

Assignment 4 - Transportation problem and Network Model

Fabrizio Fiorini

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Part 1 - Transportation problem

Loading the necessary libraries

```
setwd("C:/Users/Fabrizio/Desktop/MyBlackBoard/MIS 64018 - Quant. Management Modeling/Module 7  
- Network Models/Assignment 4")  
library(lpSolveAPI)
```

Introduction

Heart Start produces automated external defibrillators (AEDs) in each of two different plants (A and B). The AEDs are sold through three wholesalers. How many AEDs should be produced in each plant, and how should they be distributed to each of the three wholesaler warehouses so as to minimize the combined cost of production and shipping?

Solve the LP Model

In this assignment, we chose to write the problem formulation in the lp format, a text file created specifically for this problem, that includes all the information.

```
x <- read.lp("Heart_Start.lp")  
x
```

```
## Model name:  
##          Pa    Pb    Xa1    Xa2    Xa3    Xb1    Xb2    Xb3  
## Minimize 600    625    22     14     30     16     20     24  
## R1        0      0      1      0      0      1      0      0  >=  80  
## R2        0      0      0      1      0      0      1      0  >=  60  
## R3        0      0      0      0      1      0      0      1  >=  70  
## R4       -1      0      1      1      1      0      0      0  <=   0  
## R5        0     -1      0      0      0      1      1      1  <=   0  
## Kind      Std    Std    Std    Std    Std    Std    Std    Std  
## Type      Real   Real   Real   Real   Real   Real   Real   Real  
## Upper     100    120    Inf    Inf    Inf    Inf    Inf    Inf  
## Lower      0      0      0      0      0      0      0      0
```

Now that the file has been read by R, we can solve it.

```
solve(x)
```

```
## [1] 0
```

Here, zero means that R found an optimal solution.

```
get.objective(x)
```

```
## [1] 132790
```

```
get.variables(x)
```

```
## [1] 100 110 40 60 0 40 0 70
```

From the results we know that the optimal solution is reached with a minimum total cost of \$132,790. The total production from Plant A and B is, respectively, 100 and 110. Moreover, 40 AEDs are produced in Plant A and shipped to Warehouse 2, 60 are produced in plant A and shipped warehouse 2, 40 are produced in plant B and shipped to warehouse 1, and finally 70 are produced in plant B and shipped to warehouse 3.

```
get.constraints(x)
```

```
## [1] 80 60 70 0 0
```

These are the RHS values for our 5 constraints.

Part 2 - Network Model

Since there is a difference between total supply (276TBD) and total demand(274TBD), we will introduce a dummy variable, Refinery 6 (R6), that will demand 2TBD.

Objective Function (minimization problem):

$$Z = 1.52 Xw1pa + 1.60 Xw1pb + 1.40 Xw1pc + 1.70 Xw2pa + 1.63 Xw2pb + 1.55 Xw2pc + 1.45 Xw3pa + 1.57 Xw3pb + 1.30 Xw3pc + 5.15 Xpar1 + 5.69 Xpar2 + 6.13 Xpar3 + 5.63 Xpar4 + 5.80 Xpar5 + 5.12 Xpbr1 + 5.47 Xpbr2 + 6.05 Xpbr3 + 6.12 Xpbr4 + 5.71 Xpbr5 + 5.32 Xpcr1 + 6.16 Xpcr2 + 6.25 Xpcr3 + 6.17 Xpcr4 + 5.87 Xpcr5$$

Constraints:

Supply/Capacity Constraints:

$$Xw1pa + Xw1pb + Xw1pc = 93 \quad Xw2pa + Xw2pb + Xw2pc = 88 \quad Xw3pa + Xw3pb + Xw3pc = 95$$

Demand/Requirement Constraints:

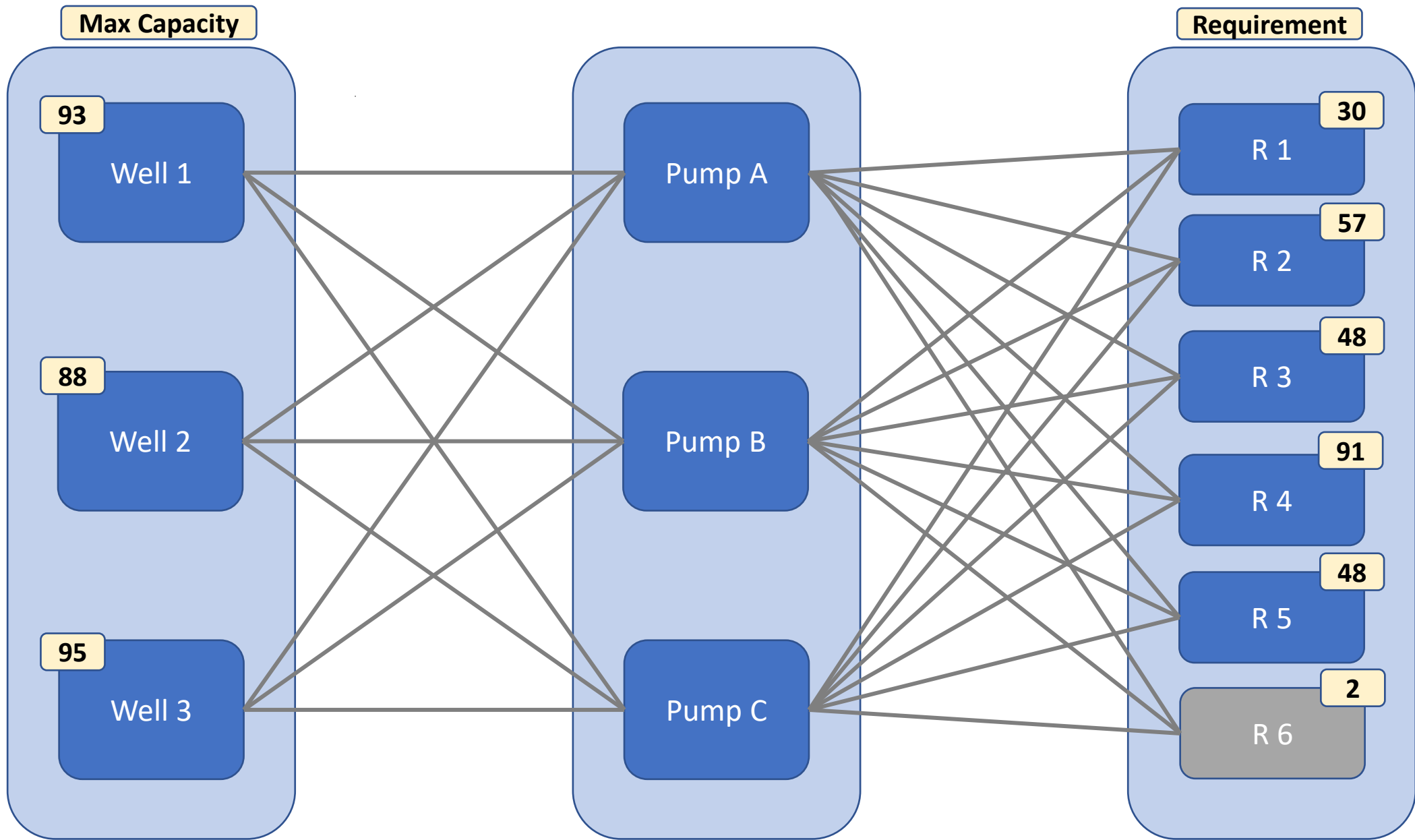
$$Xpar1 + Xpbr1 + Xpcr1 = 30 \quad Xpar2 + Xpbr2 + Xpcr2 = 57 \quad Xpar3 + Xpbr3 + Xpcr3 = 48 \quad Xpar4 + Xpbr4 + Xpcr4 = 91 \quad Xpar5 + Xpbr5 + Xpcr5 = 48 \quad Xpar6 + Xpbr6 + Xpcr6 = 2$$

Constraints from pumps to the refineries:

$$Xw1pa + Xw2pa + Xw3pa + Xpar1 + Xpar2 + Xpar3 + Xpar4 + Xpar5 + Xpar6 = 0 \quad Xw1pb + Xw2pb + Xw3pb + Xpbr1 + Xpbr2 + Xpbr3 + Xpbr4 + Xpbr5 + Xpbr6 = 0 \quad Xw1pc + Xw2pc + Xw3pc + Xpcr1 + Xpcr2 + Xpcr3 + Xpcr4 + Xpcr5 + Xpcr6 = 0$$

Non-negativity constraints:

$$X_{ij} \geq 0 \quad (i = w1, w2, w3 ; j = pa, pb, pc) \quad X_{jk} \geq 0 \quad (j = pa, pb, pc ; k = r1, r2, r3, r4, r5, r6)$$



$$\begin{aligned} \min Z &= \sum_i^n \sum_j^n c_{ij} * x_{ij} \\ \text{subject to } \sum_j^n x_{ij} - \sum_j^n x_{ji} &= b_i \end{aligned}$$

$$\begin{aligned} \min \quad & Z = 1.52x_{W1Pa} + 1.6x_{W1Pb} + 1.4x_{W1Pc} + \dots + 1.3x_{W3Pc} + 5.15x_{PaR1} + \dots + 5.87x_{PcR5} \\ \text{subject to} \end{aligned}$$

$$\begin{aligned} x_{W1Pi} + x_{W2Pi} + x_{W3Pi} + x_{PiR1} + x_{PiR2} + x_{PiR3} + x_{PiR4} + x_{PiR5} &= 0 && (\text{for } i = a, b, c) \\ x_{W1Pa} + x_{W1Pb} + x_{W1Pc} &= 93 && (\text{max capacity W1}) \\ x_{W2Pa} + x_{W2Pb} + x_{W2Pc} &= 88 && (\text{max capacity W2}) \\ x_{W3Pa} + x_{W3Pb} + x_{W3Pc} &= 95 && (\text{max capacity W3}) \\ x_{PaR1} + x_{PbR1} + x_{PcR1} &= 30 && (\text{requirement R1}) \\ x_{PaR2} + x_{PbR2} + x_{PcR2} &= 57 && (\text{requirement R2}) \\ x_{PaR3} + x_{PbR3} + x_{PcR3} &= 48 && (\text{requirement R3}) \\ x_{PaR4} + x_{PbR4} + x_{PcR4} &= 91 && (\text{requirement R4}) \\ x_{PaR5} + x_{PbR5} + x_{PcR5} &= 48 && (\text{requirement R5}) \\ x_{PaR6} + x_{PbR6} + x_{PcR6} &= 2 && (\text{requirement R6}) \end{aligned}$$