Sygnals and Systems - Dr. Akhavan

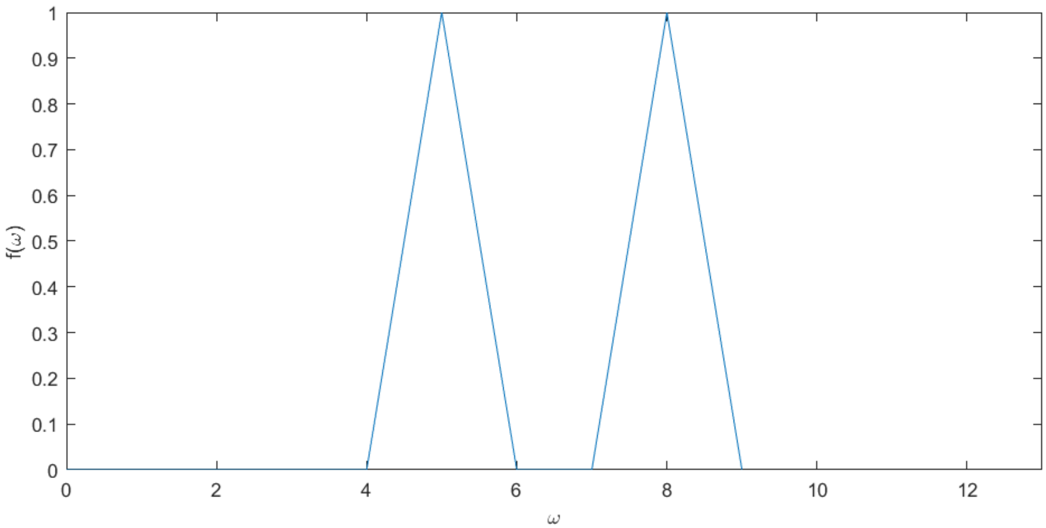
CA5 - Matin Bazrafshan

Part 0

if we consider a signal with $f_s = 20\,\mathrm{Hz}$, $T = 1\ s$ and $N = 100$ the frequency resolution will be $\delta_s = \frac{f_s}{N} = 1$, so we can not detect picks with distance less than 1.

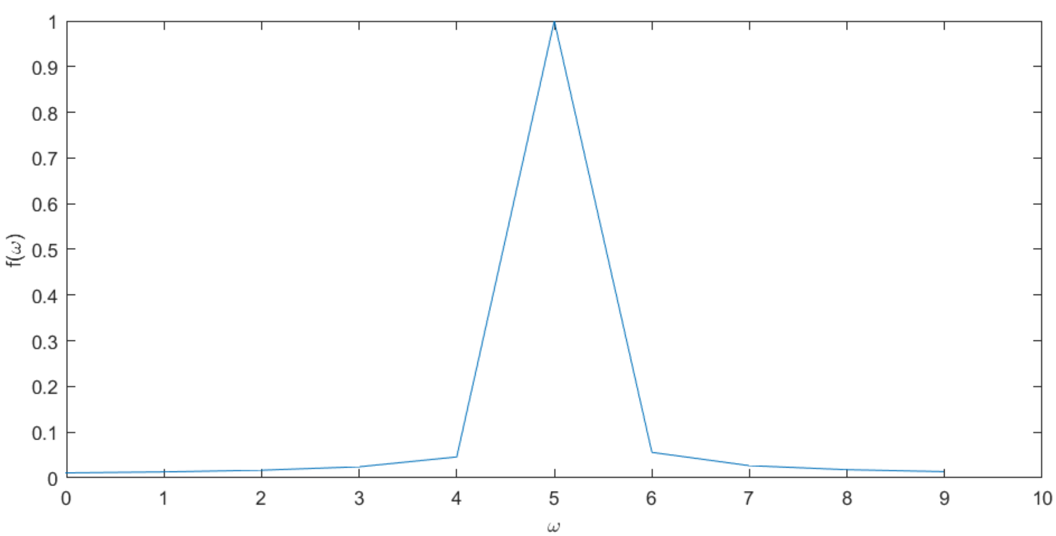
for example signal $x_1(t) = e^{2\pi j(5t)} + e^{2\pi j(8t)}$ have the picks on 5 and 8 respectively as the fourier transform of it is $\mathscr{F}(e^{2\pi j(5t)} + e^{2\pi j(8t)}) = \delta(\omega - 5) + \delta(\omega - 8)$. the distance between two pulse is 3 so it will be shown well.

```
clearvars;
fs = 20;
ts = 1 / fs;
tstart = 0;
tend = 1;
t = tstart : ts : tend - ts;
T = tend - tstart;
N = T * fs;
f = -fs/2 : fs/N : fs/2 - fs/N;


x1 = exp(1i * 2 * pi * 5 * t) + exp(1i * 2 * pi * 8 * t);
y1 = fftshift(fft(x1));
y1 = y1 / max(y1);

figure('Position', [0 0 900 400]);
plot(f,abs(y1));
xlim([0 13]);
xlabel('\omega');
ylabel('f(\omega)');
```



but in signal $x_2(t) = e^{2\pi j(5t)} + e^{2\pi j(5.1t)}$ we have $\mathscr{F}(e^{2\pi j(5t)} + e^{2\pi j(8t)}) = \delta(\omega - 5) + \delta(\omega - 5.1)$ so the distance is 0.1 and can not be detected.

```
x2 = exp(1i * 2 * pi * 5 * t) + exp(1i * 2 * pi * 5.1 * t);
y2 = fftshift(fft(x2));
y2 = y2 / max(y2);

figure('Position', [0 0 900 400]);
plot(f,abs(y2));
xlim([0 10]);
xlabel('\omega');
ylabel('f(\omega)');
```



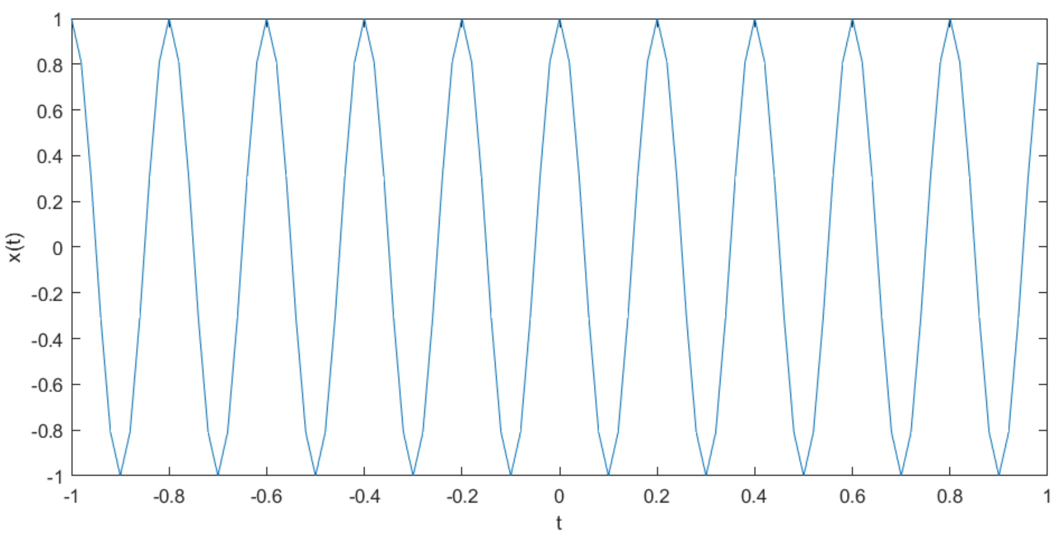
Part 1

1.1

first we plot $x_1(t) = \cos(10\pi t)$ with parameters:

- $f_s = 50$
- $t = (-1, 1)$
- $T = 2$
- $N = T \times f_s$

```
clearvars;
fs = 50;
ts = 1 / fs;
tstart = -1;
tend = 1;
t = tstart : ts : tend - ts;
T = tend - tstart;
N = T * fs;
f = -fs/2 : fs/N : fs/2 - fs/N;


figure('Position', [0 0 900 400]);
x1 = cos(10 * pi * t);
plot(t,x1);
xlabel('t');
ylabel('x(t)')
```



first we need to calculate the fourier transform:

- $x_1(t) = \cos(10\pi t) \ \rightarrow \mathscr{F}(x_1(t)) = \pi(\delta(\omega - 10\pi) + \delta(\omega + 10\pi))$

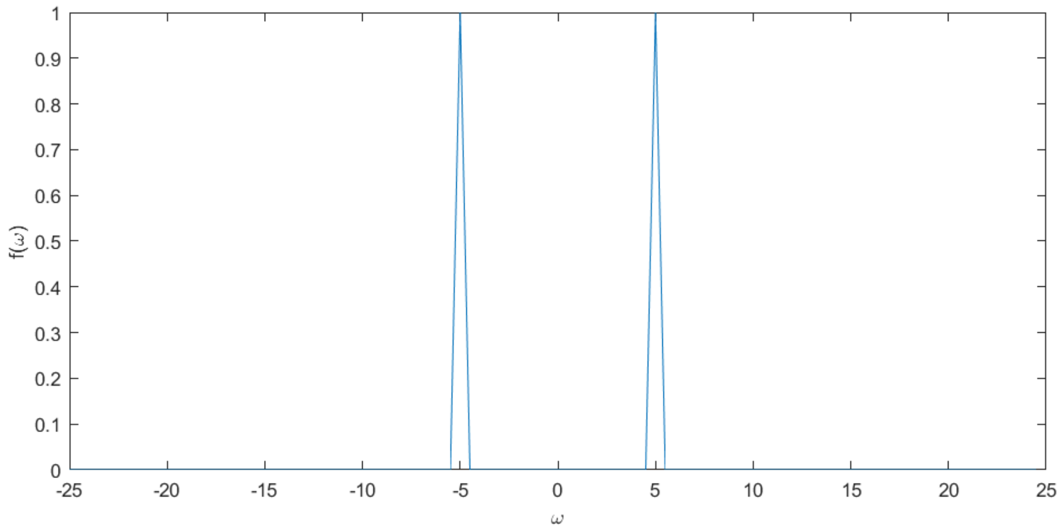
after that we normalize the output by dividing it to the maximum absolute value of it:

- $\hat{x}_1(\omega) = \dfrac{\hat{x}_1(\omega)}{\max(|\hat{x}_1(\omega)|)}$

then we consider the $f = \dfrac{\omega}{2\pi}$ so:

- $\hat{x}_1(f) = \delta(f-5) + \delta(f+5)$

```
y1 = fftshift(fft(x1));
y1 = y1 / max(abs(y1));

figure('Position', [0 0 900 400]);
plot(f,abs(y1));
xlabel('\omega');
ylabel('f(\omega)')
```



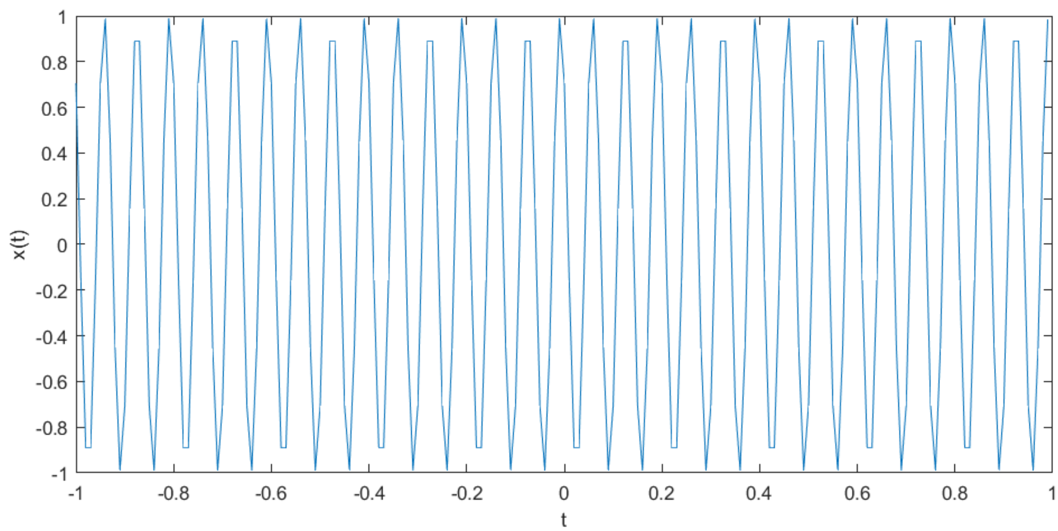
1.2

first we plot $x_2(t) = \cos\left(30\pi t + \frac{\pi}{4}\right)$ with parameters:

- $f_s = 100$
- $t = (0, 1)$
- $T = 1$
- $N = T \times f_s$

```
clearvars;
fs = 100;
ts = 1 / fs;
tstart = -1;
tend = 1;
```

```
t = tstart : ts : tend - ts;
T = tend - tstart;
N = T * fs;
f = -fs/2 : fs/N : fs/2 - fs/N;


figure('Position', [0 0 900 400]);
x2 = cos(30 * pi * t + pi / 4);
plot(t,x2);
xlabel('t');
ylabel('x(t)')
```



first we need to calculate the fourier transform:

- $x_2(t) = \cos\left(30\pi t + \frac{\pi}{4}\right) \rightarrow \mathscr{F}(x_1(t)) = \pi\left(e^{\frac{j\pi}{4}\omega}\delta(\omega - 30\pi) + e^{-\frac{j\pi}{4}\omega}\delta(\omega - 30\pi)\right)$

after that we normalize the output by dividing it to the maximum absolute value of it:

- $\hat{x}_2(\omega) = \dfrac{\hat{x}_1(\omega)}{\max\left(|\hat{x}_1(\omega)|\right)}$

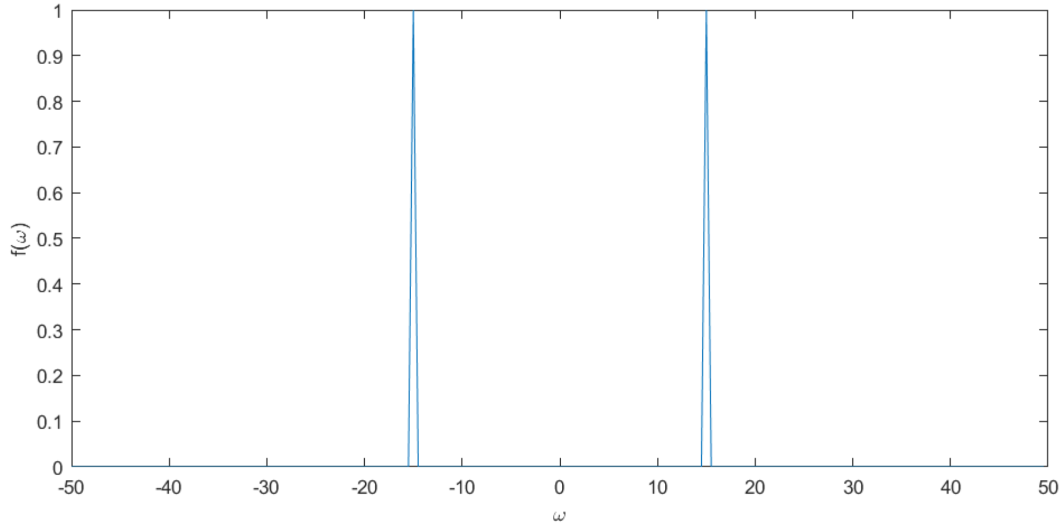
then we consider the $f = \frac{\omega}{2\pi}$ so:

- $\hat{x}_2(f) = e^{\frac{j\pi}{4}}\delta(f - 15) + e^{-\frac{j\pi}{4}}\delta(f + 15)$

We expect two picks on ± 15, as we plot $|\hat{x}_2(f)|$

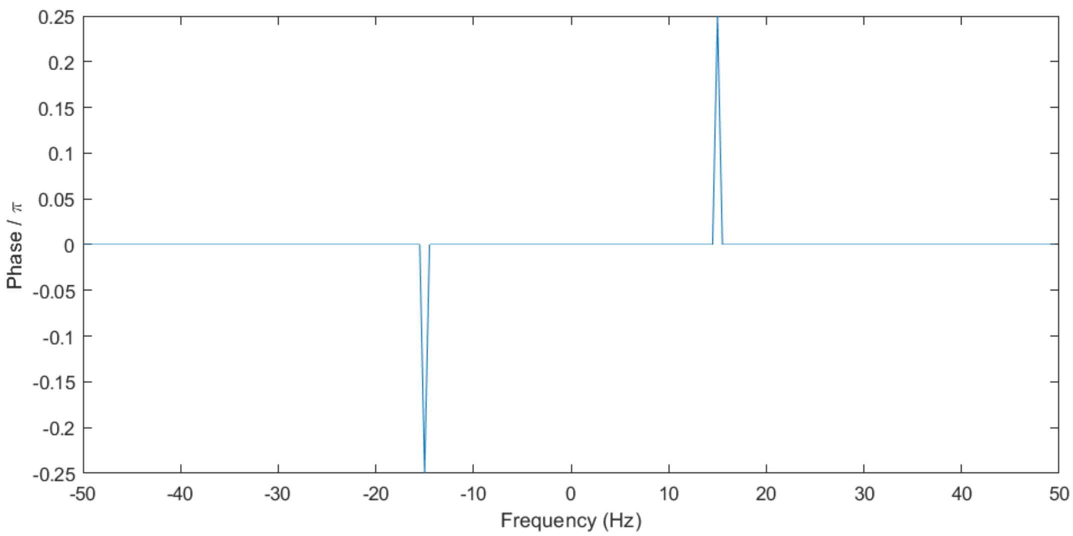
```
y2 = fftshift(fft(x2));
y2 = y2 / max(abs(y2));
```

```
figure('Position', [0 0 900 400]);
plot(f,abs(y2));
xlabel('\omega');
ylabel('f(\omega)')
```



And in plotting phase of $\hat{x}_2(f) = e^{\frac{j\pi}{4}}\delta(f-15) + e^{-\frac{j\pi}{4}}\delta(f+15)$, we consider y-axis ticks is π, so there should be a $-\frac{1}{4}$ on -15 and a $\frac{1}{4}$ on 15.

```
threshold = 1e-6;
y2(abs(y2) < threshold) = 0;
theta = angle(y2);
plot(f,theta/pi);
xlabel('Frequency (Hz)')
ylabel('Phase / \pi')
```



Part 2

2.1 Creating Mapset

```
clearvars;
mapset = create_mapset()
```

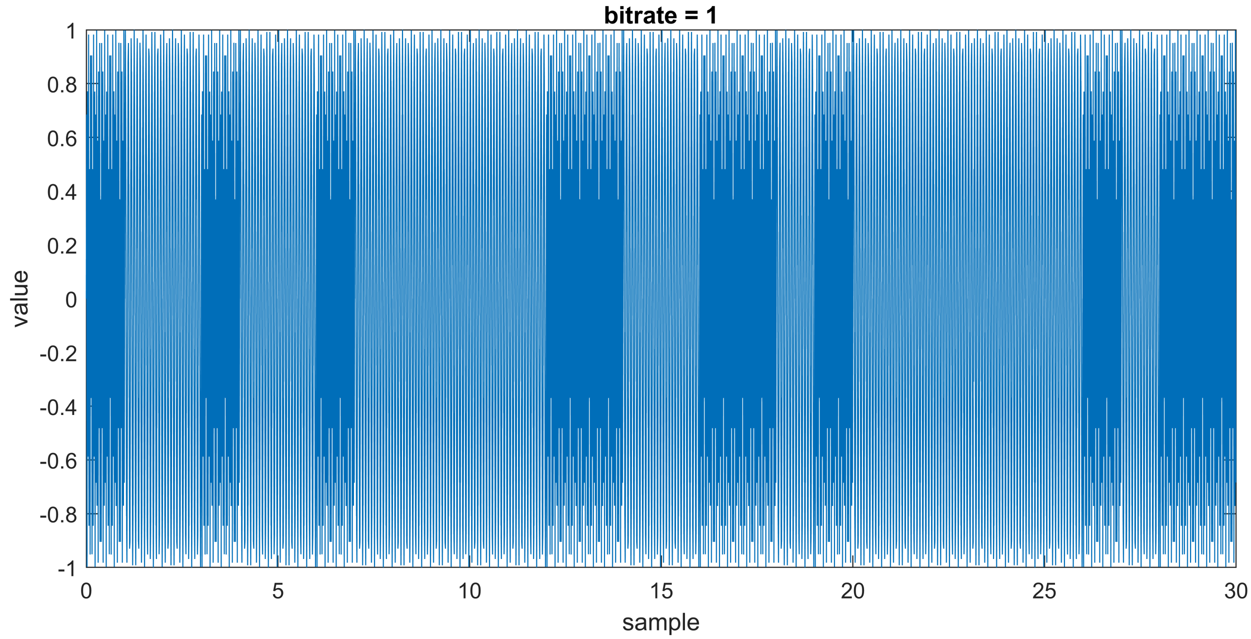
```
mapset = 2×32 cell
'a'        'b'        'c'        'd'        'e'        'f'        'g'···
'00000'    '00001'    '00010'    '00011'    '00100'    '00101'    '0
```

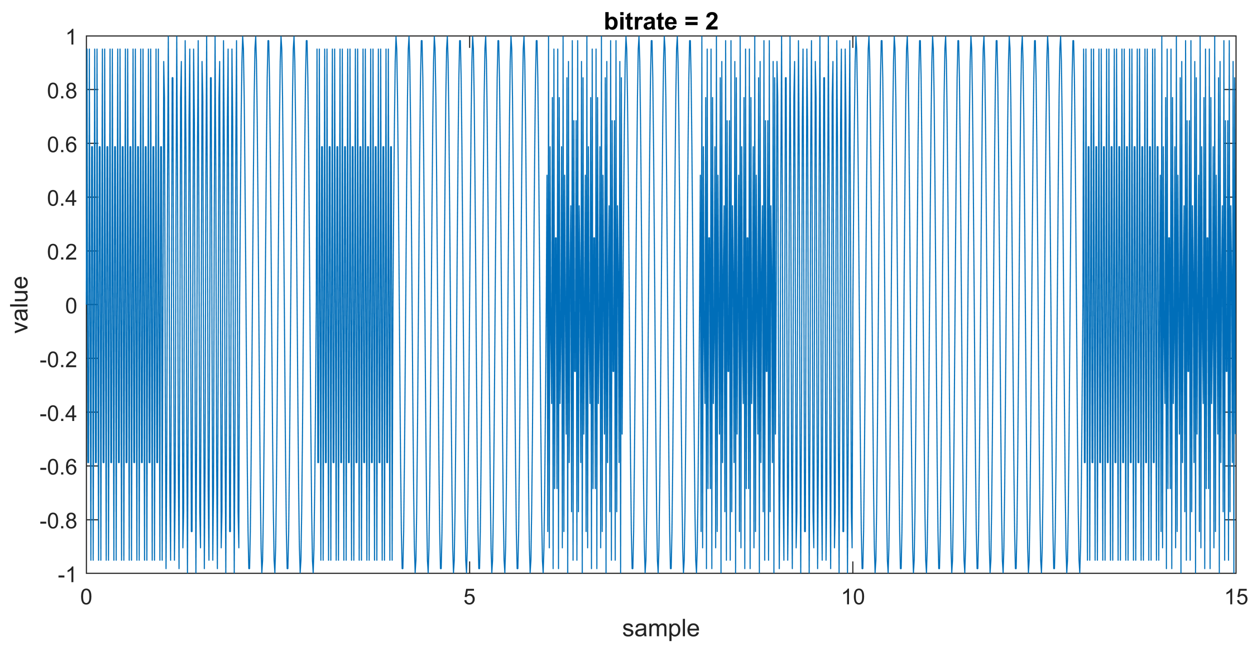
2.2 - 2.3 Encoding message

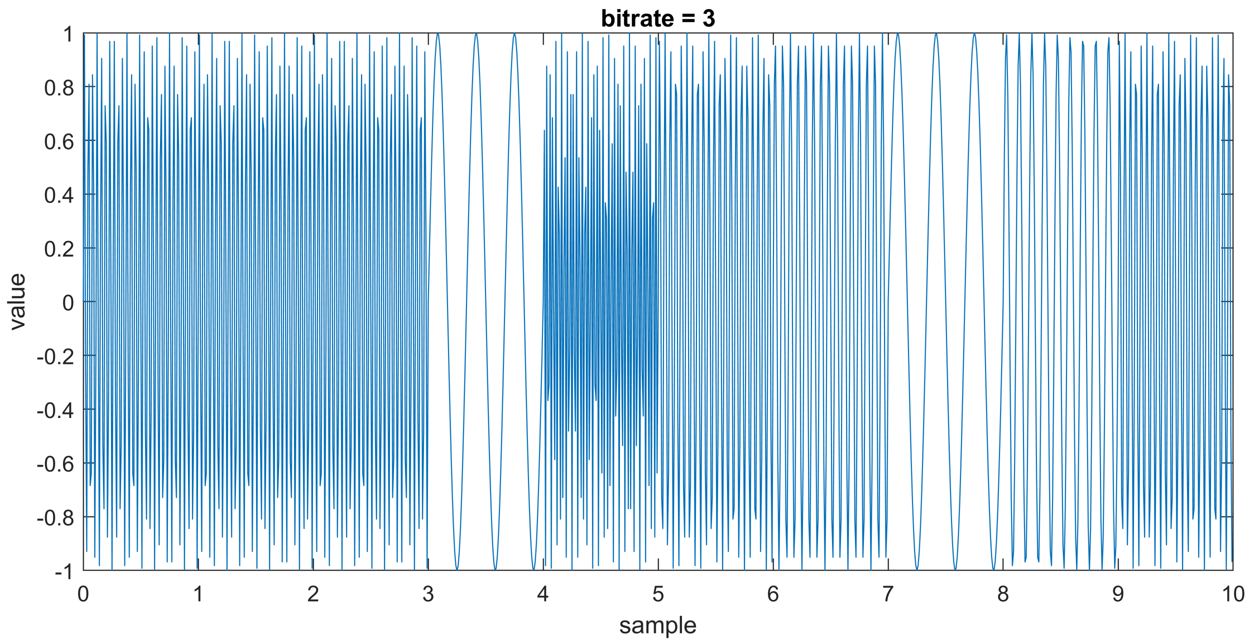
To encode message we need to find a proper f to encode bitstrings. Consider $\text{bitrate} = n$ so there is 2^n possibilites for our bitstrings. So we divide our valid frequency range by 2^n. Valide range for frequency is $(0, \frac{f_s}{2}]$, because sine function has two symmetric pulses, there is no need to use negative frequencies. so each distance(we call them step) between two used frequencies is $f_{\text{step}} = \frac{\frac{f_s}{2}}{2^n}$, and initial frequncy is $\frac{f_{\text{step}}}{2}$. so $f = f_{\text{init}} + i \times f_{\text{step}}$ where $i \in [0, 2^n - 1]$.

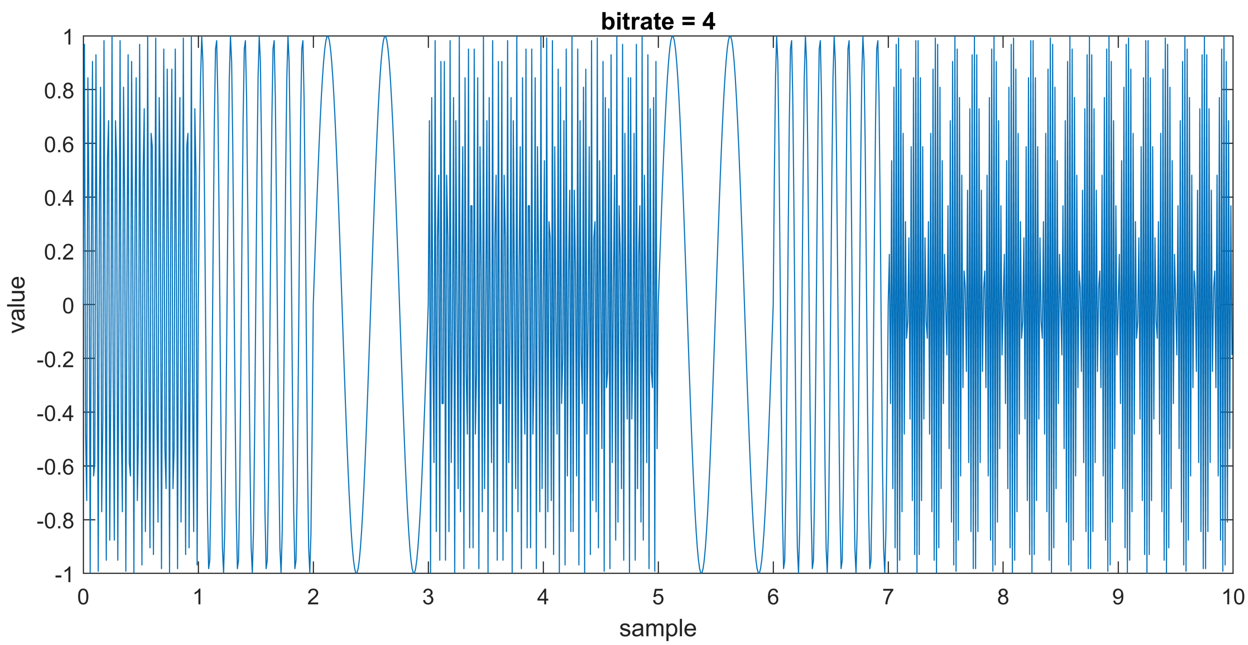
Another thing I consider is that if length of a message is not dividable by bitrate, I add as many as needed semicolons to fix that issue,so if you see semicolons at the end of decoded messages, it is not noise or miscoding.

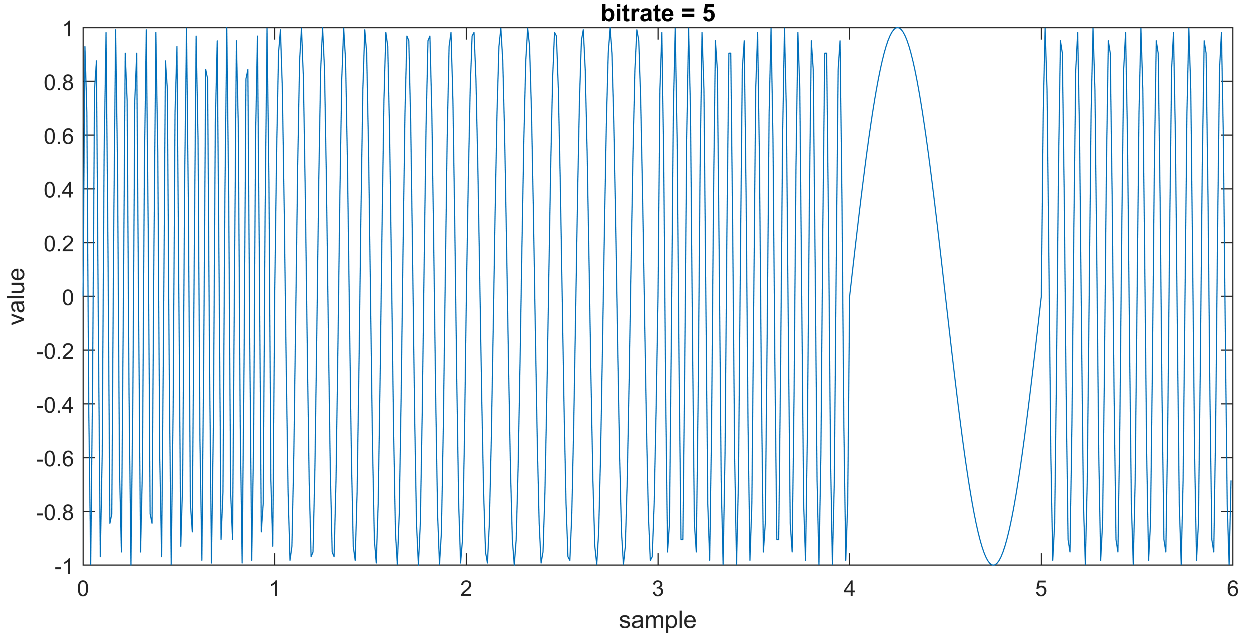
```
fs = 100;
ts = 1 / fs;

message = 'signal';

bit_rates = [1,2,3,4,5];
for i = 1:length(bit_rates)
    y = coding_freq(message, bit_rates(i), mapset, fs);
    t = 0:ts:length(y)/fs - ts;
    figure('Position', [0 0 900 400]);
    plot(t, y);
    xlabel('sample');
    ylabel('value');
    title("bitrate = " + num2str(bit_rates(i)));
end
```


bitrate = 1
value
sample
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
5
10
15
20
25
30


bitrate = 2
value
sample
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
5
10
15


bitrate = 3
value
sample


bitrate = 4
value
sample



2.4 Decoding message

To decode message, first we slice our bitstrings to strings with length of bitrate, and then we do `fft` and `fftshift` respectively, and then find max index, but the fftshift default is $\left[-\frac{f_s}{2}, \frac{f_s}{2} - 1\right]$ the expected frequency is $|\text{maxarg} - \frac{f_s}{2} - 1|$.

Another thing I consider is that if length of a message is not dividable by bitrate, I add as many as needed semicolons to fix that issue,so if you see semicolons at the end of decoded messages, it is not noise or miscoding.

```
for i = 1:length(bit_rates)
    y = coding_freq(message, bit_rates(i), mapset, fs);
    decoded = decoding_freq(y, bit_rates(i), mapset, fs);
    fprintf("bitrate: %d - decoded message: %s\n", bit_rates(i), decoded);
end
```

```
bitrate: 1 - decoded message: signal
bitrate: 2 - decoded message: signal
bitrate: 3 - decoded message: signal
bitrate: 4 - decoded message: signal;;
bitrate: 5 - decoded message: signal
```

2.5 Adding noise with variance 1e-4

The variance is 10^{-4} so the standard deriviation is 10^{-2}, we need to multiply the `std` to `randn.` and then add it to the main signal to simulate noise.

```
std = 0.01;
bit_rates = [1,2,3,4,5];
for i = 1:length(bit_rates)
    y = coding_freq(message, bit_rates(i), mapset, fs);
```

```
    y = y + std * randn(size(y));
    decoded = decoding_freq(y, bit_rates(i), mapset, fs);
    fprintf("bitrate: %d - decoded message: %s\n", bit_rates(i), decoded);
end
```

```
bitrate: 1 - decoded message: signal
bitrate: 2 - decoded message: signal
bitrate: 3 - decoded message: signal
bitrate: 4 - decoded message: signal;;
bitrate: 5 - decoded message: signal
```

2.6 Increasing noise and comparing bitrates

we do increase the noise in order to see the maximum amount of standard deriviation of noise added to signal where signal still is decode correctly.

Note: I tested multiple times with many different settings and different hyperparameters, but all resulted that bitrate 5 works better than bitrate 1 in $f = 100\,\mathrm{Hz}$ but we get expected results in $f = 150\,\mathrm{Hz}$. Also I repeat every test for every bitrate 100 times(I also tested up to 500 times but also got the same result).

fs = 100 Hz

```
bit_rates = [1,2,3,4,5];
maxstd = zeros(size(bit_rates));
fs = 100;
times = 100;
for i = 1:length(bit_rates)
    for t = 1:times
        for variance = 0.1:0.1:2
            y = coding_freq(message, bit_rates(i), mapset, fs);
            y = y + variance * randn(size(y));
            decoded = decoding_freq(y, bit_rates(i), mapset, fs);
            if(decoded ~= "signal" && decoded ~= "signal;;")
                maxstd(i) = maxstd(i) + variance - 0.1;
                break;
            end
        end
    end
    maxstd(i) = maxstd(i) / times;
end
fprintf("");
fprintf("maximum std where signal is decoded correctly: \n\n\t bitrate 1: %.3f \t bitrate 2: %.
    " %.3f \t bitrate 4: %.3f \t bitrate 5: %.3f", ...
        maxstd(1), maxstd(2), maxstd(3), maxstd(4), maxstd(5));
```

```
maximum std where signal is decoded correctly:

  bitrate 1: 1.361    bitrate 2: 1.395    bitrate 3: 1.436    bitrate 4: 1.421    bitrate 5: 1.471
```

fs = 150 Hz

```
fs = 150;
```

```
for i = 1:length(bit_rates)
    for t = 1:times
        for variance = 0.1:0.1:2
            y = coding_freq(message, bit_rates(i), mapset, fs);
            y = y + variance * randn(size(y));
            decoded = decoding_freq(y, bit_rates(i), mapset, fs);
            if(decoded ~= "signal" && decoded ~= "signal;;")
                maxstd(i) = maxstd(i) + variance - 0.1;
                break;
            end
        end
    end
    maxstd(i) = maxstd(i) / times;
end
fprintf("");
fprintf("maximum std where signal is decoded correctly: \n\n\t bitrate 1: %.3f \t bitrate 2: %.
    " %.3f \t bitrate 4: %.3f \t bitrate 5: %.3f", ...
        maxstd(1), maxstd(2), maxstd(3), maxstd(4), maxstd(5));
```

```
maximum std where signal is decoded correctly:

  bitrate 1: 1.651    bitrate 2: 1.573    bitrate 3: 1.557    bitrate 4: 1.565    bitrate 5: 1.463
```

We can see the output of signals with $f_s = 150\,\mathrm{Hz}$ shows what we expected, the more noise the get, the less signal is proofed to noise.

2.8 - 2.9 Increasing frequency(fs) and make signal more persistant

Increasing bandwidth can help the signals to be more persistant against noise, also increasing f_s can help us doing that. because when f_s is increased, we have a larger rnge of frequency to choose for encoding messages, and that will result in increasing distance between two picks after doing `fft` and `fftshif` and that makes the decoding easier and less sensetive to noises.

Note that increasing f_s without increasing bandwidth does not improve the noise resistancy of signal.

```
bit_rates = [1,2,3,4,5];
maxstd = zeros(size(bit_rates));
frequencies = [80,90,100,110,120,130,140,150];
times = 200;
for freq = (1:length(frequencies))
    for i = 1:length(bit_rates)
        for t = 1:times
            for variance = 0.1:0.1:2
                y = coding_freq(message, bit_rates(i), mapset, frequencies(freq));
                y = y + variance * randn(size(y));
                decoded = decoding_freq(y, bit_rates(i), mapset, frequencies(freq));
                % explained why "signal;;" before
                if(decoded ~= "signal" && decoded ~= "signal;;")
                    maxstd(i) = maxstd(i) + variance - 0.1;
```

```
                    break;
                end
            end
        end
        maxstd(i) = maxstd(i) / times;
    end
    fprintf("frequency(fs): %d", frequencies(freq));
    fprintf("maximum std: \n");
    fprintf("\t bitrate 1: %.3f \t bitrate 2: %.3f \t bitrate 3: %.3f \t bitrate 4: %.3f \t bit
        maxstd(1), maxstd(2), maxstd(3), maxstd(4), maxstd(5));
    fprintf("-----------------------------------------------------------------------------------

end
```

```
frequency(fs): 80
maximum std:
  bitrate 1: 1.236   bitrate 2: 1.267   bitrate 3: 1.274   bitrate 4: 1.291   bitrate 5: 1.363
---------------------------------------------------------------------------------------------------------------------
frequency(fs): 90
maximum std:
  bitrate 1: 1.293   bitrate 2: 1.343   bitrate 3: 1.382   bitrate 4: 1.367   bitrate 5: 1.421
---------------------------------------------------------------------------------------------------------------------
frequency(fs): 100
maximum std:
  bitrate 1: 1.366   bitrate 2: 1.401   bitrate 3: 1.447   bitrate 4: 1.440   bitrate 5: 1.494
---------------------------------------------------------------------------------------------------------------------
frequency(fs): 110
maximum std:
  bitrate 1: 1.431   bitrate 2: 1.459   bitrate 3: 1.504   bitrate 4: 1.504   bitrate 5: 1.543
---------------------------------------------------------------------------------------------------------------------
frequency(fs): 120
maximum std:
  bitrate 1: 1.482   bitrate 2: 1.522   bitrate 3: 1.551   bitrate 4: 1.527   bitrate 5: 1.541
---------------------------------------------------------------------------------------------------------------------
frequency(fs): 130
maximum std:
  bitrate 1: 1.528   bitrate 2: 1.534   bitrate 3: 1.589   bitrate 4: 1.590   bitrate 5: 1.586
---------------------------------------------------------------------------------------------------------------------
frequency(fs): 140
maximum std:
  bitrate 1: 1.578   bitrate 2: 1.606   bitrate 3: 1.587   bitrate 4: 1.563   bitrate 5: 1.481
---------------------------------------------------------------------------------------------------------------------
frequency(fs): 150
maximum std:
  bitrate 1: 1.625   bitrate 2: 1.578   bitrate 3: 1.606   bitrate 4: 1.578   bitrate 5: 1.395
---------------------------------------------------------------------------------------------------------------------
```

Functions

implemented functions:

```
function [mapset] = create_mapset()
    mapset = cell(2, 32);
    list = char('a':'z');
    list = [list ,' ', '.', ',','!', '"', ';'];

    for i = 1:32
        character = list(i);
        bit_string =  dec2bin(i - 1, 5);
```

```
        mapset{1, i} = character;
        mapset{2, i} = bit_string;
    end
end

function [coded_string] = encode_string(message, mapset)
    coded_string = '';
    for i = 1:length(message)
        bit_string = mapset{2, strcmp(mapset(1,:), message(i))};
        coded_string = [coded_string, bit_string];
    end
end

function [coded_freq] = coding_freq(message, bit_rate, mapset, fs)
    while(mod(length(message) * 5, bit_rate) ~= 0)
        message = [message, ';'];
    end


    ts = 1 / fs;
    tstart = 0;
    tend = 1;
    t = tstart : ts : tend - ts;

    fstep = floor((fs / 2) / (2 ^ bit_rate));
    fstart = ceil(fstep / 2);
    bit_string = encode_string(message, mapset);
    coded_freq = [];
    for i = (1:bit_rate:length(bit_string))
        f = fstart + fstep * bin2dec(bit_string(i:i+bit_rate-1));
        new_signal = sin(2 * pi * f * t);
        coded_freq = [coded_freq, new_signal];
    end
end

function [message] = decoding_freq(coded_freq, bit_rate, mapset, fs)
    code_string = '';
    fstep = floor((fs / 2) / (2 ^ bit_rate));
    fstart = ceil(fstep / 2);
    for i = (1:fs:size(coded_freq,2))
        sliced_freq = coded_freq(i:i+fs-1);
        [~, fmax] = max(abs(fftshift(fft(sliced_freq))));
        fmax = abs(fmax - fs / 2 - 1);
        for j = 0:(2 ^ bit_rate - 1)
            th_left =  j * fstep + fstart - fstep / 2;
            th_right = j * fstep + fstart + fstep / 2;
            if(fmax >= th_left && fmax <= th_right)
                code_string = [code_string, dec2bin(j, bit_rate)];
                break;
            elseif(j == 2 ^ bit_rate - 1)
                code_string = [code_string, dec2bin(j, bit_rate)];
            end
        end
    end
```

```
    message = '';
    for i = 1:5:length(code_string)
        char = mapset{1, strcmp(mapset(2,:), code_string(i:i+4))};
        message = [message, char];
    end
end
```