Workshop Report 2

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1. The plain vanilla schedule

Define Sets:

T= set of all teams W = set of all weeks

Constants:

E_{ij} = distance between the locations of team i and team j stadiums.

Decision variables:

x_{ijk} = 1 if a team i is playing a game at home against team j during week k, otherwise x_{ijk} = 0

Objective is to "minimize the total travel distance of all teams":

$$min \sum_{i\in T} \sum_{j\in T\setminus i} \sum_{k\in W} 2x_{ijk}E_{ij}$$

The constraints are:

1. Each team i plays 12 games (home or away) during the entire season:
$$\sum_{j\in T\setminus i} \sum_{k\in W} (x_{ijk} + x_{jik}) = 12 \ \forall i \in T$$
2. Each team i plays during each week k one game (home or away):
$$\sum_{j\in T\setminus i} (x_{ijk} + x_{jik}) = 1 \ \forall i \in T, k \in W$$
3. Each team i plays with another team j at most once during the entire season:
$$\sum_{k\in W} (x_{ijk} + x_{jik}) \leq 1 \ \forall i \in T, j \in T\setminus i$$
4. Each team i plays at most six home games during the entire season:
$$\sum_{j\in T\setminus i} \sum_{k\in W} x_{ijk} \leq 6 \ \forall i \in T$$

The minimum total distance travelled by all teams should be 165908 miles.

The optimal schedule for Cleveland Browns should be:

```
Week 1: Home vs. Cincinnati Bengals

Week 2: Away vs. Green Bay Packers

Week 3: Away vs. Detroit Lions

Week 4: Home vs. Baltimore Ravens

Week 5: Home vs. Buffalo Bills

Week 6: Home vs. Indianapolis Colts

Week 7: Away vs. New England Patroits

Week 8: Away vs. Pittsburgh Steelers

Week 9: Away vs. Chicago Bears

Week 10: Away vs. Minnesota Vikings

Week 11: Home vs. New York Jets

Week 12: Home vs. New York Giants
```

The home and away game pattern seem to contain many consecutive “home” or “away”.

2. Game pattern and bye week constraints

New Constraint:
No team would play more than (i) two consecutive games at home and (ii) two consecutive games away, the mathematical formulation will be:

$$\sum_{j \in T \setminus i} \left(x_{ijk} + x_{ij(k+1)} + x_{ij(k+2)}\right) \leq 2 \; \forall i \in T, k \in (W-2)$$

$$\sum_{j \in T \setminus i} \left(x_{ijk} + x_{ij(k+1)} + x_{ij(k+2)}\right) \geq 1 \; \forall i \in T, k \in (W-2)$$

The game pattern constraint implicate that for every three consecutive matches for any team, there will be no more than two matches at home or away.
The minimum total distance travelled by all teams remains unchanged.
The optimal schedule for Cleveland Browns changes drastically:

```
Week 1: Away vs. Green Bay Packers

Week 2: Away vs. Chicago Bears

Week 3: Home vs. Cincinnati Bengals

Week 4: Home vs. Indianapolis Colts

Week 5: Away vs. Detroit Lions

Week 6: Home vs. Pittsburgh Steelers

Week 7: Home vs. New York Jets

Week 8: Away vs. Buffalo Bills

Week 9: Home vs. New York Giants

Week 10: Away vs. Minnesota Vikings

Week 11: Home vs. New England Patroits

Week 12: Away vs. Baltimore Ravens
```

New Constraint:
No games at week 9 (and that the whole season is extended by a week up to 13 weeks)
For all mathematical formulation, the range of k should increase by 1 and all the previous constraints should be skipped for k = 8 (which represents week 9).

$$\sum_{i \in T} \sum_{j \in T \setminus i} x_{ijk} = 0 \;\; k = 8$$

The minimum total distance travelled by all teams remains unchanged.
The optimal schedule for Cleveland Browns changes drastically.

```
Week 1: Home vs. New York Jets

Week 2: Away vs. Detroit Lions

Week 3: Home vs. Chicago Bears

Week 4: Home vs. Buffalo Bills

Week 5: Away vs. Cincinnati Bengals

Week 6: Home vs. Pittsburgh Steelers

Week 7: Away vs. Baltimore Ravens

Week 8: Away vs. Green Bay Packers

Week 10: Home vs. Minnesota Vikings

Week 11: Away vs. Indianapolis Colts

Week 12: Home vs. New England Patroits

Week 13: Away vs. New York Giants
```

I would recommend adding the game pattern and bye week constraints since they will not increase the total distance travelled by all teams while having the benefits of reducing the fatigue and increasing fairness.

3. Fairness constraints

The team travels the most is Miami Dolphins with 11676 miles, the team travels the least is New York Giants with 3210 miles. The distance difference between them is 8466 miles.

Let the total distance constraint be S, the mathematical formulation will be:

$$\sum_{j \in T \setminus i} \sum_{k \in W} 2x_{jik}E_{ji} \leq S \; \forall i \in T$$

If the maximum allowed traveling distance is 11,000 miles:

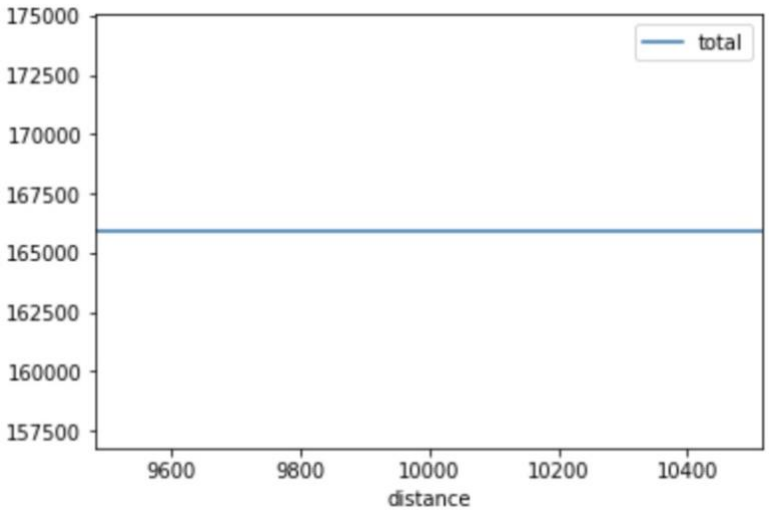
The team travels the most is New Orleans Saints with 10516 miles, the team travels the least is Pittsburgh Steelers with 4242 miles. The distance difference between them is 6274 miles.

If the maximum allowed traveling distance is 10,990 miles:

The team travels the most is Minnesota Vikings with 10892 miles, the team travels the least is Chicago Bears with 3912 miles. The distance difference between them is 6980 miles.

The distance difference increases.

The minimum of the maximum allowed travelling distance should be 9500 miles so that a feasible solution could be obtained.



The graph shows that the total amount travelled by all teams remains unchanged despite the change in the distance travelled by the team which travels the most. The scheduling logic behind the reduction is to relocate the distance travelled by each team so that they are more even so that the maximum distance is lower.

4. Scheduling to please broadcasting companies

The mathematical formulation will be:

$$\sum_{j \in D1 \setminus i} \sum_{k \in W} x_{ijk} + x_{jik} = 6 \ \forall i \in D1$$

$$\sum_{j \in D2 \setminus i} \sum_{k \in W} x_{ijk} + x_{jik} = 6 \ \forall i \in D2$$

The optimal total travelling distance for all teams will be 167776 miles.

```
Distance and schedule for Chicago Bears:
Week 1: Away vs. Indianapolis Colts, Distance 368.0 miles
Week 2: Home vs. Atlanta Falcons, Distance 0 mile
Week 3: Away vs. Green Bay Packers, Distance 414.0 miles
Week 4: Home vs. Cincinnati Bengals, Distance 0 mile
Week 5: Home vs. Tennessee Titans, Distance 0 mile
Week 6: Away vs. Pittsburgh Steelers, Distance 920.0 miles
Week 7: Home vs. New Orleans Saints, Distance 0 mile
Week 8: Away vs. Dalls Cowboys, Distance 1934.0 miles
Week 10: Away vs. Minnesota Vikings, Distance 818.0 miles
Week 11: Away vs. New York Giants, Distance 1574.0 miles
Week 12: Home vs. Cleveland Browns, Distance 0 mile
Week 13: Home vs. Detroit Lions, Distance 0 mile
```

Total travelling distance for Chicago Bears is 6028 miles.

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Distance and schedule for Jacksonville Jaguars:
Week 1: Home vs. Carolina Panthers, Distance 0 mile
Week 2: Away vs. Baltimore Ravens, Distance 1502.0 miles
Week 3: Home vs. Miami Dolphins, Distance 0 mile
Week 4: Away vs. Dalls Cowboys, Distance 1984.0 miles
Week 5: Home vs. New York Jets, Distance 0 mile
Week 6: Away vs. New England Patroits, Distance 2262.0 miles
Week 7: Away vs. Atlanta Falcons, Distance 692.0 miles
Week 8: Home vs. Washington Football Team, Distance 0 mile
Week 10: Away vs. Tennessee Titans, Distance 1190.0 miles
Week 11: Home vs. Tampa Bay Buccaneers, Distance 0 mile
Week 12: Away vs. Houston Texans, Distance 1740.0 miles
Week 13: Home vs. New Orleans Saints, Distance 0 mile
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Total travelling distance for Jacksonville Jaguars is 9370 miles.

The results are quite good as matches are divided between their own conference and the other conference, and there is only a slight increase in total distance travelled by all teams. However, compared with the result in part3, the distance difference between team travels the most and the least increases a lot, which means the fairness of this scheduling decreases.