Notes on linear attenuation coefficient

Method note

MCT-033

This document summarizes two issues: how to assess reconstruction results in terms of "linear attenuation coefficient" (LAC, also denoted as μ), and where the negative values in a reconstruction may come from. The two topics are actually unrelated.

1. How to assess reconstruction results in terms of "linear attenuation coefficient" (LAC)?

1.1 Theoretical explanation: which x-ray energy to look up

Indeed, X-ray CT is based on the fact that different materials have different attenuation coefficient μ, which depends also on the x-ray photon energy E used for measurement. The reconstructed images are represented in terms of μ (1/mm). However, micro-CT scanners, especially the ones with laboratory x-ray tubes, are not the idea tools to measure LAC due to a number of reasons. A major problem is that the x-ray source is polychromatic. Therefore, the measured μ-values with micro-CT are only roughly at correct range.

When one tries to find the relation between the measured μ -values with micro-CT and the theoretical μ values, one has to be aware of the energy of x-ray photons being used for measurement, as μ is in function of x-ray photon energy. An example of μ(E) for aluminum (Al) can be found in figure 1.

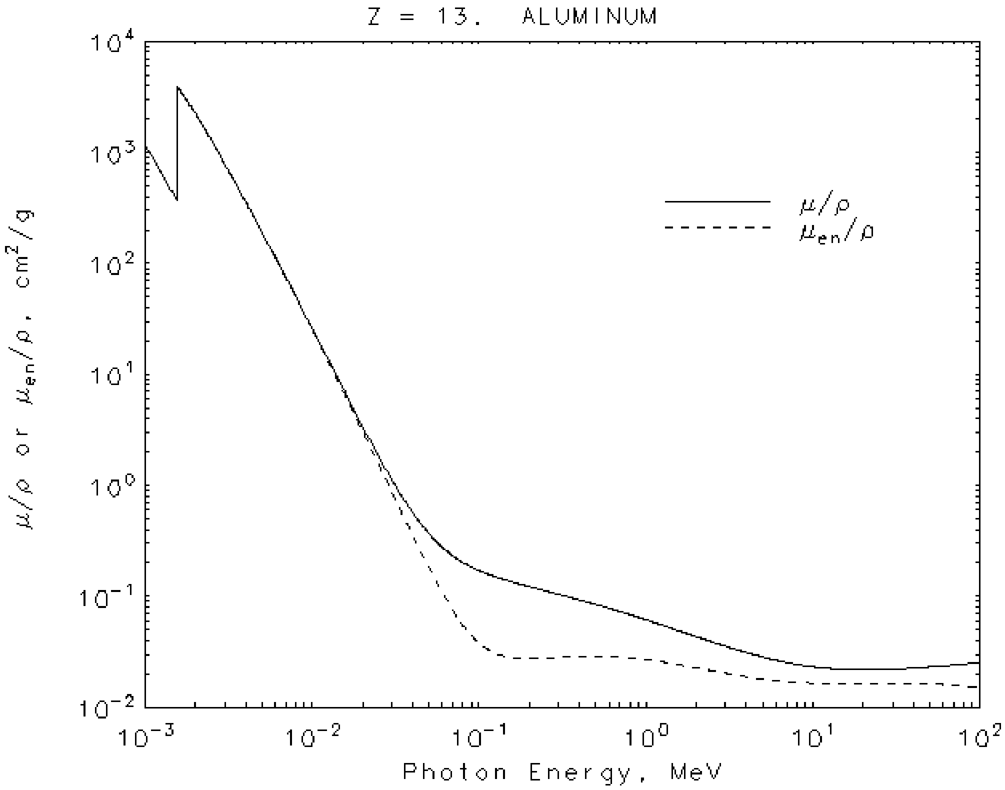

Figure 1. Illustration of μ in function of x-ray photon energy. Copied from http://physics.nist.gov/PhysRefData/XrayMassCoef/ElemTab/z13.html.

For micro-CT measurement, one should take care not to confuse the tube voltage with x-ray photon energy. The tube voltage determines only the maximum x-ray photon energy. It is the "effective x-ray energy" of the spectrum which determines the measured value. Simple explained: the effective energy is the (weighted) mean energy of x-rays which have been used for tomography imaging. This depends not only on the tube spectrum (which is also altered differently in each angular direction), but also on the detector efficiency (also a function of x-ray energy). And the very low-energy x-rays which are totally absorbed in the sample would not contribute to imaging and thus has to be excluded. In practice, it is fairly difficult to determine the effective energy of a certain setup. The characteristic energy of the target material in the source is only a reasonable approximation, and this is much lower than the tube voltage would suggest! To illustrate this, see figure 2 for x-ray spectrum of a tungsten-target tube and figure 3 for energy detection efficiency of a typical scintillator often used for micro-CT.

Please note: this is illustration only, thus not necessarily corresponds to a particular scanner type from Bruker microCT.

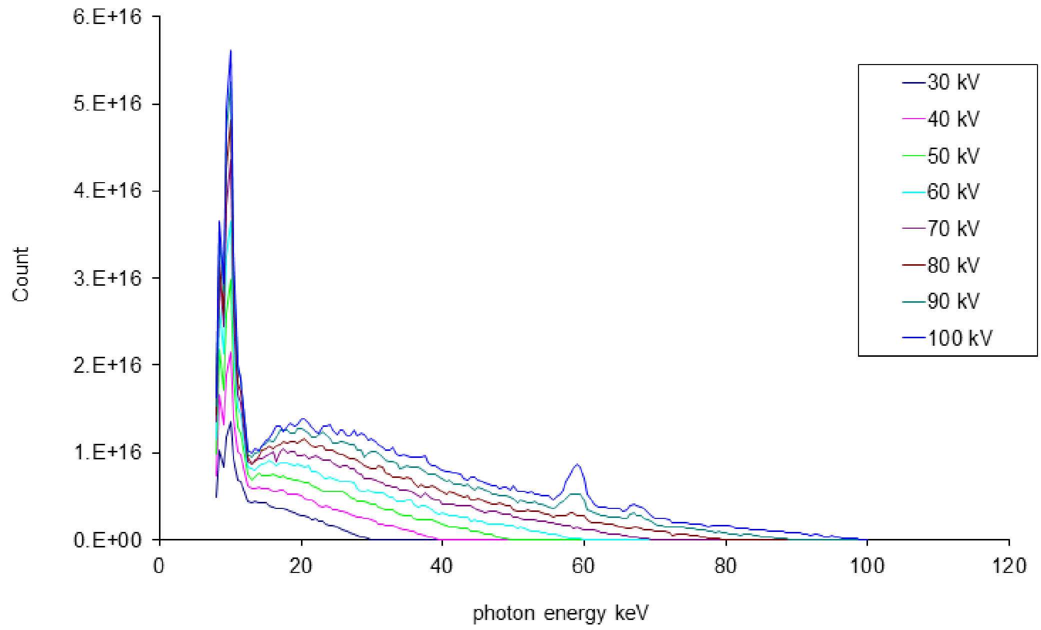

Figure 2. Photon energy spectrum emitted from tungsten target at different tube voltages. (Peter Hammersberg, etc., Linkoping Sweden, 1998).

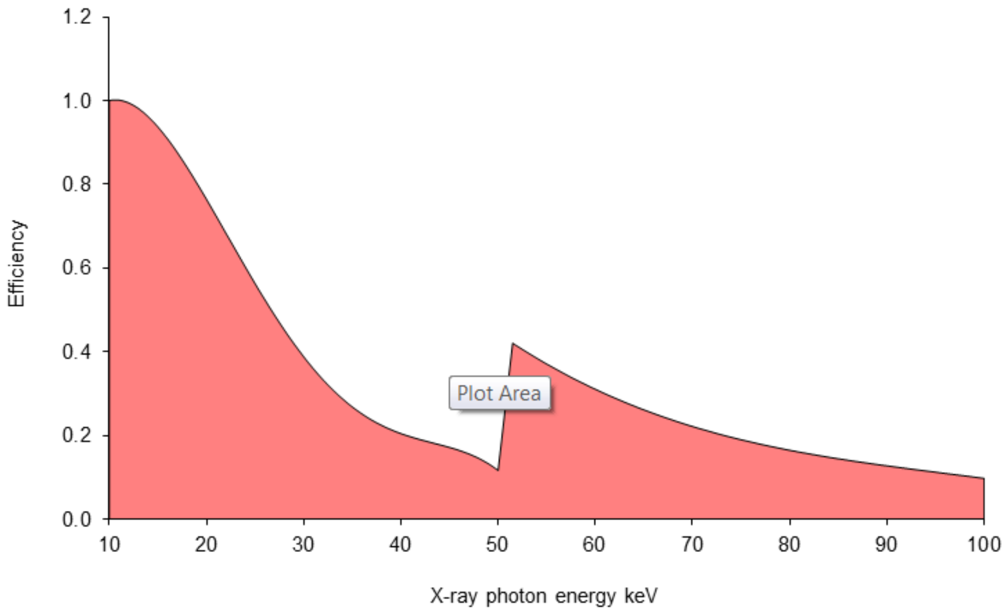

Figure 3. Gadox scintillator efficiency in function of x-ray energy.

In short, when one wants to compare the measured µ-value with a tube with spectrum in figure 2 (effective energy should be in the range roughly between 15 and 30 KeV), one should look up the theoretical values (such as that shown in figure 1 for aluminum) at this effective energy.

1.2. Practical issue: how to convert reconstructed images from grey value to µ-values in 1/mm

More specifically, for a reconstruction done with NRecon, a few steps have to be done before one gets the measured µ-values in 1/mm. The grey values have to be converted back to real numbers using the conversion ranges used during reconstruction, see the FAQ list in NRecon HLP file. As explained in the HLP file, do not forget to apply the 2π factor to obtain the measured µ-values in 1/mm! This scaling factor is omitted in NRecon (in the convolution kernel), consequently, also in other software (DataViewer, CTan) when the term "attenuation coefficient" is used. As we do not really use the AC as an absolute term, for backwards compatibility reasons and to avoid confusions, this factor is omitted.

1.3 Yet other factors which influence the measured µ-values

The reconstruction software NRecon calculates faithfully the µ-values according to the input. This can be understood as such: if one simulates a digital object with certain µ values by calculating mathematically exact shadow images, then do a reconstruction, the output will bring the original µ values back (except the 2π factor!).

However, a few factors will still distort the accuracy. They are listed here without further explanation: most of them are quite evident. If the air intensity cannot be determined properly (e.g., saturation due to too long exposure, or large object), or the object is larger than field-of-view (this is called truncation, and it causes data inconsistency in terms of reconstruction, resulting in biased quantities in the output), or severe beam-hardening effect, and etc., the reconstructed results are then distorted.

1.4. Why are there negative values after reconstruction?

Indeed, there shouldn't be any negative values, as a negative attenuation coefficient wouldn't have a physical meaning.

To understand it, one has to go back to the filtered back-projection algorithm (FBP). A brief description of FBP without going deep into details: the original projections are first pre-processed (determine air intensity for applying logarithm, smoothing, ring correction, etc.), then filtered by a well-defined kernel (ramp filter with hamming window, this process is also known as convolution), then back-projected to form an image volume. During pre-processing, the positivity is ensured (force to zero if any negative values would occur due to one of the corrections). The back-projection is an additive process, which also guarantees positivity if the input has no negative values. The negative values come from the filtering (convolution) process. To understand it, it is better to look at the ramp filter (also called convolution kernel) in real space, as shown in figure 4. As one can see, it has negative undershoots at both sides of the central peak. This can potentially introduce negative values.

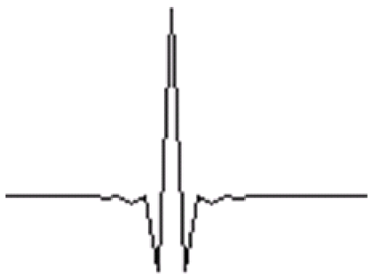

Figure 4. Central part of the ramp filter (convolution kernel): note the negative undershoots at both sides of the central peak.

The ramp filter comes from strictly exact mathematical derivations. If data are consistent, the negative values would be completely cancelled out and the reconstruction will not contain negative values at all.

Figure 5 illustrates what we mean with consistent data. As one can easily understand, inconsistency is however the reality: all measured data contain statistical errors and other errors such as inaccurate mechanical parts, mismatched flat-field correction, beam hardening effect, sample movement, spot movement, etc. These inconsistencies can cause negative values not being cancelled out completely. Looking at the shape of the ramp filter in figure 4, it is not difficult to understand that the negative values often appear at the boundaries of an object (e.g., the boundary of a circular object). Another extreme case is if the object is larger than the field-of-view: severe data inconsistency, as part of the object is seen in some views, but not in other views (coming in and out of sight in the projections).

(For those who want to know: the hamming window mentioned above, is used to reduce the effects of inconsistencies. This window is built in the convolution step, and it gives much lower weights to high-frequency components, which often correspond to measurement noise.)

Fortunately, the negative values tend to appear just outside of the object, or at the boundary of the filed-of-view. The negative values are not removed from reconstruction as they occasionally do give more indications. For most analysis, these negative values can be safely ignored. One can remove the negative values altogether in NRecon: Options->Preferences, and check the option “Default dynamic range: positive values only”.

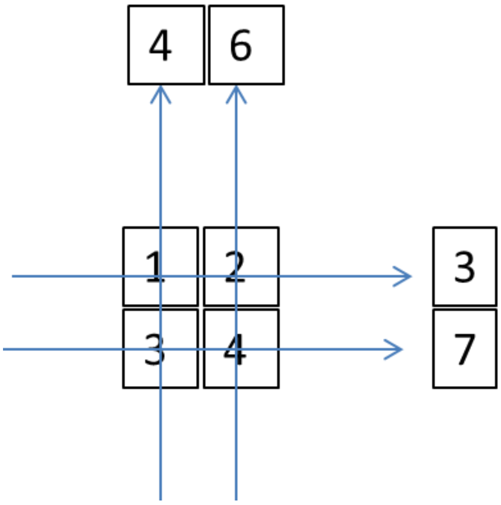

Figure 5. An illustration of consistent projection data: image is a 4-pixel matrix, 2 consistent projections (sum of pixels along the line) are depicted. If any pixel on any of the projection has a different value due to noise, the projections become inconsistent.