# Homework Assignment 5.1

Max Green TIF155

December 22, 2024

## Exercise a

The box-counting dimension $D$ of the fractal is given by:

$$D = \lim_{n \to \infty} \frac{\log N_n}{\log \left( \frac{1}{\epsilon_n} \right)}$$

where:

$$N_n = 4^n \quad \text{and} \quad \epsilon_n = \frac{1}{3^n}$$

$$D_0 = \frac{\log(4^n)}{\log(3^n)} = \frac{n \log 4}{n \log 3} = \frac{\log 4}{\log 3}$$

## Exercise b

The formula for the number of boxes is given by:

$$N_{\text{box}}(\epsilon) = N_S \left( \frac{\epsilon}{\epsilon_S} \right)^{-D_0} + N_L \left( \frac{\epsilon}{\epsilon_L} \right)^{-D_0}$$

where $N_S = 4$, $N_L = 1$, $\epsilon_S = \frac{1}{4}$, and $\epsilon_L = \frac{1}{2}$.
Substitute $\epsilon = 1$ to simplify the expression:

$$1 = 4 \left( \frac{1}{1/4} \right)^{-D_0} + 1 \left( \frac{1}{1/2} \right)^{-D_0}$$

$$1 = 4 \cdot 4^{-D_0} + 2^{-D_0} = 4 \cdot 2^{-2D_0} + 2^{-D_0}$$

Let $x = 2^{-D_0}$:

$$1 = 4x^2 + x$$

$$4x^2 + x - 1 = 0$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(4)(-1)}}{2(4)} = \frac{-1 \pm \sqrt{17}}{8}$$

$$x = \frac{-1 + \sqrt{17}}{8}$$

$$D_0 = -\log_2\left(\frac{-1+\sqrt{17}}{8}\right)$$