Programming III

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UADE

4 de noviembre de 2021

Programme

1. Review of the Previous Class

2. Branch and Bound
    - FIFO-BB
    - LIFO-BB
    - LC-BB

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2. Branch and Bound
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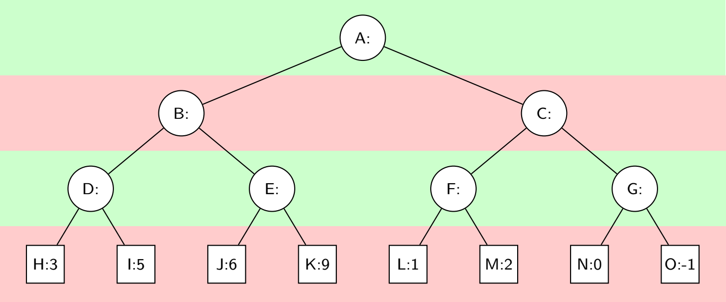
   - LIFO-BB
   - LC-BB

The Min-Max Algorithm. An Example



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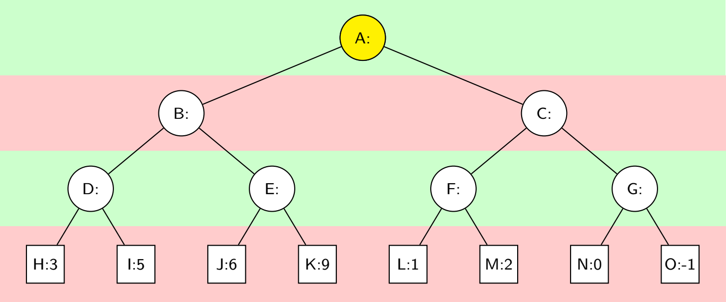
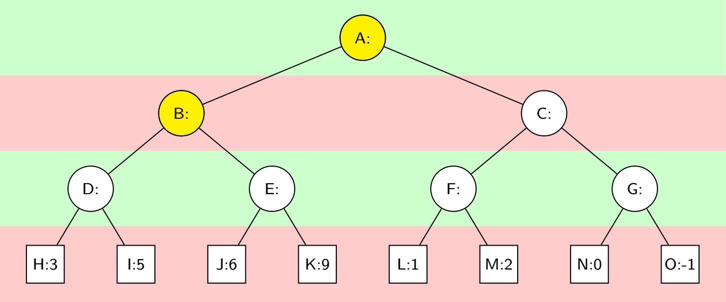


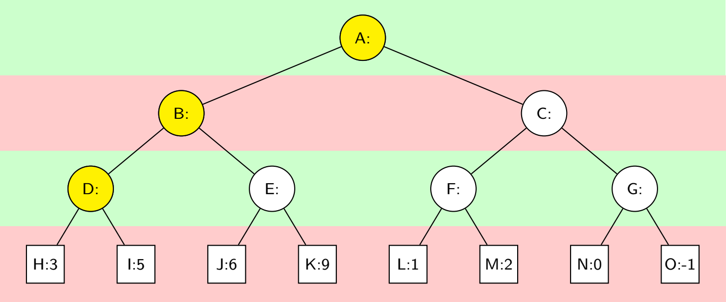
Max chooses the maximum between *B* and *C*; it calls first *B*.

The Min-Max Algorithm. An Example



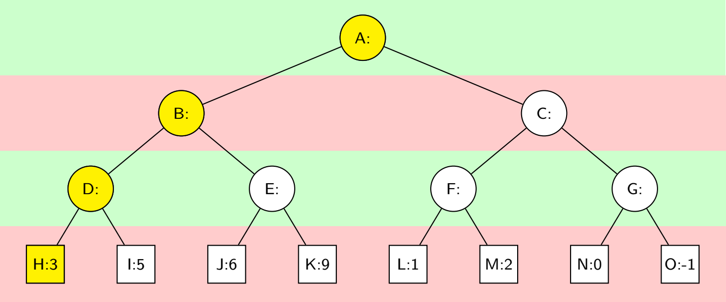
Min chooses the minimum between *D* and *E*; it calls first *D*.

The Min-Max Algorithm. An Example



Max chooses the maximum between *H* and *I*; it calls first *H*.

The Min-Max Algorithm. An Example



Node *H* yields 3.

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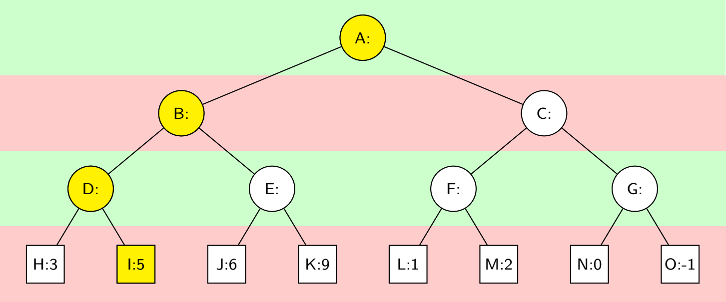

Node *I* yields 5.

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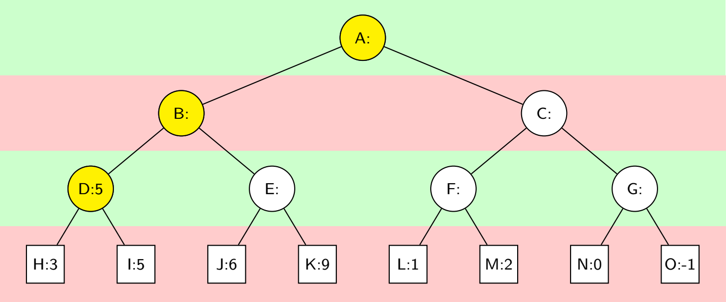
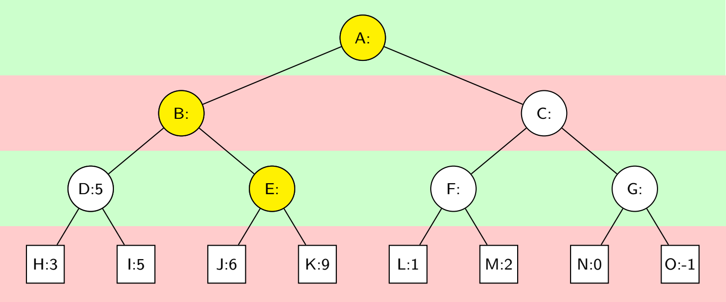


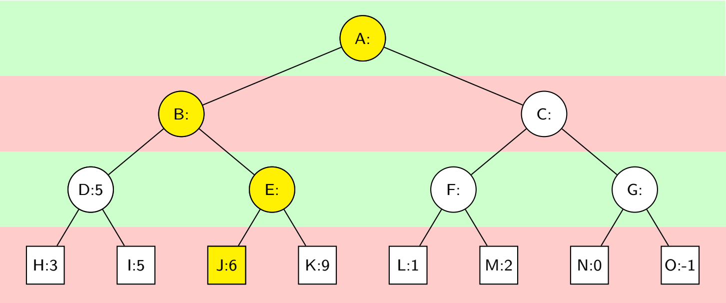
Node D yields $\texttt{max}(3,5) = 5$.

The Min-Max Algorithm. An Example



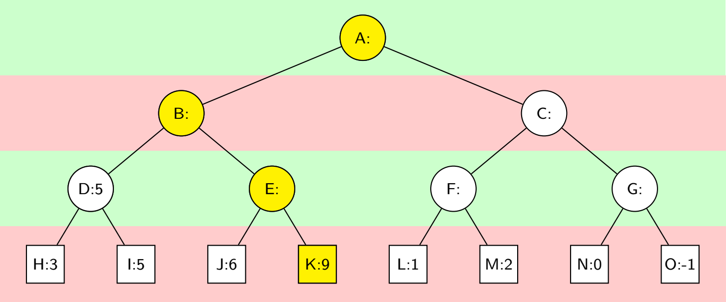
Max chooses the maximum between *J* and *K*; it calls first *J*.

The Min-Max Algorithm. An Example



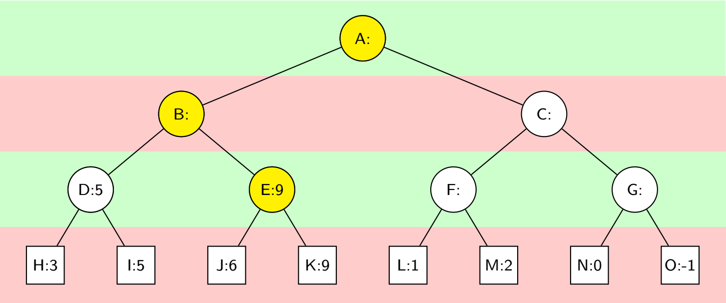
Node J yields 6.

The Min-Max Algorithm. An Example



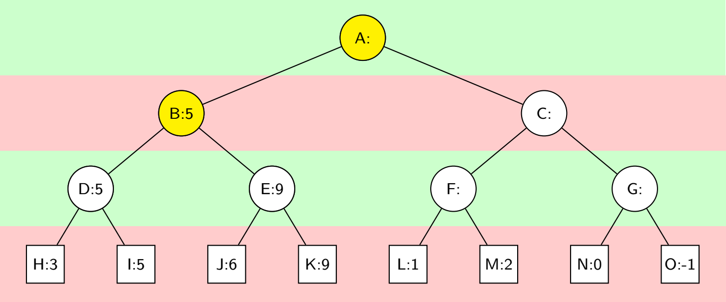
Node K yields 9.

The Min-Max Algorithm. An Example



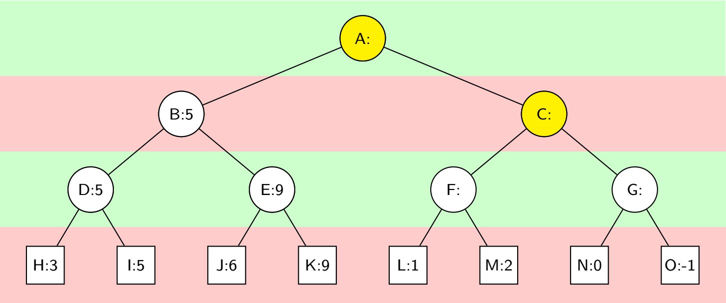
Node E yields $\mathtt{max}(6, 9) = 9$.

The Min-Max Algorithm. An Example



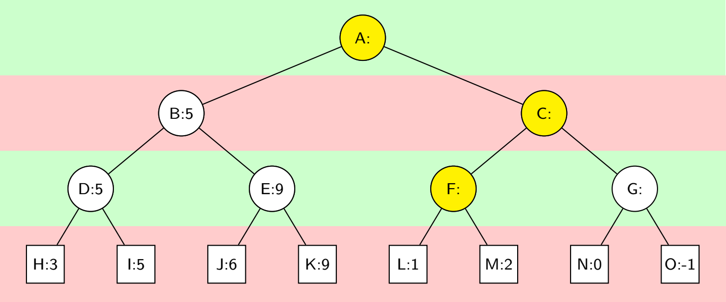
Node B yields $\mathtt{min}(5, 9) = 5$.

The Min-Max Algorithm. An Example



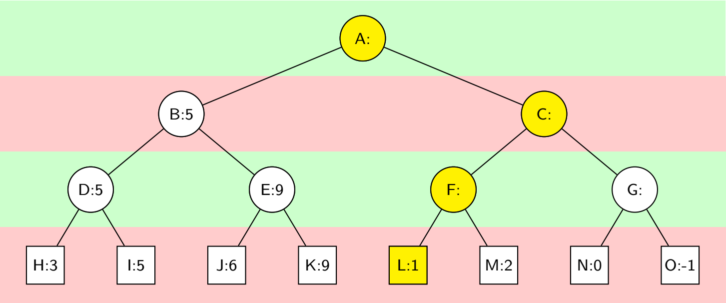
Min chooses the minimum between *F* and *G*; it calls first *F*.

The Min-Max Algorithm. An Example



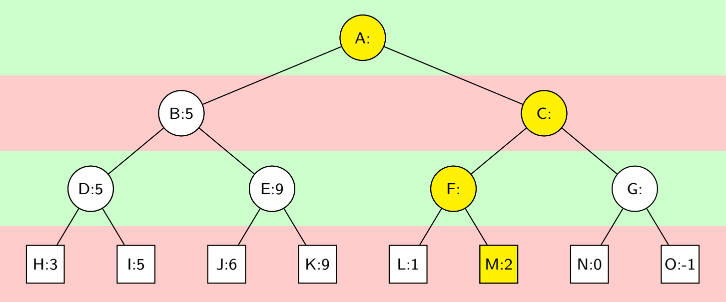
Max chooses the maximum between *L* and *M*; it calls first *L*.

The Min-Max Algorithm. An Example



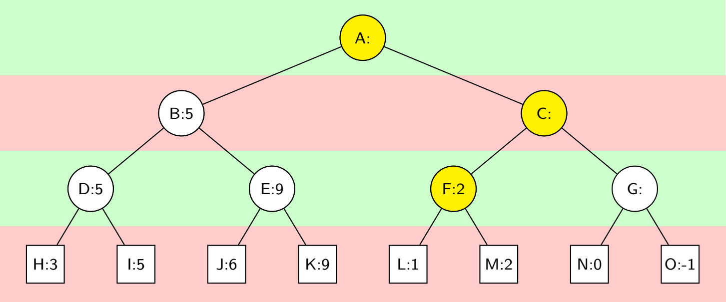
Node L yields 1.

The Min-Max Algorithm. An Example



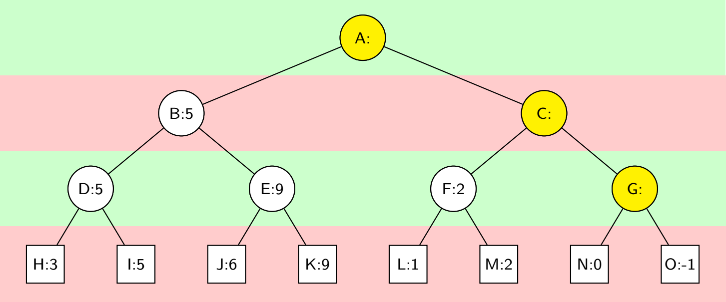
Node M yields 2.

The Min-Max Algorithm. An Example



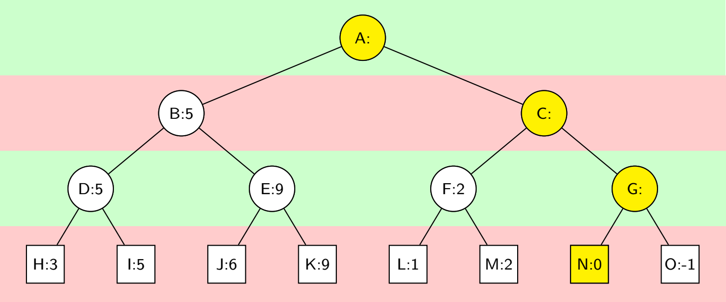
Node F yields $\mathtt{max}(1, 2) = 2$.

The Min-Max Algorithm. An Example



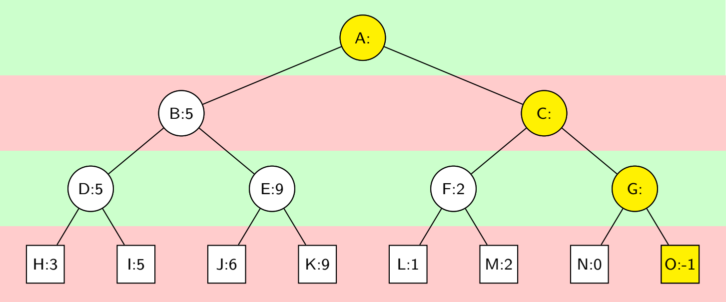
Max chooses the maximum between *N* and *O*; it calls first *N*.

The Min-Max Algorithm. An Example



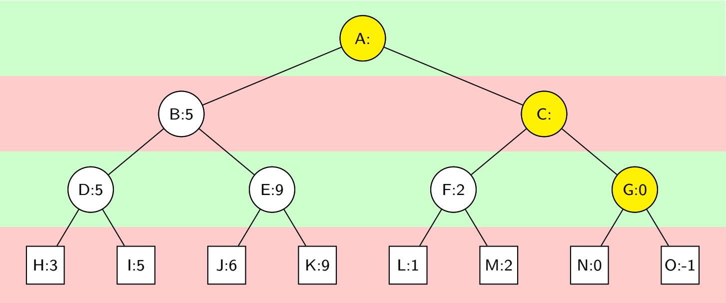
Node N yields 0.

The Min-Max Algorithm. An Example



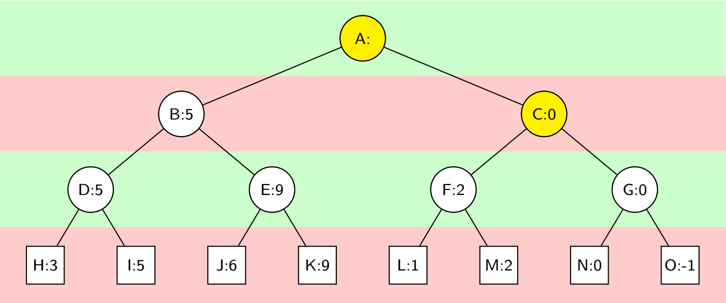
Node *O* yields -1.

The Min-Max Algorithm. An Example



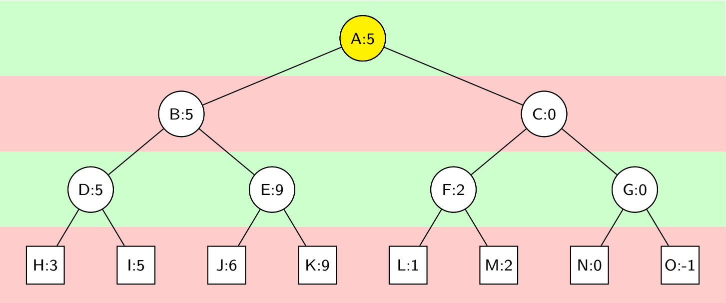
Node F yields $\max(0, -1) = 0$.

The Min-Max Algorithm. An Example



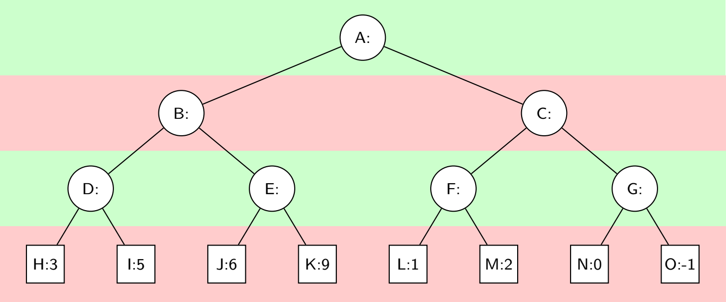
Node C yields $\mathtt{min}(2, 0) = 0$.

The Min-Max Algorithm. An Example



Node A yields $\texttt{min}(5, 0) = 5$.

Alpha–Beta Pruning. An Example



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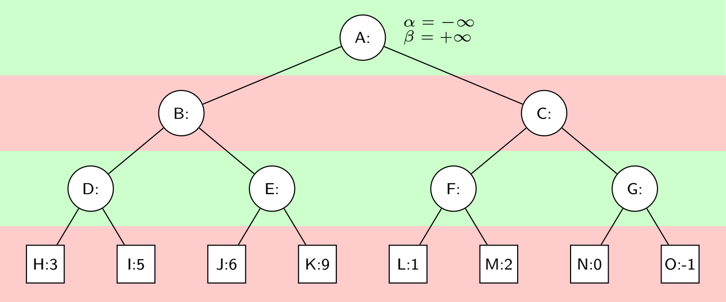
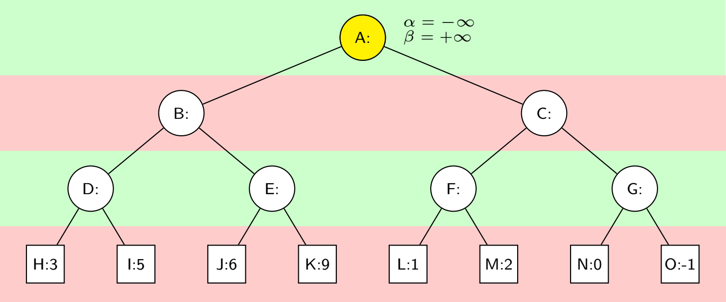


The process starts at A
The initial values are $\alpha = -\infty$ and $\beta = \infty$.

Alpha–Beta Pruning. An Example



Max chooses the maximum between *B* and *C*; it calls first *B*.

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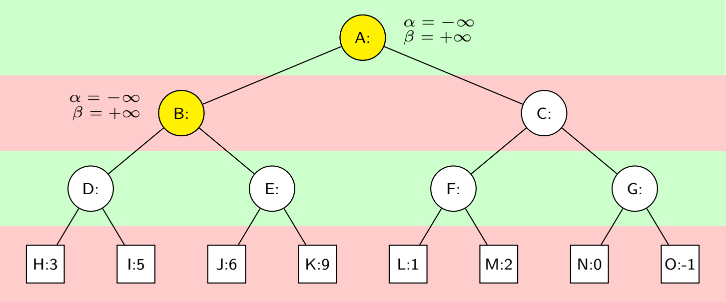
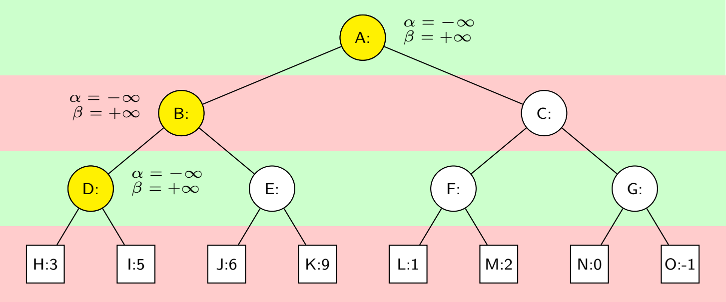


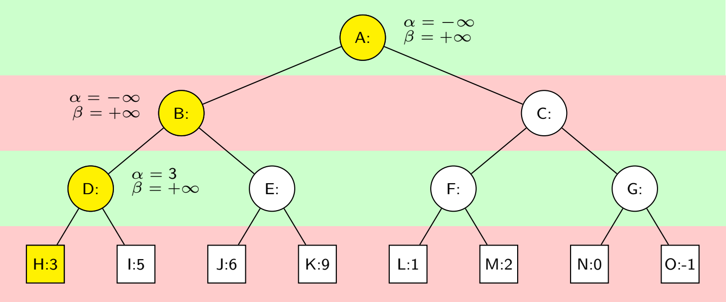
The values of α and β are passed downwards to *B*.
Min chooses the minimum between *D* and *E*; it calls first *D*.

Alpha–Beta Pruning. An Example



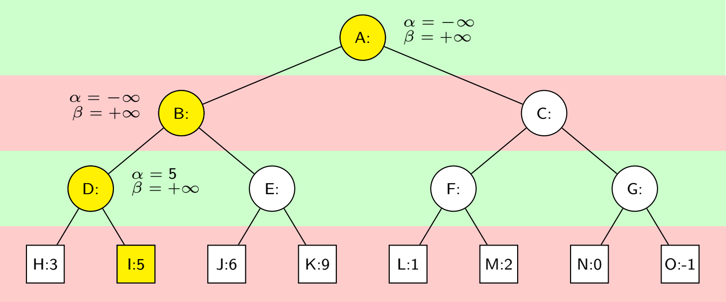
The values of α and β are passed downwards to *D*.
Max chooses the minimum between *H* and *I*; it calls first *H*.

Alpha–Beta Pruning. An Example



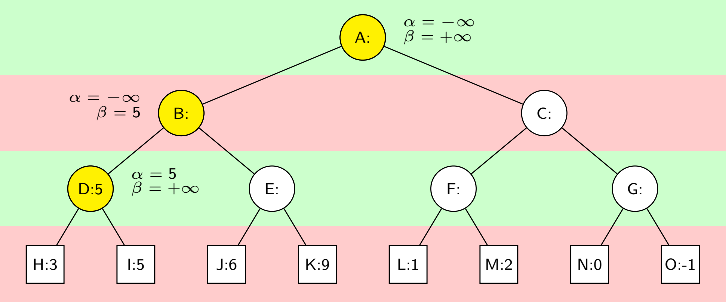
Node H yields 3.
The value of α at D is updated to 3.

Alpha–Beta Pruning. An Example



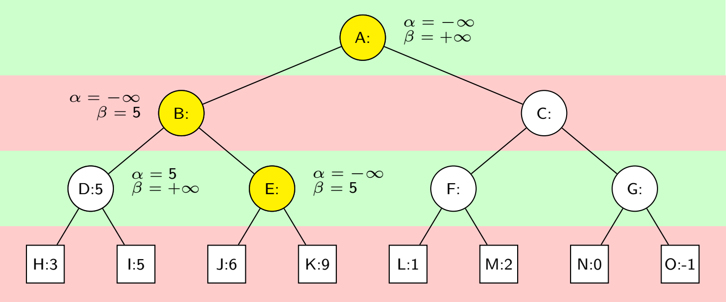
Node *I* yields 5.
The value of α at *D* is updated to 5.

Alpha–Beta Pruning. An Example



Node D yields $\texttt{max}(3, 5) = 5$.
The value of β at B is updated to 5.

Alpha–Beta Pruning. An Example



The values of α and β are passed downwards to *E*.
Max chooses the maximum between *J* and *K*; it calls first *J*.

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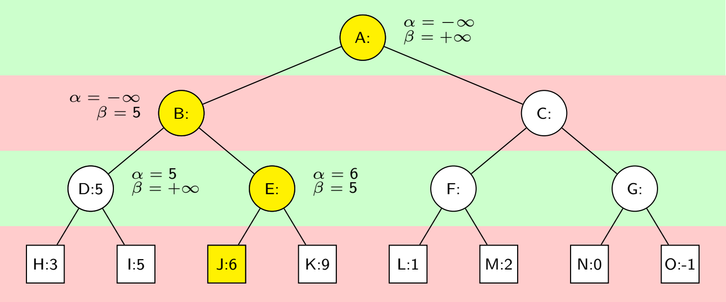
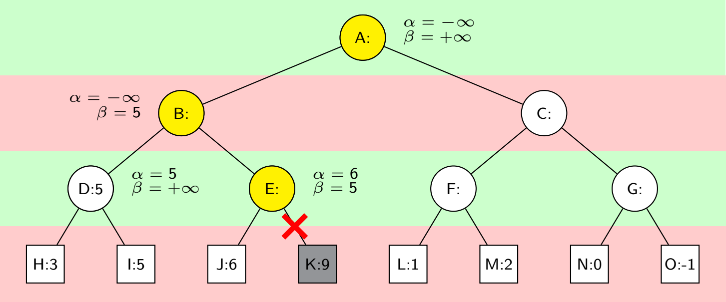


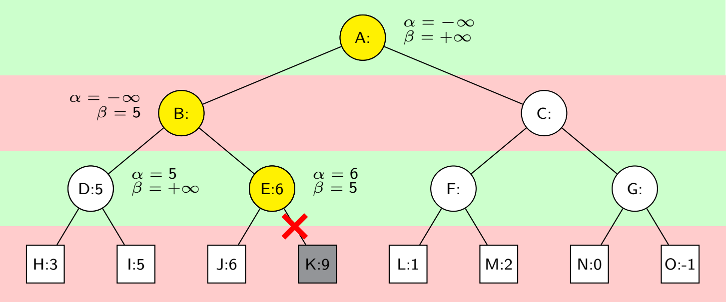
Node J yields 6.
The value of α at E is updated to 6.

Alpha–Beta Pruning. An Example



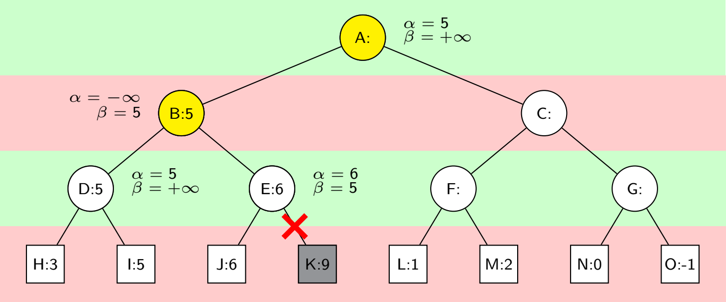
We have now $\alpha \geq \beta$
The exploration finishes. The branch is pruned.

Alpha–Beta Pruning. An Example



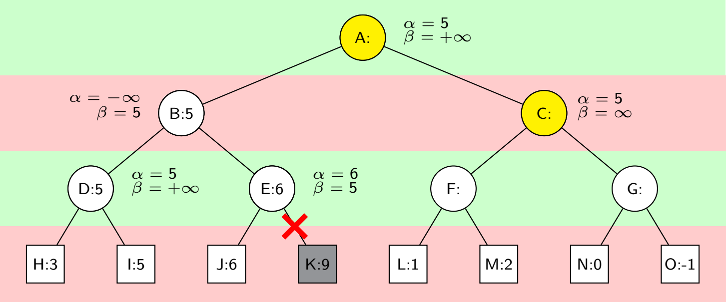
Node *E* yields 6.

Alpha–Beta Pruning. An Example



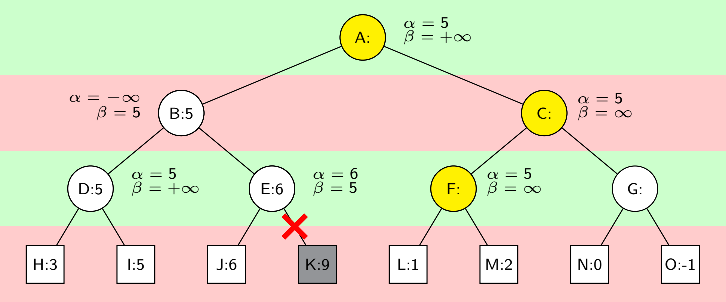
Node B yields $\mathtt{min}(5, 6) = 5$.
The value of α at A is updated to 5.

Alpha–Beta Pruning. An Example



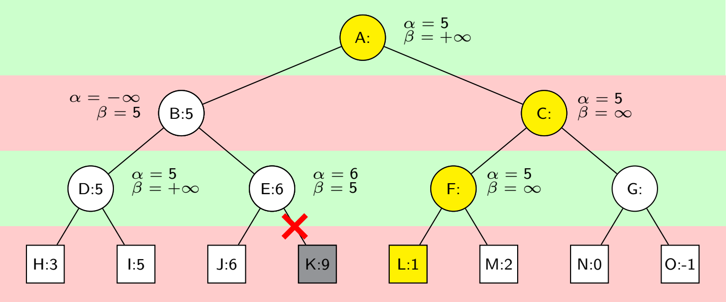
The values of α and β are passed downwards to *C*.
Min chooses the minimum between *F* and *G*; it calls first *F*.

Alpha–Beta Pruning. An Example



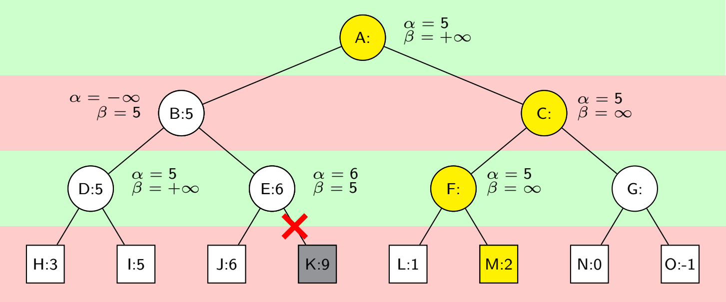
The values of α and β are passed downwards to F.
Max chooses the maximum between L and M; it calls first L.

Alpha–Beta Pruning. An Example



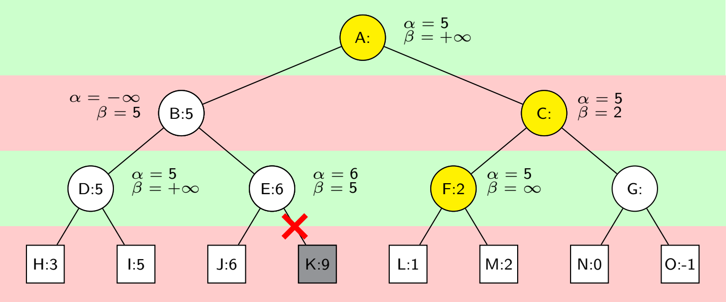
Node L yields 1.
The value of α does not change at F.

Alpha–Beta Pruning. An Example



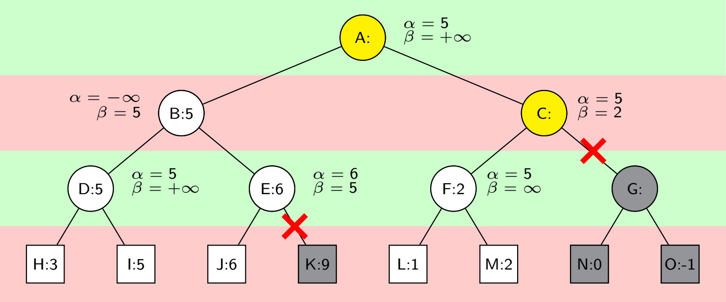
Node M yields 2.
The value of α does not change at F.

Alpha–Beta Pruning. An Example



Node F yields $\texttt{min}(1, 2) = 2$.
The value of β at C is updated to 2.

Alpha–Beta Pruning. An Example



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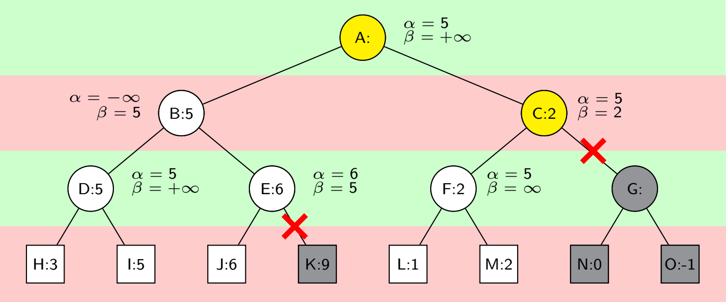
Alpha–Beta Pruning. An Example



Node C yields 2.

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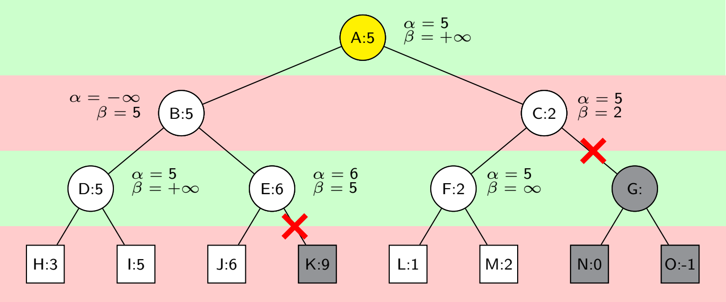

Node A yields $\mathtt{max}(5, 2) = 5$.

Branch and Bound

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- The number c represents the cost to reach the vertex; the top u is the minimum score that we have guaranteed at that point. It could improve but not get worse.
- if $u < upper$, the variable *upper* is updated to the new minimum guaranteed result.
- If $c > upper$, the branch is killed, since it not worth further exploring.

Branch and Bound: the Central Idea

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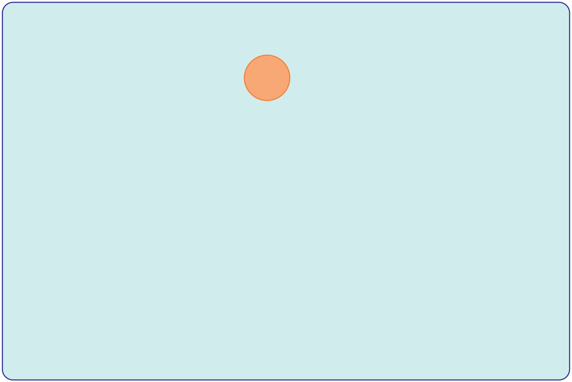

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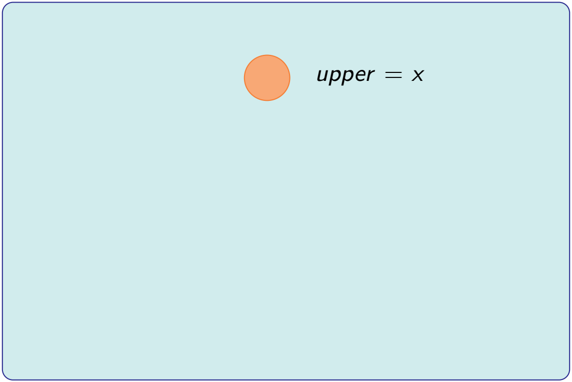

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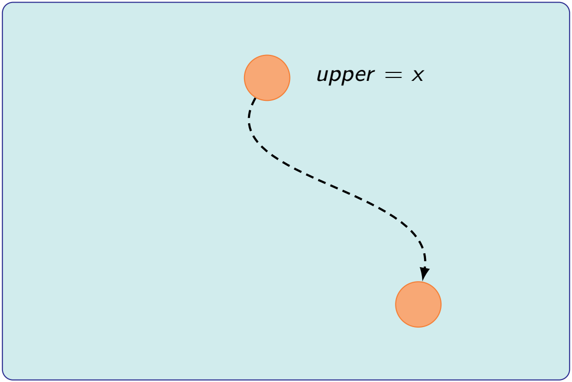

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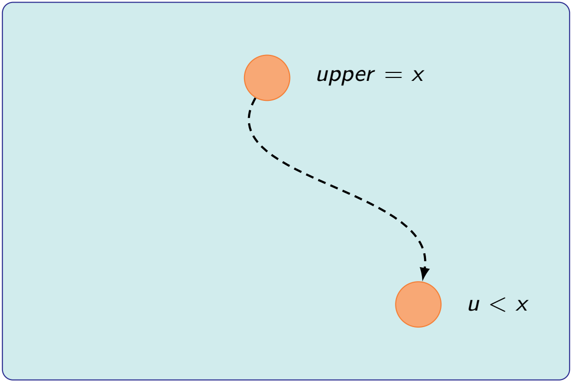

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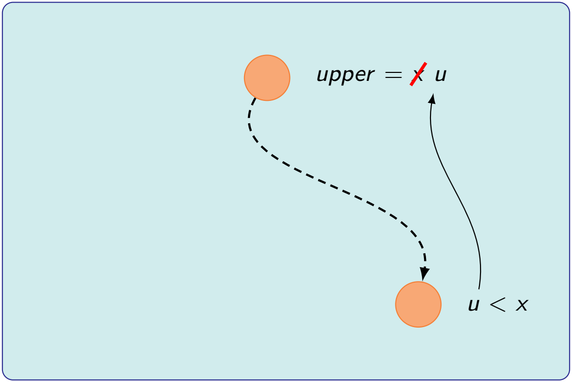

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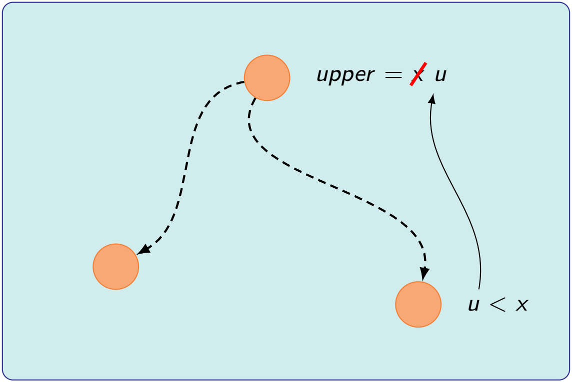

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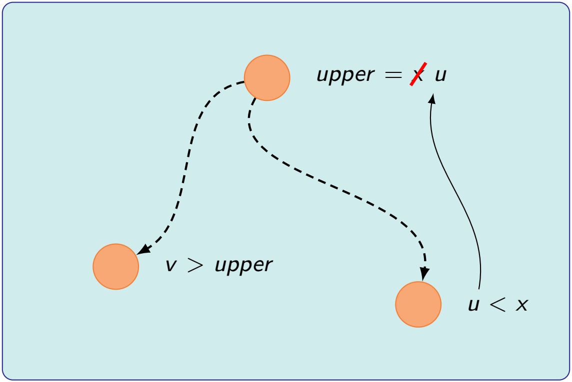

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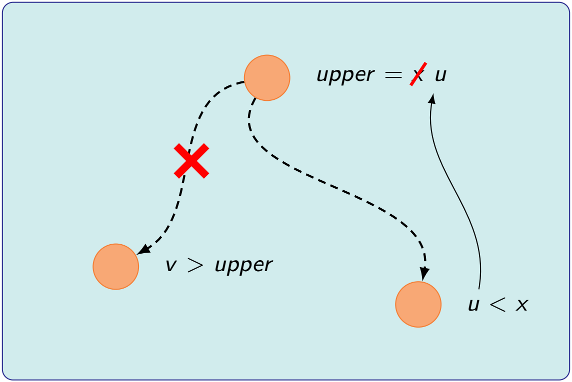

Types of Branch and Bound

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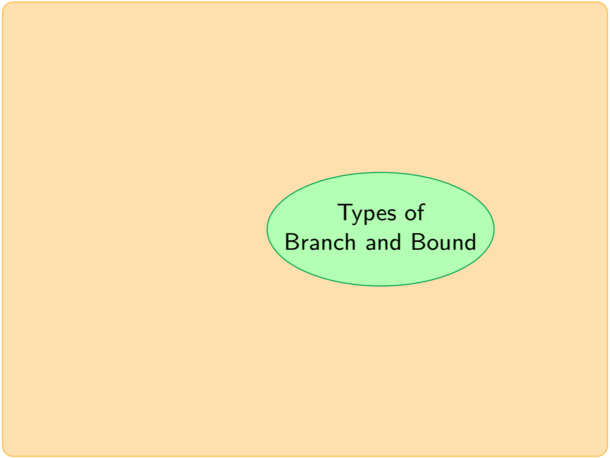

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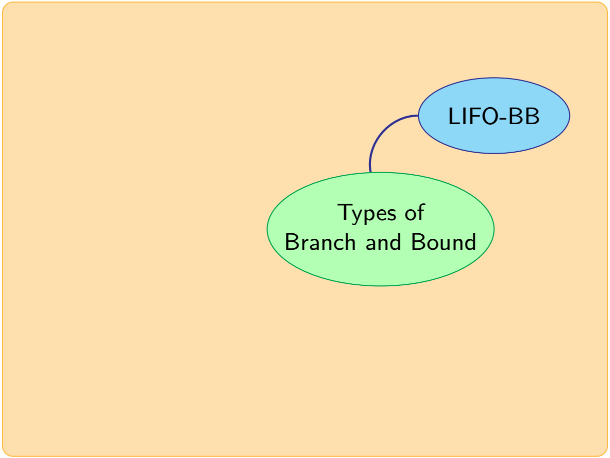

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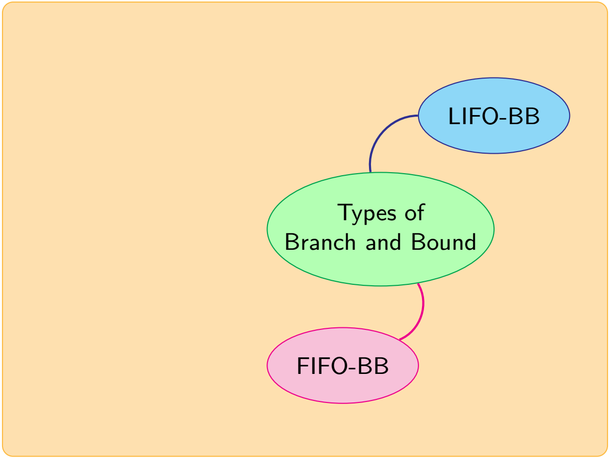

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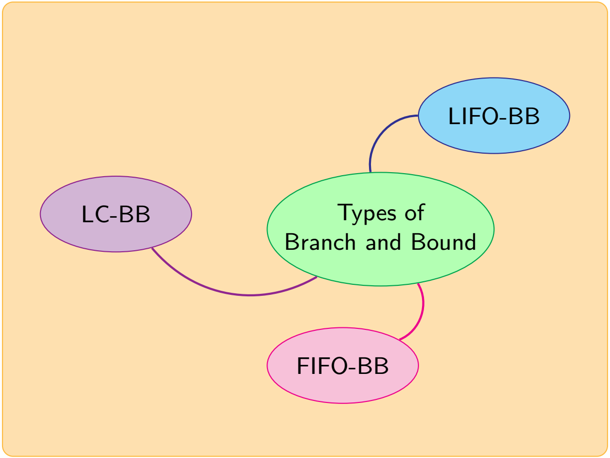

1. Review of the Previous Class

2. Branch and Bound
   - FIFO-BB
   - LIFO-BB
   - LC-BB

First-In-First-Out-Branch and Bound (FIFO-BB)

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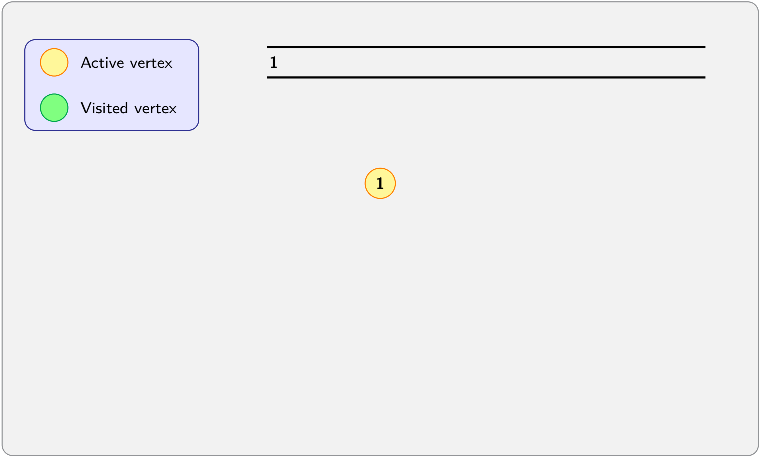
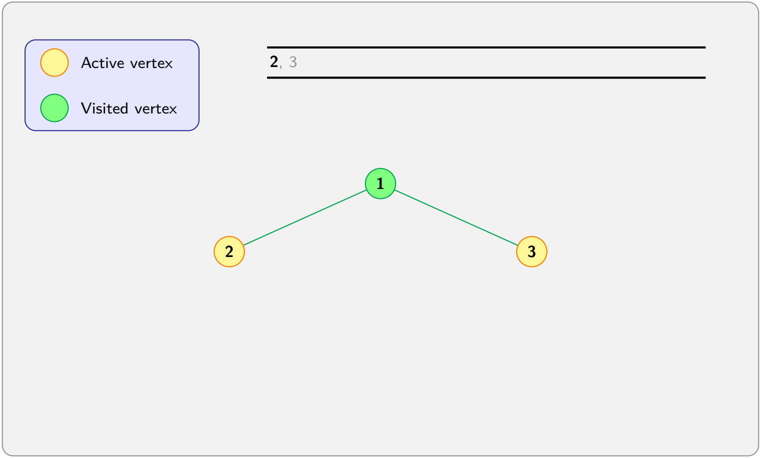
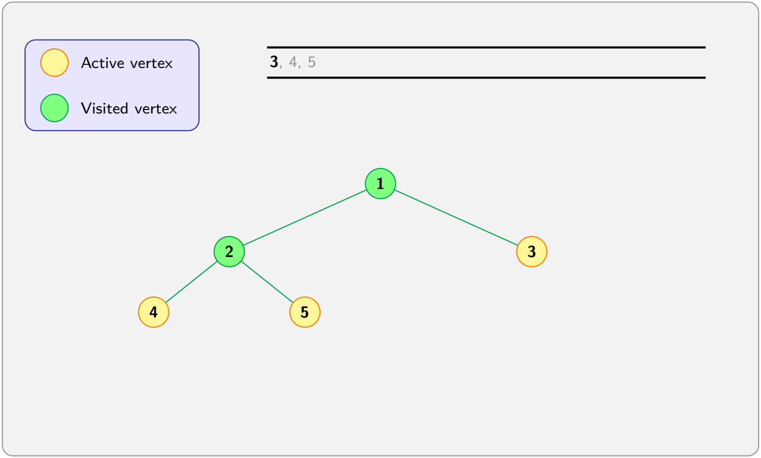


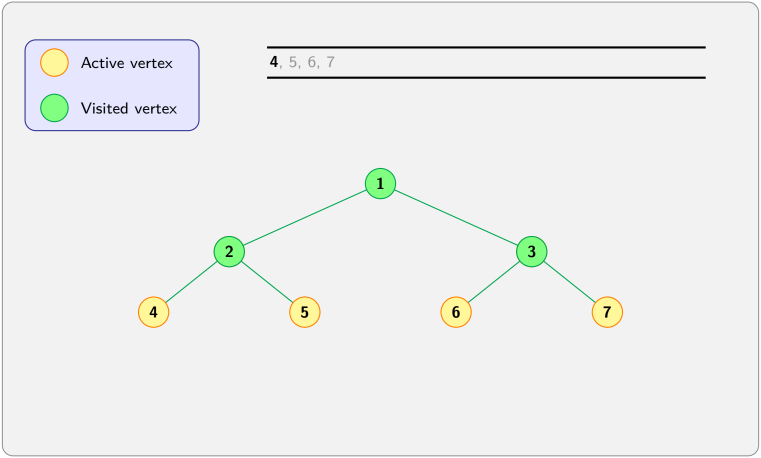
First-In-First-Out-Branch and Bound (FIFO-BB)



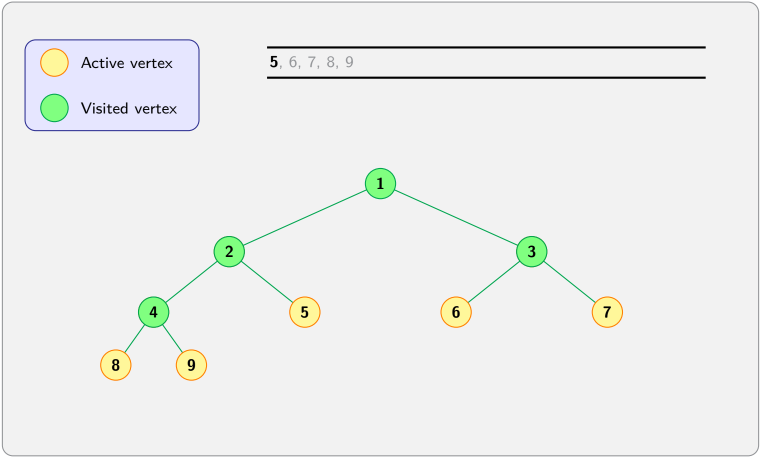
First-In-First-Out-Branch and Bound (FIFO-BB)



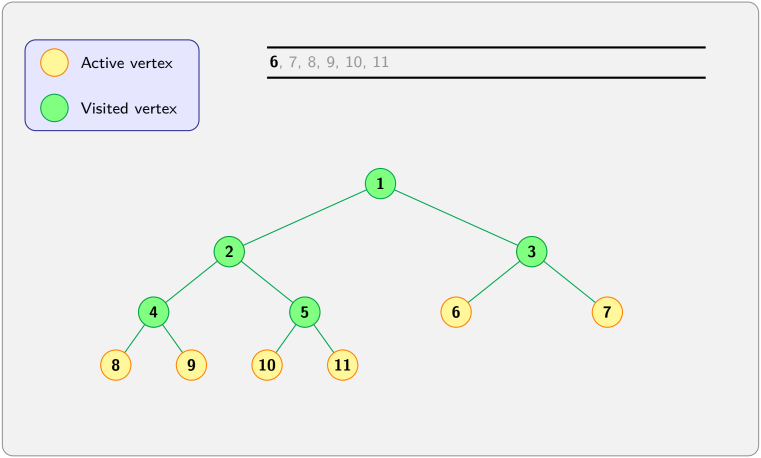
First-In-First-Out-Branch and Bound (FIFO-BB)



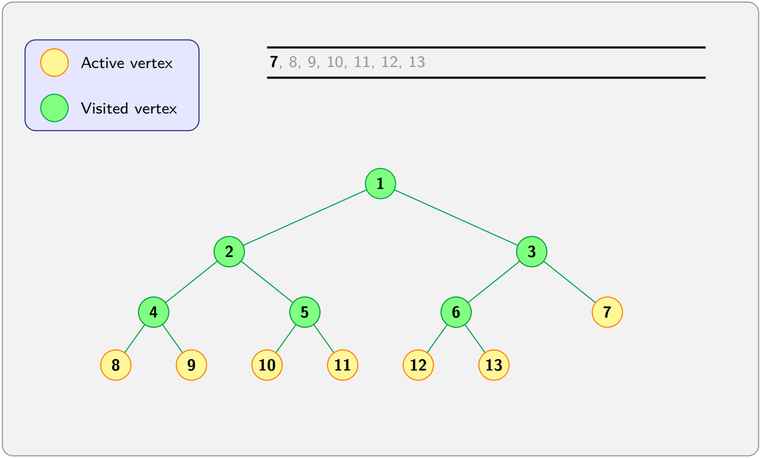
First-In-First-Out-Branch and Bound (FIFO-BB)



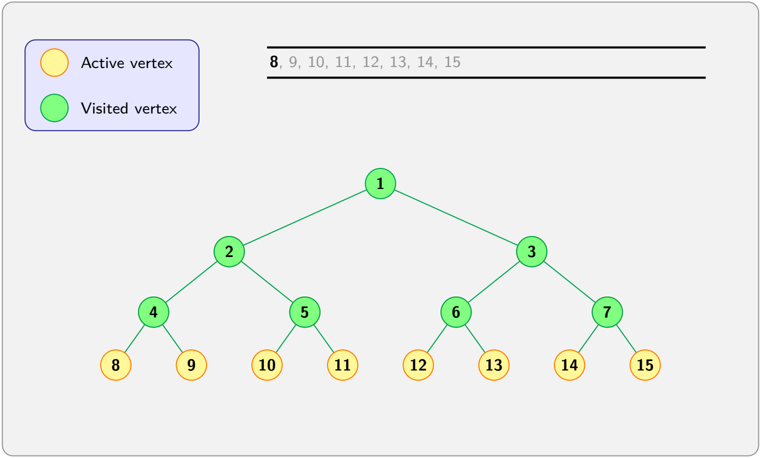
First-In-First-Out-Branch and Bound (FIFO-BB)



First-In-First-Out-Branch and Bound (FIFO-BB)



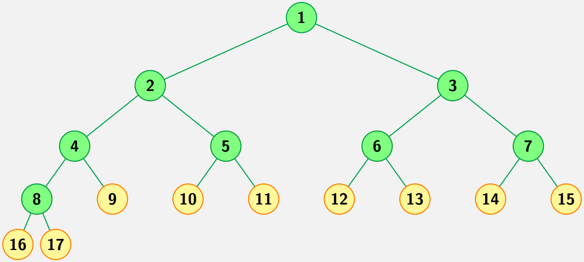
First-In-First-Out-Branch and Bound (FIFO-BB)



First-In-First-Out-Branch and Bound (FIFO-BB)

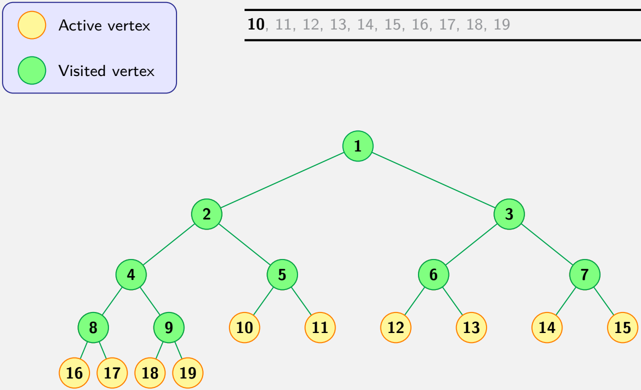
Active vertex
Visited vertex

9, 10, 11, 12, 13, 14, 15, 16, 17

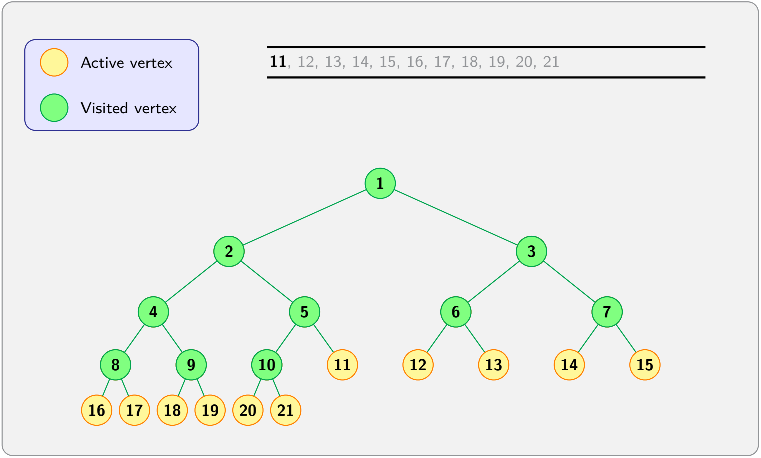


First-In-First-Out-Branch and Bound (FIFO-BB)

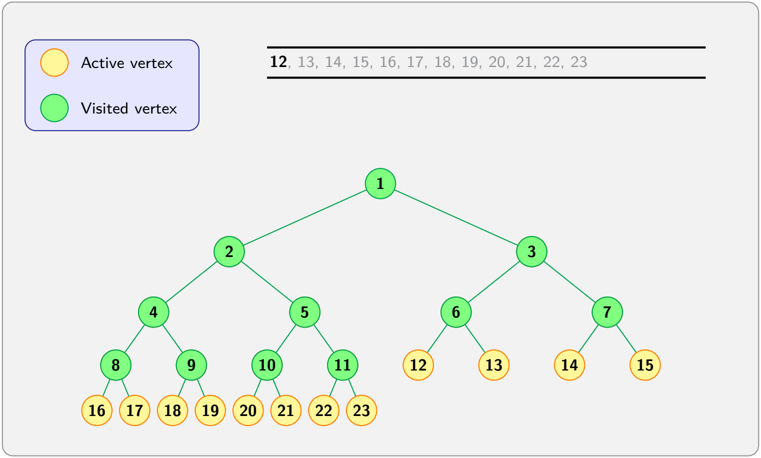
10, 11, 12, 13, 14, 15, 16, 17, 18, 19



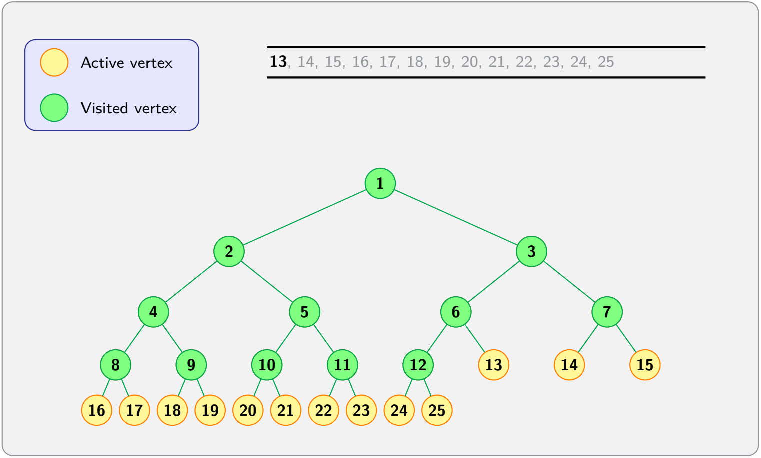
First-In-First-Out-Branch and Bound (FIFO-BB)



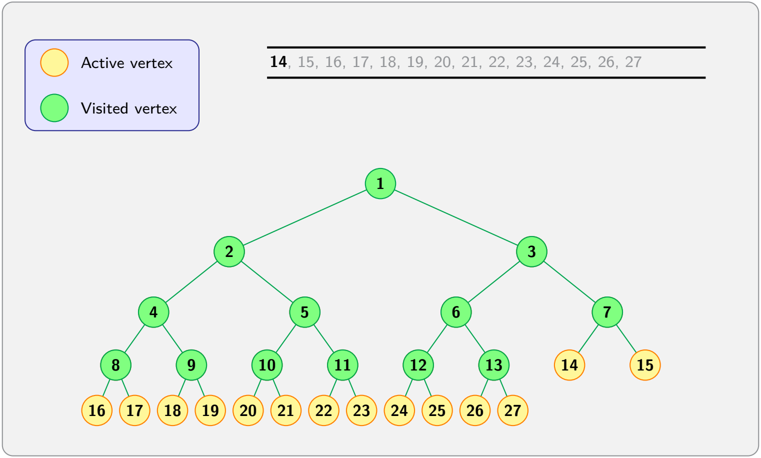
First-In-First-Out-Branch and Bound (FIFO-BB)



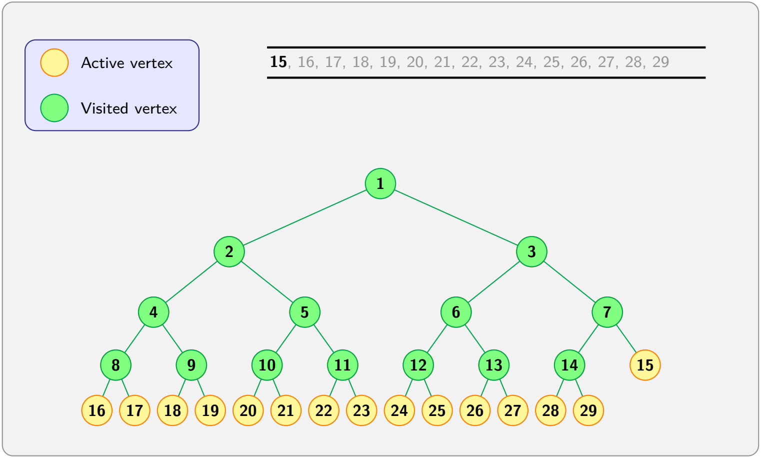
First-In-First-Out-Branch and Bound (FIFO-BB)



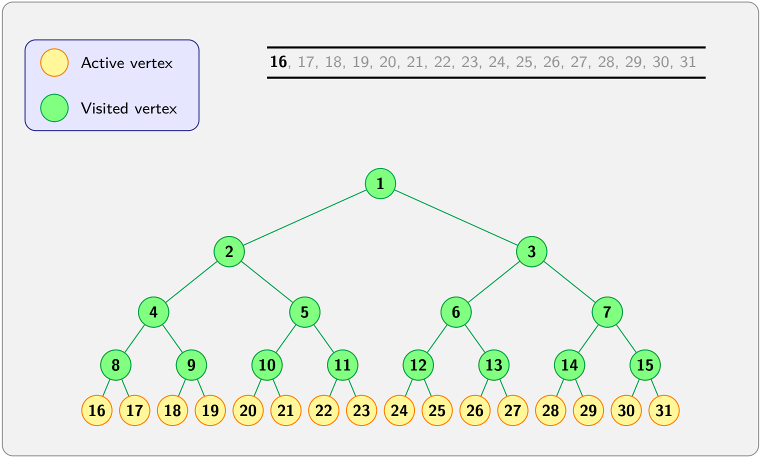
First-In-First-Out-Branch and Bound (FIFO-BB)



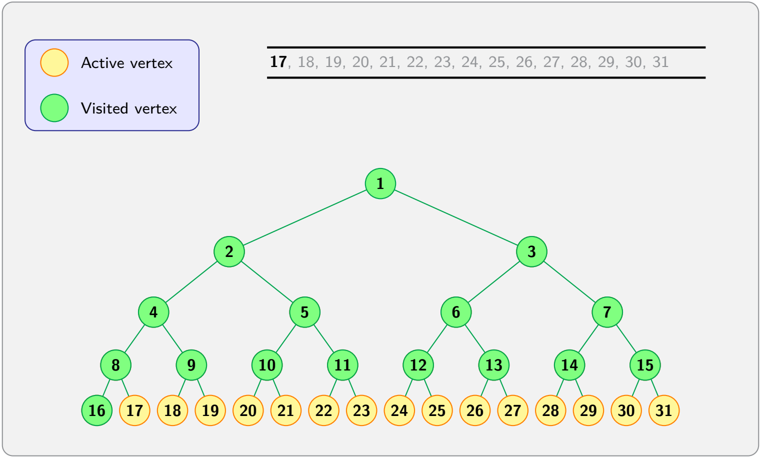
First-In-First-Out-Branch and Bound (FIFO-BB)



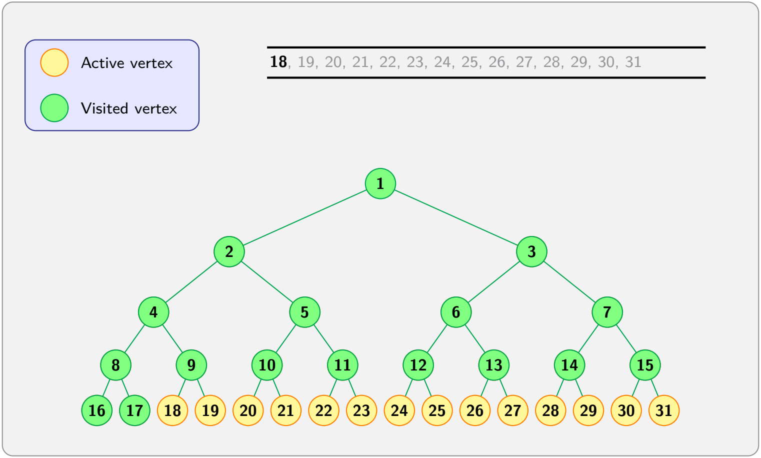
First-In-First-Out-Branch and Bound (FIFO-BB)



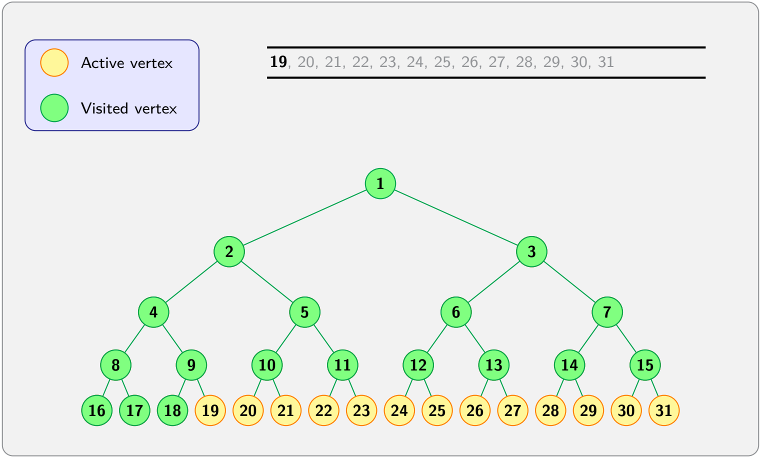
First-In-First-Out-Branch and Bound (FIFO-BB)



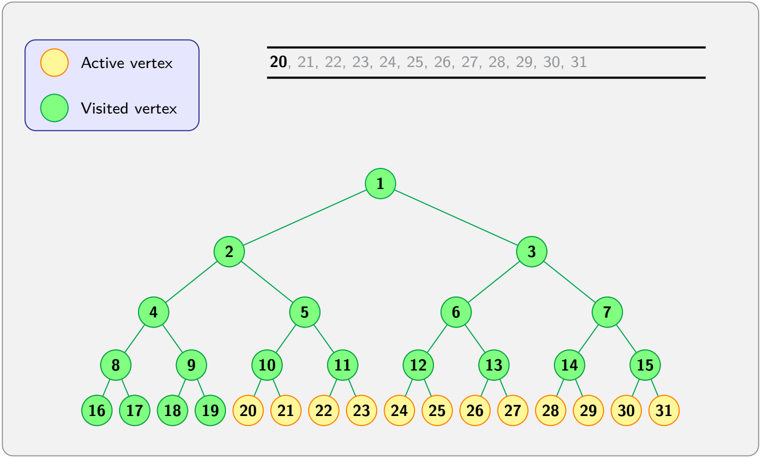
First-In-First-Out-Branch and Bound (FIFO-BB)



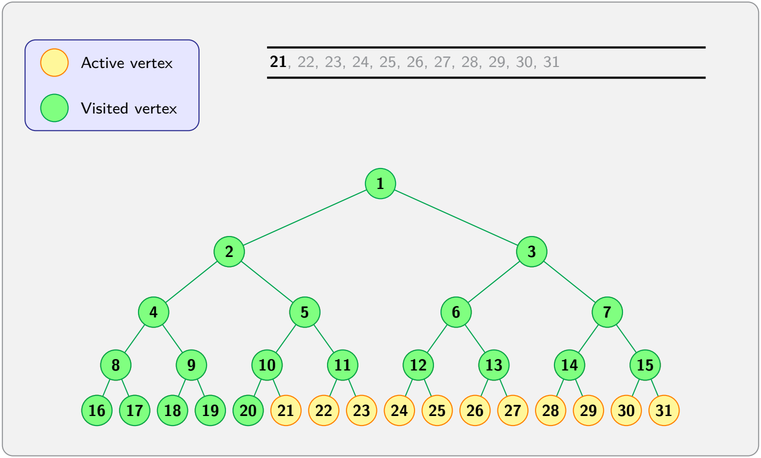
First-In-First-Out-Branch and Bound (FIFO-BB)



First-In-First-Out-Branch and Bound (FIFO-BB)



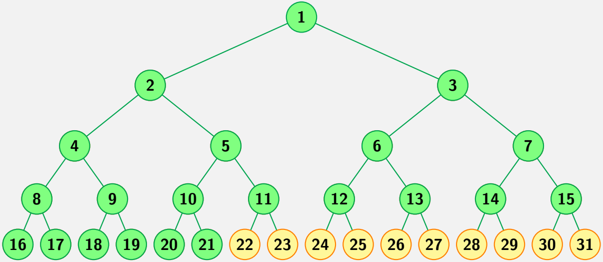
First-In-First-Out-Branch and Bound (FIFO-BB)



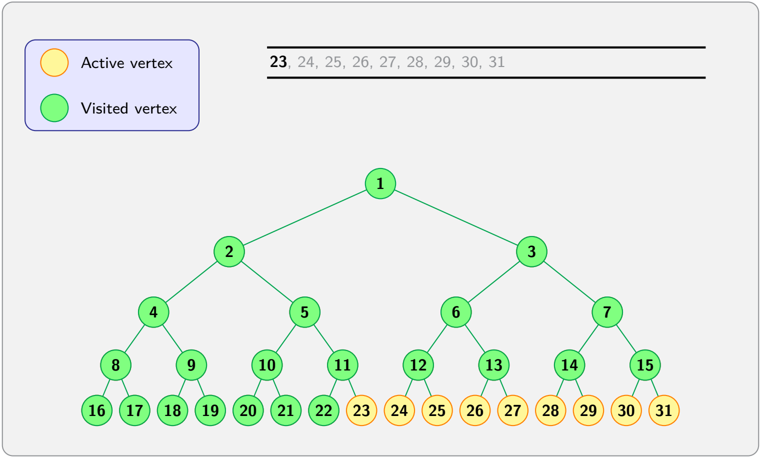
First-In-First-Out-Branch and Bound (FIFO-BB)

Active vertex
Visited vertex

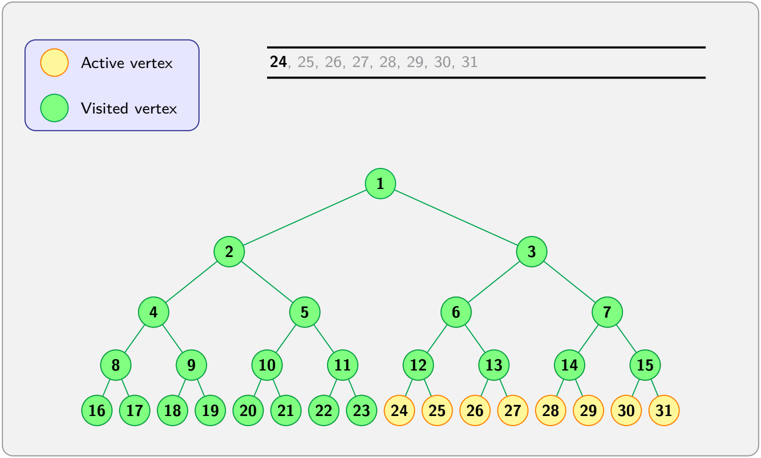
22, 23, 24, 25, 26, 27, 28, 29, 30, 31



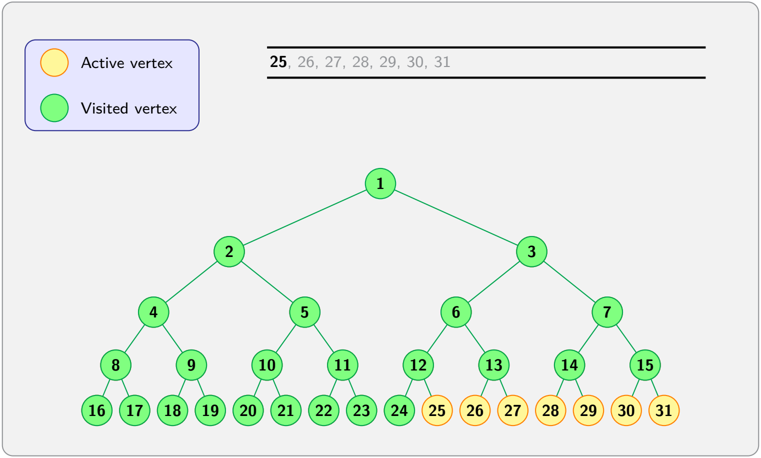
First-In-First-Out-Branch and Bound (FIFO-BB)



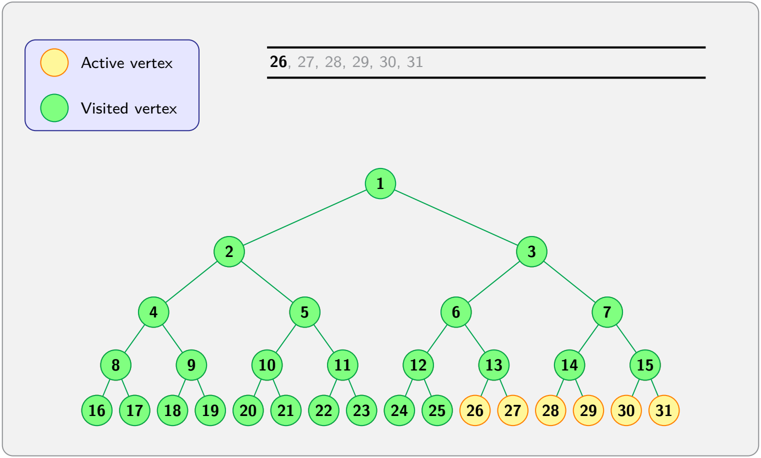
First-In-First-Out-Branch and Bound (FIFO-BB)



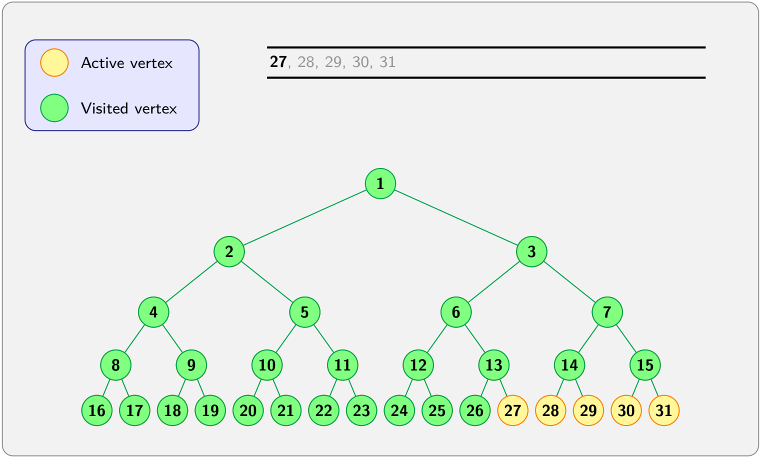
First-In-First-Out-Branch and Bound (FIFO-BB)



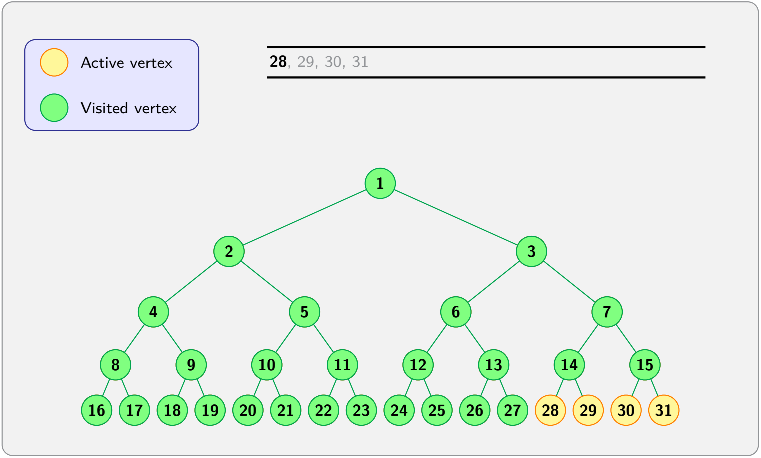
First-In-First-Out-Branch and Bound (FIFO-BB)



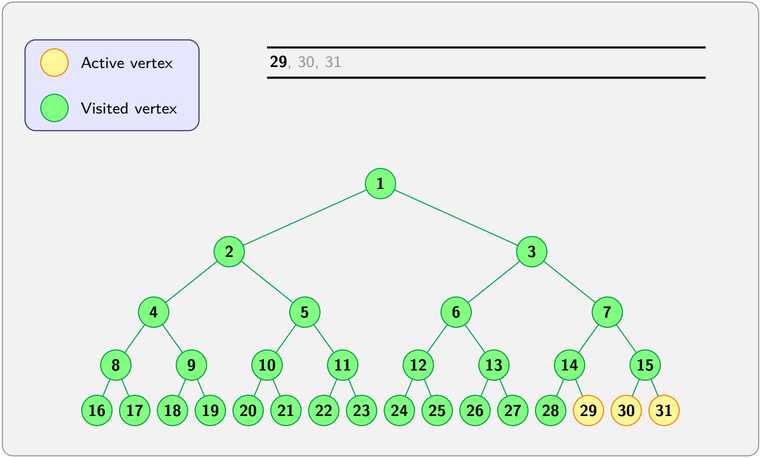
First-In-First-Out-Branch and Bound (FIFO-BB)



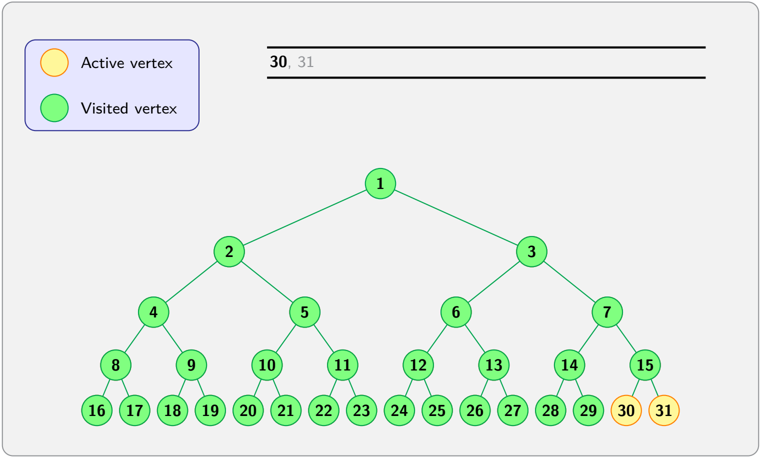
First-In-First-Out-Branch and Bound (FIFO-BB)



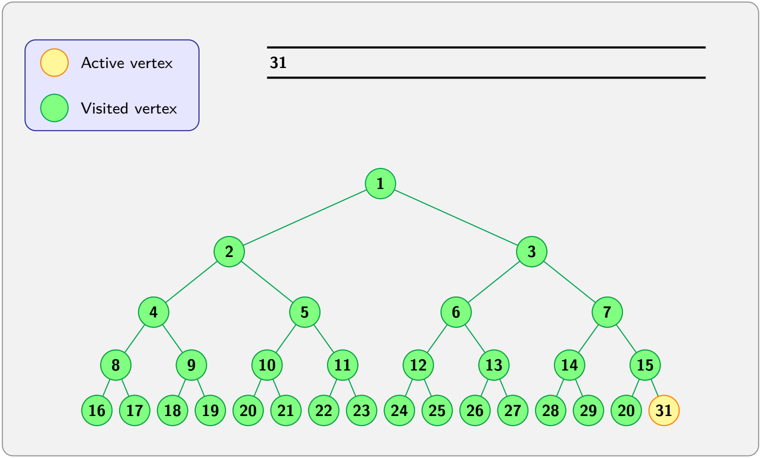
First-In-First-Out-Branch and Bound (FIFO-BB)



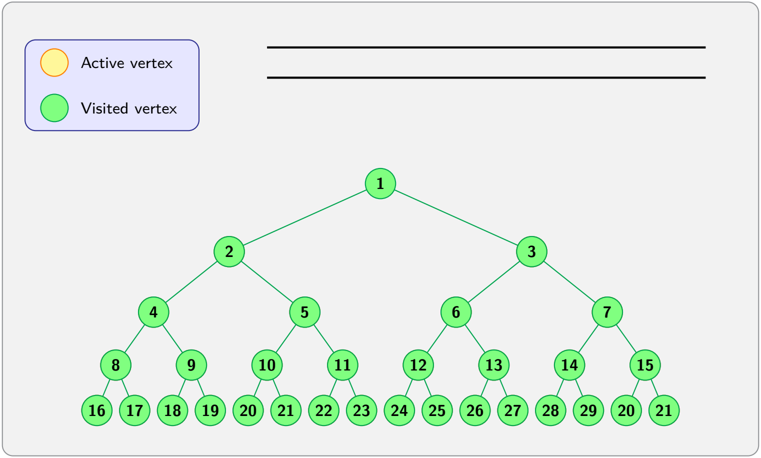
First-In-First-Out-Branch and Bound (FIFO-BB)



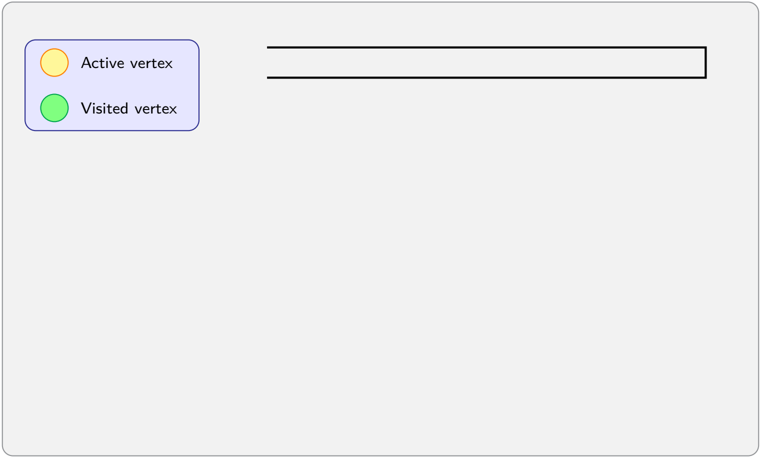
First-In-First-Out-Branch and Bound (FIFO-BB)



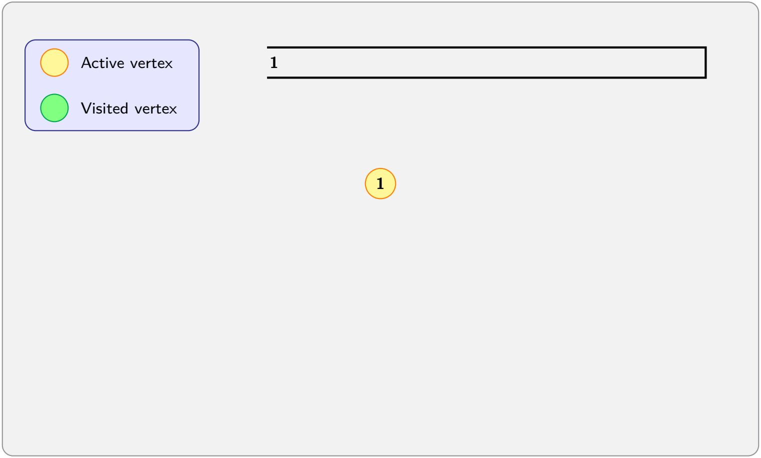
First-In-First-Out-Branch and Bound (FIFO-BB)



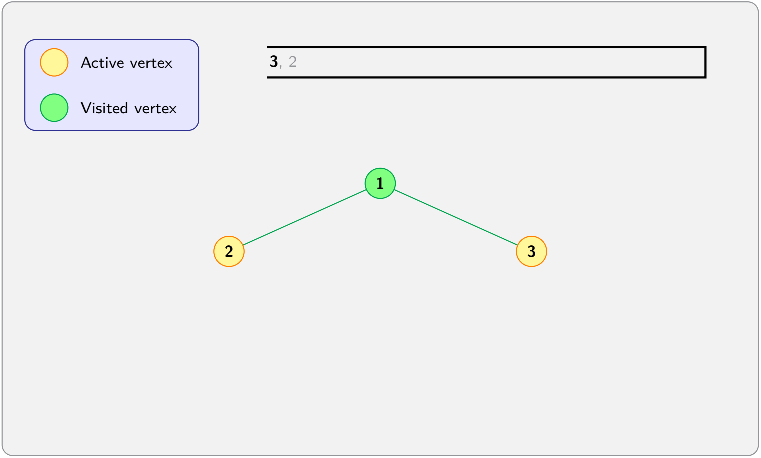
Last-In-First-Out-Branch and Bound (LIFO-BB)



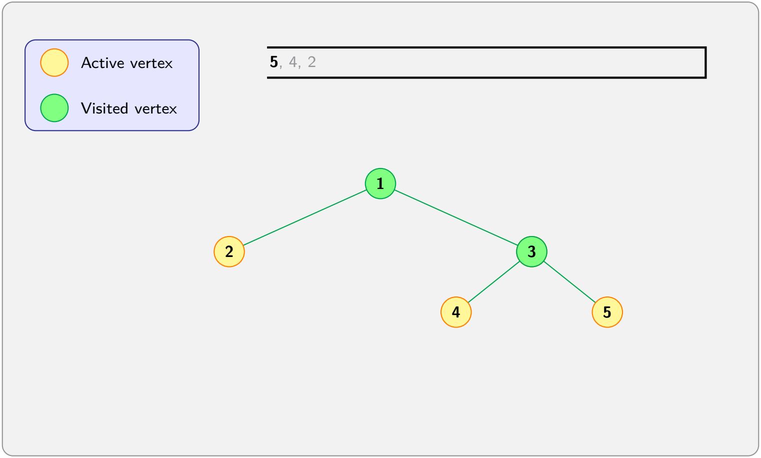
Last-In-First-Out-Branch and Bound (LIFO-BB)



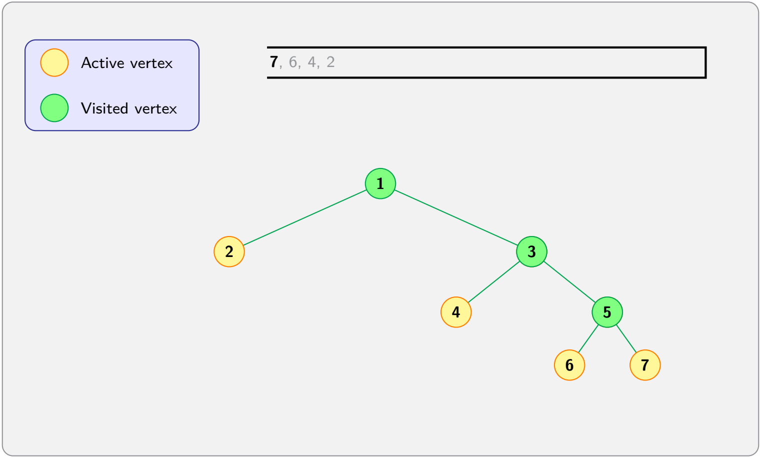
Last-In-First-Out-Branch and Bound (LIFO-BB)



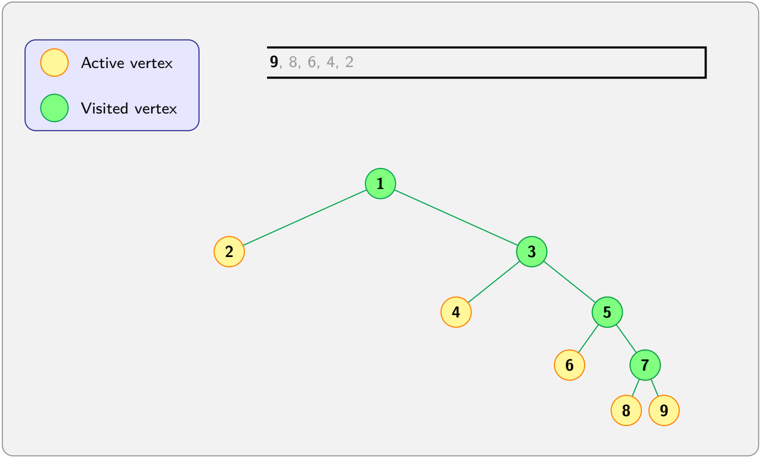
Last-In-First-Out-Branch and Bound (LIFO-BB)



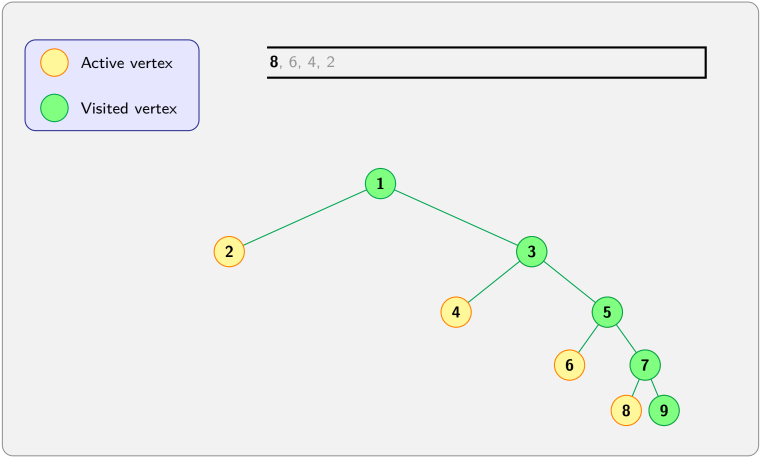
Last-In-First-Out-Branch and Bound (LIFO-BB)



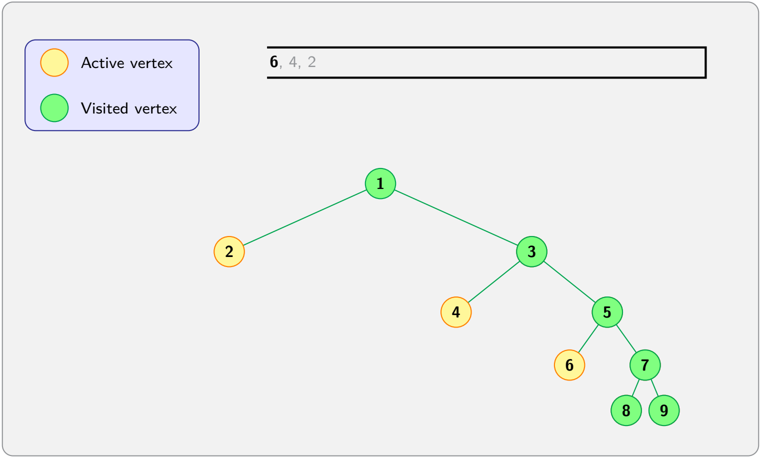
Last-In-First-Out-Branch and Bound (LIFO-BB)



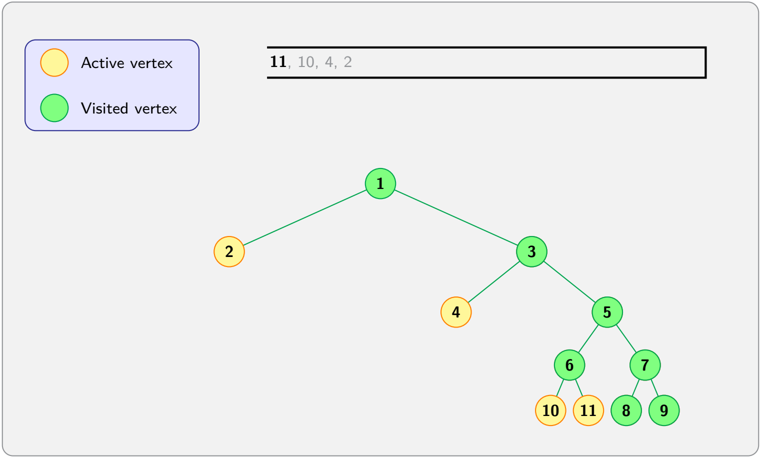
Last-In-First-Out-Branch and Bound (LIFO-BB)



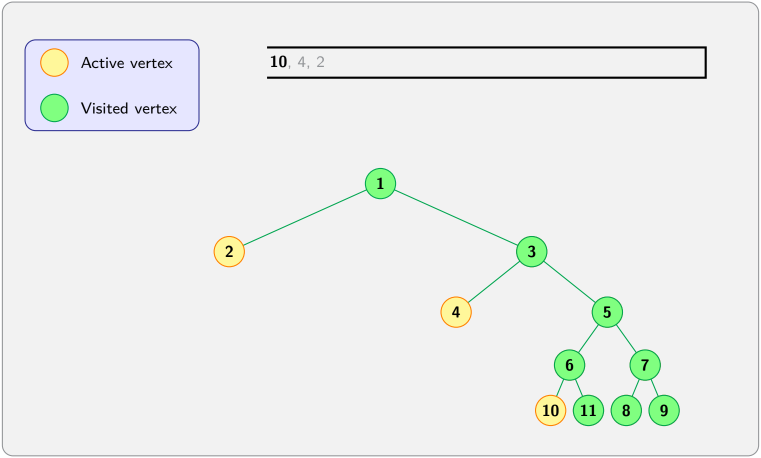
Last-In-First-Out-Branch and Bound (LIFO-BB)



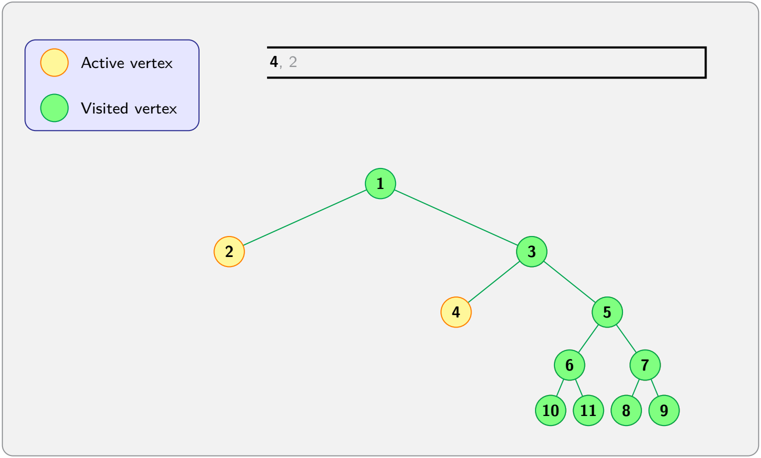
Last-In-First-Out-Branch and Bound (LIFO-BB)



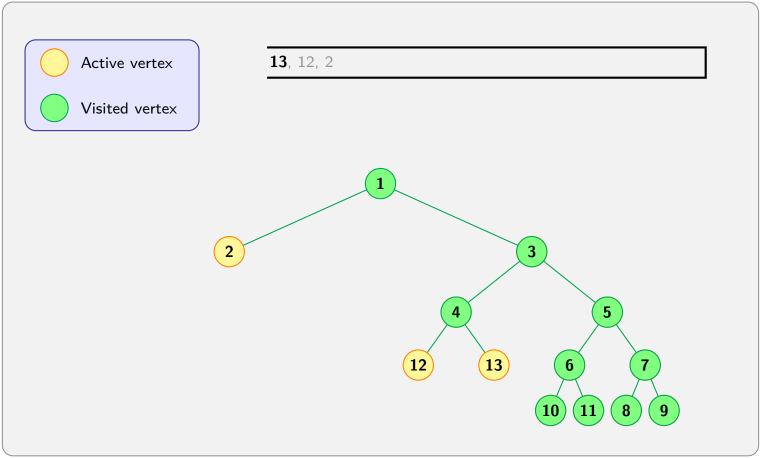
Last-In-First-Out-Branch and Bound (LIFO-BB)



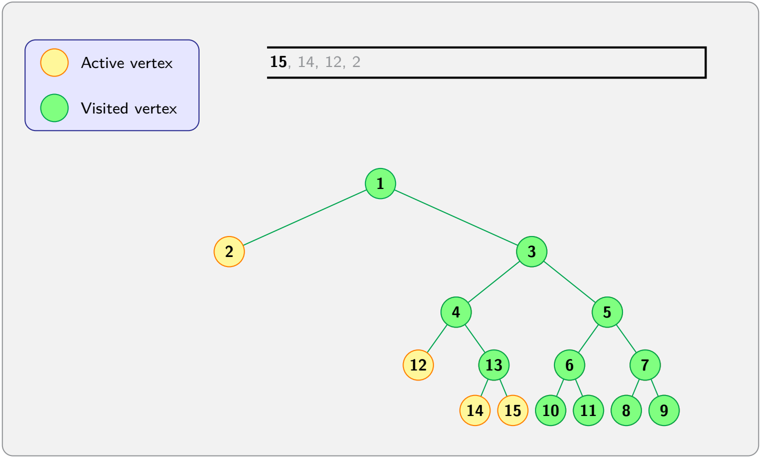
Last-In-First-Out-Branch and Bound (LIFO-BB)



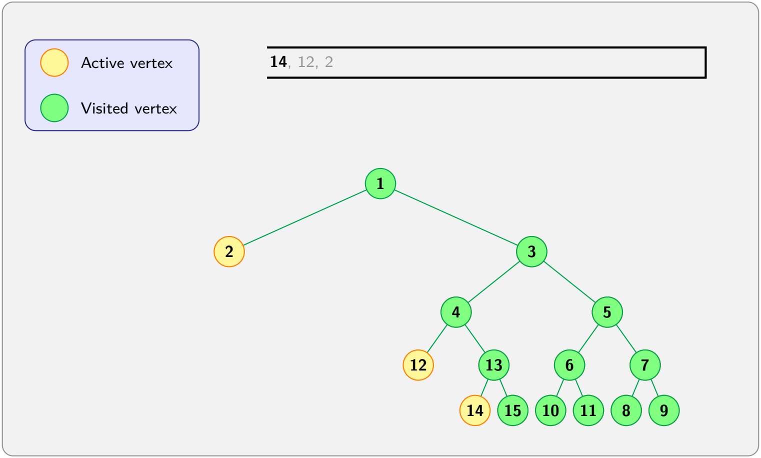
Last-In-First-Out-Branch and Bound (LIFO-BB)



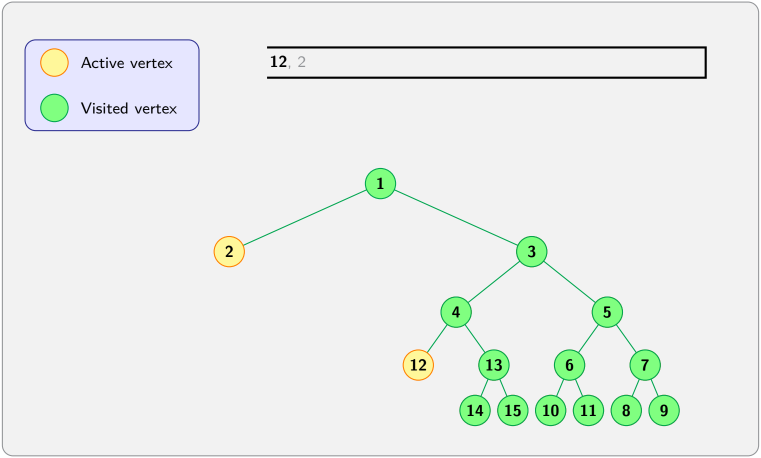
Last-In-First-Out-Branch and Bound (LIFO-BB)



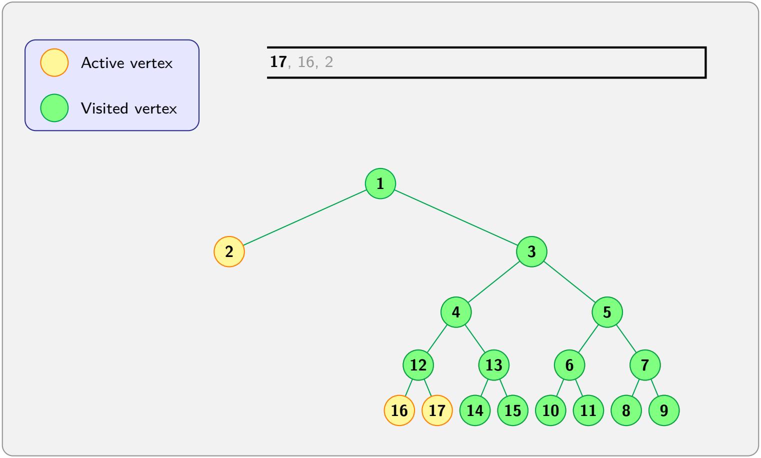
Last-In-First-Out-Branch and Bound (LIFO-BB)



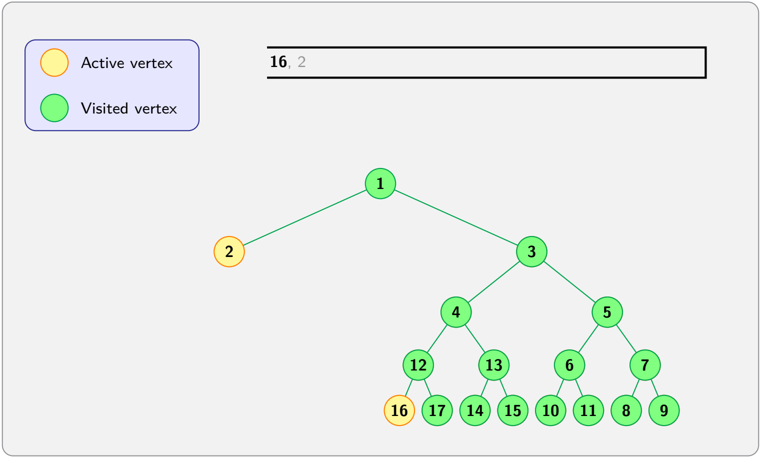
Last-In-First-Out-Branch and Bound (LIFO-BB)



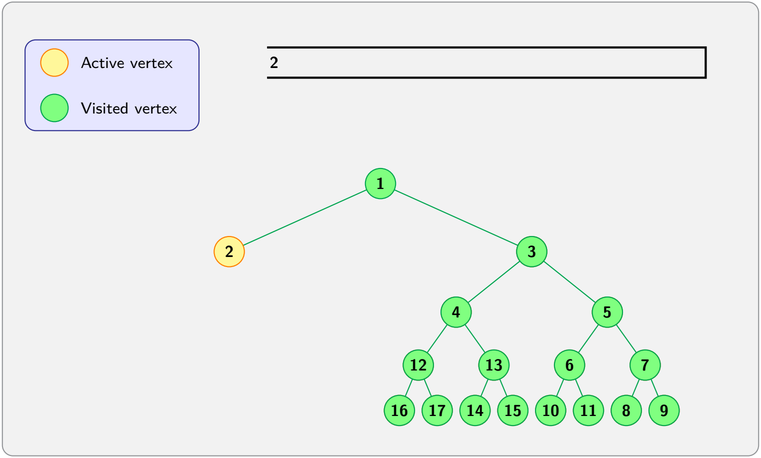
Last-In-First-Out-Branch and Bound (LIFO-BB)



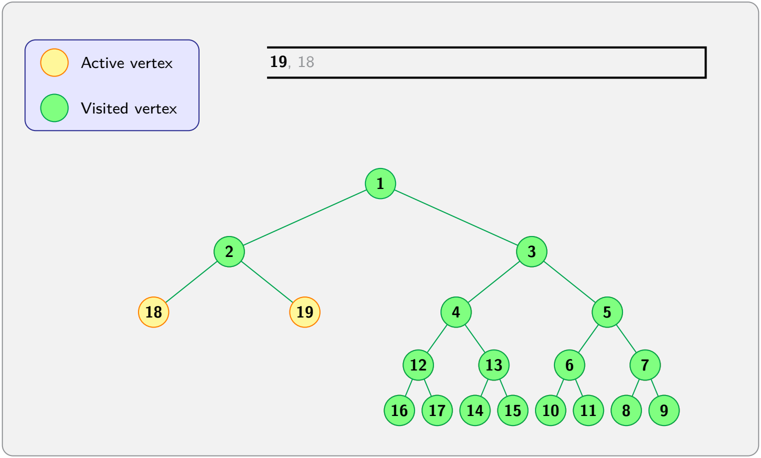
Last-In-First-Out-Branch and Bound (LIFO-BB)



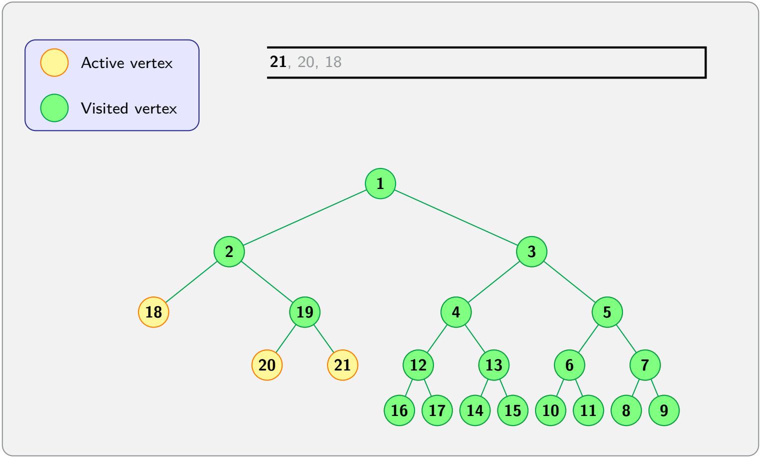
Last-In-First-Out-Branch and Bound (LIFO-BB)



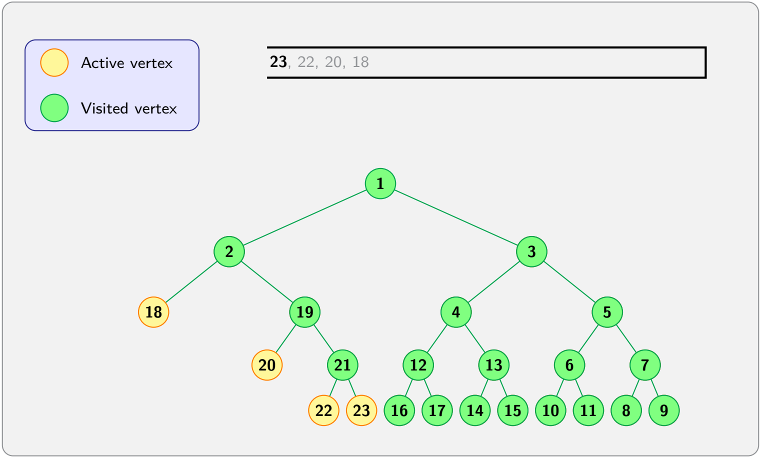
Last-In-First-Out-Branch and Bound (LIFO-BB)



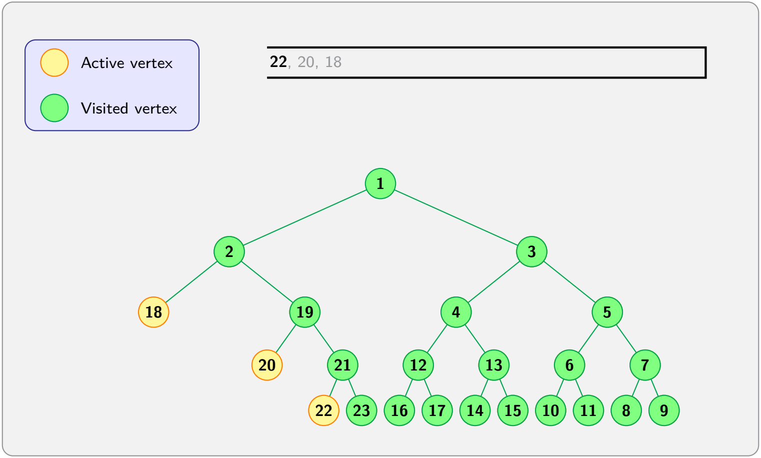
Last-In-First-Out-Branch and Bound (LIFO-BB)



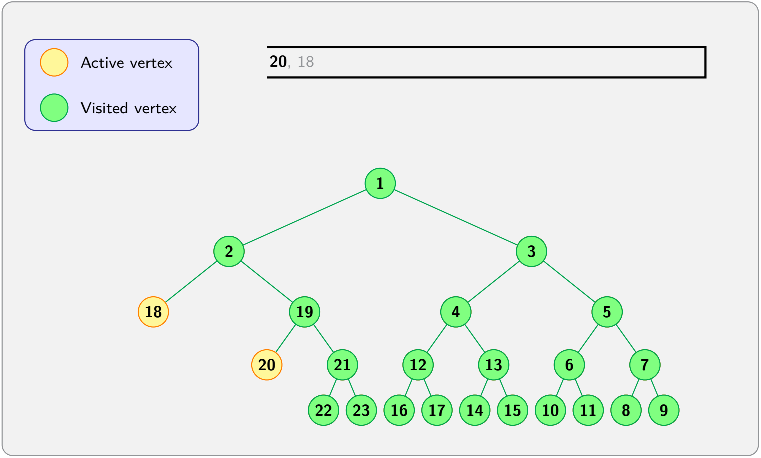
Last-In-First-Out-Branch and Bound (LIFO-BB)



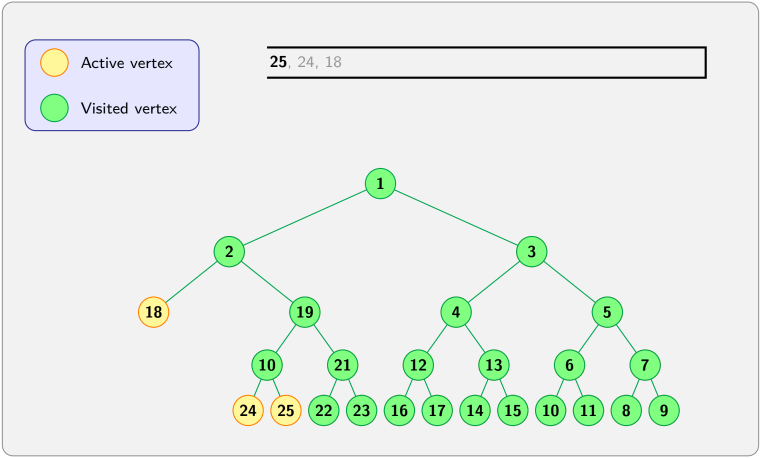
Last-In-First-Out-Branch and Bound (LIFO-BB)



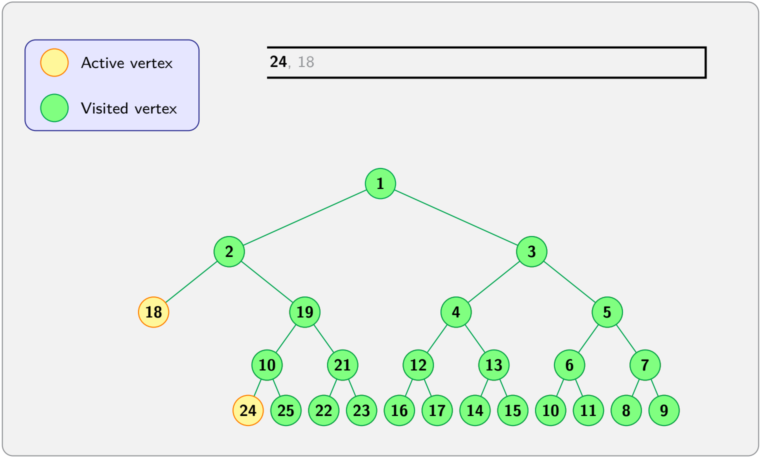
Last-In-First-Out-Branch and Bound (LIFO-BB)



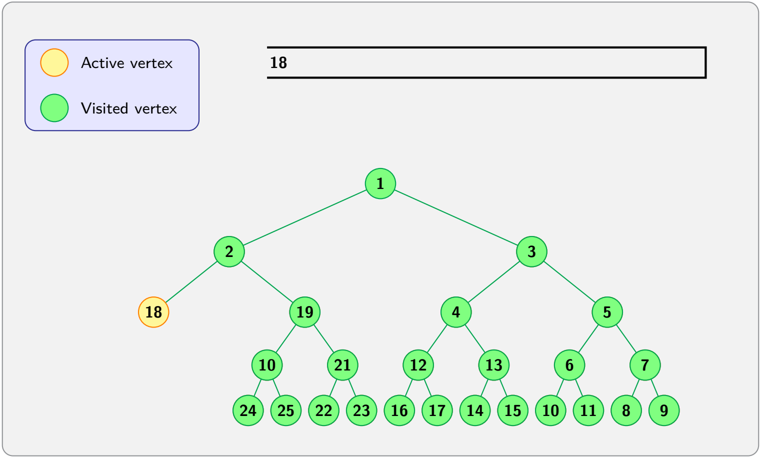
Last-In-First-Out-Branch and Bound (LIFO-BB)



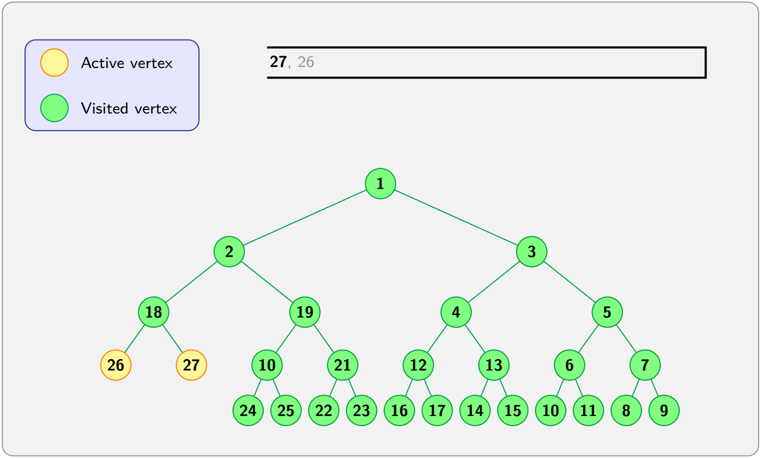
Last-In-First-Out-Branch and Bound (LIFO-BB)



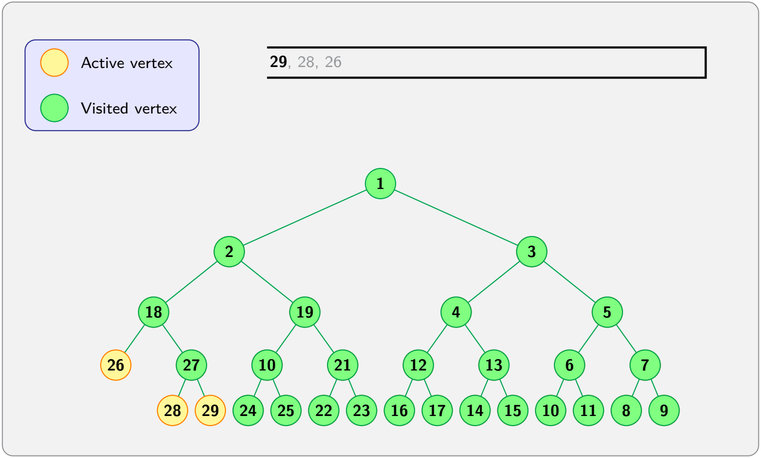
Last-In-First-Out-Branch and Bound (LIFO-BB)



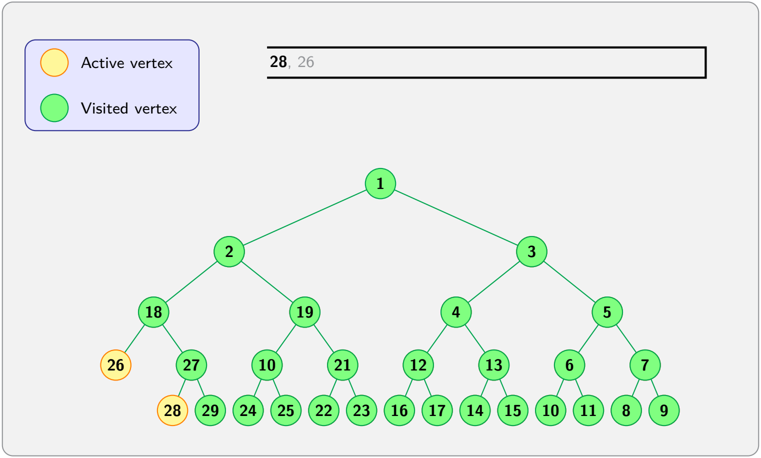
Last-In-First-Out-Branch and Bound (LIFO-BB)



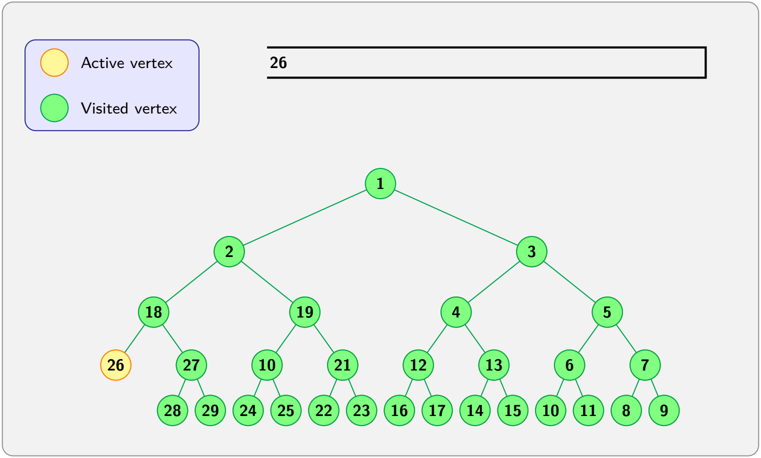
Last-In-First-Out-Branch and Bound (LIFO-BB)



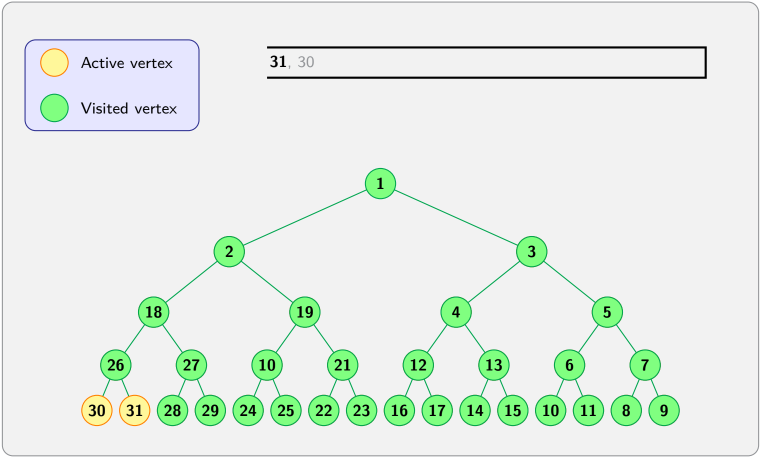
Last-In-First-Out-Branch and Bound (LIFO-BB)



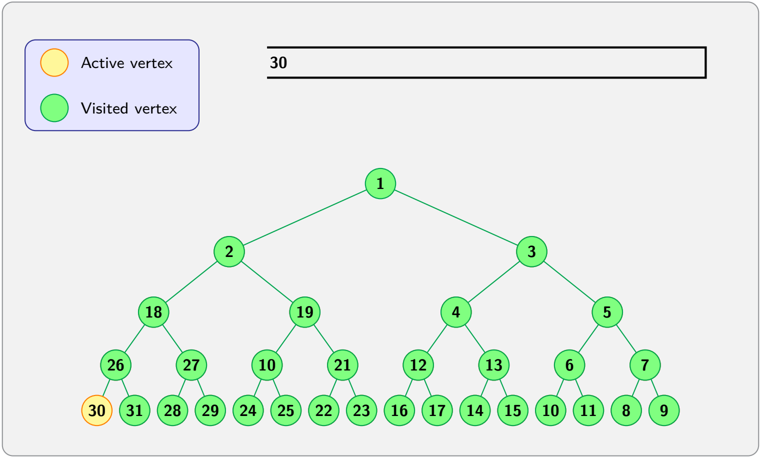
Last-In-First-Out-Branch and Bound (LIFO-BB)



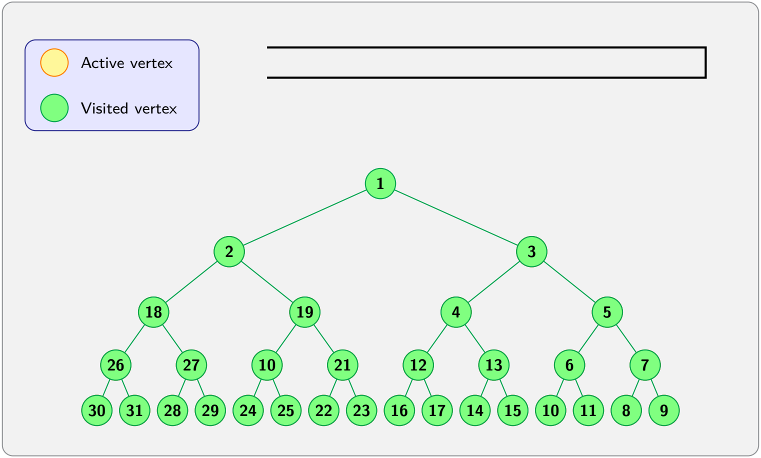
Last-In-First-Out-Branch and Bound (LIFO-BB)



Last-In-First-Out-Branch and Bound (LIFO-BB)



Last-In-First-Out-Branch and Bound (LIFO-BB)



1. Review of the Previous Class
2. Branch and Bound
   - FIFO-BB
   - LIFO-BB
   - LC-BB

The Problem of Job Sequencing with Deadlines

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- For each node we define two values: cost (c) and upper bound (u).
- Cost is defined as the sum of all penalties up to the last job considered.
- The upper bound will be the sum of all penalties except those already included in the solution.

Least Cost Branch and Bound (LC-BB)

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Penalty	4	11	7	2
Deadline	1	3	2	1
Duration	1	2	1	1

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Least Cost Branch and Bound (LC-BB)

Optimal vertex

Active vertex

Visited vertex

Killed vertex

Infeasible vertex

Penalty	4	11	7	2
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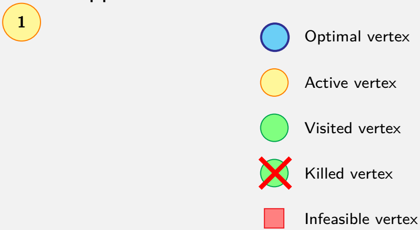
Least Cost Branch and Bound (LC-BB)

upper $= 24$



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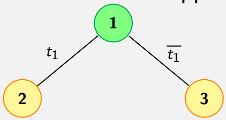

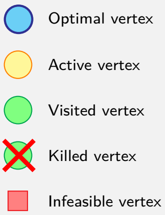

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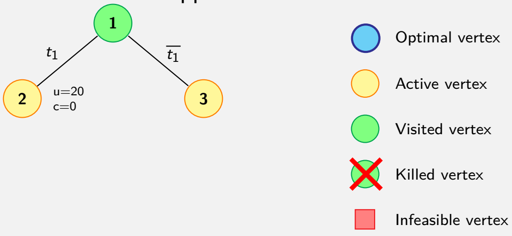

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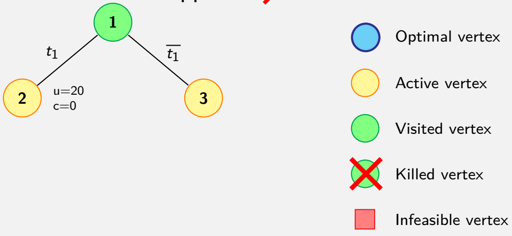
Least Cost Branch and Bound (LC-BB)

upper = $\cancel{24}$ 20



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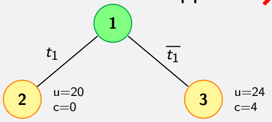

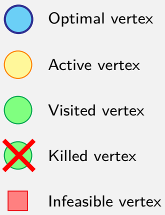

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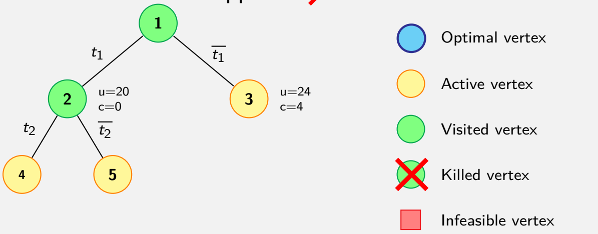

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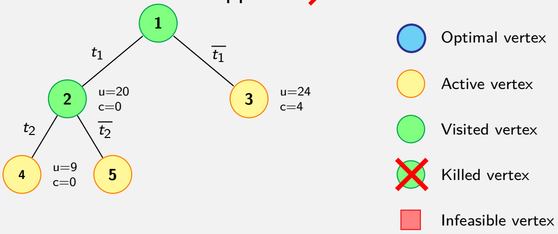

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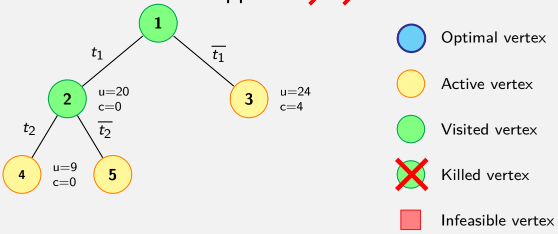
Least Cost Branch and Bound (LC-BB)

upper $= \cancel{24}\ \cancel{20}\ 9$



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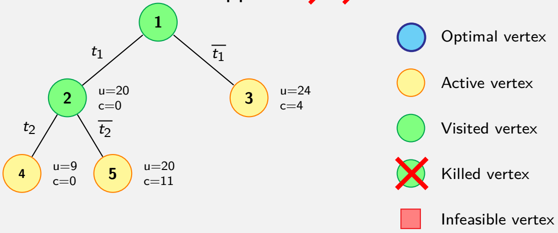

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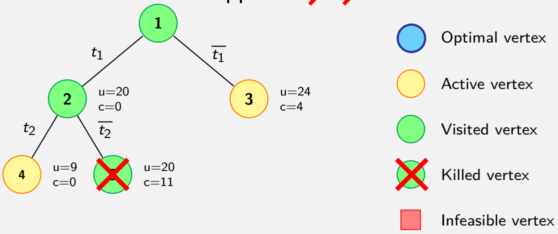

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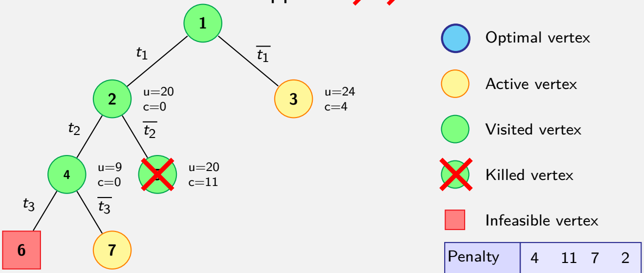

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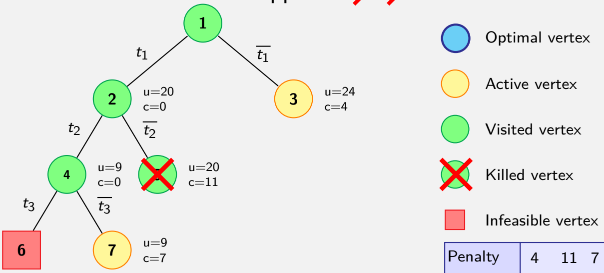

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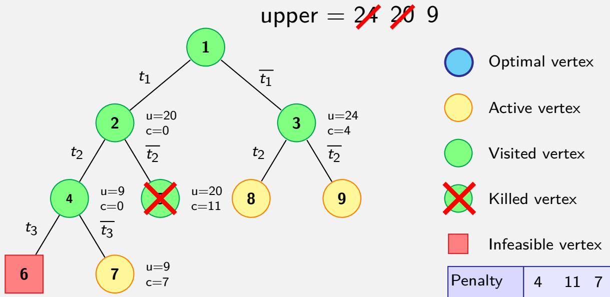

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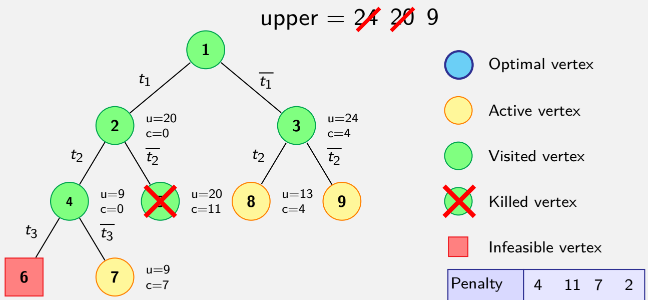

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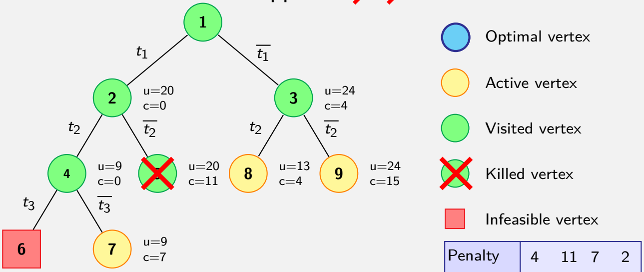

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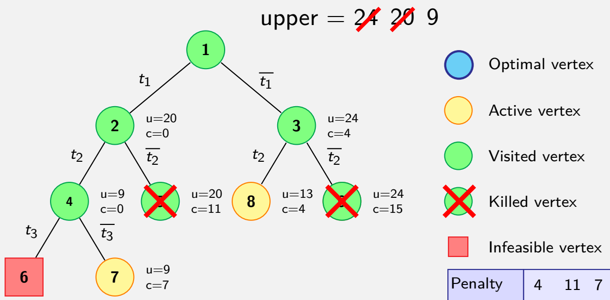

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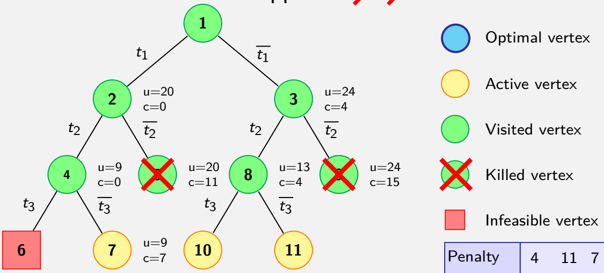

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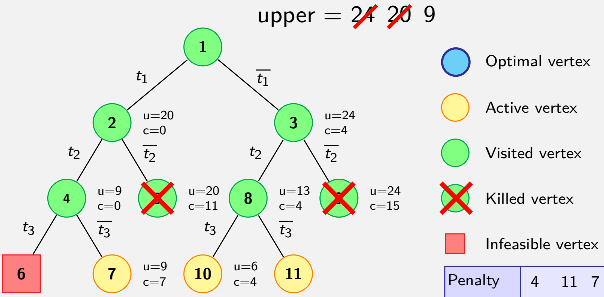

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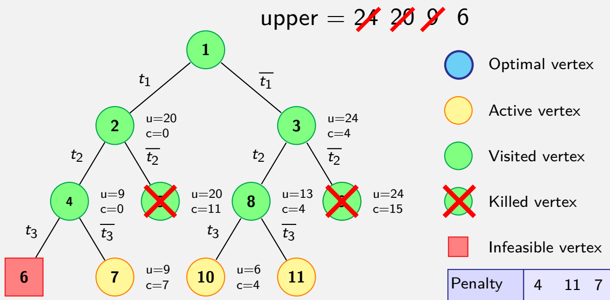

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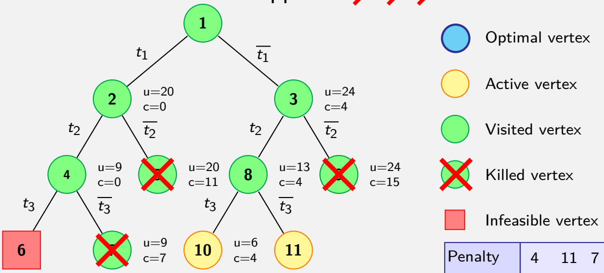
$\text{upper} = \cancel{24}\ \cancel{20}\ \cancel{9}\ 6$



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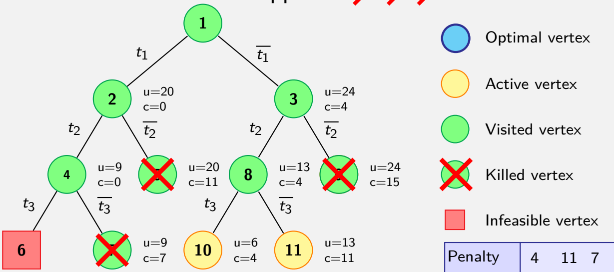

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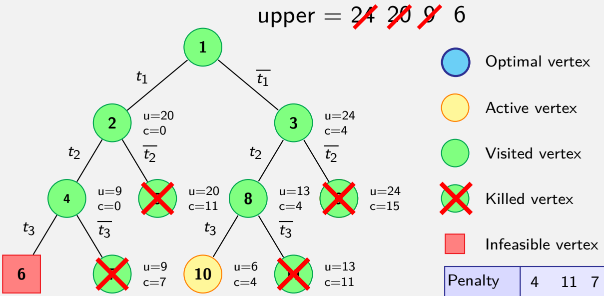

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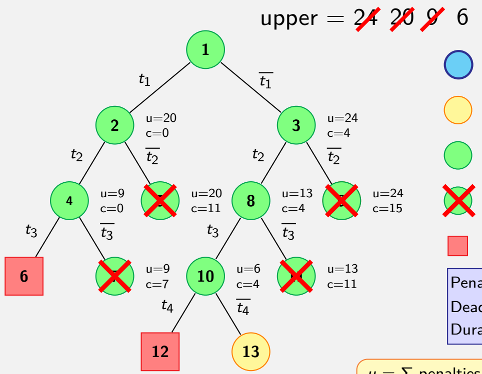

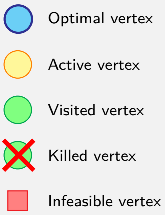

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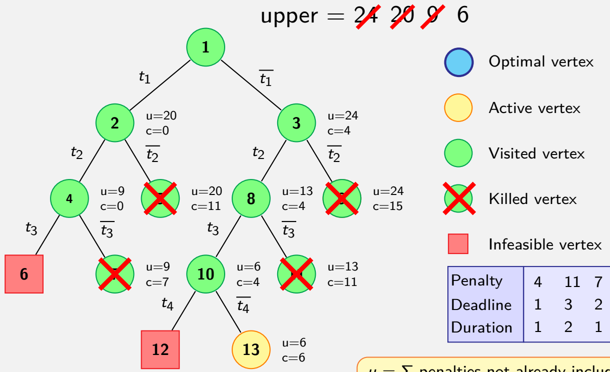

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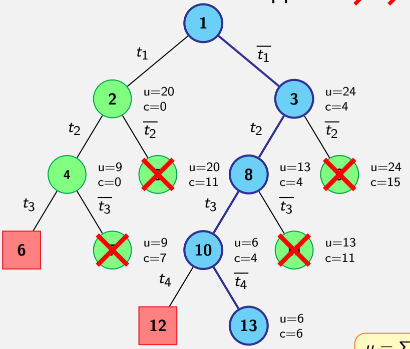

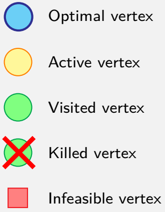

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