Portfolio Problem at Zinca Inc.

Matthew Gurnani had just been hired as a business development analyst for a large pharmaceutical company, Zinca Inc. The first task that was entrusted on him was to prepare an analysis and a recommendation regarding what drug development projects the company should fund and carry on during the coming year. This was not a task to be taken lightly since most of the company's revenues came from the drugs that were successfully developed and launched on market. Also, the funding used this year for drug development was significant adding up to $1 billion.

After discussing with Zinca's chief financial officer (CFO) George Pradalos, Matthew learned that whereas the primarily goal was to maximize the overall ENPV of the company, risk management was also getting increasingly more important. In particular, George felt that the company should forgo some expected net present value (ENPV) if that reduced risks significantly. However, Matthew was not sure whether that was possible and felt that a thorough analysis was needed. With this thought in mind, Matthew started his first assignment at Zinca.

To assist with the task, Matthew is given access to the company's drug development data. This data is summarized in two files. The first file is labelled as "drugs.csv" and it shows for each drug (i) the therapeutic area it belongs to, (ii) the time its completion takes, (iii) ENPV in $M to be obtained if the drug's development is completed, and (iv) the development cost in $M for the coming year. The second file is labelled as "drugs_cov.csv" and it contains the covariance matrix for the drugs' ENPVs.

Matthew was aware of the optimal portfolio selection and allocation of funds in financial sector and he realized that the process was rather similar. However, the main difference was that it was not possible to fund a partial drug development project. In other words, a drug development project was either fully funded and selected into the portfolio or it was not funded at all and it's development was postponed. An additional piece of information that George Pradalos was able to offer for Matthew, was that any unused funds (i.e., those not spent on drug development) could be invested risk free with a return of 3%.

Assignment

Make a report of about 3 pages (excluding figures, graphs and appendices) summarizing your results, your interpretation of the results and your conclusions and recommendations. You can include graphs and outputs in an appendix, but make sure to summarize and interpret the results in your report as well.

1. Framing the portfolio selection problem

Ignore risk management requirements for the time being. Suppose that the objective is to maximize the ENPV of the drug portfolio. The constraints that the selected portfolio has to satisfy relate to *therapeutic area budget constraints* and *drug development pipeline constraints*. The therapeutic area budget constraints limit the maximum total drug development cost in the concerning therapeutic area. The budgets are as follows:

- $100 million for oncology
- $200 million for cardiovascular
- $150 million for respiratory and dermatology
- $100 million for transplantation
- $300 million for rheumatology and hormone therapy
- $100 million for central nervous system
- $50 million for opthalmics

The pipeline constraints are required to balance the drug development so that there is a steady state of drugs completed each coming future year. In particular, Zinca wants to enforce that at least (i) 15% of the funded drugs have 1 year time-to-market, (ii) 20% of the funded drugs have 2 or 3 years time-to-market, and (iii) 25% of the funded drugs have 4 or 5 years time-to-market.

- What is the mathematical formulation of the drug development portfolio selection problem?
- Copy the file “drug.py” that can be found on blackboard. The python file contains a template for the model. Build the developed mathematical formulation using Python and Gurobi. **Save this model as “drug1.py”.** Make a recommendation on which drugs to continue developing and which drugs to put on hold.
- What percent of the overall budget is used for drug development?

2. Incorporating risk management in the analysis

- How does the mathematical formulation for the drug development need to be appended to incorporate a constraint on the portfolio's variance? Update the model "**drug1.py**" to include a constraint on the portfolio's variance. **Save this model as "drug2.py".**
- Illustrate an efficient frontier for the drug development portfolio problem. In the figure, show the entire efficient frontier with ENPV on y-axis and standard deviation of the ENPV on the x-axis.
- Use the efficient frontier to make a recommendation on how much risk to take and what drugs will be developed under this scenario.
- If the portfolio risk were to be minimized, what drugs would be chosen for development?

Suppose that Zinca receives a donation of $50 million with the condition that this money has to be allocated to advance drug development in one of the therapeutic areas.

- Which therapeutic area you would recommend to allocate the additional $50 million if the goal is to maximize profit? Hint: Update the model to have a binary variable associated with each therapeutic area, which is 1 if the additional funds are allocated on that therapeutic area.
- How does the allocation of the additional $50 million depend on the willingness to take risk at Zinca? Interpret the results.

3. Company-wide budgeting

Once Matthew had completed his preliminary analysis, George Pradalos came to see him. He was wondering whether Zinca should change the way they budgeted the drug development. In particular, George asked if Matthew could look into the implications of having a company-wide budgeting rather than the current department-wide budgeting. In other words, the company-wide budget meant that the entire $1000 million could be allocated across all drug development projects without specific constraints on different therapeutic areas. Ignore the availability of the additional $50 million.

- How does the mathematical formulation change when the therapeutic area budget constraints are replace by the company-wide budget?
- Develop the company-wide budget model. **Save this model as "drug3.py".** Consider first the risk neutral case (i.e., profit maximization). What changes company-wide

budget implies (compared to the therapeutic area budgets) in terms of (i) selected projects, (ii) portfolio's ENPV, (iii) portfolio's standard deviation, and (iv) percent of funds allocated on projects?

- How does the efficient frontier change if the company-wide budget is used?
- Would you recommend Zinca to use therapeutic area budgets or the company-wide budget? Explain your reasoning.

4. Managing extreme risks

Just when Matthew thought that he had completed the analysis, George dropped by. George told that he had heard from the grapewine that some of the competitors were selecting their drug development projects whilst accounting for the "extreme risk". George clarified that by extreme risk he meant the portfolio's ENPV at the 5th percentile of its probability distribution. It is commonly referred as the portfolio's Value at Risk (VaR) at the 5th percentile and can be interpreted as the portfolio's ENPV that occurs in the worst 5th percentile of the outcomes (or once in every 20th outcome). Thereby, George added, VaR at the 5th percentile is considered to measure the extreme risk. With this information, Matthew figured out that the portfolio's value at the 5th percentile can be constrained with the following constraint

$$\sum_{i=1}^{n} z_i \mu_i - 1.645\sigma_P \geq r,$$

where z_i are binary decision variables (z_i=1 if and only if a project i is selected), μ_i is the ENPV of a drug i, σ_P is the standard deviation of the portfolio's ENPV, and r is the minimum accepted VaR for the portfolio's ENPV at the 5th percentile. However, the problem was not computationally tractable in this format. Therefore, Matthew spent further time to study how to reformulate this VaR constraint and found out the following second-order cone programming formulation that made the optimization problem computationally amenable

$$\sum_{i=1}^{n} z_i \mu_i - 1.645h \geq r, \quad (1)$$

$$h^2 \geq \sum_{i=1}^{n} \tau_i^2, \quad (2)$$

$$\boldsymbol{\tau} = \boldsymbol{L}^T \boldsymbol{z}, \quad (3)$$

$$h \geq 0, \quad (4)$$

where h is an auxiliary variable, $\boldsymbol{\tau}$ is a $n \times 1$ vector of auxiliary variables, and matrix $\boldsymbol{L}$ is the lower triangular matrix obtained from the Cholesky factorization $\boldsymbol{L}\boldsymbol{L}^T = \boldsymbol{\Lambda}$ when $\boldsymbol{\Lambda}$ is the covariance matrix among the drugs.

- Replace the constraint on variance included in model **"drug2.py"** by the VaR constraints (1) – (4). Ignore the availability of the additional $50 million. **Save this model as "drug4.py".** What is the minimum risk measured in the highest attainable VaR of the portfolio's ENPV at the 5th percentile? What is the expected return of that portfolio?
- Explain whether Zinca should be concerned about the extreme risk of its current drug development portfolio?