Sygnals and Systems - Dr. Akhavan

CA7 - Matin Bazrafshan

Part 1

1.1

$v_R(t) + v_L(t) + v_C(t) = v_{\text{in}}(t)$

- $v_R(t) = R.\, i(t)$
- $v_L(t) = L.\frac{\text{di}(t)}{\text{dt}}$
- $$v_C(t) = \frac{1}{C}\int_{-\infty}^{t} i(z)\, d\, z$$

$$v_R(t) + v_L(t) + v_C(t) = v_{\text{in}}(t) \rightarrow \frac{d(v_R(t) + v_L(t) + v_C(t))}{\text{dt}} = \frac{d(v_{\text{in}}(t))}{\text{dt}}$$

$$\rightarrow R.\frac{d(i(t))}{\text{dt}} + L.\frac{d^2 i(t)}{\text{dt}} + \frac{1}{C} i(t) = \frac{d(v(t))}{t}$$

1.2

$$\text{RsI}(s) + \text{Ls}^2 I(s) + \frac{1}{C} I(s) = V(s) \rightarrow I(s) = \frac{V(s)}{\text{Rs} + \text{Ls}^2 + \frac{1}{C}}$$

1.3

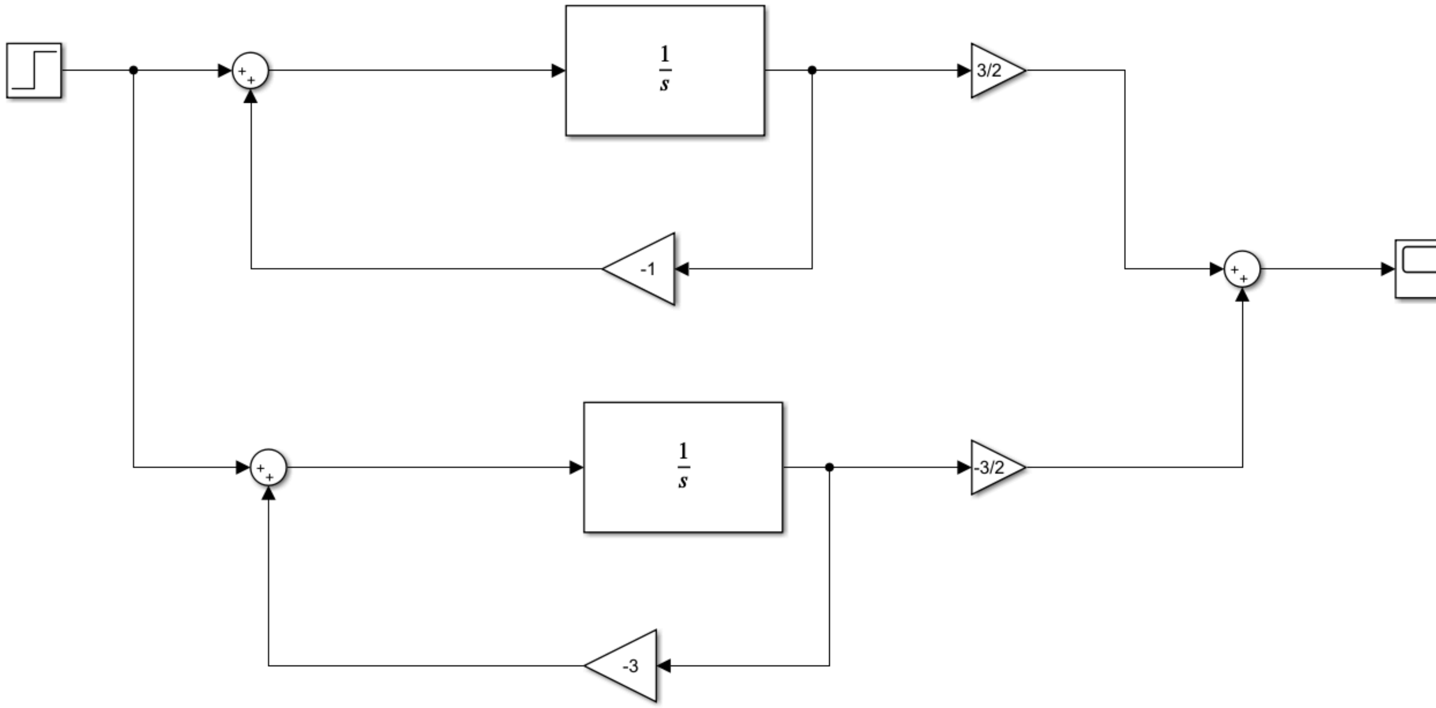
- $X(s) = V(s)$
- $Y(s) = \frac{1}{c} I(s)$

$$\rightarrow Y(s) = \frac{X(s)}{\text{RsC} + \text{Ls}^2 C + 1}$$

1.4

- $R = 1$
- $L = 0.25$
- $C = \frac{4}{3}$

$$Y(s) = \frac{X(s)}{\frac{1}{3}s^2 + \frac{4}{3}s + 1} = X(s)\left(\frac{3}{2}\frac{1}{s+1} - \frac{3}{2}\frac{1}{s+3}\right)$$



1.5

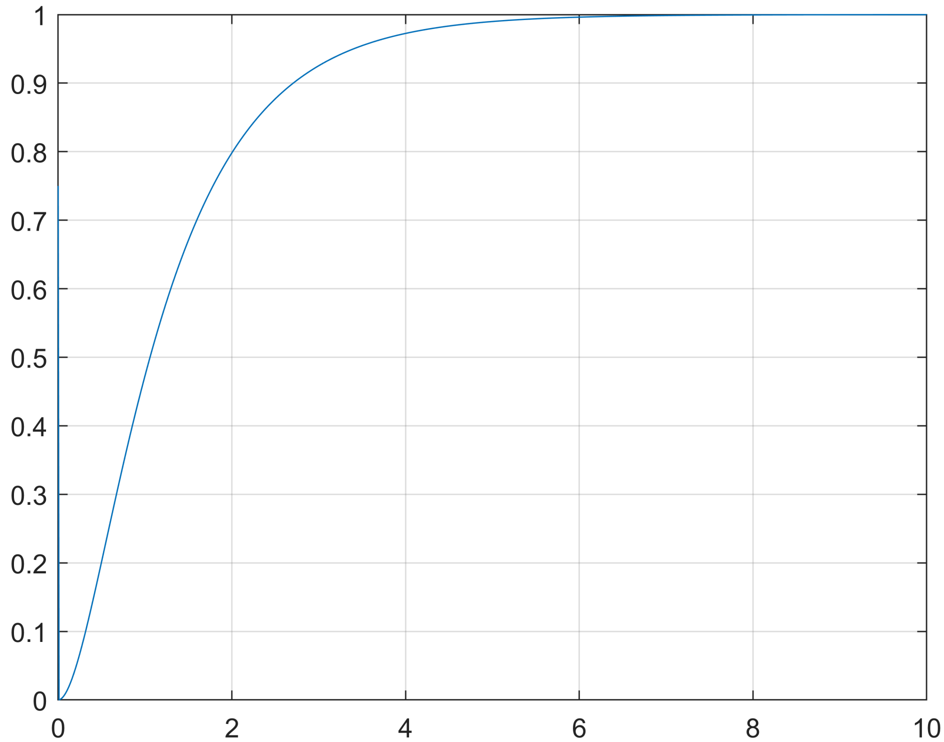
- $x(t) = u(t) \rightarrow X(s) = \frac{1}{s}$

$$\rightarrow Y(s) = \frac{3}{s^3 + 4s^2 + 3s} = \frac{1}{s} + \frac{1}{2(s+3)} + -\frac{3}{2(s+1)} \rightarrow y(t) = \left(1 + \frac{e^{-3t}}{2} - \frac{3e^{-t}}{2}\right)u(t)$$

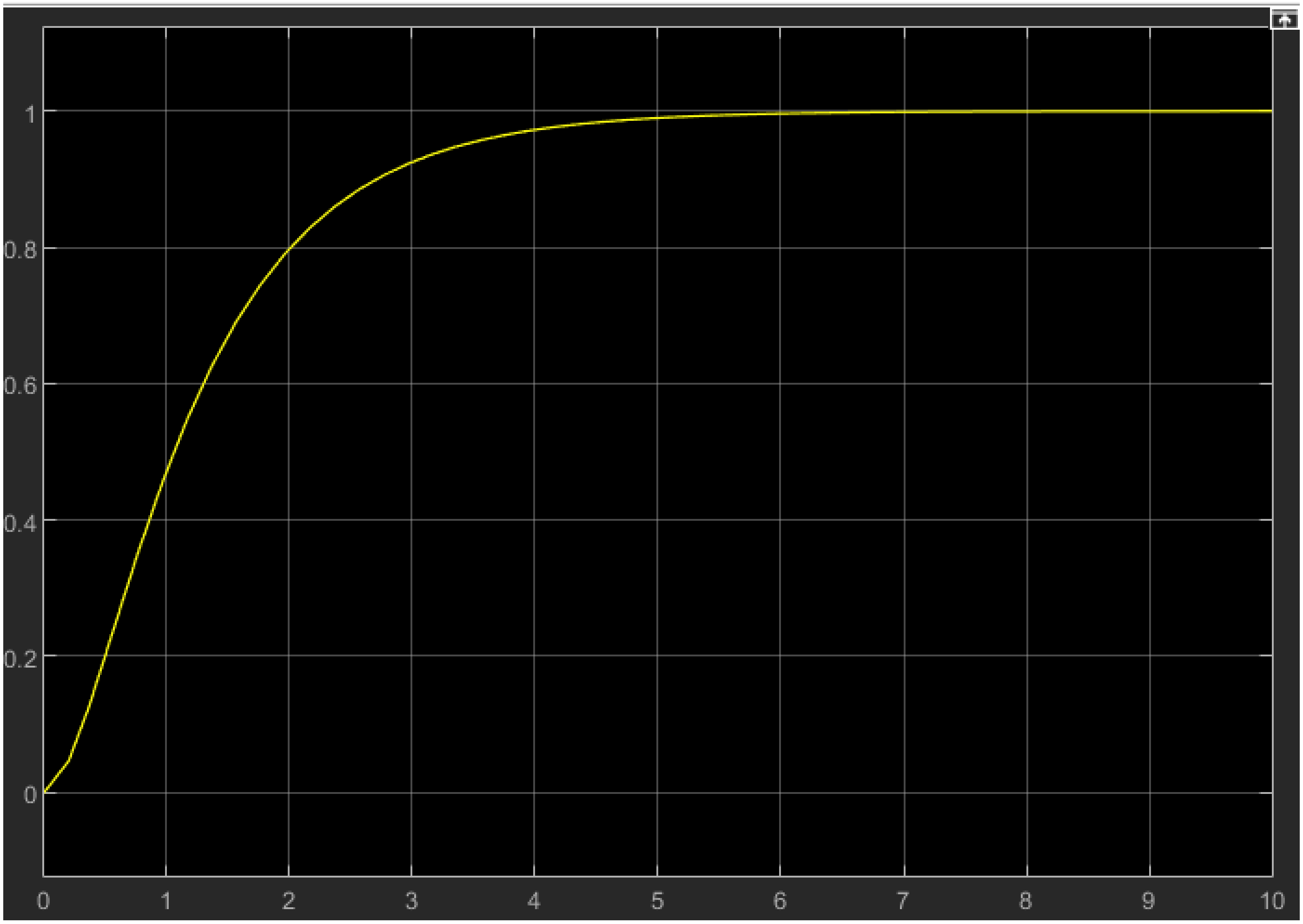
1.6

yes, they are similar.

```
t = 0:0.01:10;
plot(t, 1 + exp(-3 * t) / 2 - 3 * exp(-t) / 2 .* heaviside(t));
grid on;
```

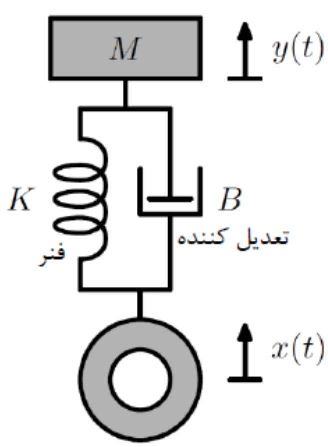


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
2
4
6
8
10



Part 2

2.1



$$K(x(t) - y(t)) = B\left(\frac{\mathrm{dx}(t)}{\mathrm{dt}} - \frac{\mathrm{dy}(t)}{\mathrm{dt}}\right) = M\,\frac{d^2y(t)}{\mathrm{dt}^2}$$

- $K = 1$
- $M = 1$

$$B\,\frac{\mathrm{dx}(t)}{\mathrm{dt}} + x(t) = \frac{d^2y(t)}{\mathrm{dt}} + B\frac{\mathrm{dy}(t)}{\mathrm{dt}} + y(t)$$

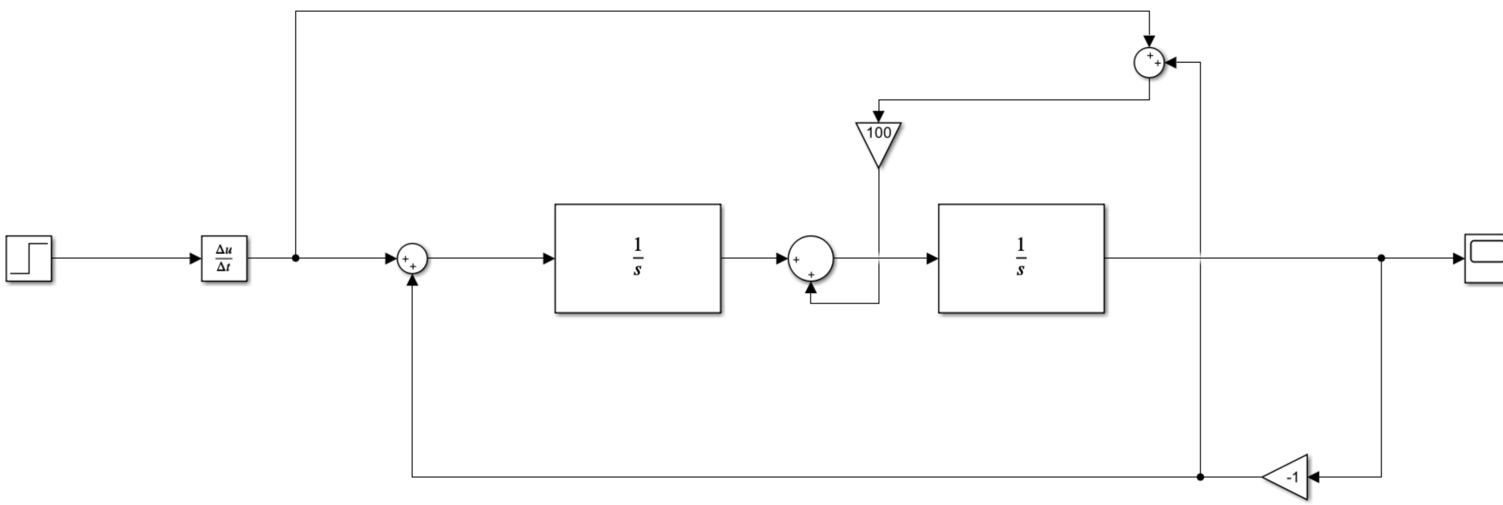
2.2

notation 1:

$$\mathrm{BsX}(s) + X(s) = s^2Y(s) + \mathrm{BsY}(s) + Y(s) \rightarrow Y(s) = \frac{X(s)(\mathrm{Bs}+1)}{s^2+\mathrm{Bs}+1}$$

notation 2:

$$\mathrm{BsX}(s) + X(s) = s^2Y(s) + \mathrm{BsY}(s) + Y(s) \rightarrow Y(s) = \frac{1}{s^2}(X(s) - Y(s)) + \frac{B}{s}(X(s) - Y(s))$$



2.3

- $x(t) = \delta(t) \rightarrow X(s) = 1$

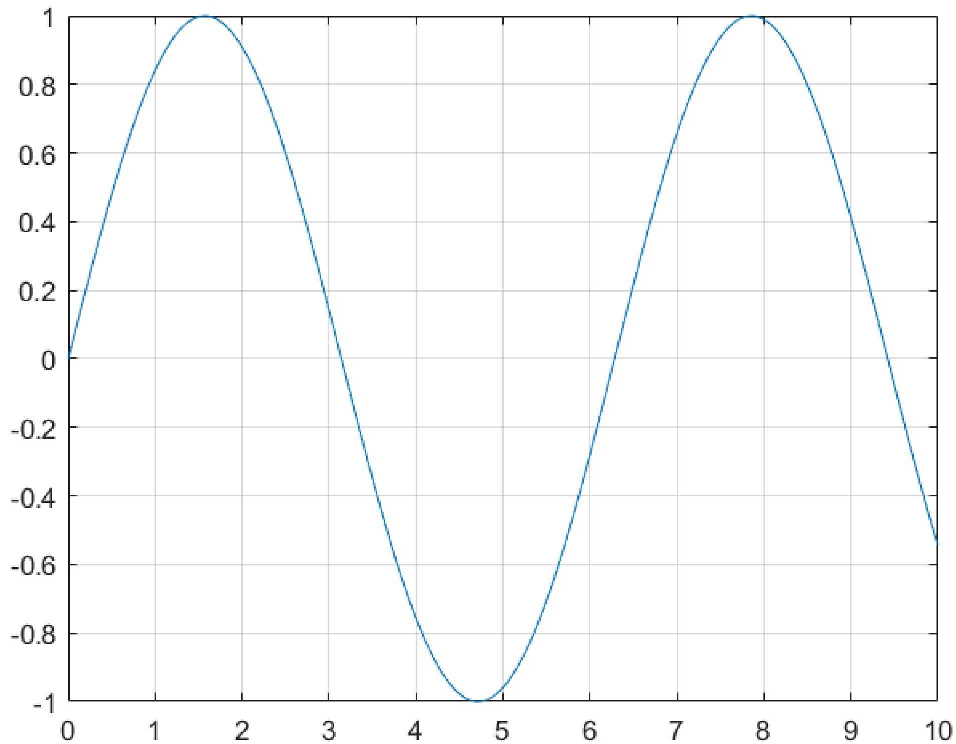
$$\rightarrow Y(s) = \frac{(\mathrm{Bs}+1)}{s^2+\mathrm{Bs}+1}$$

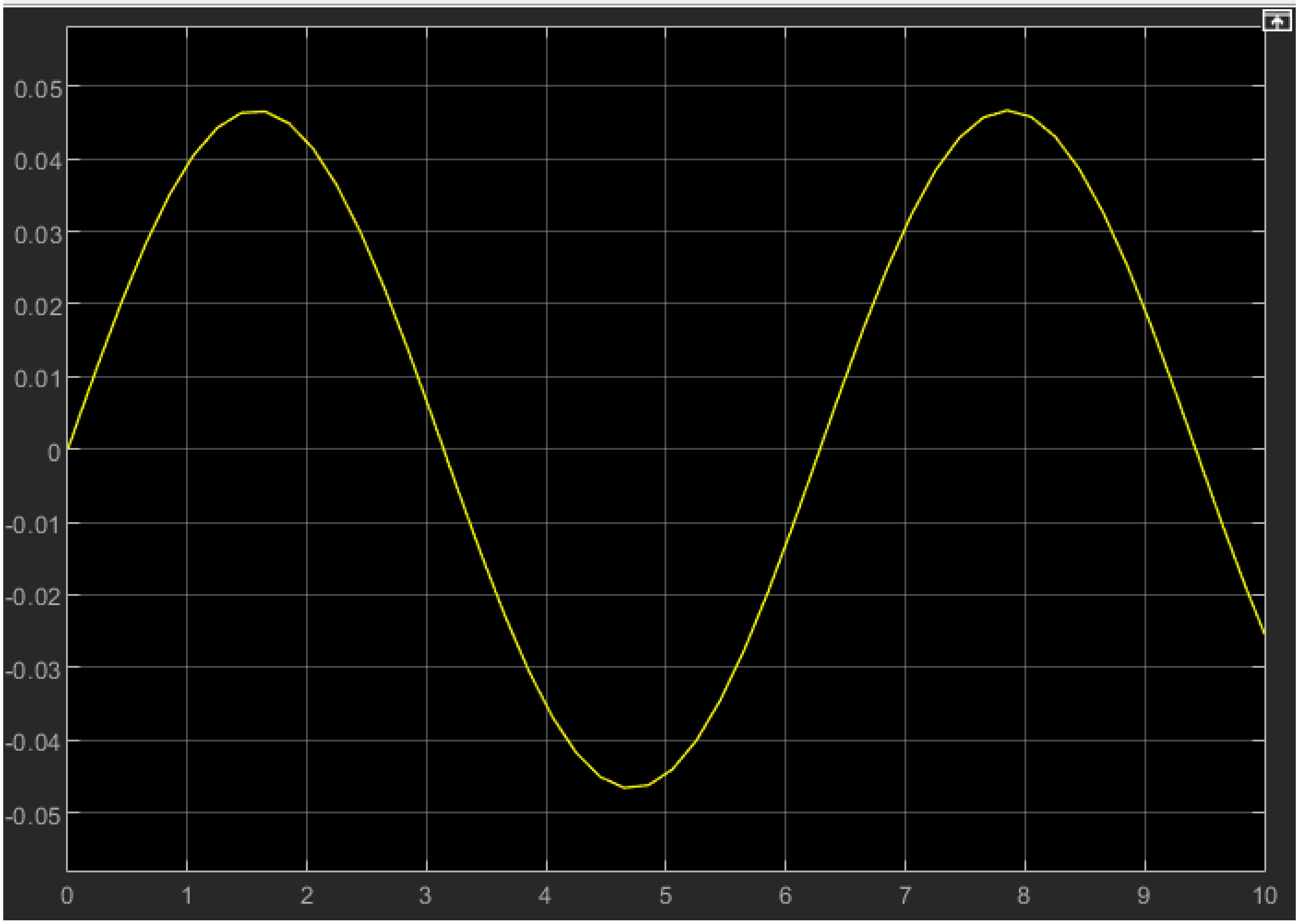
- $B = 0$

$$\rightarrow Y(s) = \frac{1}{s^2+1}$$

$$\rightarrow y(t) = \sin(t)$$

```
t = 0:0.01:10;
plot(t, sin(t));
grid on;
```





due to imaginary part, the cabin will oscillate forever, and never stops(sine function).

2.4

To avoid imaginary part. the roots of denominator must be all real, according to the quadric equation, the delta should be greater or equal to zero:

$$\Delta \geq 0 \to B^2 - 4 \geq 0 \to B \geq 2 \to \min(B) = 2$$

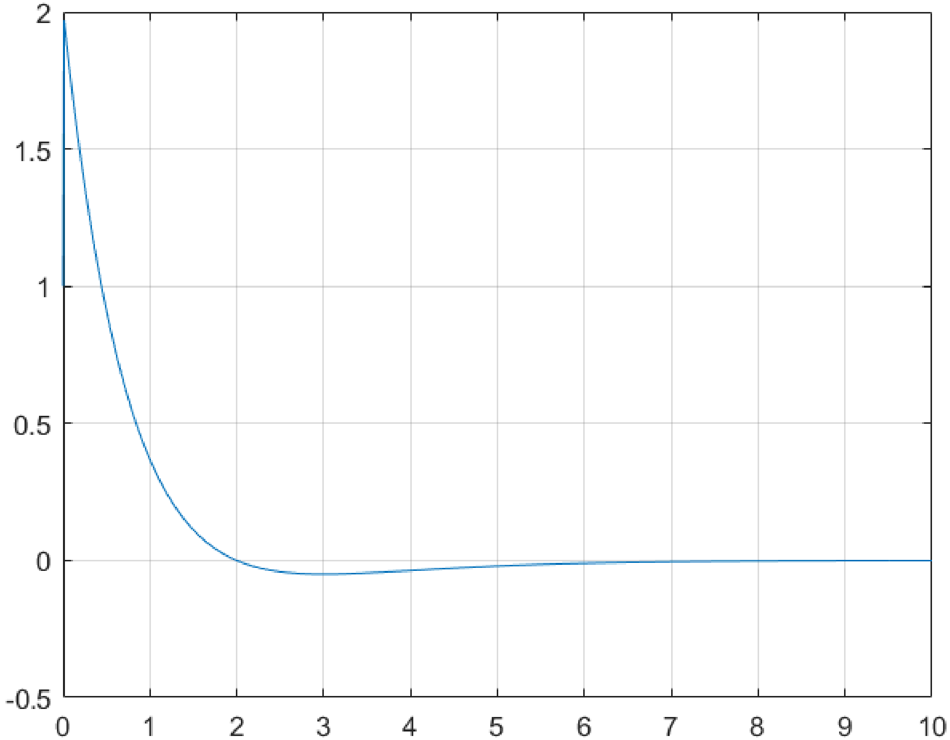
$$\to Y(s) = \frac{2s+1}{s^2+2s+1} = \frac{2}{s+1} - \frac{1}{(s+1)^2}$$

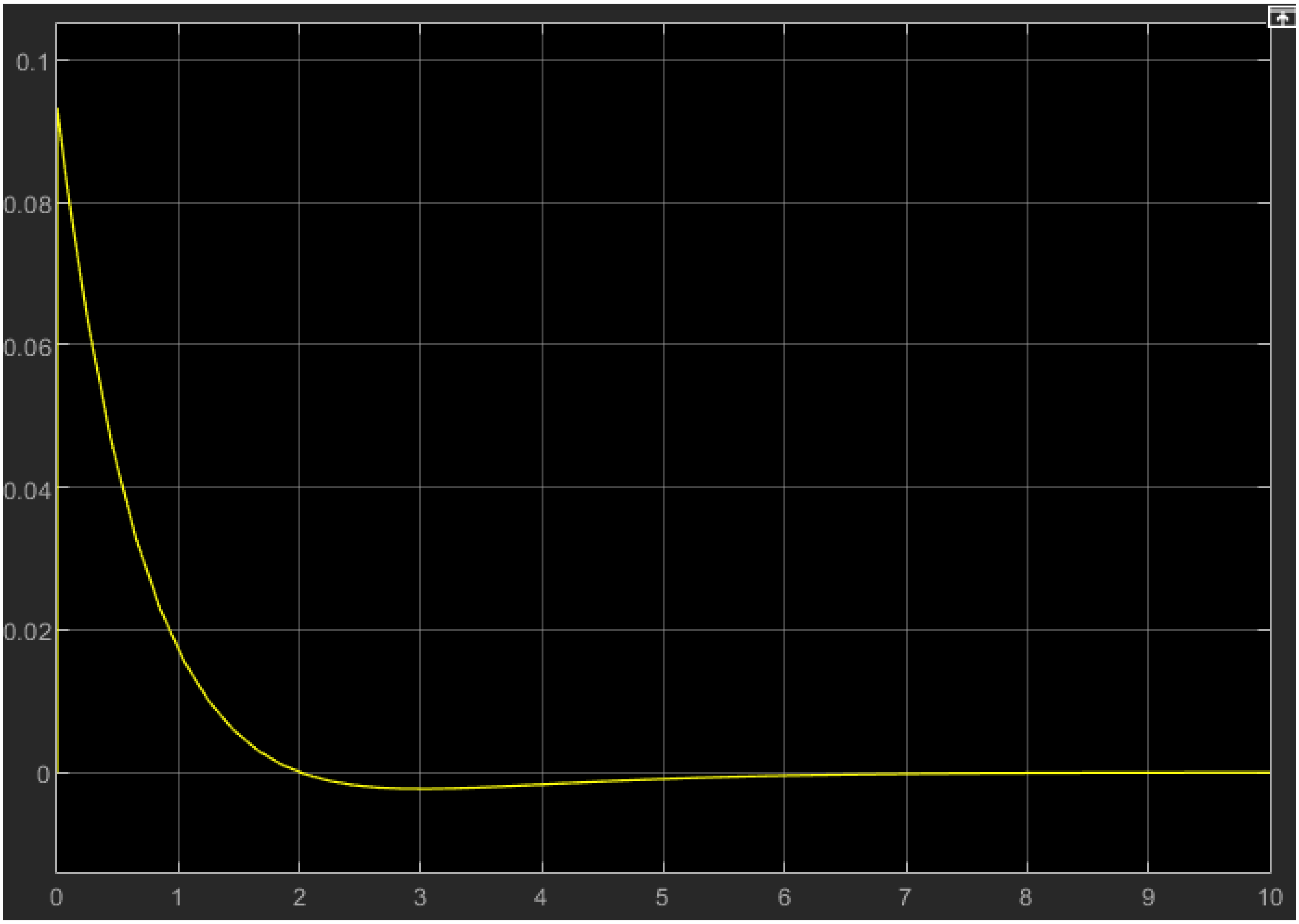
- $\frac{d}{\mathrm{ds}} \frac{1}{s+1} = -\frac{1}{(s+1)^2}$
- $\frac{1}{s+1} \to e^{-t}u(t)$

$$\to y(t) = -e^{-t}(t-2)u(t)$$

```
t = 0:0.01:10;
plot(t, -exp(-t) .* (t - 2) .* heaviside(t));
```

```
grid on;
```





The cabin smothly stops over time.

2.5

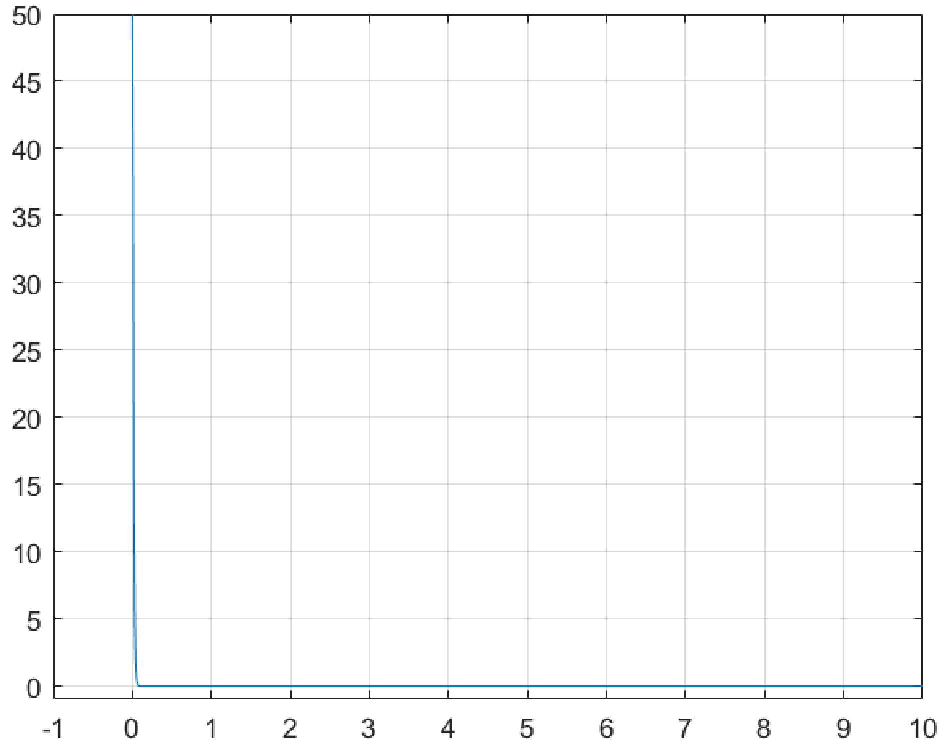
- $B = 100$

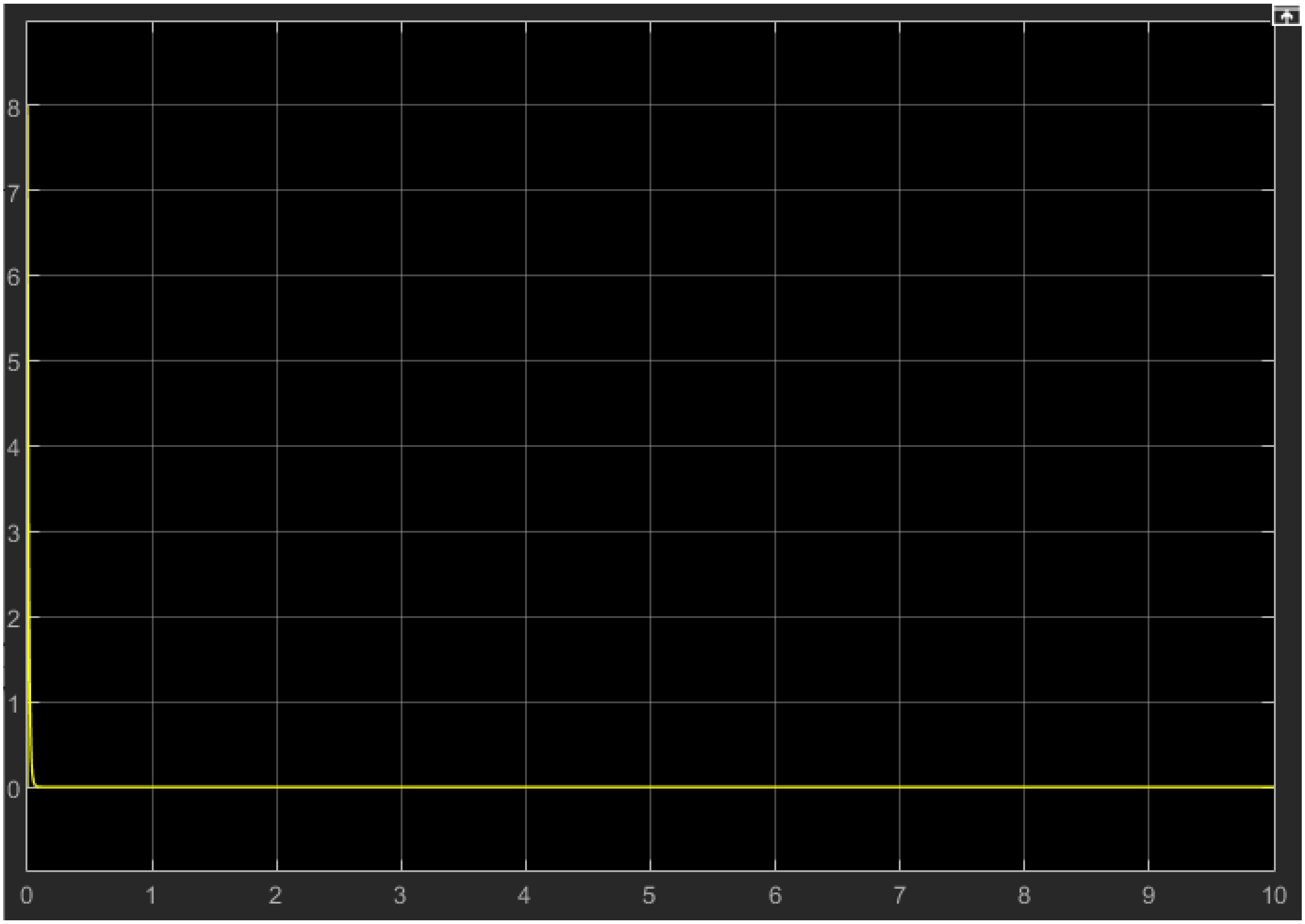
$$\rightarrow Y(s) = \frac{100s+1}{s^2+100s+1} \approx \frac{100s+1}{s^2+100.1s+1} = \frac{100s+1}{(s+100)(s+0.01)} = \frac{100}{s+100}$$

- $\frac{\alpha}{\beta + s} \rightarrow \alpha e^{-\beta t} u(t)$

$$\rightarrow y(t) = 100e^{-100t}u(t)$$

```
t = 0:0.01:10;
plot(t, 100 * exp(-100 * t) .* heaviside(t));
xlim([-1, 10]);
ylim([-1, 50]);
grid on;
```


50
45
40
35
30
25
20
15
10
5
0
-1
0
1
2
3
4
5
6
7
8
9
10



There will be a huge impluse(which take the most energy of cabin in a small part of time) and then the cabin rests.

2.6

It is obvious that $B = 2$ is the best choise.

- $B = 0$ will result in sine and the car never stops.
- $B = 100$ will cause a great impulse at the beginning.
- $B = 2$ will smoothly stops so it is the nest choise.

Physical interpretation:

- because the change in momentum of cabin is constant ($v_2 = 0 \text{ and so } m\Delta v = C$) so the impulse is equal in three given situtations, as we increase Δt, the stoping will be more smooth.

part 3

3.1

$$\frac{d^2 y(t)}{\mathrm{dt}^2}\, 3\,\frac{\mathrm{dy}(t)}{\mathrm{dt}} + 2y(t) = (t)$$

- $x(t) = 5u(t)$
- $y(0^-) = 1$
- $y'(0^-) = 1$

$$\to s^2 Y(s) - s - 1 + 3(\mathrm{s}\mathrm{Y}(s) - 1) + 2Y(s) = X(s)$$

$$\to Y(s) = \frac{X(s) + s + 4}{s^2 + 3s + 2}$$

- $X(s) = \dfrac{5}{s}$

$$\to Y(s) = \frac{s^2 + 4s + 5}{s^3 + 3s^2 + 2s}$$

$$\to Y = \frac{-2}{s+1} + \frac{-1}{2(s+2)} + \frac{5}{2s}$$

$$\to -\frac{1}{2}e^{-2t} - 2e^{-t} + \frac{5}{2}$$

3.2

```
clearvars;
sys = tf(1,1);

syms y(t)
Dy = diff(y);

ode = (diff(y,t,2) + 3 * diff(y,t,1) + 2 * y) == 5 * step(sys);
cond1 = y(0) == 1;
cond2 = Dy(0) == 1;

conds = [cond1 cond2];
ySol(t) = dsolve(ode,conds);
ySol = simplify(ySol)
t = 0:0.01:10;

ySolFun = matlabFunction(ySol);

ySolValues = ySolFun(t);

plot(t, ySolValues)
xlabel('t')
```

```
ylabel('y(t)')
```

yes, we can see the results are similar.