Assignment3

- Zitong Huang, 12432670, Computer Science and Engineering
- 3d Gaussian Splatting for Scene Reconstruction
- Prof. Feng Zheng

Task 3-1

Example: 5 Machine and 3 Jobs

- $N(Machinese) = 5$
- Jobs: t_i = {3, 1, 2}

Example: 6 Machine and 4 Jobs

- $N(Machine) = 3$
- Jobs: t_i = {1, 6, 2, 5}

Task 3-2: Example 1

- $N(Machine) = 3$
- Jobs: t_i = {1, 2, 5, 6}

Example:

- allocate order t_1, t_2, t_3, t_4:

1. allocate $t_1 = 1$ to machine 1. $Load$: {1, 0, 0}
2. allocate $t_2 = 2$ to machine 2. $Load$: {1, 2, 0}
3. allocate $t_3 = 5$ to machine 3. $Load$: {1, 2, 5}
4. allocate $t_4 = 6$ to machine 1. $Load$: {7, 2, 5}

Maximum load: 7

Order of allocation affects the final load of each machine.

- allocate order t_4, t_3, t_2, t_1:

1. allocate t_4 to machine 1. $Load$: {6, 0, 0}
2. allocate t_3 to machine 2. $Load$: {6, 5, 0}
3. allocate t_2 to machine 3. $Load$: {6, 5, 2}
4. allocate t_1 to machine 3. $Load$: {6, 5, 3}

Maximum load: 6

Task 3-2: Example 2

- $N(Machine) = 3$
- Jobs: t_i = {5, 5, 5, 5, 10}

Example:

- allocate order t_1, t_2, t_3, t_4, t_5:

1. allocate $t_1 = 5$ to machine 1. $Load$: {5, 0, 0}
2. allocate $t_2 = 5$ to machine 2. $Load$: {5, 5, 0}
3. allocate $t_3 = 5$ to machine 3. $Load$: {5, 5, 5}
4. allocate $t_4 = 5$ to machine 1. $Load$: {10, 5, 5}
5. allocate $t_5 = 10$ to machine 2. $Load$: {10, 15, 5}

Maximum load: 15

- allocate order t_5, t_4, t_3, t_2, t_1:

1. allocate $t_5 = 10$ to machine 1. $Load$: {10, 0, 0}
2. allocate $t_4 = 5$ to machine 2. $Load$: {10, 5, 0}
3. allocate $t_3 = 5$ to machine 3. $Load$: {10, 5, 5}
4. allocate $t_2 = 5$ to machine 2. $Load$: {10, 10, 5}
5. allocate $t_1 = 5$ to machine 3. $Load$: {10, 10, 10}

Maximum load: 10

Task 3-3: 2 Machine and 3 Jobs

- Proof:
  - Assume t_i = {a, b, c}, where $a > b > c$
  - Worst case of greedy algorithm: $T_{worst} = max(a, b, c) + min(b, c) = a + c$
  - Optimal solution: $T = max(a, b + c)$
  - We have

$$\frac{T}{T*} = \begin{cases} a/(a+c) & \text{if}(a \geq b + c) \\ (b+c)/(a+c) & \text{if } (a < b + c) \end{cases} \quad (1)$$

  Simplify, we have $\frac{T}{T*} >= \frac{2}{3}$

- Example:
  - t_i = {10, 5, 5}
  - **Greedy algorithm:** $T_{worst} = 15$, **Optimal solution:** $T* = 10$

Task 3-3: 4 Machine and 7 Jobs

- Example:
  - t_i = {10, 10, 10, 10, 10, 10, 30}
  - $T_{worst}: 20, 20, 50 = 50$
  - $T*: 30, 30, 30 = 30$

Task 3-3: m Machine and n Jobs

- Assume t_i = {t_1, t_2, ..., t_n}, where $t_1 \geq t_2 \geq \ldots \geq t_n$, and $n \gg m$
- Worst case of greedy algorithm: $T_{worst} = t_1 + min_{S_{n-1}}$, since minimum value in solution of $T_{n-1} = (t_2, t_3, \ldots, t_n)$ on m machine **satisfied** $min_{S_{n-1}} \leq \frac{sum(t_2,t_3,\ldots,t_n)}{m}$
- Optimal solution: $T^* \geq \frac{sum(t_1,t_2,t_3,\ldots,t_n)}{m}$

We have $\frac{T_{worst}}{T^*} = \frac{t_1+S_{n-1}}{T^*}$

- For some spetial cases that $t_1 = \frac{sum(t_2,\ldots,t_n)}{(m-1)}$ (like examples above), $T^* = \frac{sum(t_1,t_2,t_3,\ldots,t_n)}{m} = t_1$, $T_{worst} = t_1 + min_{S_{n-1}} \geq t_1 + \frac{sum(t_2,t_3,\ldots,t_n)}{m}$, We have:

$$Ratio_{max} = \frac{t_1 + \frac{sum(t_2,t_3,\ldots,t_n)}{m}}{t_1} = \frac{t_1 + \frac{(m-1)t_1}{m}}{t_1} = \frac{2m-1}{m} \rightarrow 2(when\ m \rightarrow \infty)$$

Thus, when $t_1 = \frac{sum(t_2,\ldots,t_n)}{m-1}$, and the ratio is maximum close to 2.