An $n-K$ Contigency Constrained Unit Commitment Model via Robust Optimization

Fabrício Oliveira [1] Alexandre Street[1] José Manuel Arroyo [2]

Pontifical Catholic University[1]

Universidad de Castilla- La Mancha[2]

May 3, 2010

Outline

Introduction

Current reliability policy and associated security standards in power systems mainly focus on events such as the random outage of a single or at most two transmission or generation assets - $n-1$ and $n-2$ security criteria [1].

Recent blackouts caused by a coincidence in in time of several independent system outages [2] made questionable the adequacy of such reliability framework. [2, 3, 4, 5, **?**]

From a deterministic viewpoint, reserves have been traditionally modeled in the generation scheduling - the so-called security constrained unit commitment (SCUC).

In these models, pre-specified requirements for different types of reserve are imposed.

Introduction

Main drawback of such approaches: their dependence on an priori determination of a system-wide or local reserve requirements that might lead to suboptimal or even infeasible solutions once the contigency occurs.

In contrast to the SCUC, contigency-constrained unit commitment (CCUC) determine individual reserve contributions so that the power balance is explicity imposed under both normal and contigency states.

Current computing capabilities may allow incorporating $n-1$ and $n-2$ security criteria in the CCUC models presented in [6, 7, 8, 9] for practical power systems. However, the extension to tighter security levels would lead to intractability, due to the huge number of contigency states that should be considered.

Introduction

How to deal with that: CCUC models only consider a limited set of credible contigencies, which is determined based on experience and engineering judgment.

Our main contribution: we present an alternative approach that efficiently incorporates a deterministic $n - K$ security criterion into the CCUC problem ($n - K$ CCUC).

Unlike previously reported CCUC models [6, 7, 8, 9] relying on a reduced set of credible contigencies, we propose a joint energy and reserve dispatch model based on robust optimization that allows considering all combination of at most K unit outages, i.e., $\sum_i^K \binom{n}{i}$ in a computationally efficient manner.

$n - K$ Contigency Constrained Unit Commitment

The $n - K$ CCUC determines the optimal generation schedule and reverse allocation so that the power demand is supplied both normal and contigency states.

We propose a explicit consideration of an $n - K$ security criterion by which all combinations up K unit outages are modeled.

For the sake of simplicity a single-bus is adopted, comprising a single period.

Notwithstanding, the extension of the proposed model for network consideration, as well as multiperiod settings is straightfoward.

Based on [6] and [9] the $n - K$ CCUC can be formulated as follows:

$n-K$ Contigency Constrained Unit Commitment

$$\min_{p_i, p_i^k, r_i, v_i} \sum_{i \in N} C_i^P(p_i, v_i) + C_i^R(r_i) \tag{1}$$

$$s.t. \sum_{i \in N} p_i = D \tag{2}$$

$$\sum_{i \in N} p_i^k = D \quad \forall k \in \mathcal{C} \tag{3}$$

$$\underline{P}_i v_i \leq p_i \leq \overline{P}_i v_i \quad \forall i \in N \tag{4}$$

$$A_i^k \underline{P}_i v_i \leq p_i^k \leq A_i^k \overline{P}_i v_i \quad \forall i \in N, k \in \mathcal{C} \tag{5}$$

$$p_i^k \leq A_i^k (p_i + r_i) \quad \forall i \in N, k \in \mathcal{C} \tag{6}$$

$$p_i + r_i \leq \overline{P}_i v_i \quad \forall i \in N \tag{7}$$

$$0 \leq r_i \leq \overline{R}_i v_i \quad \forall i \in N \tag{8}$$

$$v_i \in \{0, 1\} \quad \forall i \in N \tag{9}$$

$n-K$ Contigency Constrained Unit Commitment

Since the redispatch cost is not included in the objective function, the problem can be equivalently reformulated by dropping the post-contigency variables p_i^k:

$$\sum_{i \in N} p_i^k \leq \sum_{i \in N} A_i^k (p_i + r_i) \Rightarrow D \leq \sum_{i \in N} A_i^k (p_i + r_i) \tag{10}$$

Thus, replacing constraints (3), (5), and (6) by (10), we have a model with no post-contigency variables.

$n-K$ Contigency Constrained Unit Commitment

$$\min_{p_i, p_i^k, r_i, v_i} \sum_{i \in N} C_i^P(p_i, v_i) + C_i^R(r_i) \tag{11}$$

$$s.t. \sum_{i \in N} p_i = D \tag{12}$$

$$\sum_{i \in N} A_i^k (p_i + r_i) \geq D \quad \forall k \in \mathcal{C} \tag{13}$$

$$\underline{P}_i v_i \leq p_i \leq \overline{P}_i v_i \quad \forall i \in N \tag{14}$$

$$p_i + r_i \leq \overline{P}_i v_i \quad \forall i \in N \tag{15}$$

$$0 \leq r_i \leq \overline{R}_i v_i \quad \forall i \in N \tag{16}$$

$$v_i \in \{0, 1\} \quad \forall i \in N \tag{17}$$

Robust $n - K$ Contigency Constrained Unit Commitment

Problem (11)-(17) explicitly models $n - K$ security criterion by considering all $\binom{n}{K}$ combinations of unit outages - constraint (13)

The problem stated before can be viewed as a particular instance of a robust optimization problem [10].

The uncertain parameters are the A_i^k representing the availabitlity of generation units under each contigency scenario.

Based on this fact, we propose a robust optimization approach to solve the $n - K$ CCUC problem, where K is the robustness parameter used to adjust the conservatism level.

Robust Bilevel Counterpart

The contigency dependence is introduced by constraint (10): The

$$\sum_{i \in N} A_i^k (p_i + r_i) \geq D \quad \forall k \in \mathcal{C}$$

Since this requirement must hold for all contigencies $k \in \mathcal{C}$, it is sufficient to guarantee that it also holds for the worst case.

The worst-case contigency - the one with the tightest left-hand side - is known as the umbrella contigency cite25.

Robust Bilevel Counterpart

The contigency dependence constraint can be expressed as:

$$d^{WC*}(p,r,K) \geq D$$
$$d^{WC*}(p,r,K) = \inf_{k\in\mathcal{C}}\{\sum_{i\in N} A_i^k (p_i + r_i)\} \tag{18}$$

Where $d^{WC*}(p,r)$ denotes de the maximum power that can be supplied under the worst-case contigency for a given power and up-spinning reserve.

(18) can be formulated as a optimization problem by defining a decision variable vector $a = [a_1, ..., a_n]$ which is 0 if generatir i is unavailable in the worst-contingency state, being 1 otherwise.

Robust Bilevel Counterpart

The robust bilevel counterpart for the $n-K$ CCUC is formulated as follows:

$$\min_{p_i, p_i^k, r_i, v_i} \sum_{i \in N} C_i^P(p_i, v_i) + C_i^R(r_i) \tag{19}$$

$$s.t.\ \underline{P}_i v_i \leq p_i \leq \overline{P}_i v_i \quad \forall i \in N \tag{20}$$

$$p_i + r_i \leq \overline{P}_i v_i \quad \forall i \in N \tag{21}$$

$$0 \leq r_i \leq \overline{R}_i v_i \quad \forall i \in N \tag{22}$$

$$v_i \in \{0, 1\} \quad \forall i \in N \tag{23}$$

$$d^{WC*}(p, r, K) \geq D \tag{24}$$

$$d^{WC*}(p, r, K) = \min_{a_i \in [0,1]} \{\sum_{i \in N} a_i (p_i + r_i) \,|\, \sum_{i \in N} a_i \geq n - K\} \tag{25}$$

Robust Bilevel Counterpart

One can notice that:

$$\min_{a_i} \sum_{i \in N} a_i \left(p_i + r_i\right) \tag{26}$$

$$s.t. \sum_{i \in N} a_i \geq n - K \tag{27}$$

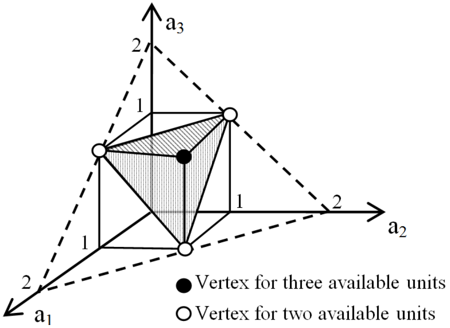
$$a_i \in [0, 1] \tag{28}$$

presents a unimodular matrix structure, which implies that, for integer values of K, the lower-level problem stated above always provides integer (binary) optimal soluiton for a.

We can define a parameter $A^k = [A_1^k, ..., A_n^k], \forall k \in \mathcal{C}$ which represents the vertexes of the polyhedral set defined by (27) and (28).

Robust Bilevel Counterpart

Figure 1 bellow ilustrates both the polyhedral set defined by (27) and (28) and ${A^k}_k \in \mathcal{C}$ for 3 unit system with $K = 1$



The dashed volume represents our polyhedral uncertainty set.

Robust Single-Level Counterpart

Based on the robust optimization approach presented by Bertsimas and Sim [18], we propose an efficient single-level equivalent formulation for the bilevel problem stated before.

In order to reach uch formulation, we need to:

- replace $d^{WC*}(p, r, K)$ by the dual obj. function of the lower-level optimization problem
- replace the lower-level optimization problem by its dual feasibility constraints

Robust Single-Level Counterpart

The single-level MIP equivalent model for the robust bilevel $n-K$ *CCUC* is as follows:

$$\min_{p_i, p_i^k, r_i, v_i} \sum_{i \in N} C_i^P(p_i, v_i) + C_i^R(r_i) \tag{29}$$

$$s.t. \sum_{i \in N} p_i = D \tag{30}$$

$$\underline{P}_i v_i \leq p_i \leq \overline{P}_i v_i \quad \forall i \in N \tag{31}$$

$$p_i + r_i \leq \overline{P}_i v_i \quad \forall i \in N \tag{32}$$

$$0 \leq r_i \leq \overline{R}_i v_i \quad \forall i \in N \tag{33}$$

$$v_i \in \{0, 1\} \quad \forall i \in N \tag{34}$$

$$(n-K)\, y - \sum_{i \in N} z_i \geq D \tag{35}$$

$$y - z_i \leq p_i + r_i \quad \forall i \in N \tag{36}$$

$$z_i \geq 0 \quad \forall i \in N \tag{37}$$

$$y \geq 0 \tag{38}$$

Robust Single-Level Counterpart

Comparing the model size of the proposed robust equivalent and the original contigency-dependent model:

Model	# Constraints	# Cont. Var.	# Bin. Var.
Contigency-dep.	$3n\|\mathcal{C}\| + \|\mathcal{C}\| + 4n + 1$	$2n + n\|\mathcal{C}\|$	n
Robust	$5n + 2$	$3n + 1$	n

Where $|\mathcal{C}| = \sum_{i=1}^{K} \binom{n}{K}$.

As can be seen, even for the simpliest case of $K = 1$, the number of constraints and continuous varibles of the robust model grows linearly with the number of the generating units whereas this increase is quadratic for the contigency dependent model.

Real-Size Case Study

In order to evaluate the performance of the proposed formulation, a base test system comprising ten units was used.

The system demand for the base test system is equal to 700MW.

Other 9 additional case studies were generated by replicating the original base test system and scaling the demand accordingly.

Optimization stopping criteria:

- Optimality GAP: 0.1%
- Time: 1000s

System Data

Unit	$\underline{P}_i\,(MW)$	$\overline{P}_i\,(MW)$	$c_i^f\,(\$)$	$c_i^t\,(\$)$	$C_i^R\,(\$)$
1	150	455	2550	16.19	1.62
2	150	455	2550	17.26	1.73
3	70	180	1300	16.60	1.66
4	70	180	1300	16.50	1.65
5	50	165	1620	19.70	1.97
6	30	90	800	22.26	2.23
7	40	85	850	27.74	2.77
8	20	60	550	25.92	2.59
9	20	60	550	27.27	2.73
10	20	60	550	27.79	2.78

System Demand: 700MW

Numerical Results

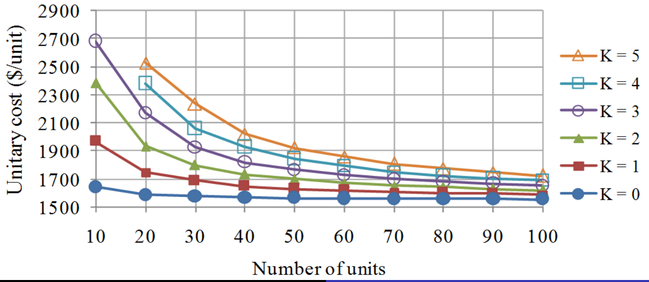
The table bellow summarizes the impact of security on costs.

# Units	$K = 0$ (\$)	$K = 1$ (%)	$K = 2$ (%)	$K = 3$ (%)	$K = 4$ (%)	$K = 5$ (%)
10	16433.4	19.7	45.0	63.0	Infeasible	Infeasible
20	31693.4	10.2	21.9	36.9	50.2	59.5
30	47311.8	7.0	14.1	22.0	30.5	41.6
40	62620.2	5.3	10.7	16.1	23.1	29.0
50	78152.1	4.3	8.5	12.8	18.0	22.9
60	93684.1	3.6	7.1	10.7	14.8	18.9
70	108992.0	3.1	6.1	9.2	12.2	16.0
80	124524.0	2.7	5.4	8.0	10.7	13.8
90	140056.0	2.4	4.8	7.1	9.5	12.3
100	155365.0	2.1	4.3	6.4	8.6	10.7

Numerical Results

Relationship between total cost and n for different K:

	$K = 0$	$K = 1$	$K = 2$	$K = 3$	$K = 4$	$K = 5$
Angular ($/security)	1545.9	1546.7	1544.2	1532.2	1518.3	1514.8
Linear ($)	843.2	4118.7	7649.2	11921.7	16510.9	20586.8



Numerical Results

Comparison of computing times for real size-systems:

# Units	1 R	1 CD	2 R	2 CD	3 R	3 CD	4 R	4 CD	5 R	5 CD
10	0.1	0.1	0.1	0.5	0.1	0.1	I	I	I	I
20	0.1	0.3	0.1	1.3	0.1	1.3	0.1	13.9	0.1	146.9
30	0.9	1.0	0.5	1.7	0.9	20.3	0.1	314.6	0.3	1000.0
40	1.4	2.0	1.1	8.1	0.4	30.0	0.6	1000.0	0.2	OM
50	3.0	2.5	3.1	22.6	0.3	96.9	9.5	OM	1.4	OM
60	11.3	14.8	6.4	97.5	2.5	704.3	68.7	OM	4.1	OM
70	12.2	14.8	25.7	390.5	8.3	1000.0	0.8	OM	65.8	OM
80	6.6	9.7	10.1	136.8	3.1	1000.0	0.7	OM	108.7	OM
90	6.7	11.2	1.7	1000.0	73.2	OM	6.0	OM	195.1	OM
100	18.7	38.9	33.7	1000.0	21.2	OM	3.3	OM	0.8	OM

Conclusions

This paper presented a novel framework for dealing with unit commitment $n - K$ security criteria based on robust optimization.

Major contribution: The proposed approach allows system operators to schedule power and reserve while considering all combinations of up to K generation units outages.

How we achieved that:

- Formulation of the original contigency-dependent model as a robust bilevel program;
- Formulation of the robust bilevel model as a equivalent single-level mixed-integer that is efficiently solved.

Conclusions

Numerical results show that the proposed robust model outperforms the contigency dependent formulation.

Next steps:

- Currently Underway: multiperiod setting and considering line outages ina network-constrained system;
- Further Research: Pricing energy and reserve under the $n - K$ security criterion.

References I

A. Wood and B. Wollenberg, *Power generation, operation, and control.*
Wiley New York et al., 1984.

G. Oliveira, S. Binato, M. Pereira, P. Inc, and R. de Janeiro, "Value-based transmission expansion planning of hydrothermal systems under uncertainty," *IEEE Transactions on Power Systems*, vol. 22, no. 4, pp. 1429–1435, 2007.

M. Carrión, J. Arroyo, and N. Alguacil, "Vulnerability-constrained transmission expansion planning: A stochastic programming approach," *IEEE Transactions on Power Systems*, vol. 22, no. 4, pp. 1436–1445, 2007.

J. Choi, T. Mount, and R. Thomas, "Transmission expansion planning using contingency criteria," *IEEE Transactions on Power Systems*, vol. 22, no. 4, pp. 2249–2261, 2007.

References II

- N. Alguacil, M. Carrión, and J. Arroyo, "Transmission network expansion planning under deliberate outages," *International Journal of Electrical Power & Energy Systems*, vol. 31, no. 9, pp. 553–561, 2009.
- J. Arroyo and F. Galiana, "Energy and reserve pricing in security and network-constrained electricity markets," *IEEE Transactions on Power Systems*, vol. 20, no. 2, pp. 634–643, 2005.
- F. Galiana, F. Bouffard, J. Arroyo, and J. Restrepo, "Scheduling and pricing of coupled energy and primary, secondary, and tertiary reserves," *Proceedings of the IEEE*, vol. 93, no. 11, pp. 1970–1983, 2005.
- J. Wang, N. Redondo, and F. Galiana, "Demand-side reserve offers in joint energy/reserve electricity markets," *IEEE Transactions on Power Systems*, vol. 18, no. 4, pp. 1300–1306, 2003.

References III

- P. Ruiz, C. Philbrick, E. Zak, K. Cheung, and P. Sauer, "Uncertainty management in the unit commitment problem," *IEEE Transactions on Power Systems*, vol. 24, no. 2, 2009.
- D. Bertsimas and M. Sim, "The price of robustness," *Operations Research*, vol. 52, no. 1, pp. 35–53, 2004.