

# Laws of Equivalence

Given any statement variables $p$ , $q$ , and $r$ , a tautology $t$ and a contradiction $c$ , the following logical equivalences hold:		
1. Commutative laws:	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2. Associative laws:	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. Distributive laws:	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. Identity laws:	$p \wedge t \equiv p$	$p \vee c \equiv p$
5. Negation laws:	$p \vee \sim p \equiv t$	$p \wedge \sim p \equiv c$
6. Double negative law:	$\sim(\sim p) \equiv p$	
7. Idempotent laws:	$p \wedge p \equiv p$	$p \vee p \equiv p$
8. DeMorgan's laws:	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$
9. Universal bounds laws:	$p \vee t \equiv t$	$p \wedge c \equiv c$
10. Absorption laws:	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11. Negations of $t$ and $c$ :	$\sim t \equiv c$	$\sim c \equiv t$
12. Definition of $\rightarrow$ :	$p \rightarrow q \equiv \sim p \vee q$	
13. Definition of $\leftrightarrow$ :	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$	

## Rules of Inference

<u>Modus Ponens</u>		<u>Modus Tollens</u>		<u>Conjunction</u>		<u>Transitivity</u>	
$p \rightarrow q$		$p \rightarrow q$		$p$		$p \rightarrow q$	
$\underline{p}$		$\underline{\sim q}$		$\underline{q}$		$\underline{q \rightarrow r}$	
$\therefore q$		$\therefore \sim p$		$\therefore p \wedge q$		$\therefore p \rightarrow r$	
<u>Elimination</u>				<u>Generalization</u>			
$p \vee q$		$p \vee q$		$p$		$q$	
$\underline{\sim q}$		$\underline{\sim p}$		$\therefore p \vee q$		$\therefore p \vee q$	
$\therefore p$		$\therefore q$					
<u>Specialization</u>				<u>Contradiction rule</u>		<u>Proof by division into cases</u>	
$\underline{p \wedge q}$		$\underline{p \wedge q}$		$\underline{\sim p \rightarrow c}$		$p \vee q$	
$\therefore p$		$\therefore q$		$\therefore p$		$p \rightarrow r$	
						$\underline{q \rightarrow r}$	
						$\therefore r$	