Assignment 3 (revised)

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Post-Optimality and Sensitivity Analysis

Loading the necessary libraries

```
setwd("C:/Users/Fabrizio/Desktop/MyBlackBoard/MIS 64018 - Quant. Management Modeling/Module 5
- Post Optimality Analysis and Duality Theory/Assignment 3")
library(lpSolveAPI)
```

Introduction

Solve the LP Model

In this assignment, we chose to write the problem formulation in the lp format, a text file created specifically for this problem, that includes all the information.

```
x <- read.lp("Weigelt_Corporation_Assignment3.lp")
x
```

```
## Model name:
##   a linear program with 9 decision variables and 11 constraints
```

Now that the file has been read by R, we can solve it.

```
solve(x)
```

```
## [1] 0
```

Here, zero means that R found an optimal solution.

```
get.objective(x)
```

```
## [1] 696000
```

```
get.variables(x)
```

```
## [1] 516.6667 177.7778   0.0000   0.0000 666.6667 166.6667   0.0000   0.0000
## [9] 416.6667
```

From the results we can understand what is the optimal production mix among the 3 product sizes and the 3 plants in order to obtain the maximum profit. According to the model solution: - in Plant 1 the company should produce 517 Large size and 178 Medium size. - in Plant 2 it should produce 667 Medium size and 167 Small size. - in Plant 3 it should produce exclusively 417 Small size. With this production levels the company would register an expected profit of $696,000.

```
get.constraints(x)
```

```
##  [1]  5.166667e+02  8.444444e+02  5.833333e+02  1.300000e+04  1.200000e+04
##  [6]  5.000000e+03  6.944444e+02  8.333333e+02  4.166667e+02 -2.037268e-10
## [11]  0.000000e+00
```

These are the RHS values for our 11 constraints.

Post-Optimality and Sensitivity Analysis

```
get.sensitivity.rhs(x) #shadow prices
```

```
## $duals
##  [1]    0.00    0.00    0.00   12.00   20.00   60.00    0.00    0.00    0.00
## [10]   -0.08    0.56    0.00    0.00  -24.00  -40.00    0.00    0.00 -360.00
## [19] -120.00    0.00
##
## $dualsfrom
##  [1] -1.000000e+30 -1.000000e+30 -1.000000e+30  1.122222e+04  1.150000e+04
##  [6]  4.800000e+03 -1.000000e+30 -1.000000e+30 -1.000000e+30 -2.500000e+04
## [11] -1.250000e+04 -1.000000e+30 -1.000000e+30 -2.222222e+02 -1.000000e+02
## [16] -1.000000e+30 -1.000000e+30 -2.000000e+01 -4.444444e+01 -1.000000e+30
##
## $dualstill
##  [1] 1.000000e+30 1.000000e+30 1.000000e+30 1.388889e+04 1.250000e+04
##  [6] 5.181818e+03 1.000000e+30 1.000000e+30 1.000000e+30 2.500000e+04
## [11] 1.250000e+04 1.000000e+30 1.000000e+30 1.111111e+02 1.000000e+02
## [16] 1.000000e+30 1.000000e+30 2.500000e+01 6.666667e+01 1.000000e+30
```

```
get.sensitivity.obj(x) #reduced costs
```

```
## $objfrom
## [1]  3.60e+02  3.45e+02 -1.00e+30 -1.00e+30  3.45e+02  2.52e+02 -1.00e+30
## [8] -1.00e+30  2.04e+02
##
## $objtill
## [1] 4.60e+02 4.20e+02 3.24e+02 4.60e+02 4.20e+02 3.24e+02 7.80e+02 4.80e+02
## [9] 1.00e+30
```

Looking at the output above, we can get the shadow prices of our constraints (duals), the valid ranges for the shadow prices (dualsfrom and dualstill), and the reduced costs for our variables (objfrom and objtill). For example, the shadow price of fourth constraint is 12, meaning that an increase of 1 unit of that resource (square feet of in-process storage space available for a day's production in Plant 1) would increase the total profit by 12. Also, we can measure the valid range for the shadow price of the fourth constraint, meaning that the final optimal solution will not change if the amount of the resource is within that range. Finally, the reduced costs for example for the first variable (Plant1Large) tell us the range in which the coefficient of the variable (profit) has to be in order to be considered in the final solution of the problem.

Solve the Dual LP Model

```
y <- read.lp("Weigelt_Corporation_Dual_Assignment3.lp")
y
```

```
## Model name:
##   a linear program with 11 decision variables and 9 constraints
```

```
#set boundaries for columns 10 and 11, default for the rest
set.bounds(y, lower = c(-Inf, -Inf), columns = 10:11)
```

Now that the file has been read by R and we set the boundary, we can solve it.

```
solve(y)
```

```
## [1] 0
```

Here, zero means that R found an optimal solution.

```
get.objective(y)
```

```
## [1] 696000
```

```
get.variables(y)
```

```
##  [1]  0.00  0.00  0.00 12.00 20.00 60.00  0.00  0.00  0.00 -0.08  0.56
```

These are the RHS values for our 11 constraints.

Post-Optimality and Sensitivity Analysis

```
get.sensitivity.rhs(y) #shadow prices
```

```
## $duals
##  [1] 516.66667 177.77778   0.00000   0.00000 666.66667 166.66667   0.00000
##  [8]   0.00000 416.66667 383.33333 355.55556 166.66667   0.00000   0.00000
## [15]   0.00000  55.55556  66.66667  33.33333   0.00000   0.00000
## 
## $dualsfrom
##  [1]  3.600000e+02  3.450000e+02 -1.000000e+30 -1.000000e+30  3.450000e+02
##  [6]  2.880000e+02 -1.000000e+30 -1.000000e+30  2.040000e+02 -4.000000e+01
## [11] -1.500000e+01 -2.400000e+01 -1.000000e+30 -1.000000e+30 -1.000000e+30
## [16] -1.000000e+30 -2.605325e+13 -1.000000e+30 -1.000000e+30 -1.000000e+30
## 
## $dualstill
##  [1] 4.60e+02 4.20e+02 1.00e+30 1.00e+30 3.75e+02 3.24e+02 1.00e+30 1.00e+30
##  [9] 1.00e+30 6.00e+01 1.50e+01 1.20e+01 1.00e+30 1.00e+30 1.00e+30 2.52e+02
## [17] 6.00e+01 4.80e+02 1.00e+30 1.00e+30
```

```r
get.sensitivity.obj(y) #reduced costs
```

```
## $objfrom
##  [1]    516.6667    844.4444    583.3333  11222.2222  11500.0000   4800.0000
##  [7]    694.4444    833.3333    416.6667 -25000.0000 -12500.0000
##
## $objtill
##  [1] 1.000000e+30 1.000000e+30 1.000000e+30 1.388889e+04 1.250000e+04
##  [6] 5.181818e+03 1.000000e+30 1.000000e+30 1.000000e+30 1.000000e+30
## [11] 1.250000e+04
```

As we saw for the primal problem, we obtained the shadow prices of our constraints (duals), the valid ranges for the shadow prices (dualsfrom and dualstill), and the reduced costs for our variables (objfrom and objtill). According to the Duality Theory, each primal LP problem has an associated dual problem, and the relationships between them can be useful. In our case, the maximization problem became a minimization problem, and our n variables and m constraints have been switched to m variables and n constraints. Notice that the optimal solution for the dual problem is the same of the primal problem. Sometimes, the dual problem is solved since it can contains less constraints than the primal problem and the solution can be used for the primal problem.