Property Demonstration on the Greedy Algorithm regarding Load Balancing Problem with Various Examples

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Recent Research Topic: Generative Model
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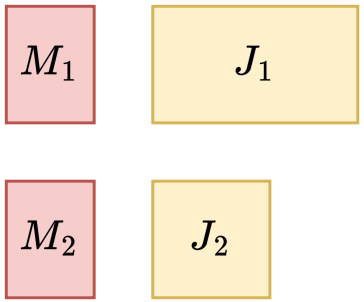
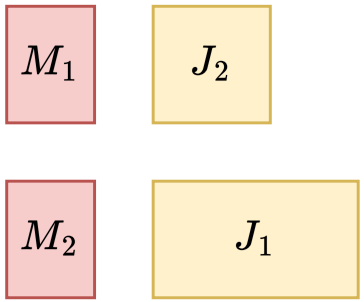
Task 3-1

2 identical machines M_1, M_2 and two jobs J_1, J_2 with processing time $t_1 > t_2$.

Possible input sequences:

$$\{(J_1, J_2), (J_2, J_1)\}$$

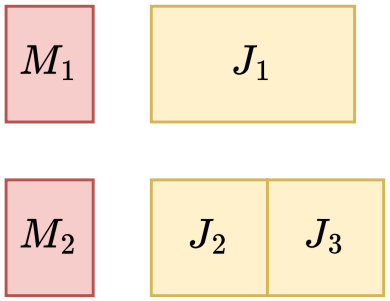


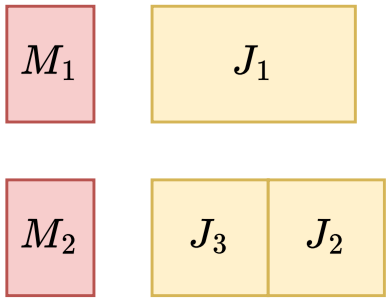


3 identical machines M_1, M_2 and three jobs J_1, J_2, J_3 with processing time $t_1 > t_2 = t_3$.

Possible input sequences:

$$\{(J_1, J_2, J_3), (J_2, J_1, J_3), (J_1, J_3, J_2), (J_3, J_1, J_2), (J_2, J_3, J_1), (J_3, J_2, J_1)\}$$

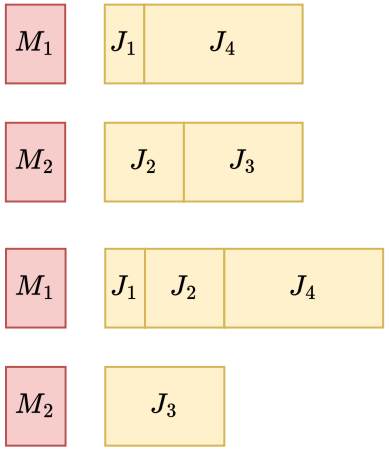




The obtained makespan T is always the same as T^*.

Task 3-2

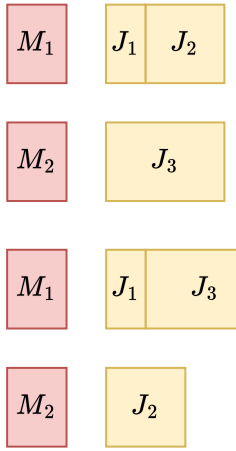
4 identical machines M_1, M_2 and three jobs J_1, J_2, J_3, J_4 with processing time $t_1 = 100, t_2 = 200, t_3 = 300, t_4 = 400$.



With input (J_1, J_2, J_4, J_3), $T = 500 = T^*$ obtained

With input (J_1, J_3, J_2, J_4), $T = 700$ obtained

3 identical machines M_1, M_2 and two jobs J_1, J_2, J_3 with processing time $t_1 = 1, t_2 = 2, t_3 = 3$.

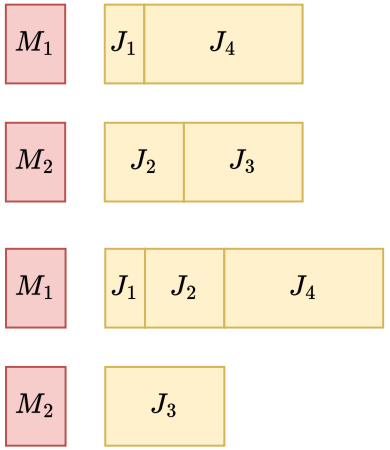


With input (J_1, J_3, J_2), $T = 3 = T^*$ obtained

With input (J_1, J_2, J_3), $T = 4$ obtained

Task 3-3

4 identical machines M_1, M_2 and three jobs J_1, J_2, J_3, J_4 with processing time $t_1 = d + 1, t_2 = 2d + 1, t_3 = 3d + 1, t_4 = 4d + 1$.



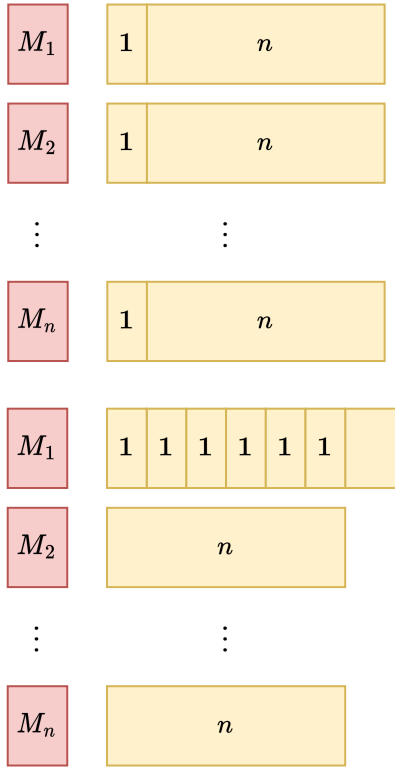
With input (J_1, J_2, J_4, J_3), $T = 5d + 2 = T^*$ obtained.

With input (J_1, J_3, J_2, J_4), $T = 7d + 2$ obtained.

$$\frac{T}{T^*} = \frac{7d + 2}{5d + 2}$$

$$\lim_{d \to +\infty} \frac{T}{T^*} = \frac{7}{5} = 1.4$$

n identical machines $\{M_i\}, i \in [0, n]$ and $2n$ jobs $\{J_i\}, i \in [0, 2n]$ with processing time $t_1 = t_2 = \cdots = t_n = 1, t_{n+1} = t_{n+2} = \cdots = t_{2n} = n$.



With input $(J_1, \cdots, J_{2n})$, $T = n + 1 = T^*$ obtained.

With input $(J_1, J_{2n}, J_2, J_3 \cdots, J_{2n-1})$, $T = 2n$ obtained

$$\frac{T}{T^*} = \frac{2n}{n + 1}$$

$$\lim_{n \to +\infty} \frac{T}{T^*} = 2$$