

## 2.1\2.1.py

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1 #%% Import libraries
2 import numpy as np
3 import matplotlib.pyplot as plt
4
5 def derivative(x, y, A_sigma):
6     return A_sigma @ np.array([x, y])
7
8 def classify_stability(eigenvalues):
9     real_parts = np.real(eigenvalues)
10    imag_parts = np.imag(eigenvalues)
11
12    if all(real_parts < 0):
13        return "Stable Node"
14    elif all(real_parts > 0):
15        return "Unstable Node"
16    elif real_parts[0] * real_parts[1] < 0:
17        return "Saddle Point"
18    elif imag_parts[0] != 0:
19        if real_parts[0] == 0:
20            return "Center"
21        elif real_parts[0] < 0:
22            return "Stable Focus"
23        else:
24            return "Unstable Focus"
25    else:
26        return "Other"
27
28 #%% Run and plot
29 sigmaList = [-1, 0, 1]
30 x = np.linspace(-2, 2, 20)
31 y = np.linspace(-2, 2, 20)
32 X, Y = np.meshgrid(x, y)
33
34 # Create subplots: 1 row, 3 columns
35 fig, axes = plt.subplots(1, 3, figsize=(18, 6))
36 fig.suptitle("Trajectories for Different Sigma Values")
37
38 for idx, sigma in enumerate(sigmaList):
39     A_sigma = np.array([[sigma + 3, 4], [-9/4, sigma - 3]])
40     dX, dY = np.zeros(X.shape), np.zeros(Y.shape)
41
42     # Calculate derivatives
43     for i in range(X.shape[0]):
44         for j in range(X.shape[1]):
45             dx, dy = derivative(X[i, j], Y[i, j], A_sigma)
46             magnitude = np.sqrt(dx**2 + dy**2)
47             if magnitude > 0:
48                 dX[i, j], dY[i, j] = dx / magnitude, dy / magnitude
49
50     # Plot vector field in the corresponding subplot
51     ax = axes[idx]
```

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52     ax.streamplot(X, Y, dX, dY)
53     ax.set_title(f"Sigma = {sigma}")
54     ax.set_xlabel("x")
55     ax.set_ylabel("y")
56     ax.grid()
57
58     # Stability analysis
59     eigenvalues, _ = np.linalg.eig(A_sigma)
60     stability = classify_stability(eigenvalues)
61     print(f"Sigma = {sigma}: {stability}")
62
63 # Adjust layout
64 plt.tight_layout()
65 plt.show()
```