ESERCIZI 29 OTTOBRE 2025

1. $$\int \underset{\substack{\uparrow\\ FD}}{x} \operatorname{arctg}\frac{1}{x}\,dx = \frac{1}{2}x^2\operatorname{arctg}\frac{1}{x} - \int \frac{1}{2}x^2 \frac{1}{1+\frac{1}{x^2}}\left(-\frac{1}{x^2}\right)dx =$$

$$= \frac{1}{2}x^2\operatorname{arctg}\frac{1}{x} + \frac{1}{2}\int \frac{x^2}{x^2+1}\,dx =$$

$$= \frac{1}{2}x^2\operatorname{arctg}\frac{1}{x} + \frac{1}{2}\int \frac{x^2+1-1}{x^2+1}\,dx =$$

$$= \frac{1}{2}x^2\operatorname{arctg}\frac{1}{x} + \frac{1}{2}\int dx - \frac{1}{2}\int\frac{dx}{x^2+1} =$$

$$= \frac{1}{2}x^2\operatorname{arctg}\frac{1}{x} + \frac{1}{2}x - \frac{1}{2}\operatorname{arctg} x + k$$

2. $$I = \int_0^2 (x+|x-1|)\log(x+1)\,dx = (\text{per la prop. additiva})$$

$$= \int_0^1 \log(x+1)\,dx + \int_1^2 (2x-1)\log(x+1)\,dx$$

$$\left(\text{infatti } |x-1| = \begin{cases} 1-x & \text{se } x<1 \\ x-1 & \text{se } x\geq 1\end{cases}\right)$$

cerchiamo $\int \log(x+1)\,dx = \int \underset{\substack{\uparrow\\ FD}}{1}\cdot\log(x+1)\,dx =$

$$= x\log(x+1) - \int\frac{x}{x+1}\,dx =$$

$$= x\log(x+1) - x + \log(x+1) + k =$$

$$= (x+1)\log(x+1) - x + k$$

cerchiamo $\int \underset{\substack{\uparrow \\ \text{FD}}}{(2x-1)} \log(x+1)\,dx = (x^2-x)\log(x+1) -$

$$- \int \frac{x^2-x}{x+1}\,dx \qquad (*)$$

$$\begin{array}{rr|l} x^2 & -x & x+1 \\ \cline{3-3} -x^2 & -x & x-2 \\ \cline{1-2} & -2x & \\ & 2x+2 & \end{array} \qquad \frac{x^2-x}{x+1} = x-2+\frac{2}{x+1}$$

quindi $(*) = (x^2-x)\log(x+1) - \frac{1}{2}x^2 + 2x - 2\log(x+1) + k$

dunque

$$I = \Big[(x+1)\log(x+1) - x\Big]_0^1 + \Big[(x^2-x)\log(x+1) - \frac{1}{2}x^2 + 2x - 2\log(x+1)\Big]_1^2 =$$

$$= 2\log 6 - \frac{1}{2}$$

3. $I = \int \frac{1+\sqrt{x}}{\sqrt{x}\,\sqrt{1-x}}\,dx \qquad \begin{array}{l} x>0 \\ 1-x>0 \end{array} \Rightarrow (a,b) = \,]0,1[$

primo modo $\quad \sqrt{x} = t \quad t \geq 0$

$x = t^2 = g(t)$

$0 < t^2 < 1 \Rightarrow 0 < t < 1 \Rightarrow (c,d) = \,]0,1[$

$g'(t) = 2t \qquad g^{-1}(x) = \sqrt{x}$

$$I = \Bigg[\underbrace{\int \frac{1+t}{t\sqrt{1-t^2}}\,2t\,dt}_{J}\Bigg]_{t=\sqrt{x}}$$

$$J = 2\int \frac{1+t}{\sqrt{1-t^2}}\,dt = 2\int \frac{dt}{\sqrt{1-t^2}} - \int \frac{-2t}{\sqrt{1-t^2}}\,dt =$$

$$= 2\arcsin t - \left[\int \frac{dz}{\sqrt{z}}\right]_{z=1-t^2} =$$

$$= 2\arcsin t - 2\sqrt{1-t^2} + k$$

$$I = 2\arcsin\sqrt{x} - 2\sqrt{1-x} + k$$

$\underline{\text{II}}$ modo $\qquad \sqrt{1-x} = t \qquad t \geq 0$

$x = 1-t^2 = g(t)$

$0 < 1-t^2 < 1 \Rightarrow \qquad (c,d) =]0,1[$

$g'(t) = -2t \qquad g^{-1}(x) = \sqrt{1-x}$

$$I = \Big[\underbrace{\int \frac{1+\sqrt{1-t^2}}{\sqrt{1-t^2}\;t}(-2t)\,dt}_{J}\Big]_{t=\sqrt{1-x}}$$

$$J = -2\int \frac{1+\sqrt{1-t^2}}{\sqrt{1-t^2}}\,dt = -2\arcsin t - 2t + k$$

$$I = -2\arcsin\sqrt{1-x} - 2\sqrt{1-x} + k$$

4. Trovare f primitiva in $]-\infty, +\infty[$ di

$$f(x) = e^{|x-1|+2x}$$

e tale che $f(-1) = \frac{1}{2e}$

$$f(x) = \begin{cases} e^{x+1} & x \leq 1 \\ e^{3x-1} & x \geq 1 \end{cases}$$

una primitiva è del tipo

$$f(x) = \begin{cases} e^{x+1} + k_1 & x \leq 1 \\ \frac{1}{3} e^{3x-1} + k_2 & x > 1 \end{cases}$$

impongo la continuità nel punto $x = 1$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$$

$$e^2 + k_1 = \frac{1}{3} e^2 + k_2 \Rightarrow k_2 = k_1 + \frac{2}{3} e^2$$

quindi tutte le primitive sono

$$f(x) = \begin{cases} e^{x+1} + k & x \leq 1 \\ \frac{1}{3} e^{3x-1} + k + \frac{2}{3} e^2 & x > 1 \end{cases}$$

impongo che $f(-1) = \frac{1}{2e}$

$$e^{-1+1} + k = \frac{1}{2e} \Rightarrow k = \frac{1}{2e} - 1$$

La funzione cercata è

$$f(x) = \begin{cases} e^{x+1} + \frac{1}{2e} - 1 & x \leq 1 \\ \frac{1}{3} e^{3x-1} + \frac{1}{2e} - 1 + \frac{2}{3} e^2 & x > 1 \end{cases}$$