ASSIGNMENT 2
SOLVING LP MODELS IN R AND GRAPHICALLY

Fall 2021

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(Computer Center Staffing) You are the Director of the Computer Center for Gaillard College and responsible for scheduling the staffing of the center. It is open from 8 am until midnight. You have monitored the usage of the center at various times of the day and determined that the following numbers of computer consultants are required.

Time of day	Minimum number of consultants required to be on duty
8 am–noon	4
Noon–4 pm	8
4 am–8 pm	10
8 am–midnight	6

Two types of computer consultants can be hired: full-time and part-time. The full-time consultants work for eight consecutive hours in any of the following shifts: morning (8 am – 4 pm), afternoon (noon – 8 pm), and evening (4 pm – midnight). Full-time consult- ants are paid $14 per hour. Part-time consultants can be hired to work any of the four shifts listed in the table. Part-time consultants are paid $12 per hour. An additional requirement is that during every time period, at least one full-time consultant must be on duty for every part-time consultant on duty.

a. <u>Determine a minimum-cost staffing plan</u> for the center. In your solution, how many consultants will be paid to work full time and how many will be paid to work part time? What is the minimum cost?
b. After thinking about this problem for a while, you have decided to <u>recognize meal breaks explicitly in the scheduling of full-time consultants</u>. In particular, full-time consultants are entitled to a one-hour lunch break during their eight-hour shift. In addition, employment rules specify that the lunch break can start after three hours of work or after four hours of work, but those are the only alternatives. Part-time consultants do not receive a meal break. Under these conditions, find a minimum-cost staffing plan. What is the minimum cost?

12am – 8am	closed	closed	closed	0
8am – 12pm	X_1+Y_1			4
12pm – 4pm	X_1+Y_2	X_2+Y_2		8
4pm – 8pm		X_2+Y_3	X_3+Y_3	10
8pm – 12am			X_3+Y_4	6

X_i = number of full-time consultants on the shift i

Y_i = number of part-time consultants on the shift i

Objective Function:
Minimize $C_{tot} = 14 \times X_1 + 14 \times X_2 + 14 \times X_3 + 12 \times Y_1 + 12 \times Y_2 + 12 \times Y_3 + 12 \times Y_4$

Subject to Constraints:

- $X_1 + Y_1 \geq 4$ (min requirement for shift 8-12)
- $X_1 + X_2 + Y_2 \geq 8$ (min requirement for shift 12-4)
- $X_2 + X_3 + Y_3 \geq 10$ (min requirement for shift 4-8)
- $X_3 + Y_4 \geq 6$ (min requirement for shift 8-12)
- $X_i \geq 0$ (nonnegativity)
- $Y_i \geq 0$ (nonnegativity)
- $X_1 \geq Y_1$ (requirement of at least 1 full-time consultant for each part-time consultant)
- $X_1 + X_2 \geq Y_2$ (requirement of at least 1 full-time consultant for each part-time consultant)
- $X_2 + X_3 \geq Y_3$ (requirement of at least 1 full-time consultant for each part-time consultant)
- $X_3 \geq Y_4$ (requirement of at least 1 full-time consultant for each part-time consultant)

Let us say that the total number of full-time consultants on the shift i is equal to the sum of full-time consultants that decide to take a break on the 4th hour and the ones that decide to take a break on the 5th.
$X_i = X^4{}_i + X^5{}_i$

The constraints will change as follow:

- $X^4{}_1 + X^5{}_1 + Y_1 \geq 4$ (min requirement for shift 8-9)
- $X^4{}_1 + X^5{}_1 + Y_1 \geq 4$ (min requirement for shift 9-10)
- $X^4{}_1 + X^5{}_1 + Y_1 \geq 4$ (min requirement for shift 10-11)
- $X^5{}_1 + Y_1 \geq 4$ (min requirement for shift 11-12, break)
- $X^4{}_1 + X^4{}_2 + X^5{}_2 + Y_2 \geq 8$ (min requirement for shift 12-1, break)
- $X^4{}_1 + X^5{}_1 + X^4{}_2 + X^5{}_2 + Y_2 \geq 8$ (min requirement for shift 1-2)
- $X^4{}_1 + X^5{}_1 + X^4{}_2 + X^5{}_2 + Y_2 \geq 8$ (min requirement for shift 2-3)
- $X^4{}_1 + X^5{}_1 + X^5{}_2 + Y_2 \geq 8$ (min requirement for shift 3-4, break)
- $X^4{}_2 + X^4{}_3 + X^5{}_3 + Y_3 \geq 10$ (min requirement for shift 4-5, break)
- $X^4{}_2 + X^5{}_2 + X^4{}_3 + X^5{}_3 + Y_3 \geq 10$ (min requirement for shift 5-6)
- $X^4{}_2 + X^5{}_2 + X^4{}_3 + X^5{}_3 + Y_3 \geq 10$ (min requirement for shift 6-7)
- $X^4{}_2 + X^5{}_2 + X^5{}_3 + Y_3 \geq 10$ (min requirement for shift 7-8, break)
- $X^4{}_3 + Y_4 \geq 6$ (min requirement for shift 8-9, break)
- $X^4{}_3 + X^5{}_3 + Y_4 \geq 6$ (min requirement for shift 9-10)
- $X^4{}_3 + X^5{}_3 + Y_4 \geq 6$ (min requirement for shift 10-11)
- $X^4{}_3 + X^5{}_3 + Y_4 \geq 6$ (min requirement for shift 11-12)

Consider the problem from the previous assignment.

Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week. <u>Solve this problem graphically</u>.

	Collegiate (X_1)	**Mini (X_2)**
Material	3 sqft	2 sqft
Sales	1,000 /week	1,200 /week
Production Time	45 min	40 min
Unit Profit	$32	$24

Resources (constraints):

- 5,000 sqft/week
- 35 laboreres working 40 hours/week each
- 1,000 max weekly quantity of X_1
- 1,200 max weekly quantity of X_2

Objective function:

$Z = 32\ X_1 + 24\ X_2$ (maximize profit)

Subject To (ST):

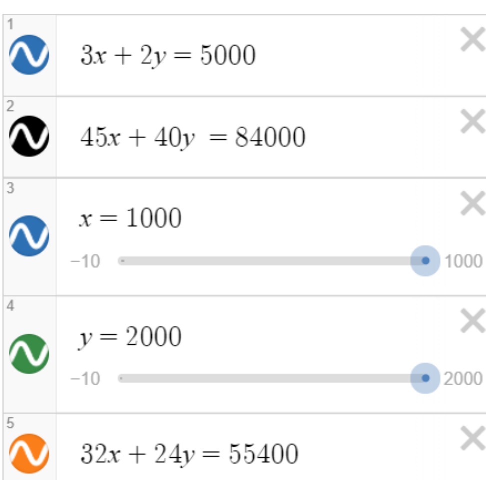
$3\ X_1 + 2\ X_2 \leq 5{,}000$ (material usage)

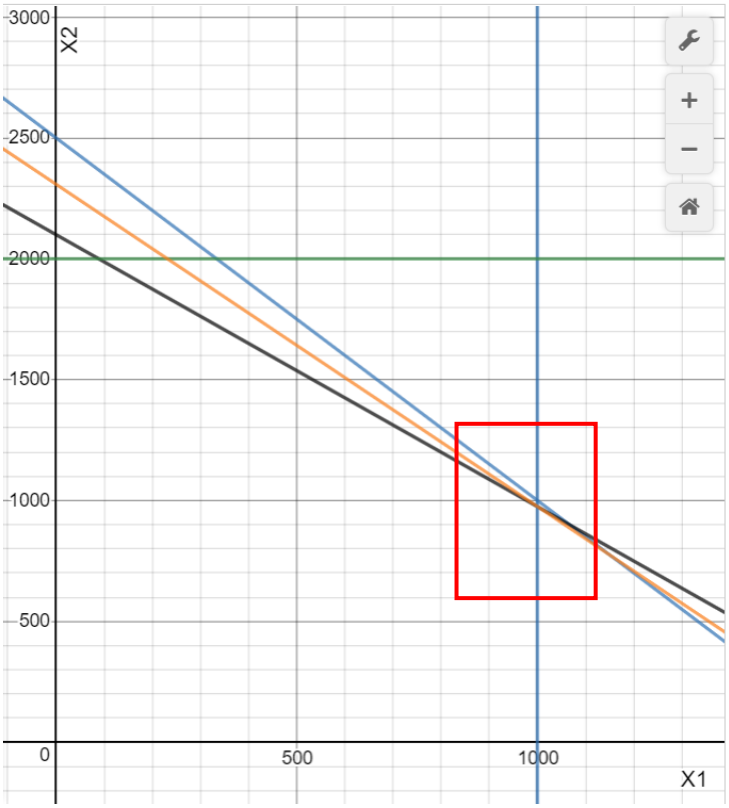
$X_1 \leq 1{,}000$ (quantity to be sold)

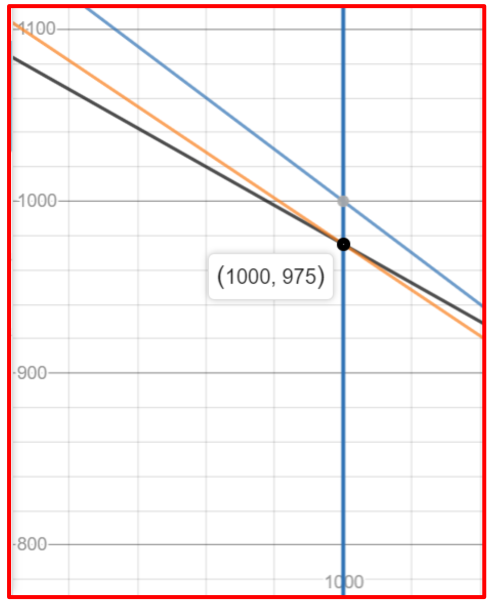
$X_2 \leq 2{,}000$ (quantity to be sold)

$45\ X_1 + 40\ X_2 \leq 84{,}000$* (labor time for production)

$X_1, X_2 \geq 0$ (quantities not negative)







Any value of X1 and X2 that lies in the green region will satisfy all the constraints of the model.
Using a web tool (https://www.transum.org/Maths/Activity/Graph/Desmos.asp) I solved the model graphically by inserting all the boundaries given by the constraints.
The result is that approximately the optimum production mix that maximize the profit for the company is reached when 1,000 Collegiate backpacks and 975 Mini backpacks are produced. It yields a total profit of around $55,400.

(Weigelt Production) The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes--large, medium, and small-- that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved. The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively. Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day. At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product. Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit. a. Define the decision variables b. Formulate a linear programming model for this problem. c. Solve the problem using lpsolve, or any other equivalent library in R.

(See the R Markdown file on the GitHub repository for the course)