Assignment7: Clustering

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- Assume we need to divide a set into n parts.
- First, find a function map points in initial partition to another low-dimantion feature space.
- Then, cut points in feature space by a simpler function(like average-divide, or get n-1 max distance)
- Finally, map divided points back to original space.

Here is an example with map function: $\phi: s \mapsto \|s\|_1$

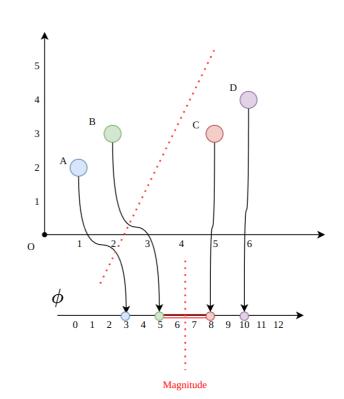
Points divided by function

$$Mag = \{x_{i,j} | d_{i,j} \in \{d_0, d_1, ..., d_n\}\}$$

- $d_{i,j}$ is the distance between x_i and x_j
- d_n is the n-th largest distance

Here, assume we need to divide 4 point into 2 parts.

0-th largest distance is $d_{B,C}=3$, so we devide set into AB and CD



Assume k = 1. Let distance function be

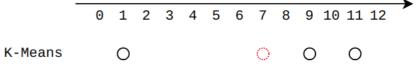
$$d(x,y) = ||x - y||$$

- The first row is the case with k-means.
- The second row is the case with k-medoids.

For k-means, mean value of point set S is 7. Cluster center is 7.

For k-medoids, point have smallest distance to others is 9, so cluster center is 9.

Obviously, k-medoids paid less attention to outlier samples when outlier like noise occurs.



K-Medoid

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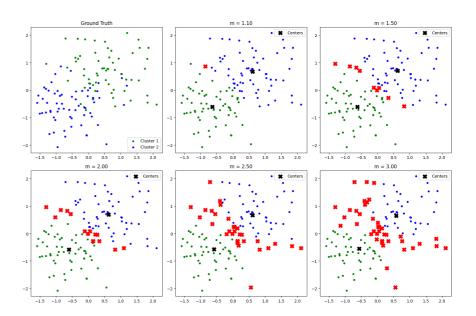
Consider two point sets:

$$S_1 \sim N([0.5, 0.5], 0.5I)$$

 $S_2 \sim N([-0.5, -0.5], 0.5I)$

Red cross represented membership value of a point lower than 0.65.

Easy to find that, with the increase of m-value, the boundaries betweem clusters will be blurer.



Consider three point sets:

$$S_1 \sim N([0, 0], I)$$

 $S_2 \sim N([3, 3], I)$
 $S_3 \sim N([1.5, 1.5], I)$

- K-Means: Each point is assigned a crisp, distinct cluster label, with no overlap between clusters.
- Fuzzy C-Means: Points are assigned a membership probability for each cluster, visualized through blended colors.
- Unlike K-means, fuzzy C-means does not enforce strict boundaries

