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Tarea 1: reducción de datos

- Problema 1:

↳ $$\Delta x = \sum_{i=1}^{n} \frac{\partial x}{\partial u_i} \Delta u_i$$

a) Sea $x = Au$, entonces $\Delta x = A \Delta u$

- Tenemos que: $\frac{\partial x}{\partial u_i} = \frac{\partial (Au)}{\partial u_i} \longrightarrow \frac{\partial x}{\partial u_i} = A \frac{\partial u}{\partial u_i}$

- Por lo tanto: $\Delta x = \sum_{i=1}^{n} \frac{\partial x}{\partial u_i} \Delta u_i \longrightarrow$

- $\Delta x = \sum_{i=1}^{n} A \frac{\partial u}{\partial u_i} \Delta u_i$
- $\Delta x = A \sum_{i=1}^{n} \frac{\partial u}{\partial u_i} \Delta u_i$
- $\Delta x = A \Delta u$ ∎

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b) Sea $x = Au + Bv$, entonces $\Delta x = A\Delta u + B\Delta v$

- Tenemos que: $\frac{\partial x}{\partial u_i} = \frac{\partial}{\partial u_i}(Au + Bv) \longrightarrow \frac{\partial x}{\partial u_i} = A \frac{\partial u}{\partial u_i} + B \frac{\partial v}{\partial u_i}$

- Por lo tanto: $\Delta x = \sum_{i=1}^{n} \frac{\partial x}{\partial u_i} \Delta u_i \longrightarrow$

- $\Delta x = \sum_{i=1}^{n} \left[A \frac{\partial u}{\partial u_i} + B \frac{\partial v}{\partial u_i} \right] \Delta u_i$
- $\Delta x = \left[A \sum_{i=1}^{n} \frac{\partial u}{\partial u_i} \Delta u_i \right] + \left[B \sum_{i=1}^{n} \frac{\partial v}{\partial u_i} \Delta u_i \right]$
- $\Delta x = A\Delta u + B\Delta v$ ∎

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c) Sea $x = uv$, entonces $\frac{\Delta x}{x} = \frac{\Delta u}{u} + \frac{\Delta v}{v}$

Tenemos que:

- $$\Delta x = \sum_{i=1}^{n}\left[v\frac{\partial u}{\partial u_i} + u\frac{\partial v}{\partial u_i}\right]\Delta u_i$$

- $$\Delta x = \left[\sum_{i=1}^{n} v\frac{\partial u}{\partial u_i}\Delta u_i + u\frac{\partial v}{\partial u_i}\Delta u_i\right]$$

- $$\Delta x = \left[v\sum_{i=1}^{n}\frac{\partial u}{\partial u_i}\Delta u_i\right] + \left[u\sum_{i=1}^{n}\frac{\partial v}{\partial u_i}\Delta u_i\right] \longrightarrow \Delta x = v\Delta u + u\Delta v$$

Notemos que u y v se puede reescribir de la siguente forma: $u = x/v$, $v = x/u$. Por lo tanto

$$\Delta x = v\Delta u + u\Delta v \longrightarrow \Delta x = \frac{x}{u}\Delta u + \frac{x}{v}\Delta v$$

$$\Delta x = x\left(\frac{\Delta u}{u} + \frac{\Delta v}{v}\right) \longrightarrow \frac{\Delta x}{x} = \frac{\Delta u}{u} + \frac{\Delta v}{v} \quad \blacksquare$$

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d) Sea $x = u/v$, entonces $\frac{\Delta x}{x} = \frac{\Delta u}{u} - \frac{\Delta v}{v}$

$$\Delta x = \frac{\partial x}{\partial u}\Delta u + \frac{\partial x}{\partial v}\Delta v$$

$$\Delta x = \frac{1}{v}\Delta u - \frac{u}{v^2}\Delta v \longrightarrow \frac{\Delta x}{x} = \left(\frac{v}{u}\right)\left(\frac{1}{v}\right)\Delta u - \frac{u}{v^2}\left(\frac{v}{u}\right)\Delta v$$

$$\frac{\Delta x}{x} = \frac{\Delta u}{u} - \frac{\Delta v}{v} \quad \blacksquare$$

Problema 2:

$$\sigma_x^2 = \lim_{N\to\infty}\left[\sum_{i=1}^{N}\frac{1}{N}(x_i-\bar{x})^2\right]$$

$$\sum_{i=1}^{N}\frac{1}{N}(x_i-\bar{x})^2 = \sum_{i=1}^{N}\frac{1}{N}\left[\frac{\partial x}{\partial u}\Delta u + \frac{\partial x}{\partial v}\Delta v\right]^2$$

Cuando expandimos el binomio obtenemos:

$$\lim_{N\to\infty}\left[\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\partial x}{\partial u}\Delta u\right)^2 + \left(2\frac{\partial x}{\partial u}\Delta u\frac{\partial x}{\partial v}\Delta v\right) + \left(\frac{\partial x}{\partial v}\Delta v\right)^2\right]$$

$$\left(\frac{\partial x}{\partial u}\right)^2\lim_{N\to\infty}\left[\sum_{i=1}^{N}\frac{1}{N}\Delta u^2\right] + \left(\frac{\partial x}{\partial v}\right)^2\lim_{N\to\infty}\left[\sum_{i=1}^{N}\frac{1}{N}\Delta v^2\right] + 2\frac{\partial x}{\partial u}\frac{\partial x}{\partial v}\lim_{N\to\infty}\left[\sum_{i=1}^{N}\frac{1}{N}\Delta v\Delta u\right]$$

Ahora notemos que: $\Delta v = (v_i - \bar{v})$

$\Delta u = (u_i - \bar{u})$

$$\sigma_x^2 = \left(\frac{\partial x}{\partial u}\right)^2\sigma_u^2 + \left(\frac{\partial x}{\partial v}\right)^2\sigma_v^2 + 2\sigma_{uv}^2\left(\frac{\partial x}{\partial u}\right)\left(\frac{\partial x}{\partial v}\right) \blacksquare$$

Problema 3:

t	6.29	6.37	6.35	6.62	6.23	6.39	6.4	6.29
d	10.06	10.02	10.09	10.05	9.78	9.99	9.69	9.85

$$\bar{t} = \frac{6.29 + 6.37 + 6.35 + 6.62 + 6.23 + 6.39 + 6.4 + 6.29}{8}$$

$$\bar{t} = 6.3675$$

$$S_t = \sqrt{\frac{1}{8-1}\left[(6.29-6.4)^2 + (6.37-6.4)^2 + \ldots + (6.29-6.4)^2\right]}$$

$$S_t = 0.117$$

$$\bar{d} = \frac{10.06 + 10.02 + 10.09 + 10.05 + 9.78 + 9.99 + 9.69 + 9.85}{8}$$

$$\bar{d} = 9.941$$

$$S_d = \sqrt{\frac{1}{8-1}\left[(10.06 - 9.9)^2 + (10.02 - 9.9)^2 + \ldots + (9.85 - 9.9)^2\right]}$$

$$S_d = 0.148$$

$$\bar{v} = \frac{\bar{d}}{\bar{t}} \longrightarrow \frac{9.94}{6.36} = 1.56$$

$$S_v = \sqrt{S_t^2\left(\frac{\partial v}{\partial t}\right)^2 + S_d^2\left(\frac{\partial v}{\partial d}\right) + 2S_{dt}^2\left(\frac{\partial v}{\partial t}\right)\left(\frac{\partial v}{\partial d}\right)}$$

$$S_v = \sqrt{S_t^2\left(-\frac{1}{t^2}\right)^2 + S_d^2\left(\frac{1}{t}\right)^2 + 2S_{dt}^2\left(\frac{1}{t}\right)\left(-\frac{1}{t^2}\right)}$$

$$S_{dt}^2 = \frac{1}{8-1}\left[(6.29 - 6.4)(10.06 - 9.9) + \ldots + (6.29 - 6.4)(10.02 - 9.9)\right]$$

$$S_{dt}^2 = \frac{1}{7}(-0.0321) = -0.00458$$

$$\sqrt{(0.117)^2\left(-\frac{1}{6.4^2}\right)^2 + (0.148)^2\left(\frac{1}{6.4}\right)^2 + 2(-0.00458)\left(-\frac{1}{6.4^2}\right)\left(\frac{1}{6.4}\right)}$$

- $S_v = 0.0233$

$$v = 1.56 \pm 0.02$$

Notemos que la varianza podemos separarla en tres partes, una contribución por parte del tiempo, otra por parte de la distancia. En cuanto al tercer termino, la contribución de este, depende de la correlación que existe entre d y t.

En el caso donde $N \to \infty$ podemos notar que S^2_{dt} se haría muy pequeño, ya que N está en el denominador. Esto significa que el termino que depende de d y t se vuelve despreciable.