Property Demonstration on the Simple Nearest Neighbor Greedy Algorithm regarding TSP Problem with Various Examples

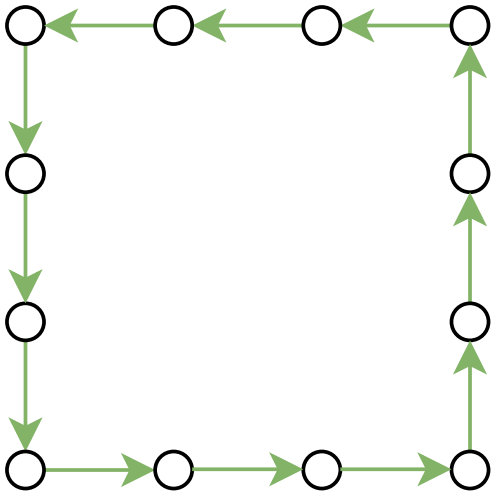
陈子蔚 Chen Ziwei

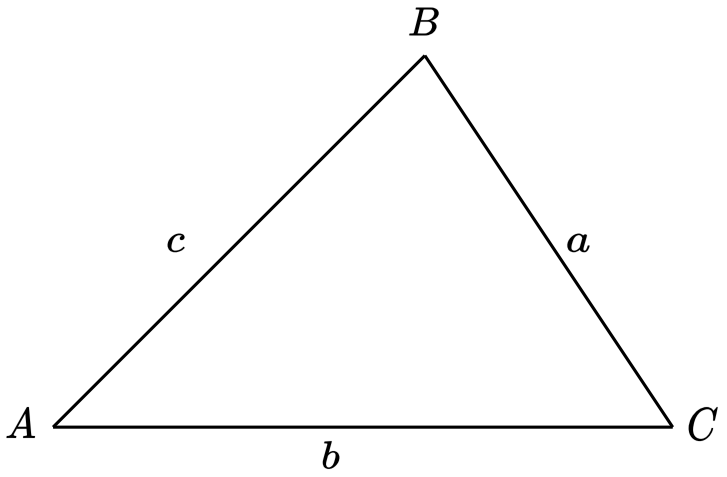
Department of Computer Science and Engineering

Recent Research Topic: Generative Model

Supervisor: 郑锋

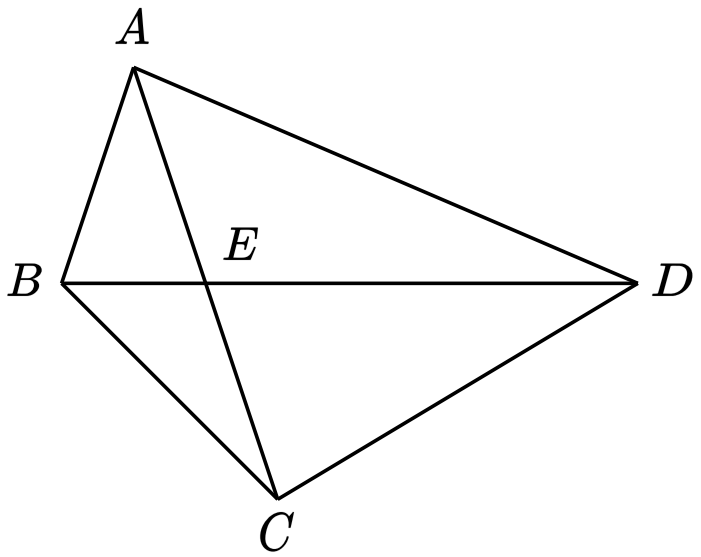
Task 1-1





Optimal solutions would be obtained by simple nearest neighbor greedy algorithm when the graph is quite simple or highly symmetric.

Task 1-2



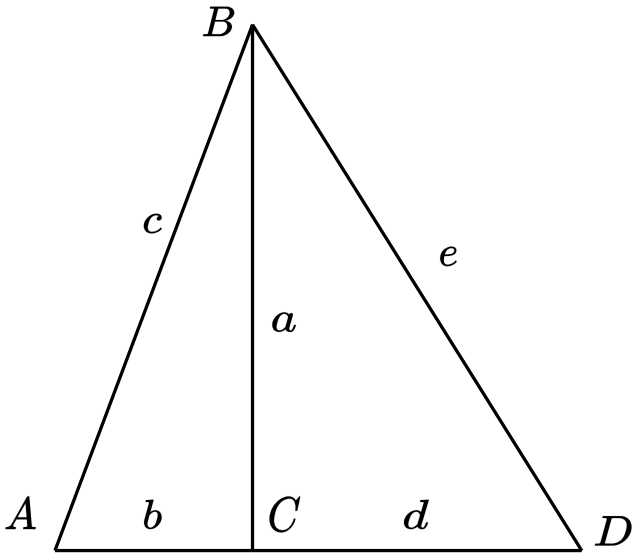
Note that 'E' is not a real node(city) in this graph, which is for explanation indeed.

$$AB < BC < CD < AC < AD < BD$$

With SNNG algorithm, if A is the start point, path $ABCDA$ would be obtained and if B is the start point, path $BACDB$ would be obtained.

$$\begin{aligned}\text{len}(BACDB) - \text{len}(ABCDA) &= AC + DB - BC - AD \\ &= AE + EC + BE + DE - BC - AD \\ &= AE + DE - AD + EC + BE - BC > 0\end{aligned}$$

, which means $BACDB$ is sub-optimal.



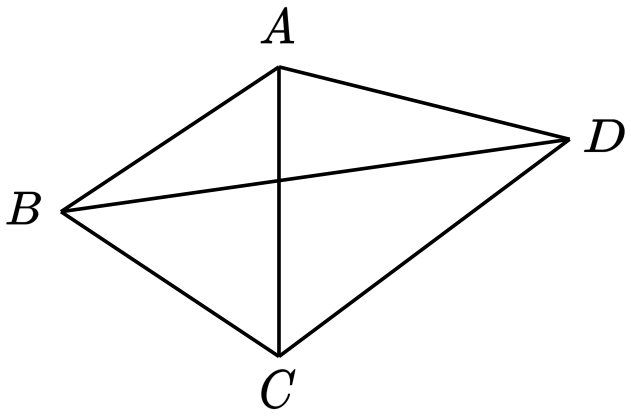
$$b < d < a < b + d < c < a + d < e$$

With SNNG algorithm, if A is the start point, path $ACDBA$ would be obtained and if B is the start point, path $BCADB$ would be obtained.

$$\text{len}(BCADB) - \text{len}(ACDBA) = a + b - c > 0$$

, which means $BCADB$ is sub-optimal.

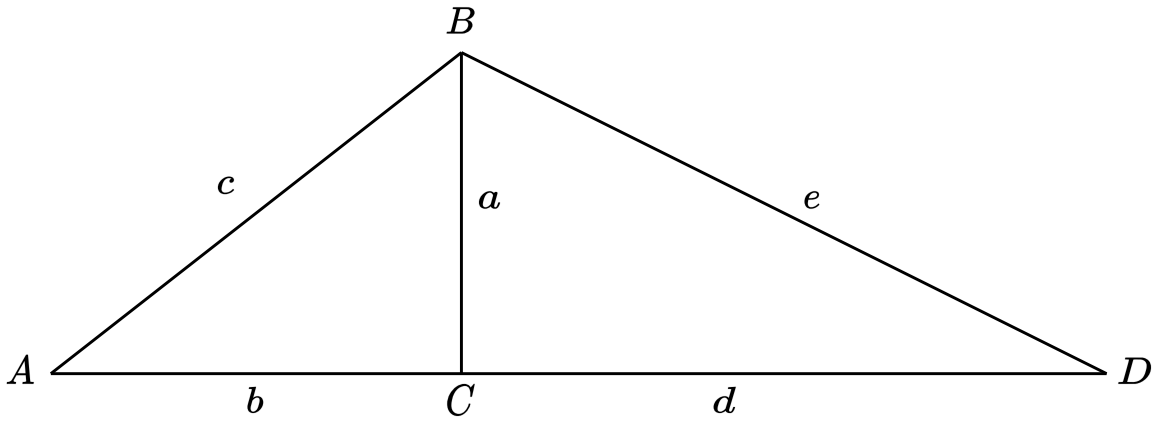
Task 1-3



$AB = 202, BC = 201, CD = 300$
$AD = 201, AC = 200, BD = 400$

Start Point	Path	Length
A	$ACBDA$	1002
B	$BCADB$	1002
C	$CADBC$	1002
D	$DACBD$	1002
NA	$\boldsymbol{ABCDA}$	**904**

, which means all solutions by SNNG are not optimal.

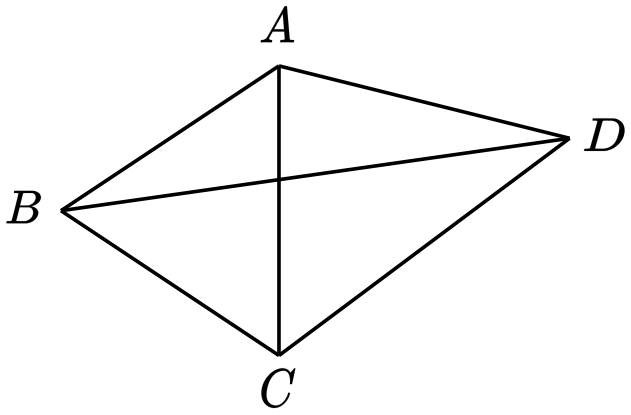


$a < b < d < e < b + d$

Start Point	Path	Length
A	$ACBDA$	$a + b + b + d + e\ (a + b > c)$
B	$BCADB$	$a + b + b + d + e\ (a + b > c)$
C	$CBADC$	$a + b + c + d + d\ (a + d > e)$
D	$DCBAD$	$a + b + c + d + d\ (a + d > e)$
NA	$\boldsymbol{ACDBA}$	$\boldsymbol{c + b + d + e}$

, which means all solutions by SNNG are not optimal.

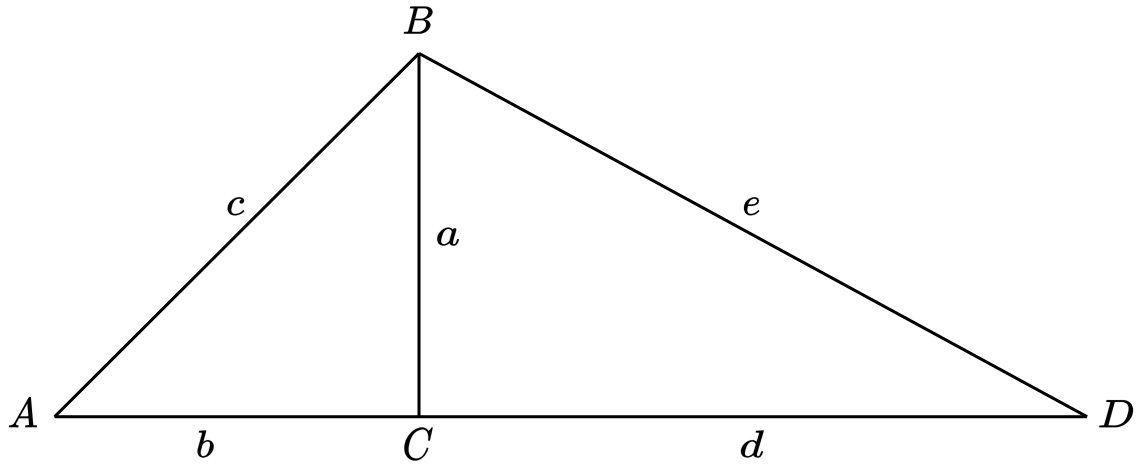
Task 1-4



$AB = 202, BC = 201, CD = 300$
$AD = 201, AC = 201, BD = 400$

Start Point	Path	Length
C	$CADBC$	1003
C	$CBADC$	904

, which means $CADBC$ is sub-optimal.



$a = b < c < d < e < b + d$

Start Point	Path	Length
C	$CBADC$	$a + b + c + d + d\ (a + d > e)$
C	$CABDC$	$b + c + d + e$

, which means $CBADC$ is sub-optimal.