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***Abstract*—Insert abstract here**

***Index Terms*—Keywords here**

NOTATION

Indices

n	node
u	generation unit
ℓ	transmission line
o	operating condition
ν	iteration

Sets

Ψ_n^G	existing generation units at node n
Ψ^L	existing transmission lines
Ψ_n^{G+}	candidate generation units at node n
Ψ^{L+}	candidate transmission lines
$\Phi^{L1/L2/L3}$	1st/2nd/3rd level decision variables
Ω	uncertainty set
Ξ	feasibility set

Parameters

W_o	the weight of operating condition o
$\tilde{D}_n$	nominal demand at node n
$\hat{D}_n$	demand increase at node n
C_u^x	investment cost of candidate unit u
C_ℓ^y	investment cost of candidate transmission line ℓ
C_u^G	generation cost of unit u
Λ^D	demand uncertainty budget
$\Lambda^{min/max}$	minimum/maximum price

Variables

d_n	uncertain demand at node n
$z_{o,n}$	auxiliary variables for linearizing $\lambda_{o,n} d_n$
$\lambda_{o,n}$	price in condition o at node n
$\tilde{\lambda}_{o,n}$	auxiliary variables for linearizing $\lambda_{o,n} d_n$
$f_{o,\ell,\nu}$	transmission flow in line ℓ in condition o at iteration ν
$g_{o,u,\nu}$	generation at unit u in condition o at iteration ν
$\bar{\beta}_{o,u}$	dual variable for maximum generation of unit u in condition o
$\underline{\beta}_{o,u}$	dual variable for minimum generation of unit u in condition o
$\bar{\mu}_{o,\ell}$	dual variable for maximum flow in line ℓ in condition o
$\underline{\mu}_{o,\ell}$	dual variable for maximum flow in line ℓ in condition o
u_n	equal 1 if demand is increased from the nominal level at node n

I. MATHEMATICAL FORMULATION

A. Stochastic robust optimization problem

The stochastic robust optimization problem is

$$\min_{\Phi^{L1}} \sum_{u \in \Psi^{G+}} C_u^x x_u + \sum_{\ell \in \Psi^{L+}} C_\ell^y y_\ell + \tag{1}$$

$$\max_{\Phi^{L2} \in \Omega} \quad \min_{\Phi^{L3} \in \Xi} \sum_o W_o \sum_u C_u^g g_{o,u}, \tag{2}$$

where $\Phi^{L1} = \{x_u \,\forall u \in \Psi^{G+}, y_\ell \,\forall \ell \in \Psi^{L+}\}$, $\Phi^{L2} = \{d_n \,\forall n\}$, and $\Phi^{L3} = \{g_{o,u} \,\forall o, u, f_{o,\ell} \,\forall o, \ell\}$. The uncertainty set Ω is given by

$$\begin{aligned} \Omega = \{ d_n = \tilde{D}_n + u_n \hat{D}_n & \qquad \forall n \\ \sum_n d_n \le \Lambda^D \}. & \end{aligned} \tag{3}$$

Given the optimal values $x_u^* \,\forall u \in \Psi^{G+}$, $y_\ell^* \,\forall \ell \in \Psi^{L+}$, and $d_n^* \,\forall n$, the feasibility set $\Xi(g_{o,u}, f_{o,\ell})$ is

$$\{ \sum_{u \in \Psi_n^G} g_{o,u} + \sum_\ell Y_{\ell,n} f_{o,\ell} = d_n^* \qquad \forall o, n \tag{4}$$

$$0 \le g_{o,u} \le G_{o,u}^{max} \qquad \forall o, u \in \Psi_n^G \tag{5}$$

$$F_{o,\ell}^{min} \le f_{o,\ell} \le F_{o,\ell}^{max} \qquad \forall o, \ell \in \Psi^L \tag{6}$$

$$0 \le g_{o,u} \le G_{o,u}^{max} x_\ell^* \qquad \forall o, u \in \Psi_n^{G+} \tag{7}$$

$$F_{o,\ell}^{min} y_\ell^* \le f_{o,\ell} \le F_{o,\ell}^{max} y_\ell^* \qquad \forall o, \ell \in \Psi^{L+}. \} \tag{8}$$

B. Master problem

The master problem at iteration ν is

$$\underset{\Phi^{L1}, \Omega_{o,\nu}^M, \theta}{\text{minimize}} \sum_{u \in \Psi^{G+}} C_u^x x_u + \sum_{\ell \in \Psi^{L+}} C_\ell^y y_\ell + \theta \tag{9}$$

subject to

$$\theta \ge \sum_o W_o \sum_u C_u^g g_{o,u,\nu'} \qquad \forall \nu' \le \nu \tag{10}$$

$$\sum_{u \in \Psi_n^G} g_{o,u,\nu'} + \sum_\ell Y_{\ell,n} f_{o,\ell,\nu'} = d_{n,\nu'}^* \qquad \forall o, n, \nu' \le \nu \tag{11}$$

$$0 \le g_{o,u,\nu'} \le G_{o,u}^{max} \qquad \forall o, u \in \Psi_n^G, \nu' \le \nu \tag{12}$$

$$F_{o,\ell}^{min} \le f_{o,\ell,\nu'} \le F_{o,\ell}^{max} \qquad \forall o, \ell \in \Psi^L, \nu' \le \nu \tag{13}$$

$$0 \leq g_{o,u,\nu'} \leq G_{o,u}^{max} x_\ell \qquad \forall o, u \in \Psi_n^{G+}, \nu' \leq \nu \tag{14}$$

$$F_{o,\ell}^{min} y_\ell \leq f_{o,\ell,\nu'} \leq F_{o,\ell}^{max} y_\ell \qquad \forall o, \ell \in \Psi^{L+}, \nu' \leq \nu, \tag{15}$$

where $\Omega_{o,\nu}^M = \{g_{o,u,\nu'}\,\forall u, f_{o,\ell,\nu'}\,\forall \ell\}, \forall o, \nu' \leq \nu$. $d_{n,\nu'}^*, \forall n, \nu' \leq \nu$ are input data obtained from all the previous solutions of the subproblem.

C. Subproblem

The subproblem is

$$\begin{aligned}\underset{\Phi^{L2},\Omega_o^S}{\text{maximize}} \sum_o \Bigg[& \sum_n \lambda_{o,n} d_n + \\ & \sum_{u\in\Psi^G} \bar{\beta}_{o,u} G_{o,u}^{max} + \\ & \sum_{\ell\in\Psi^L} \bar{\mu}_{o,\ell} F_{o,\ell}^{max} - \underline{\mu}_{o,\ell} F_{o,\ell}^{min} + \\ & \sum_{u\in\Psi^{G+}} \bar{\beta}_{o,u} G_{o,u}^{max} x_u^* + \\ & \sum_{\ell\in\Psi^{L+}} \left(\bar{\mu}_{o,\ell} F_{o,\ell}^{max} - \underline{\mu}_{o,\ell} F_{o,\ell}^{min}\right) y_\ell^* \Bigg]\end{aligned} \tag{16}$$

subject to

$$\lambda_{o,u(n)} - \bar{\beta}_{o,u} + \underline{\beta}_{o,u} = C_u^g W_o \quad \forall o, u \tag{17}$$

$$\sum_n Y_{\ell,n} \lambda_{o,n} - \bar{\mu}_{o,\ell} + \underline{\mu}_{o,\ell} = 0 \quad \forall o, \ell \tag{18}$$

$$\text{Eqs. (3)}, \tag{19}$$

where $\Omega_o^S = \{\lambda_{o,n} \forall n, \bar{\beta}_{o,u} \forall u, \underline{\beta}_{o,u} \forall u, \bar{\mu}_{o,\ell} \forall \ell, \underline{\mu}_{o,\ell} \forall \ell\}, \forall o$. $x_u^*\,\forall u \in \Psi^{G+}$ and $y_\ell^*\,\forall \ell \in \Psi^{L+}$ are input data obtained from the previous solution of the master problem. The index $u(n)$ denotes the node at which unit u is located.

The product of a continuous and a binary variable $\lambda_{o,n} d_n$ in the objective function (16) is linearized exactly with $\lambda_{o,n} d_n = z_{o,n} \hat{D}_n + \lambda_{o,n} \tilde{D}_n$ and by adding the following constraints to the subproblem

$$z_{o,n} = \lambda_{o,n} - \tilde{\lambda}_{o,n} \forall o, n \tag{20}$$

$$\Lambda^{min} u_n \leq z_{o,n} \leq \Lambda^{max} u_n \tag{21}$$

$$\Lambda^{min}(1 - u_n) \leq \tilde{\lambda}_{o,n} \leq \Lambda^{max}(1 - u_n) \tag{22}$$

Consequently, the decision variables $d_n\,\forall n$ are replaced with $z_{o,n} \forall o, n, \tilde{\lambda}_{o,n} \forall o, n$ in the subproblem.

II. Sample data

Sample data is in Table I.

Variable	Value
n	[0, 1, 2, 3]
u	[0, 1, 2, 3]
ℓ	[0, 1, 2, 3]
o	[0, 1]
Ψ_n^G	{1: 1, 3: 3}
Ψ_n^{G+}	{0: 0, 2: 2}
Ψ^L	[0, 2]
Ψ^{L+}	[1, 3]
C_u^G	[1, 5, 5, 5]
$G_{o,u}^{max}$	10
$F_{o,\ell}^{max}$	5
$F_{o,\ell}^{min}$	-5
$Y_{\ell,n}$	$\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & & 0 & -1 \end{bmatrix}$
W_o	[0.5, 0.5]
C_u^x	1
C_ℓ^y	1
$\tilde{D}_n$	3
$\bar{D}_n$	1
Λ^D	2
$\Lambda^{min/max}$	-100 / 100

TABLE I
SAMPLE DATA

References