

Clustering Algorithms and their Application to Facial Image Analysis

Hamid Sadeghi

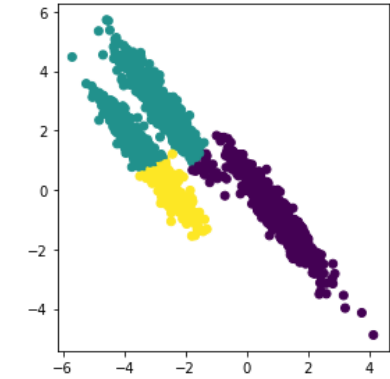
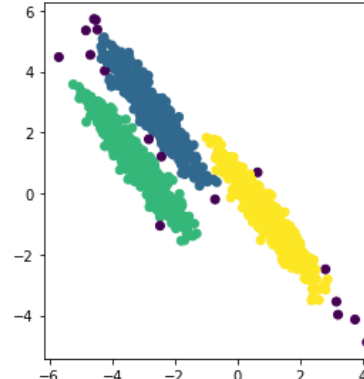
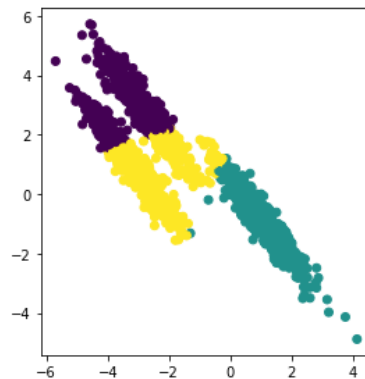
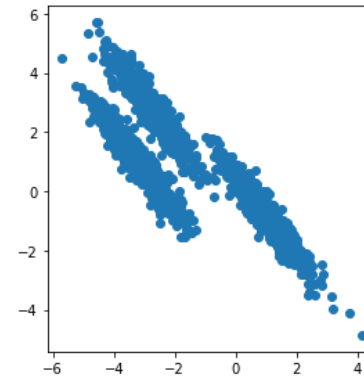
nextera, FaceCup 1400 (2022)



Contents

- ✓ Introduction
- ✓ Clustering Algorithms
 - Evaluation
 - Face Analysis

Clustering summary



Evaluation

How to evaluate the quality of clusters?

Internal Evaluation (unsupervised)

Using clustered data itself

- *Silhouette coefficient*
- *Davies-Bouldin index*
- *Dunn index*

External Evaluation (supervised)

Using ground truth or gold standard

- *Purity*
- *Rand Index*
- *Normalized Mutual Information (NMI)*
- *F-measure*



Evaluation

- Purity

$$purity(\Omega, C) = \frac{1}{N} \sum_k \max_j |\omega_k \cup c_j|$$



Evaluation

- Purity

$$\text{purity}(\Omega, C) = \frac{1}{N} \sum_k \max_j |\omega_k \cup c_j|$$

clusters

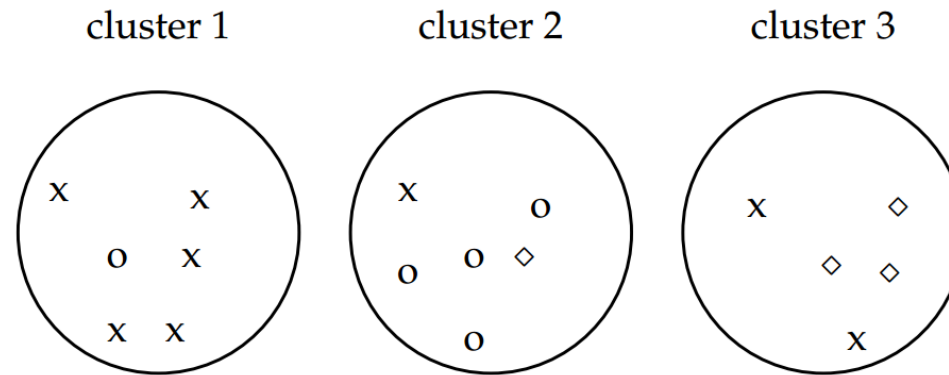
classes

Evaluation

- Purity

$$purity(\Omega, C) = \frac{1}{N} \sum_k \max_j |\omega_k \cup c_j|$$

clusters
classes



► **Figure 16.4** Purity as an external evaluation criterion for cluster quality. Majority class and number of members of the majority class for the three clusters are: x, 5 (cluster 1); o, 4 (cluster 2); and ◇, 3 (cluster 3). Purity is $(1/17) \times (5 + 4 + 3) \approx 0.71$.

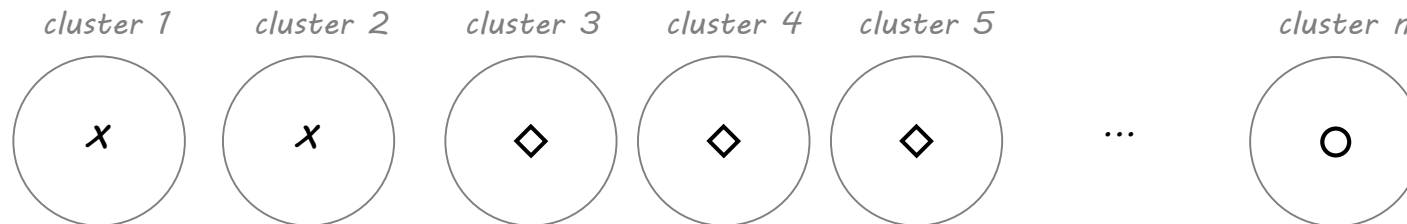
Evaluation

- Purity

$$\text{purity}(\Omega, C) = \frac{1}{N} \sum_k \max_j |\omega_k \cup c_j|$$

clusters
classes

If each data gets 1 cluster \Rightarrow purity = 1 (!)



Evaluation

- Rand Index

$$RI = \frac{TP + TN}{TP + FP + FN + TN}$$

Evaluation

- Rand Index

$$RI = \frac{TP + TN}{TP + FP + FN + TN}$$

True Positive pairs (arrow from TP to numerator)
True Negative pairs (arrow from TN to numerator)
False Positive pairs (arrow from FP to denominator)
False Negative pairs (arrow from FN to denominator)

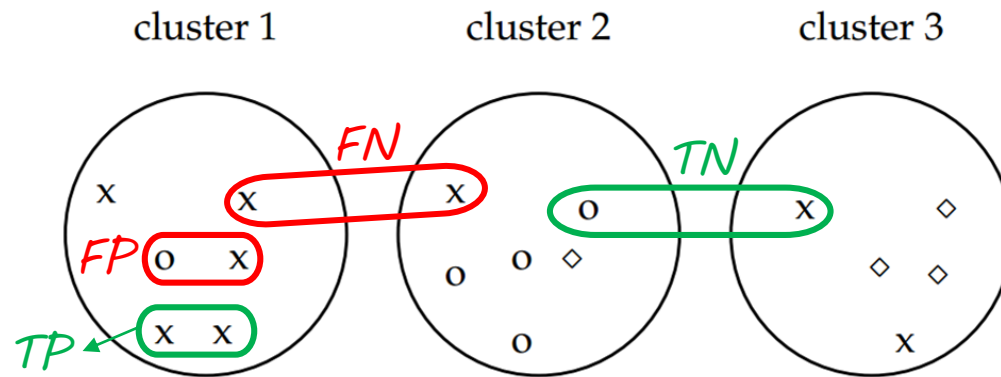
Evaluation

- Rand Index

$$RI = \frac{TP + TN}{TP + FP + FN + TN}$$

True Positive pairs
 True Negative pairs
 False Positive pairs
 False Negative pairs

Sample pairs:



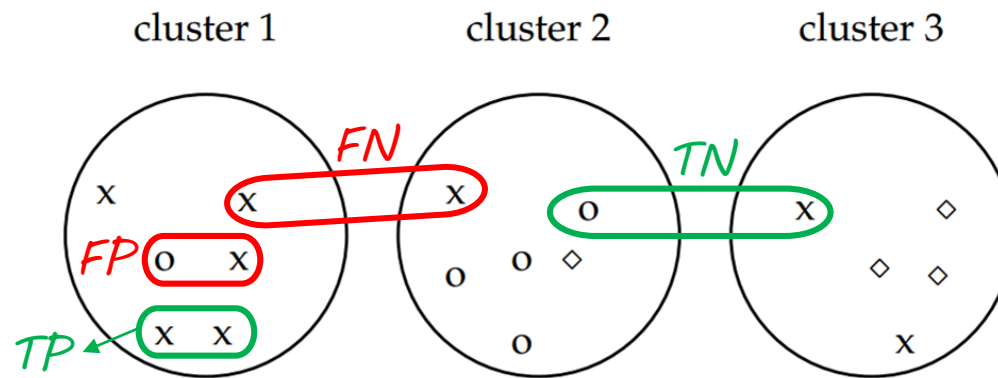
Evaluation

- Rand Index

$$RI = \frac{TP + TN}{TP + FP + FN + TN}$$

True Positive pairs (arrow to TP)
 True Negative pairs (arrow to TN)
 False Positive pairs (arrow to FP)
 False Negative pairs (arrow to FN)

Sample pairs:



$$TP + FP = \binom{6}{2} + \binom{6}{2} + \binom{5}{2} = 40 \text{ Positive pairs}$$

$$TP = \binom{5}{2} + \binom{4}{2} + \binom{3}{2} + \binom{2}{2} = 20$$

$$\Rightarrow FP = 40 - 20 = 20$$

	Same cluster	Different clusters
Same class	TP = 20	FN = 24
Different classes	FP = 20	TN = 72

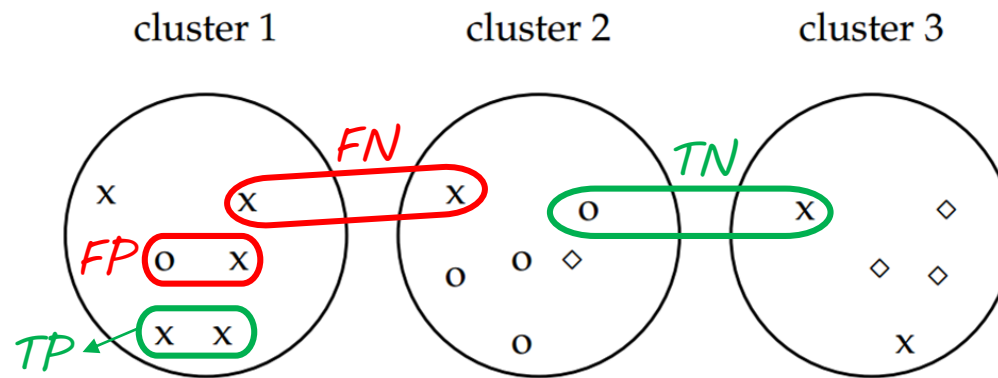
Evaluation

- Rand Index

$$RI = \frac{TP + TN}{TP + FP + FN + TN}$$

True Positive pairs (arrow to TP)
 True Negative pairs (arrow to TN)
 False Positive pairs (arrow to FP)
 False Negative pairs (arrow to FN)

Sample pairs:



	Same cluster	Different clusters
Same class	TP = 20	FN = 24
Different classes	FP = 20	TN = 72

$$RI = (20 + 72) / (20 + 20 + 24 + 72) \approx 0.68$$

Evaluation

- *Normalized Mutual Information (NMI)*

$$NMI(\Omega, C) = \frac{I(\Omega, C)}{[H(\Omega) + H(C)]/2}$$

Evaluation

- Normalized Mutual Information (NMI)

$$NMI(\Omega, C) = \frac{I(\Omega, C)}{[H(\Omega) + H(C)]/2}$$

Mutual Information ← $I(\Omega, C)$ → *clusters*
 ← $[H(\Omega) + H(C)]/2$ → *entropy* → *classes*

Evaluation

- Normalized Mutual Information (NMI)

Mutual Information \leftarrow $I(\Omega, C)$ \rightarrow clusters

$$NMI(\Omega, C) = \frac{I(\Omega, C)}{[H(\Omega) + H(C)]/2}$$

\leftarrow entropy \rightarrow classes

$$I(\Omega, C) = \sum_k \sum_j P(\omega_k \cap c_j) \log \frac{P(\omega_k \cap c_j)}{P(\omega_k) P(c_j)} = \sum_k \sum_j \frac{|\omega_k \cap c_j|}{N} \log \frac{N |\omega_k \cap c_j|}{|\omega_k| |c_j|}$$

Evaluation

- Normalized Mutual Information (NMI)

Mutual Information ← $I(\Omega, C)$ → *clusters*

$NMI(\Omega, C) = \frac{I(\Omega, C)}{[H(\Omega) + H(C)]/2}$ → *classes*

entropy ← $H(\Omega)$

probabilities of a data being in the intersection of ω_k and c_j

$$I(\Omega, C) = \sum_k \sum_j P(\omega_k \cap c_j) \log \frac{P(\omega_k \cap c_j)}{P(\omega_k) P(c_j)} = \sum_k \sum_j \frac{|\omega_k \cap c_j|}{N} \log \frac{N |\omega_k \cap c_j|}{|\omega_k| |c_j|}$$

probabilities of a data being in cluster ω_k

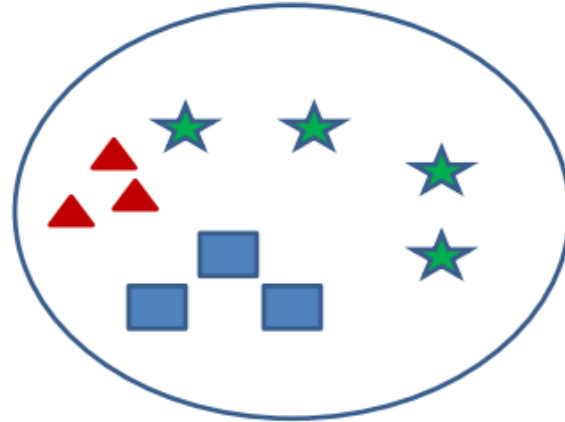
probabilities of a data being in class c_j

$$H(\Omega) = \sum_k P(\omega_k) \log P(\omega_k) = \sum_k \frac{|\omega_k|}{N} \log \frac{|\omega_k|}{N}$$

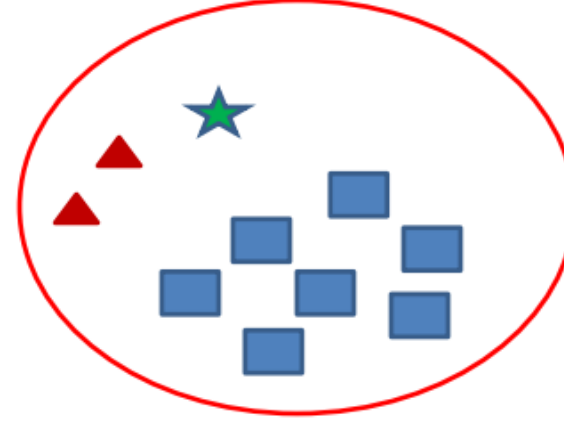
Evaluation

- NMI example

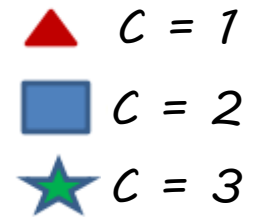
Cluster 1 ($\Omega = 1$)



Cluster 2 ($\Omega = 2$)

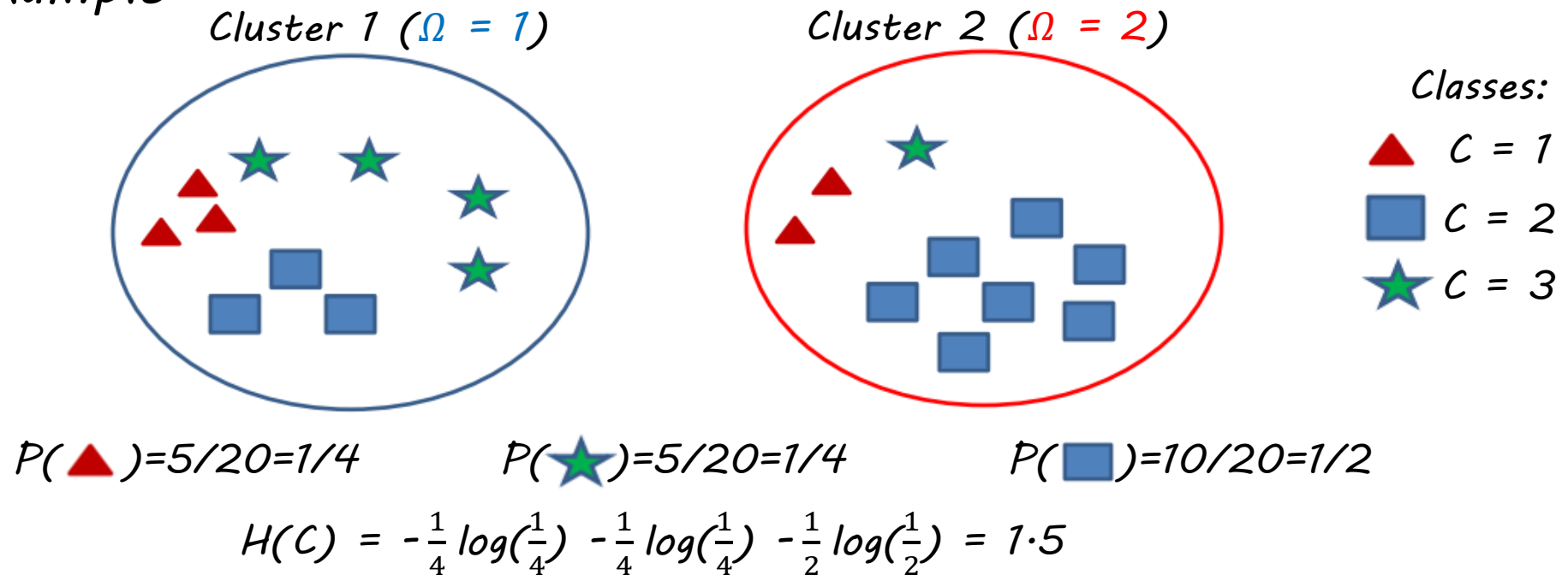


Classes:



Evaluation

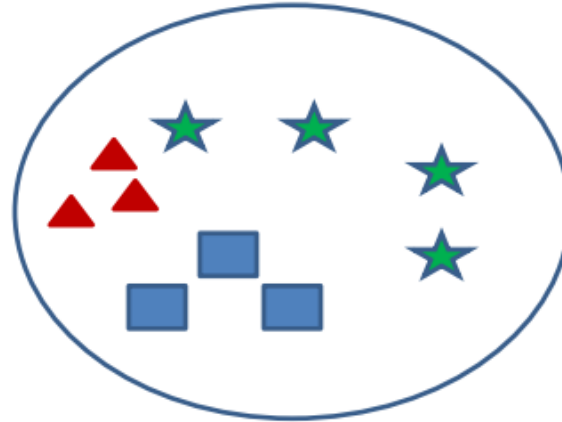
- NMI example



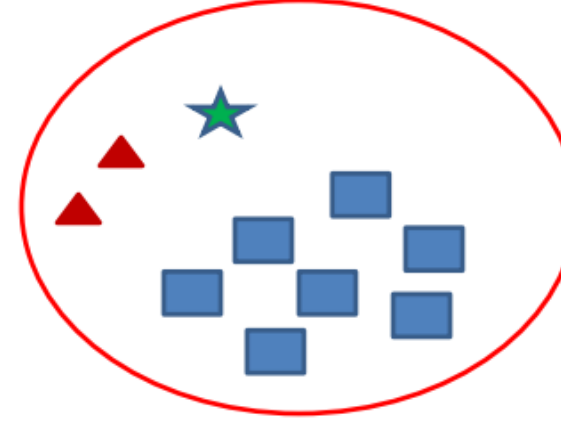
Evaluation

- NMI example

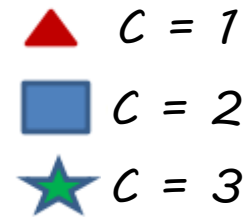
Cluster 1 ($\Omega = 1$)



Cluster 2 ($\Omega = 2$)



Classes:



$$P(\blacktriangle) = 5/20 = 1/4$$

$$P(\star) = 5/20 = 1/4$$

$$P(\blacksquare) = 10/20 = 1/2$$

$$H(C) = -\frac{1}{4} \log\left(\frac{1}{4}\right) - \frac{1}{4} \log\left(\frac{1}{4}\right) - \frac{1}{2} \log\left(\frac{1}{2}\right) = 1.5$$

$$P(\Omega = 1) = 10/20 = 1/2$$

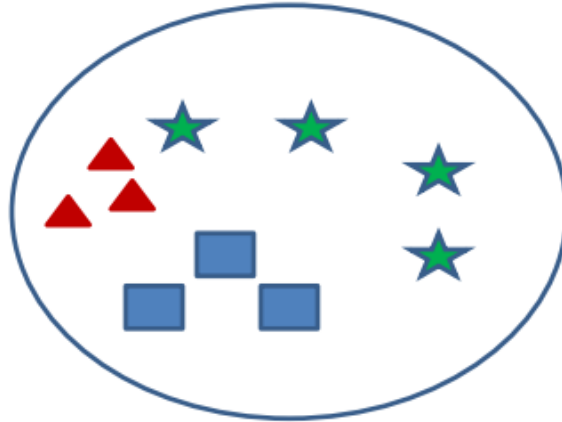
$$P(\Omega = 2) = 10/20 = 1/2$$

$$H(\Omega) = -\frac{1}{2} \log\left(\frac{1}{2}\right) - \frac{1}{2} \log\left(\frac{1}{2}\right) = 1$$

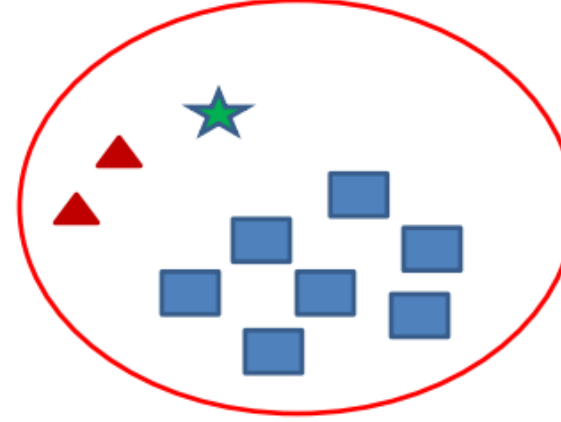
Evaluation

- NMI example

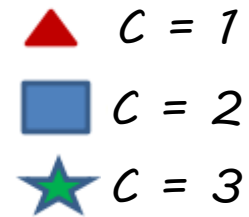
Cluster 1 ($\Omega = 1$)



Cluster 2 ($\Omega = 2$)



Classes:



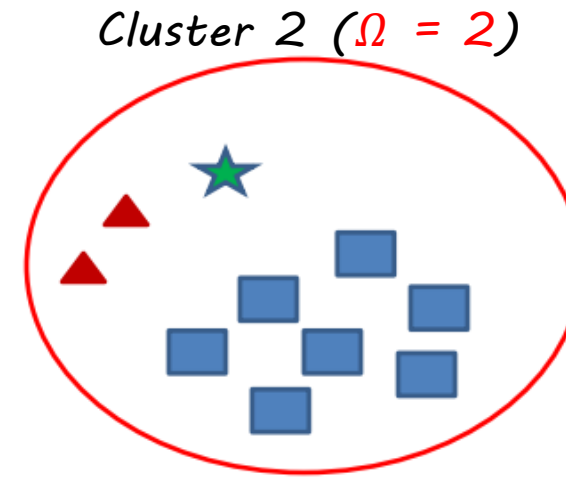
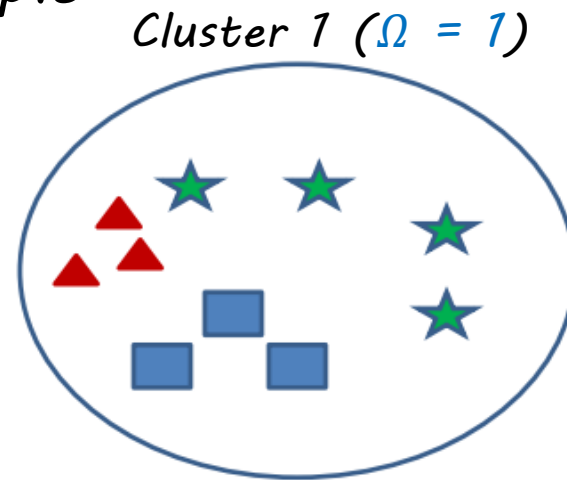
$$I(\Omega, C) = \sum_k \sum_j \frac{|\omega_k \cap c_j|}{N} \log \frac{N |\omega_k \cap c_j|}{|\omega_k| |c_j|}$$

$$= \frac{3}{20} \log \left(\frac{20 \times 3}{10 \times 5} \right) + \frac{3}{20} \log \left(\frac{20 \times 3}{10 \times 10} \right) + \frac{4}{20} \log \left(\frac{20 \times 4}{10 \times 5} \right) + \frac{2}{20} \log \left(\frac{20 \times 2}{10 \times 5} \right) + \frac{7}{20} \log \left(\frac{20 \times 7}{10 \times 10} \right) + \frac{1}{20} \log \left(\frac{20 \times 1}{10 \times 5} \right) = 0.1361$$






Evaluation

- NMI example



Classes:

-  $C = 1$
-  $C = 2$
-  $C = 3$

$$NMI(\Omega, C) = \frac{I(\Omega, C)}{[H(\Omega) + H(C)]/2} = \frac{0.1361}{[1 + 1.5]/2} = 0.1089$$

Evaluation

- *Precision*

(correct rate within clusters)

$$P = \frac{TP}{TP + FP}$$

- *Recall*

(correct rate within classes/ground truth)

$$R = \frac{TP}{TP + FN}$$

- *F-measure*

$$F_{\beta} = \frac{(\beta^2 + 1)PR}{\beta^2 P + R}$$

$\beta = 0$: $F = P$
 $\beta \uparrow$: R

Evaluation

- *implementation:*

https://colab.research.google.com/github/hamidsadeghi68/face-clustering/blob/main/clustering_kmeans.ipynb

https://colab.research.google.com/github/hamidsadeghi68/face-clustering/blob/main/clustering_dbscan.ipynb

https://colab.research.google.com/github/hamidsadeghi68/face-clustering/blob/main/clustering_agglomerative.ipynb





Face Analysis

- *Introduction*
- *Face Detection & Preprocessing*
- *Face Recognition*
- *A Complete Face Clustering Algorithm*

Face Analysis: Introduction

Face Recognition (It is different from face clustering!)

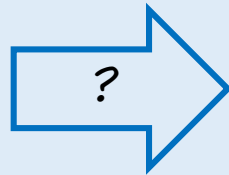
Identification (1:N)

Verification (1:1)

Face Analysis: Introduction

Face Recognition (It is different from face clustering!)

Identification (1:N)



Emilia Clarke
 Angelina Jolie
 Abbas Bouazar
 .
 .
 .

N

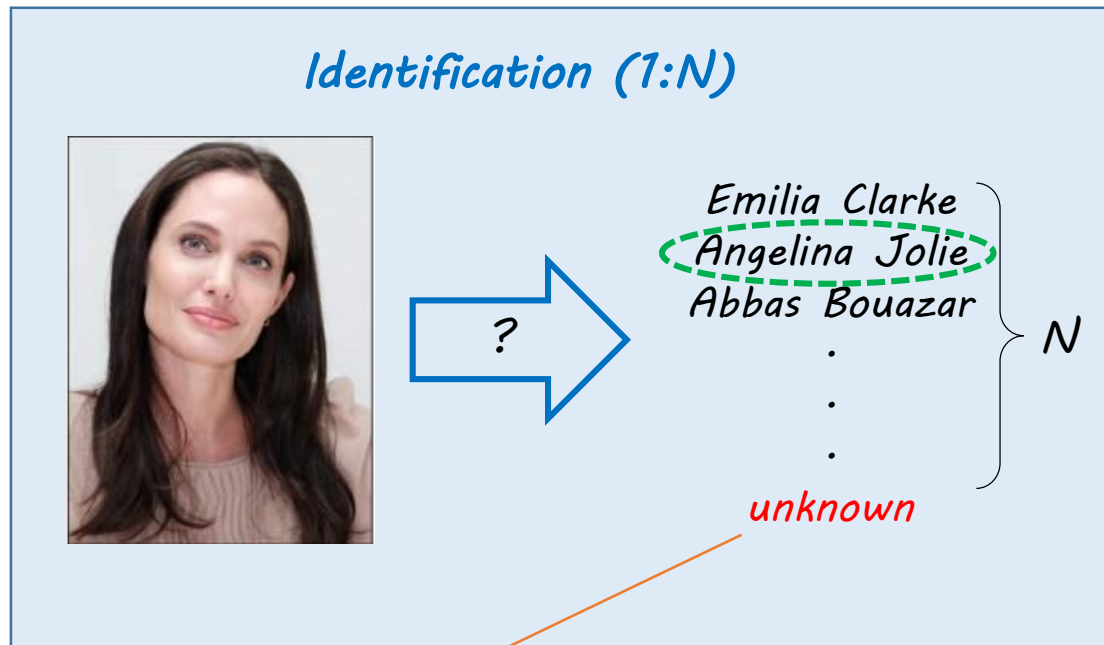
unknown

For open-set problem

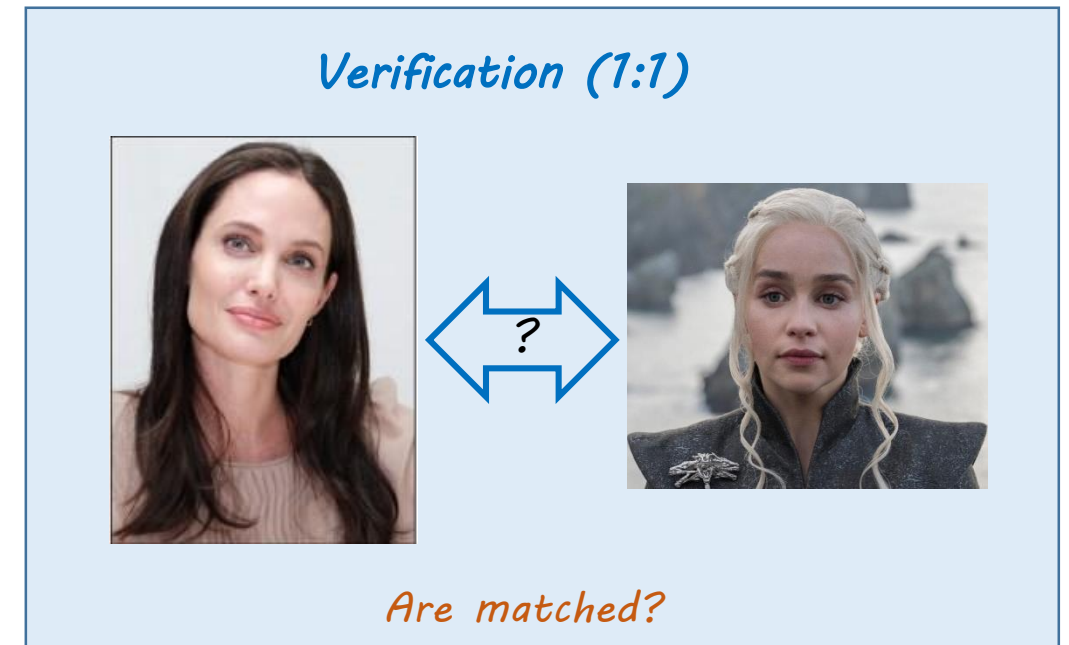
Verification (1:1)

Face Analysis: Introduction

Face Recognition (It is different from face clustering!)

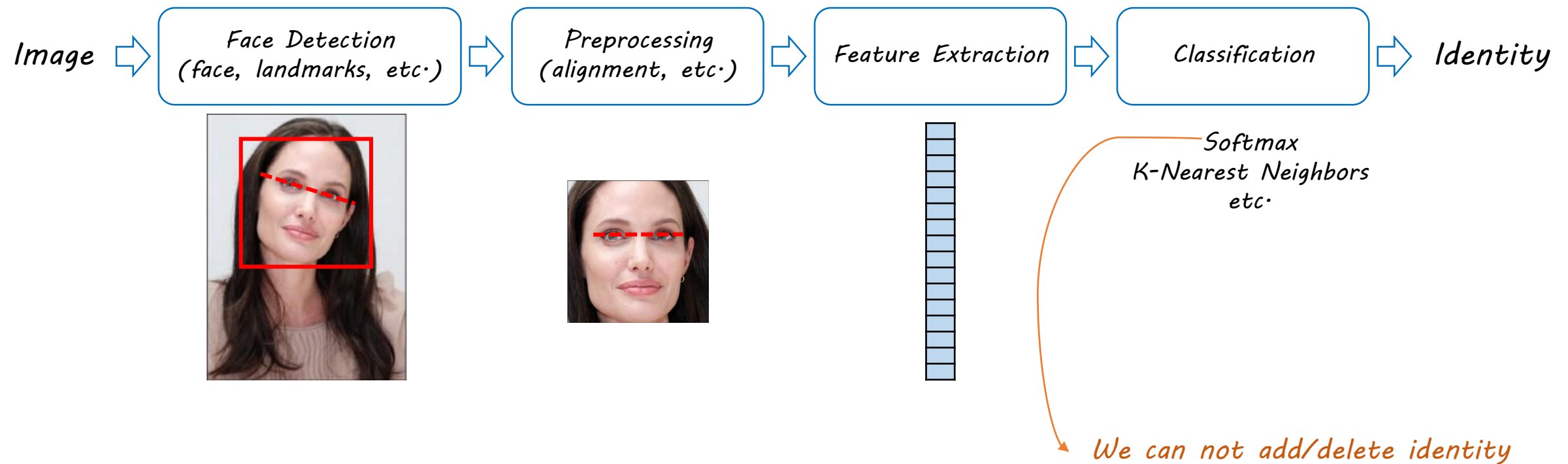


For open-set problem

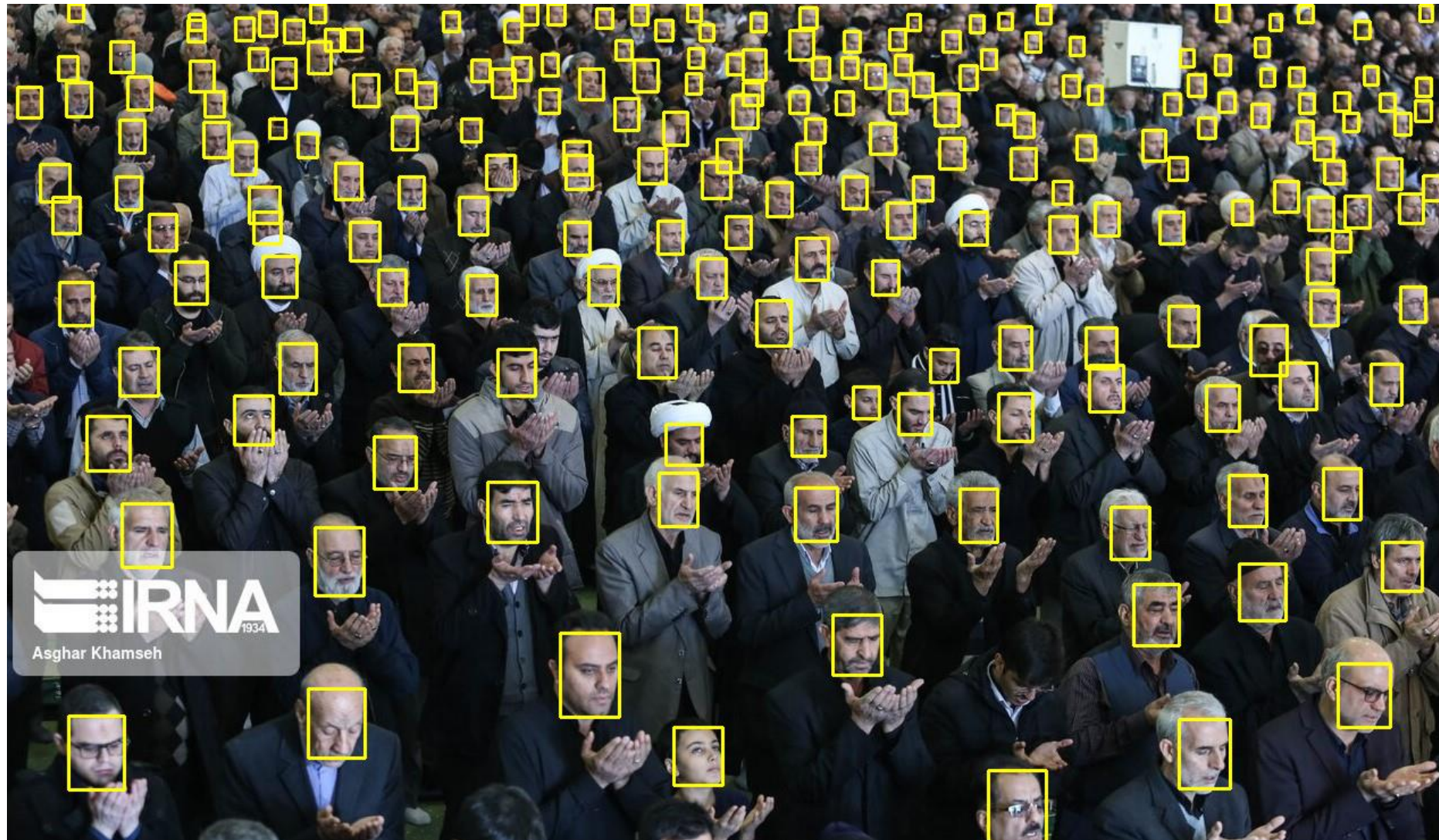


Face Analysis: Introduction

- Block diagram of a face recognition system

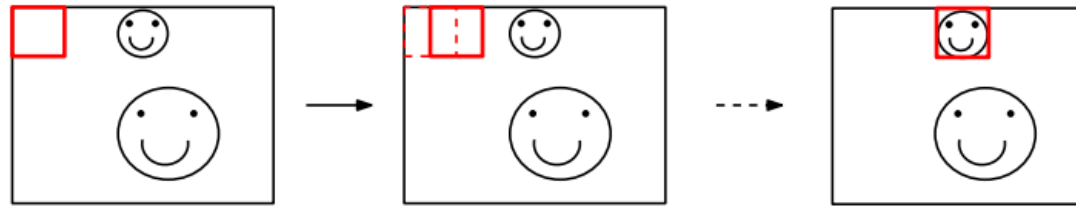


Face Analysis: Face Detection



Face Analysis: Face Detection

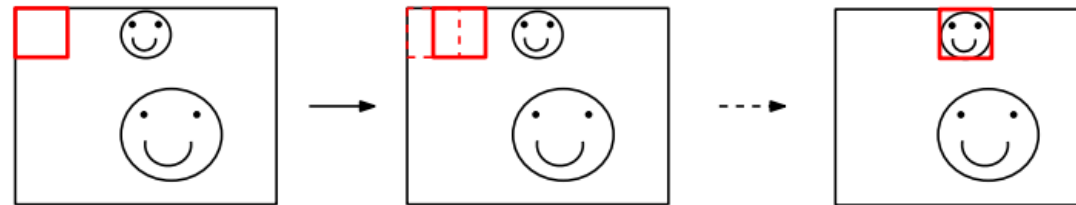
- Haar Cascade



F. Comaschi, et al., RASW: a Run-time Adaptive Sliding Window to Improve Viola-Jones Object Detection

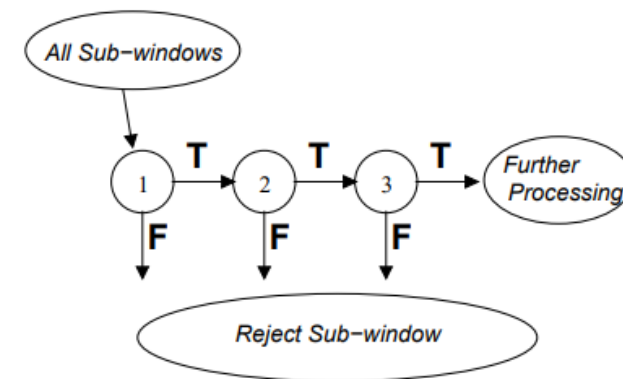
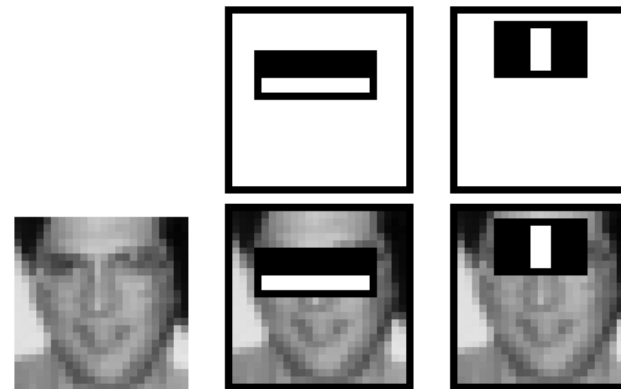
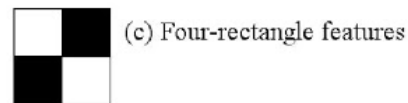
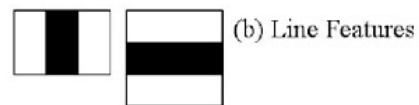
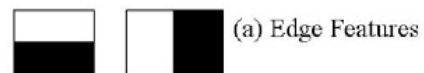
Face Analysis: Face Detection

- Haar Cascade



F. Comaschi, et al., RASW: a Run-time Adaptive Sliding Window to Improve Viola-Jones Object Detection

Each feature: $\sum(\text{pixels under black rectangle}) - \sum(\text{pixels under white rectangle})$



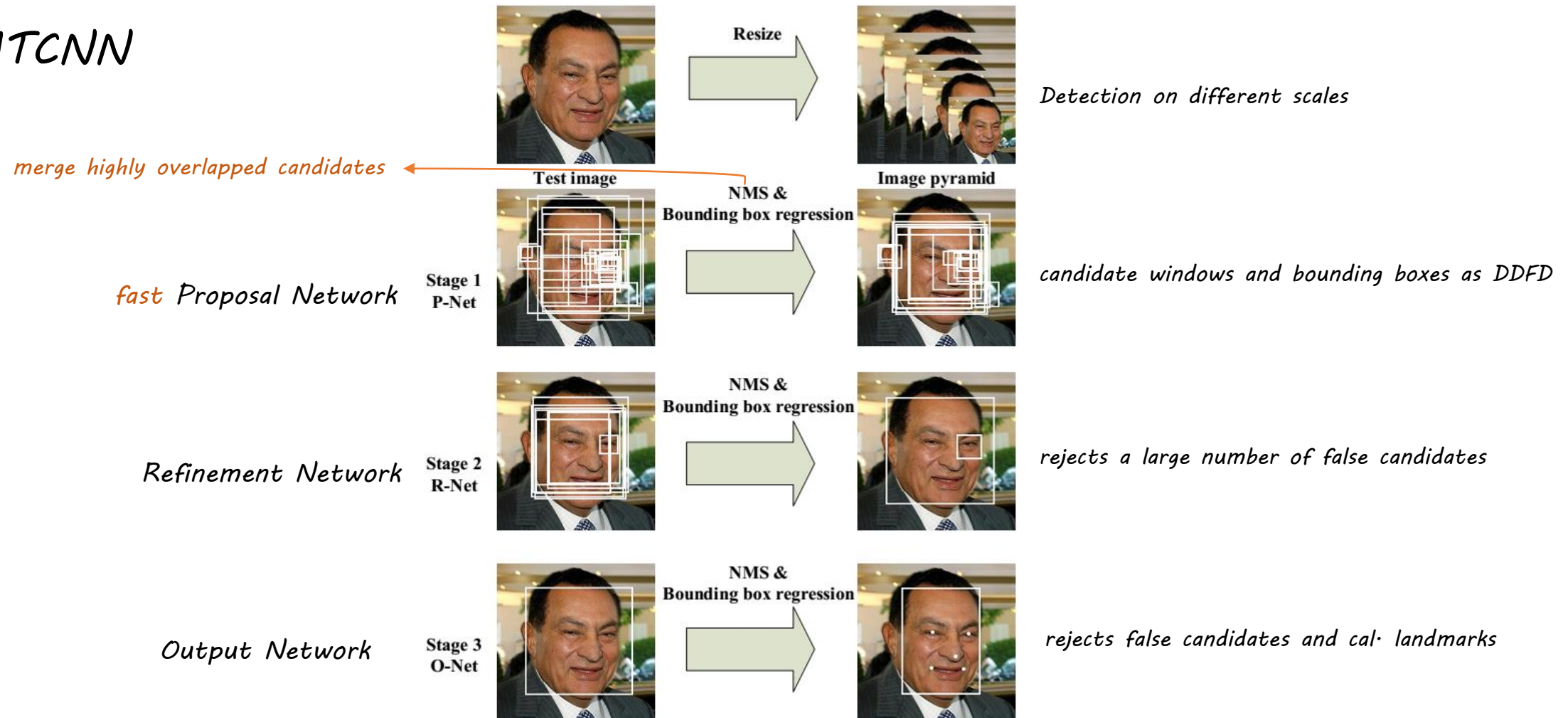
Code:

<https://towardsdatascience.com/face-detection-with-haar-cascade-727f68dafd08>

https://opencv24-python-tutorials.readthedocs.io/en/latest/py_tutorials/py_objdetect/py_face_detection/py_face_detection.html

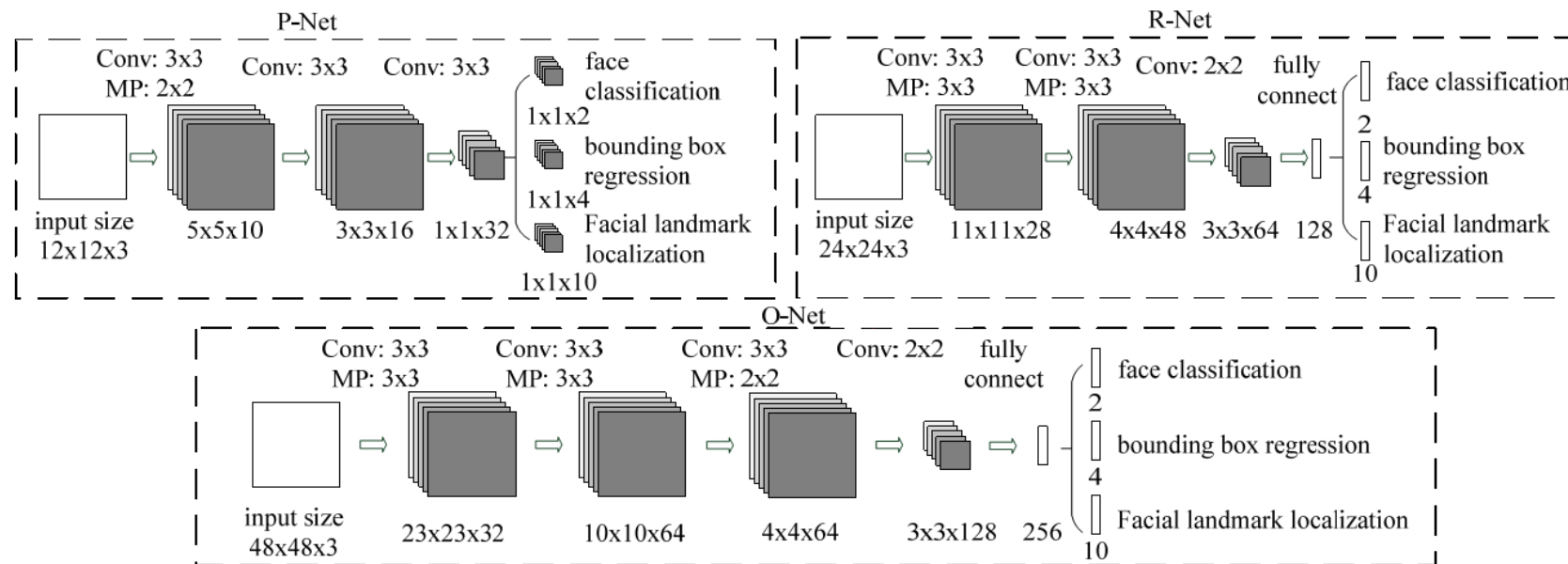
Face Analysis: Face Detection

- *MTCNN*



Face Analysis: Face Detection

- MTCNN



Code:

Python package: <https://github.com/ipazc/mtcnn>
<https://github.com/davidsandberg/facenet/tree/master/src/align>
[https://github.com/kpzhang93/MTCNN face detection alignment](https://github.com/kpzhang93/MTCNN_face_detection_alignment)

Face Analysis: Face Detection

- MTCNN

Face classification loss:
$$L_i^{det} = -\left(y_i^{det} \log(p_i) + (1 - y_i^{det})(\log(1 - p_i))\right) \quad y_i^{det} \in \{0,1\}$$

Bounding box regression loss:
$$L_i^{box} = \|\hat{y}_i^{box} - y_i^{box}\|_2^2$$

landmark localization loss:
$$L_i^{Landmark} = \|\hat{y}_i^{Landmark} - y_i^{Landmark}\|_2^2$$

overall learning target:

$$\min \sum_{i=1}^N \sum_{j \in (det, box, landmark)} \alpha_j \beta_i^j L_i^j$$

$\beta_i^j \in \{0,1\}$ (sample type indicator)

task importance:
$$\begin{cases} PNet, RNet: & \alpha_{det} = 1, \alpha_{box} = 0.5, \alpha_{landmark} = 0.5 \\ ONet: & \alpha_{det} = 1, \alpha_{box} = 0.5, \alpha_{landmark} = 1 \end{cases}$$

Face Analysis: Face Detection

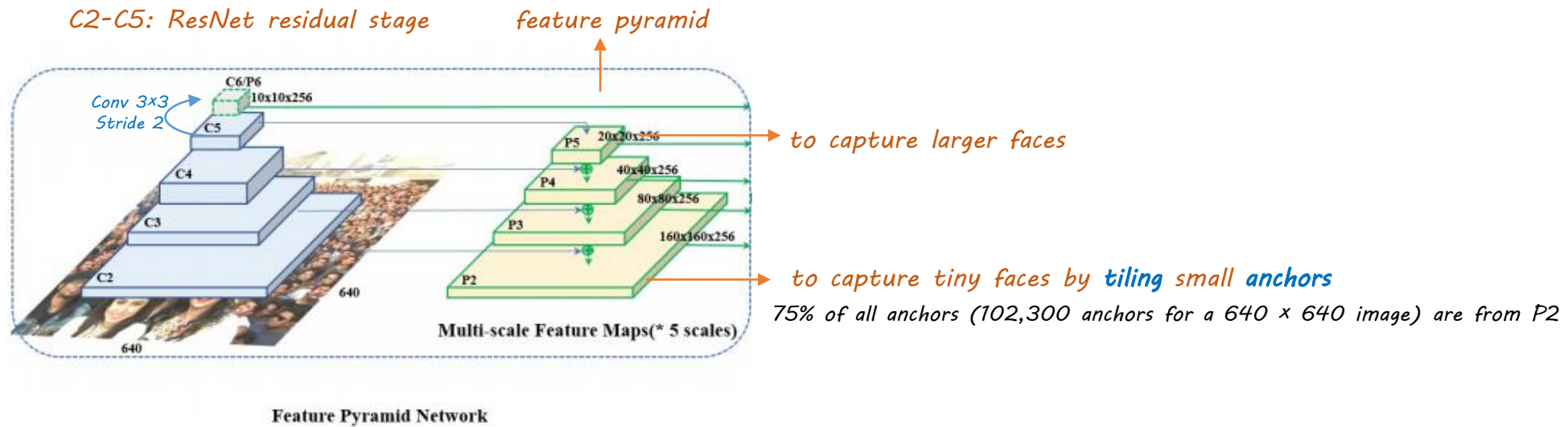
- *Implementation (MTCNN):*

https://colab.research.google.com/github/hamidsadeghi68/face-clustering/blob/main/face_detection_mtcnn.ipynb



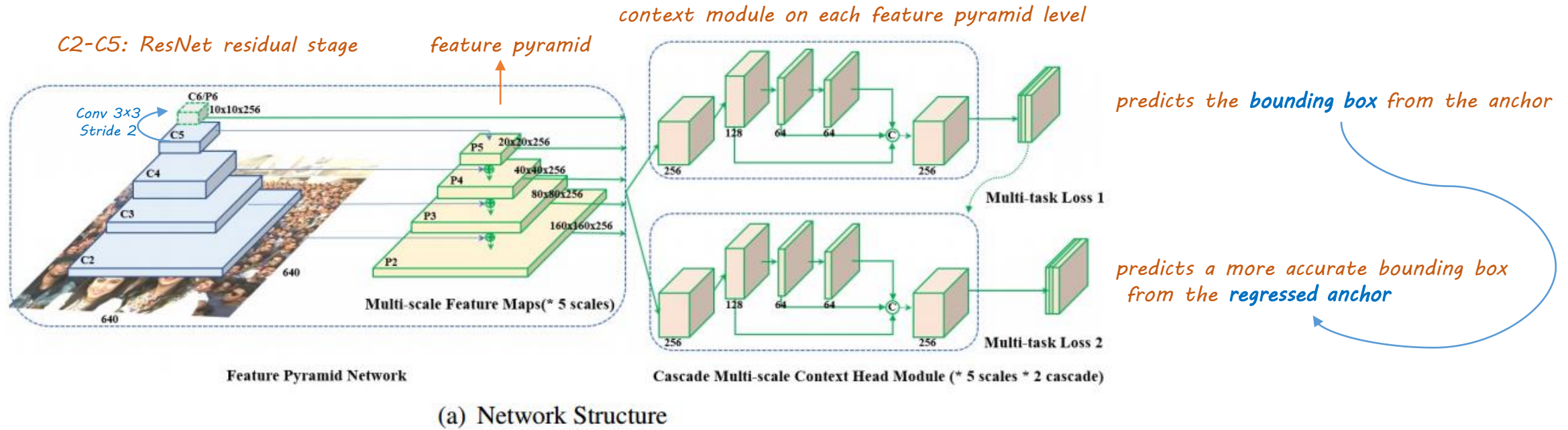
Face Analysis: Face Detection

- RetinaFace



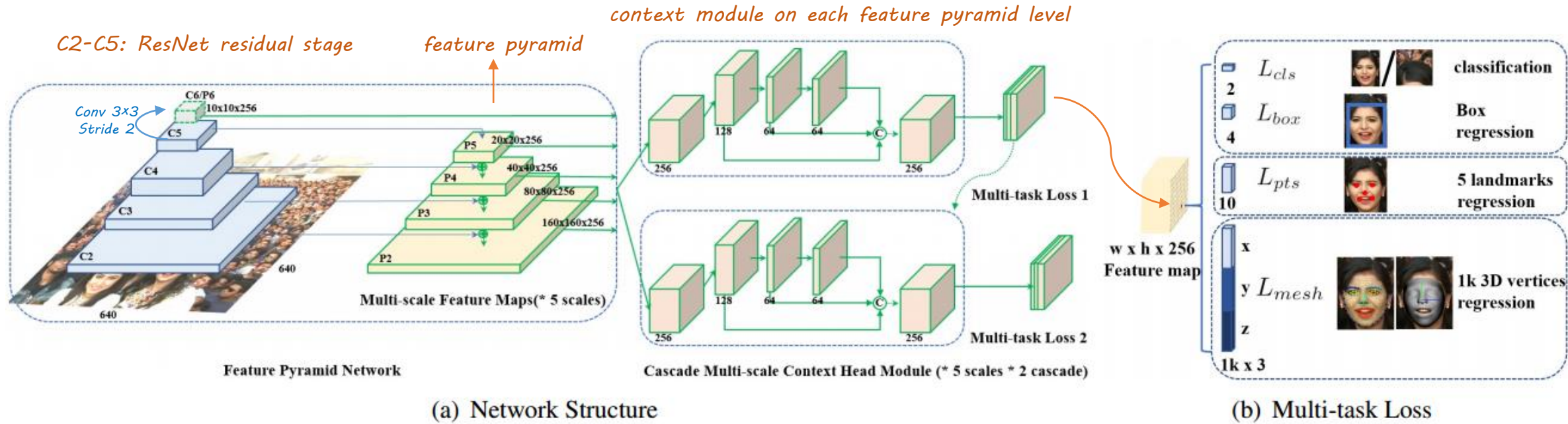
Face Analysis: Face Detection

- RetinaFace



Face Analysis: Face Detection

• RetinaFace



Code:

<https://github.com/deepinsight/insightface/tree/master/detection/retinaface>