference?
  - Ordering matters in Permutation
  - Ordering doesn’t matter in Combination

Combinatorics - Examples

- What are the number of different ways in which you can get a 5-card hand from a standard deck of cards?

- Is this an example of combination or permutation?

Permutations

$$P^{n}_{k} = \frac{n!}{(n-k)!}$$

Permutations

Example 1 - How many different ways can you arrange the seating of 5 students in the front row?

Students = ['Bridget', 'Georgina', 'Tawab', 'Danny', 'Jeremy O']

Example 2 - how many different ways can you give a 4 digit passwords on an iphone assuming that digit cannot be repeated?

Permutations

Example 3 (optional) - how many different ways can you arrange the letter **MISSISSIPPI**?

$$\frac{n!}{n_1!n_2!\ldots n_k!}$$

Combination

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Combination

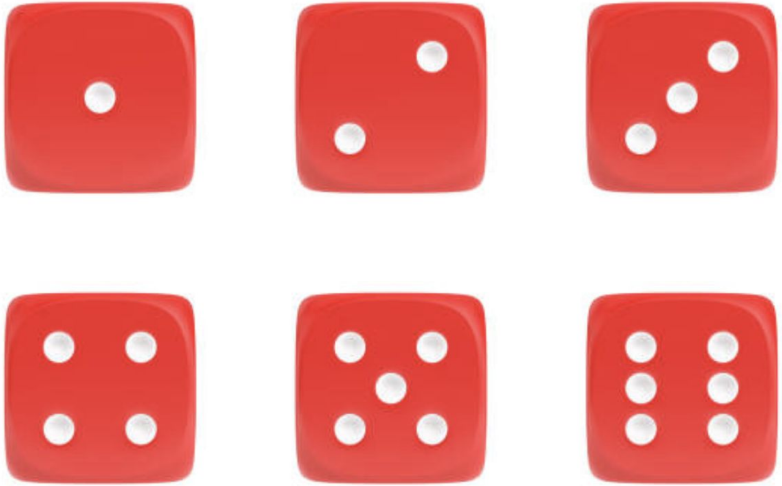
Example 1 - How many different ways can I randomly give 3 Amazon gift cards to the 14 students here?

Combinatorics

- What is the probability of getting at least 3 heads out of 6 tosses?

Combinatorics - Example

- A pair of fair, standard die are rolled. What is the probability that the results sum up to 5?



Combinatorics - Example

- In a roomful of 30 people, what is the probability that at least two people have the same birthday? Assume birthdays are uniformly distributed and there is no leap year complication.