Property Demonstration on the Simple Nearest Neighbor Greedy Algorithm regarding TSP Problem with Various Examples

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Recent Research Topic: Generative Model

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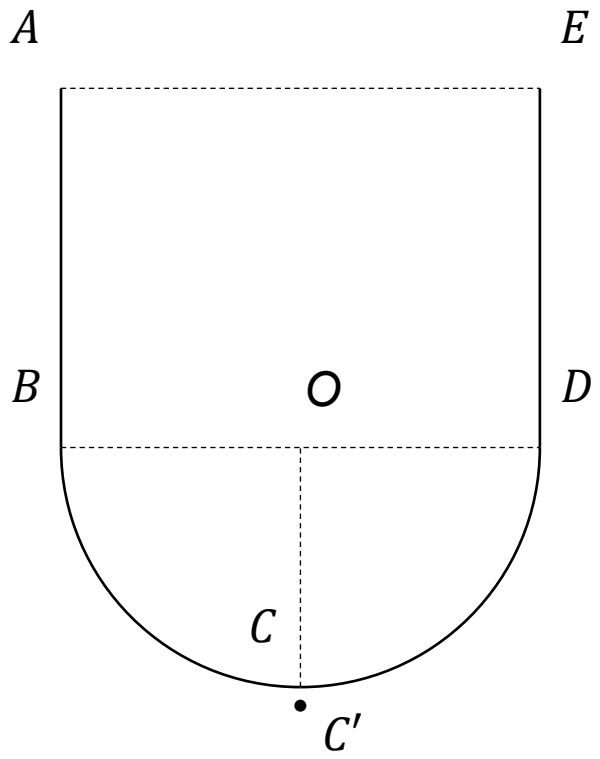
Task 2-1



Consider a graph consisting of a semi-arc connecting two line segments and an additional point, C'. There are dense points on $AB, \widehat{BC}, \widehat{CD}, DE$. $AB = CE = l, OB = OC = OD = r$ and CC' is quite mall value ϵ.

Optimal path: $A\widehat{BC}C'\widehat{CD}A$

$$\text{len}\left(A\widehat{BC}C'\widehat{CD}A\right) \cong \text{len}\left(\widehat{ABCDEA}\right) = 2l + \pi r + 2r$$

SSN greedy obtained path: $\widehat{ABCD}EC'A$

$$\text{len}\left(\widehat{ABCD}EC'A\right) \cong \text{len}\left(\widehat{ABCD}ECA\right) = 2l + \pi r + 2\sqrt{r^2 + l^2}$$

$$\text{Solution Quality Index} = \frac{2l + \pi r + 2\sqrt{r^2 + l^2}}{2l + \pi r + 2r} = \frac{2 \cdot \frac{l}{r} + \pi + 2\sqrt{1 + \frac{l^2}{r^2}}}{2 \cdot \frac{l}{r} + \pi + 2}$$

$$\lim_{t \to +\infty} \frac{2t + \pi + 2\sqrt{1 + t^2}}{2t + \pi + 2} = \lim_{t \to +\infty} \frac{2 + \frac{\pi}{t} + 2\sqrt{\frac{1}{t^2} + 1}}{2 + \frac{\pi + 2}{t}} = 2$$

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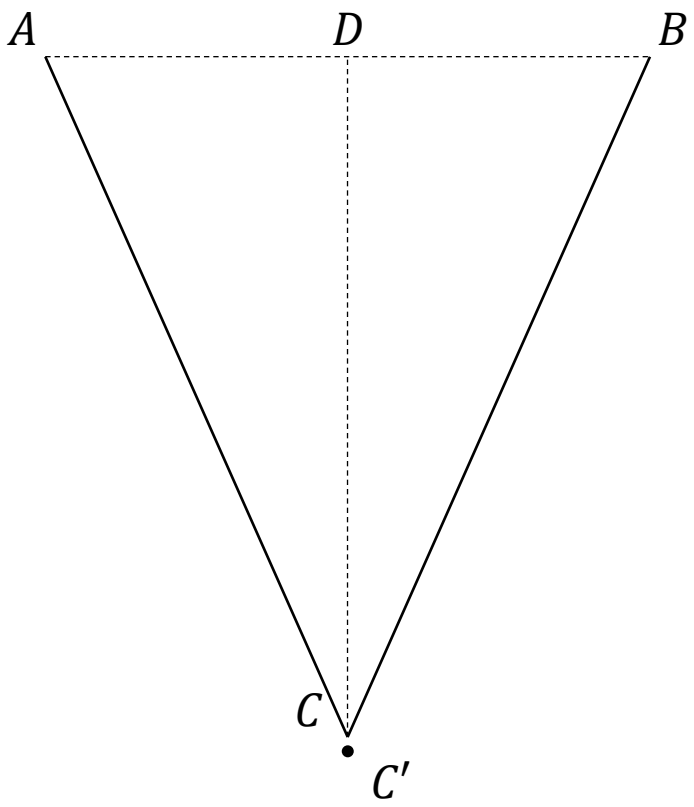

Consider a graph consisting of two line segments and an additional point, C'. There are dense points on AC, BC. $AD = BD = x, DC = y$ and CC' is quite mall value ϵ.

Optimal path: $ACC'CBA$

$$\text{len}(ACC'CBA\ ) \cong \text{len}(ACBA) = 2x + 2\sqrt{x^2 + y^2}$$

SSN greedy obtained path: $ACBC'A$

$$\text{len}(ACBC'A) \cong \text{len}(ACBCA) = 4\sqrt{x^2 + y^2}$$

$$\text{Solution Quality Index} = \frac{4\sqrt{x^2 + y^2}}{2x + 2\sqrt{x^2 + y^2}} = \frac{2\sqrt{1 + \frac{y^2}{x^2}}}{1 + \sqrt{1 + \frac{y^2}{x^2}}}$$

$$\lim_{t \to +\infty} \frac{2\sqrt{1 + t^2}}{1 + \sqrt{1 + t^2}} = 2 - \lim_{t \to +\infty} \frac{2}{1 + \sqrt{1 + t^2}} = 2$$