Programming III

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UADE

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Programme

1. A First Example: 0-1 Knapsack
2. A Second Example: Task Assignment
3. A Third Example: Travelling Salesman Problem (TSP)

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- Since this is a maximisation problem, we consider the values as negative.
- For each node we will compute two magnitudes: upper bound (u) and cost (c.)
- The upper bound of the node will be the sum of the values not already included; the cost will be the sum of the values not already included *with fractions*.

A First Example: 0-1 Knapsack

object	$o1$	$o2$	$o3$	$o4$
value	10	10	12	18
weight	2	4	6	9
$m = 15$				

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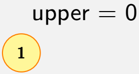

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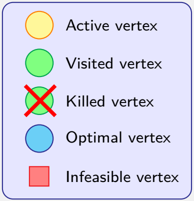

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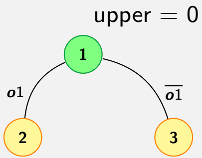

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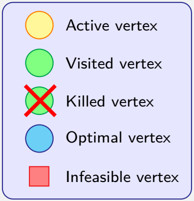
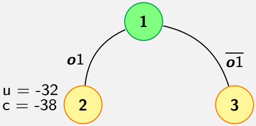


A First Example: 0-1 Knapsack

upper $= 0$



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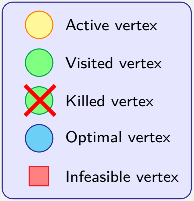

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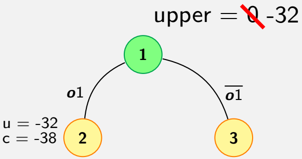

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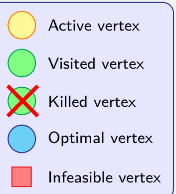

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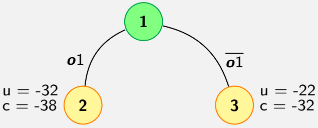

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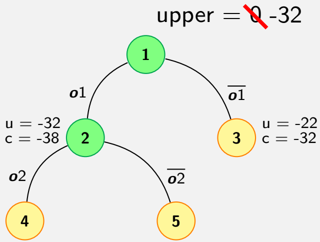

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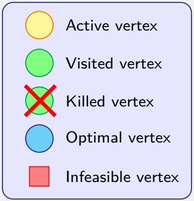

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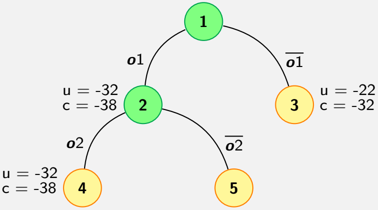

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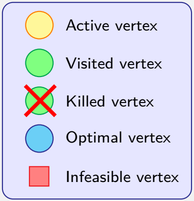

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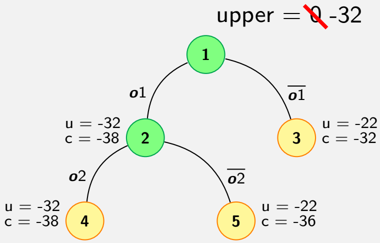

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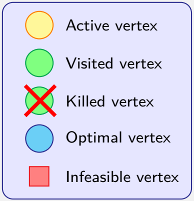

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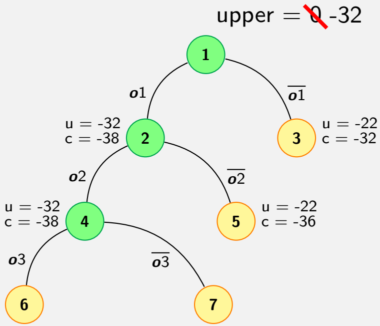

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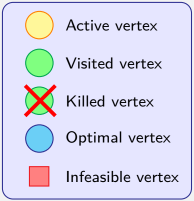

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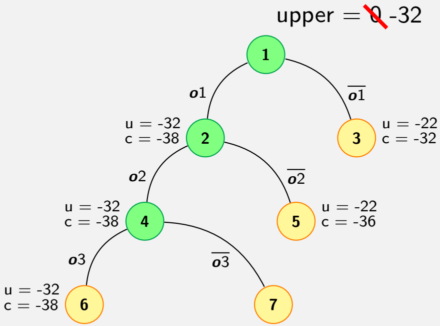

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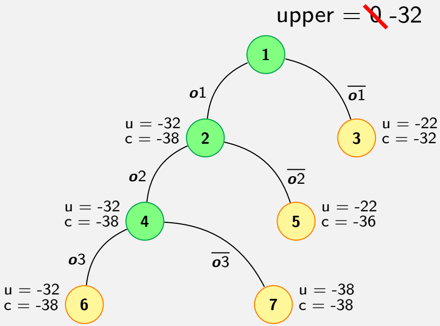

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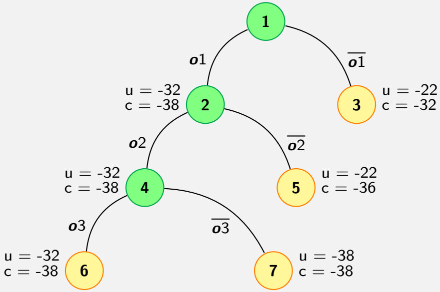

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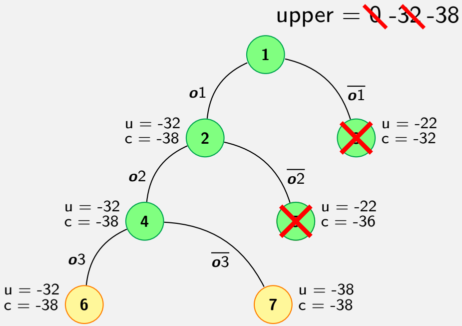

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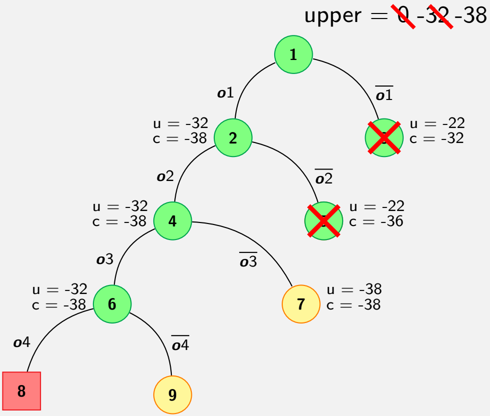

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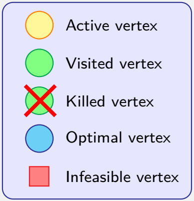

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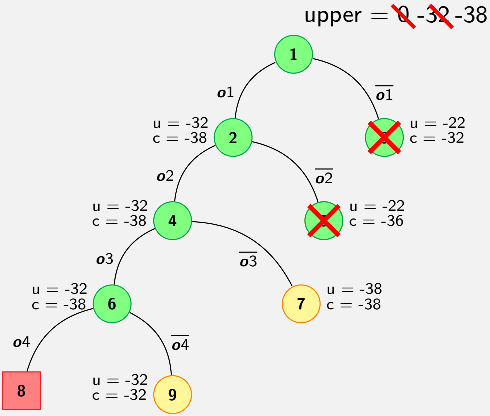

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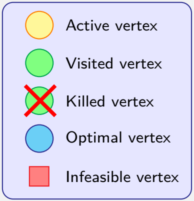

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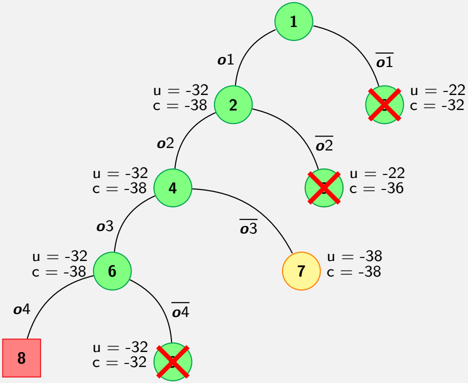

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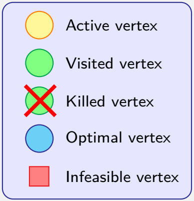

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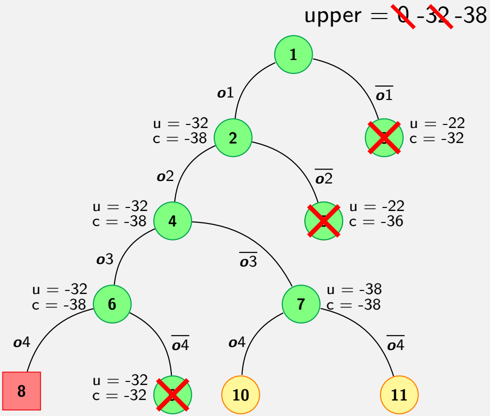

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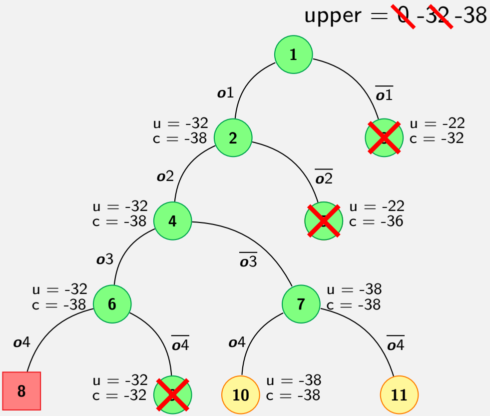

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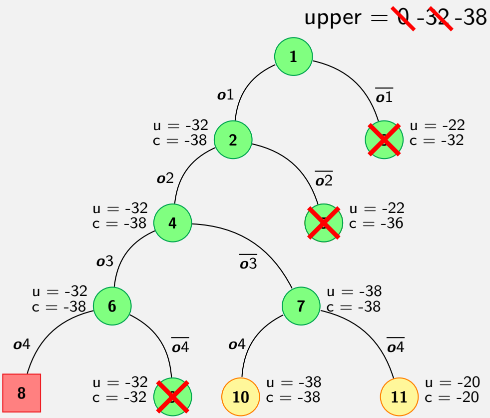

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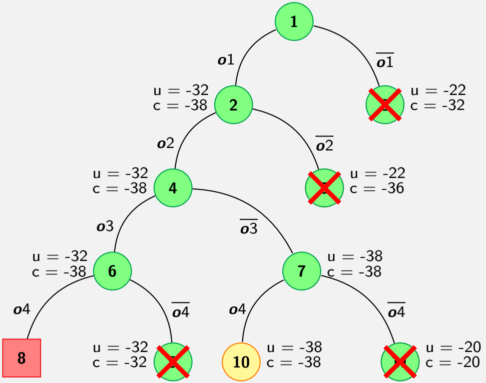

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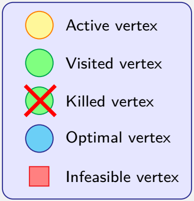

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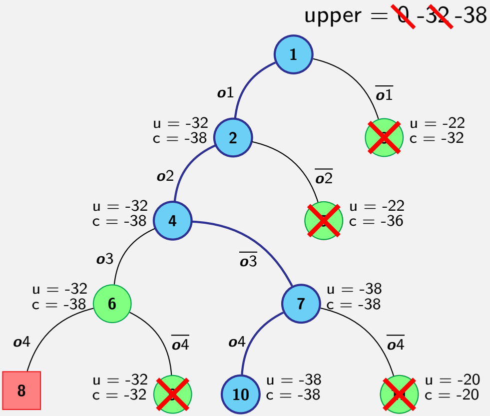

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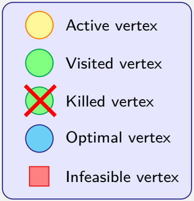

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- Each person must be assigned to exactly one job. We want to do so with the minimum cost.
- The variable *upper* will be initially assigned the cost of an arbitrary assignment.
- For each node we will calculate a cost (c), which will be the sum of the minimum costs of the tasks not yet assigned and the costs already incurred.
- The *upper* variable will be only updated when we have completed some assignment that has a better cost.

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	1	2	3	4
a	11	12	18	40
b	14	15	13	22
c	11	17	19	23
d	17	14	20	28

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Active vertex
Optimal vertex
Visited vertex
Killed vertex

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upper $= 73$

	1	2	3	4
a	11	12	18	40
b	14	15	13	22
c	11	17	19	23
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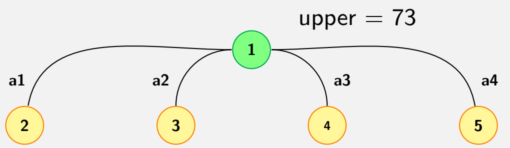

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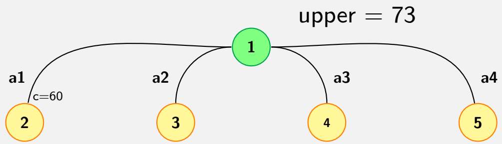

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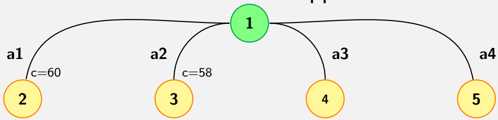

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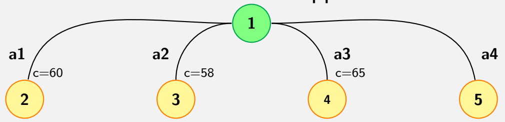

	1	2	3	4
a	11	12	18	40
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c	11	17	19	23
d	17	14	20	28

$c = \Sigma$ minimum cost of unassigned tasks

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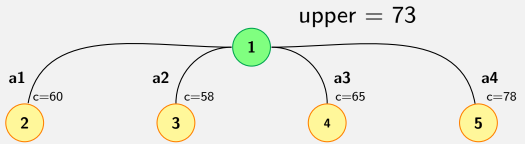

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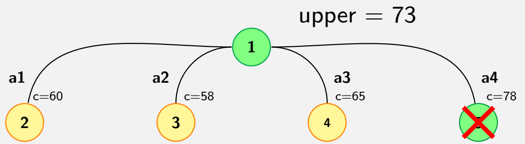

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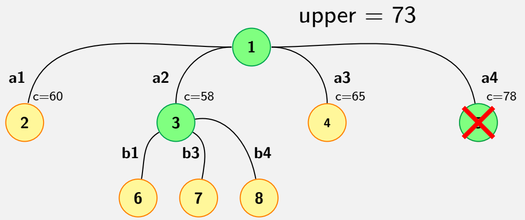

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Active vertex   Optimal vertex

Visited vertex   Killed vertex

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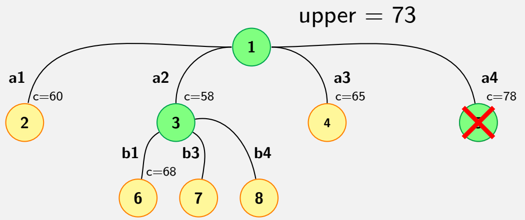

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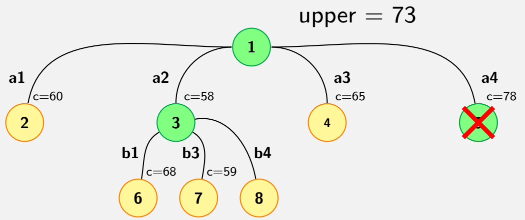

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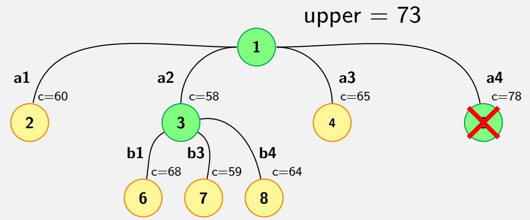

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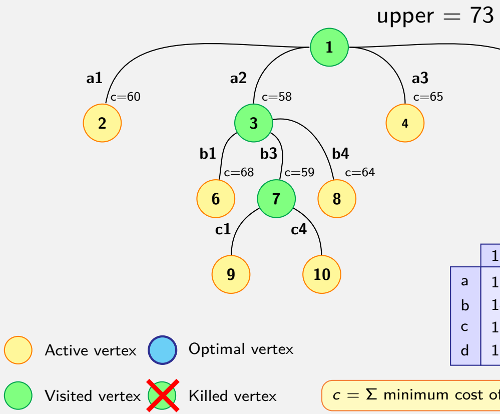

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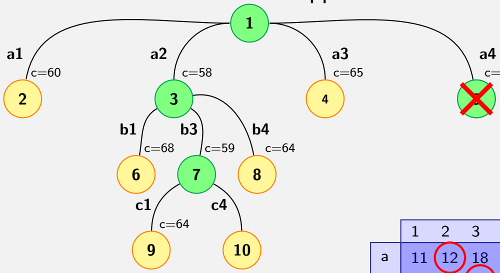

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A Second Example: Task Assignement

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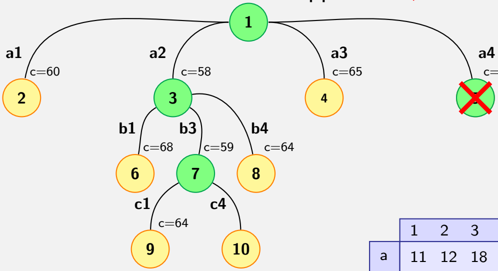

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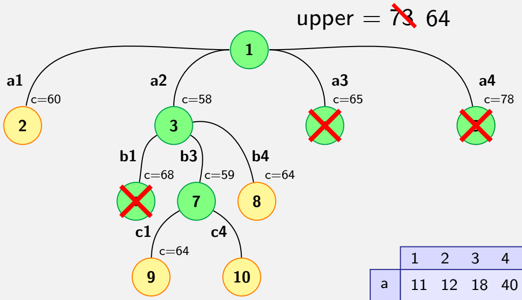

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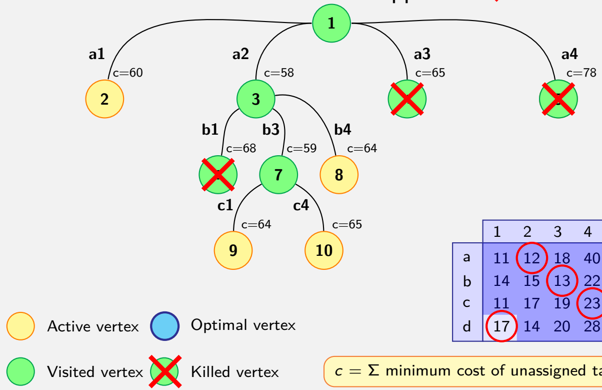

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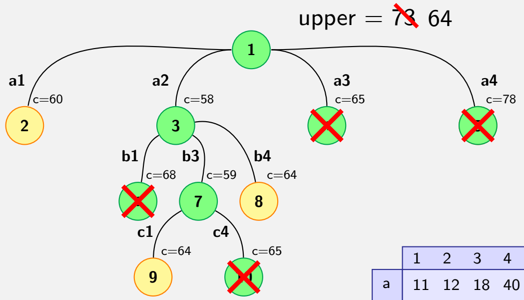

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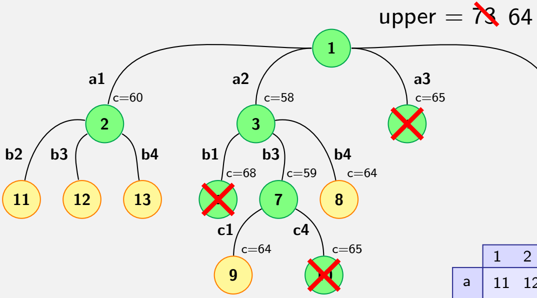

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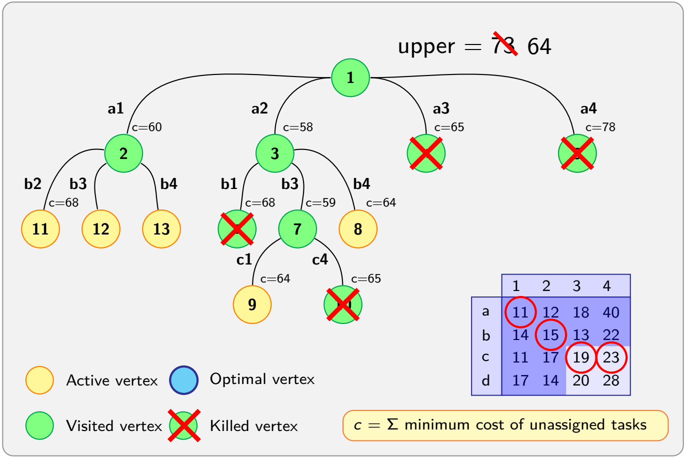

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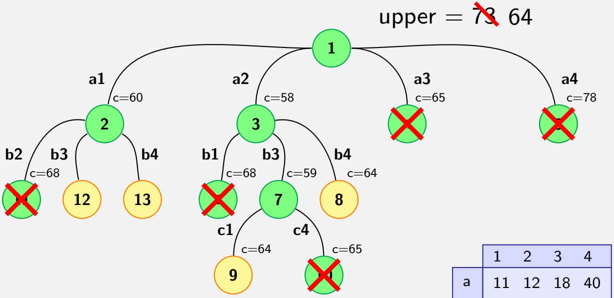

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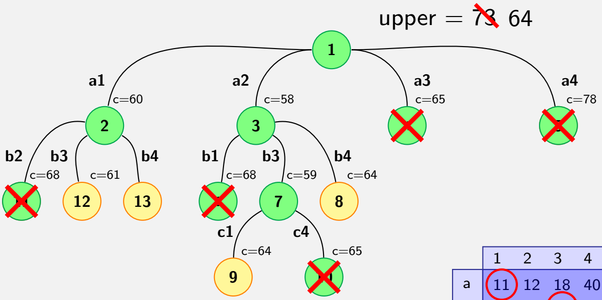

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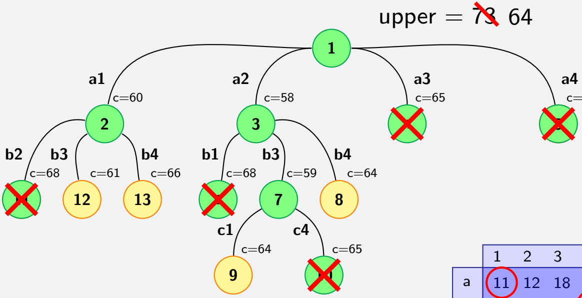

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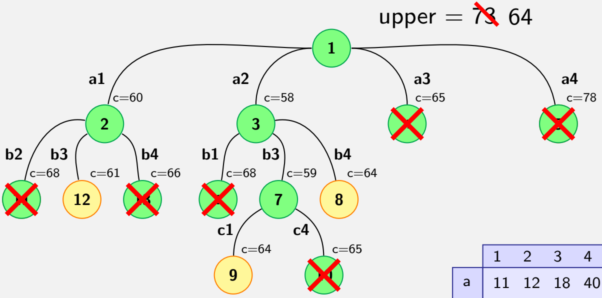

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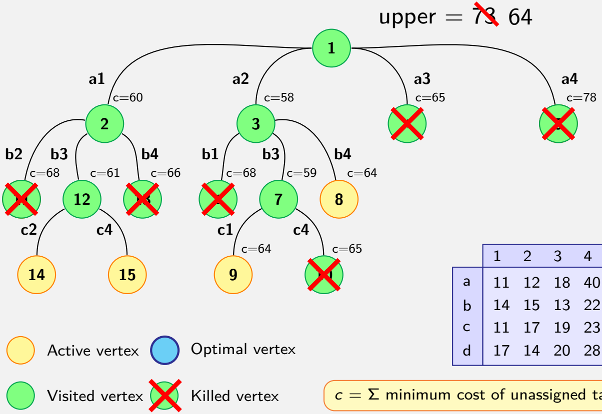

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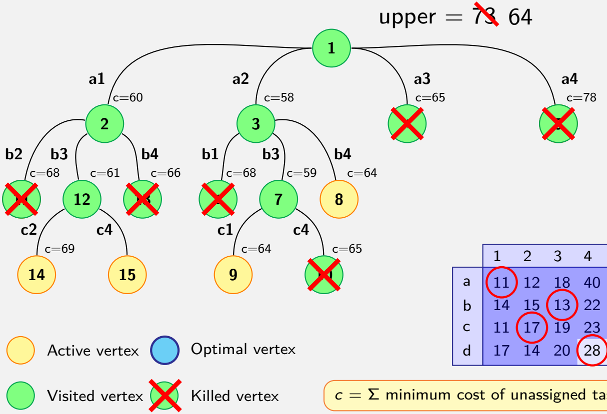

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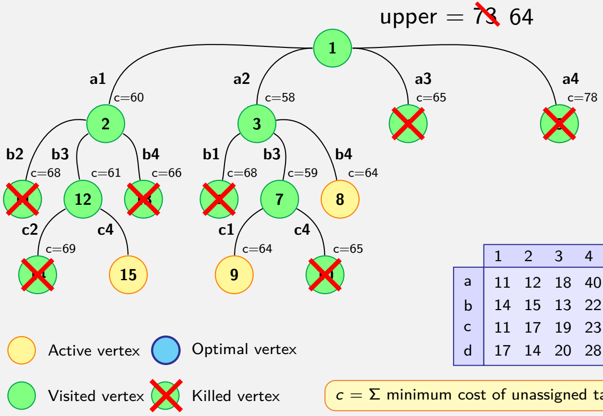

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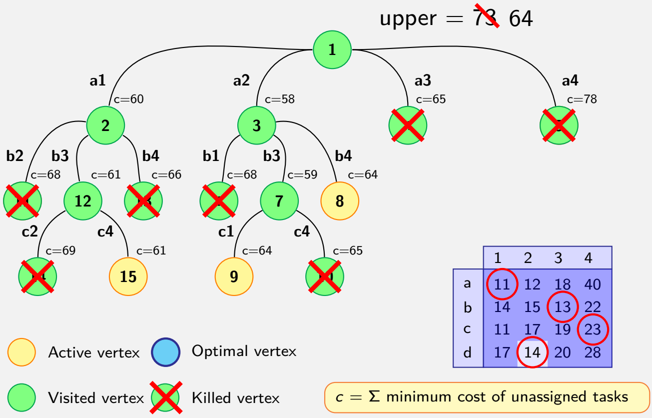

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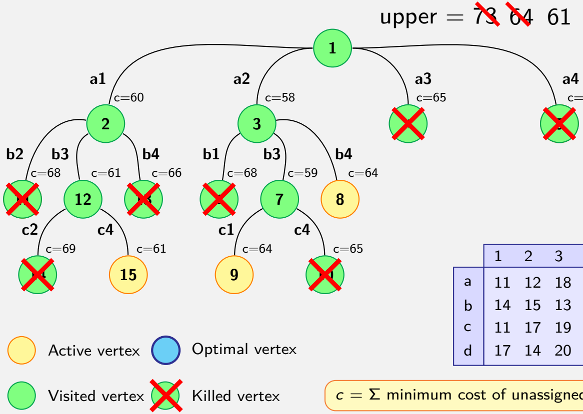

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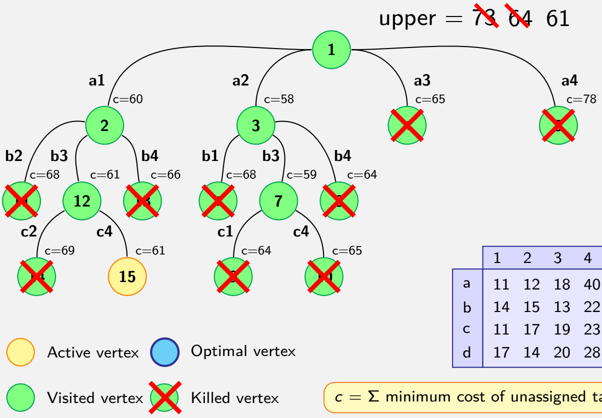

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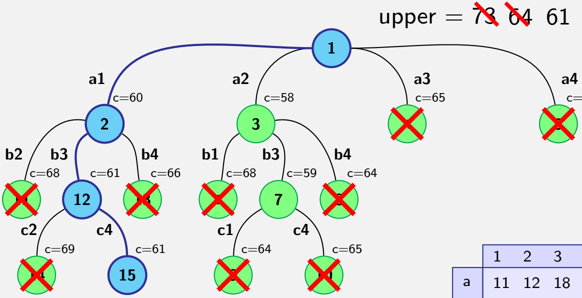

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1. A First Example: 0-1 Knapsack
2. A Second Example: Task Assignment
3. A Third Example: Travelling Salesman Problem (TSP)

Well, I got on the road, and I went north to Providence. (...) And then I went to Waterbury. Waterbury is a fine city. Big clock city, the famous Waterbury clock. Sold a nice bill there. And then Boston—Boston is the cradle of the Revolution. A fine city. And a couple of other towns in Mass., and on to Portland and Bangor and straight home!

Arthur Miller, *Death of a Salesman*

A Third Example: Travelling Salesman Problem (TSP)

- In this problem, we have a set S of connected cities. The edges of the graph have costs.

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- The total cost cannot be better than the sum of c_1 plus the minimum cost c_2 to go from the last city of V to some city in R plus the minimum cost c_3 from some city in R to the first city in V plus the cost c_4 of the MST of R.

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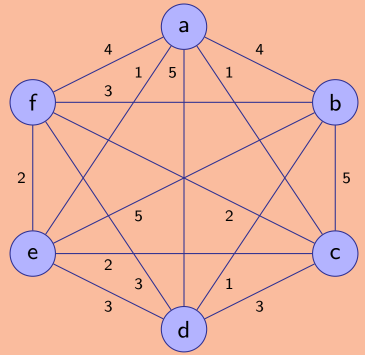
- Can you see why?

The Map for the TSP Example



The Strategy for TSP

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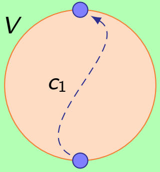

$c = c_1$

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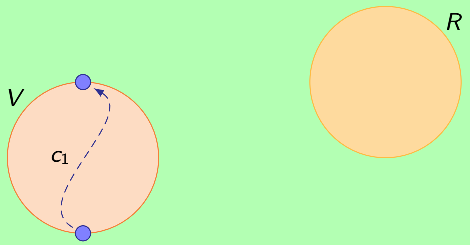

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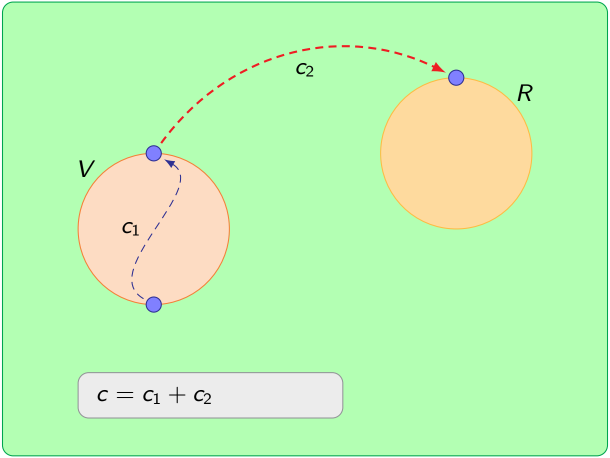

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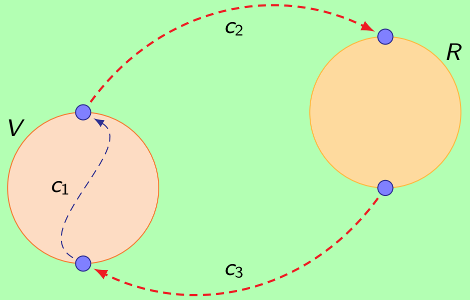
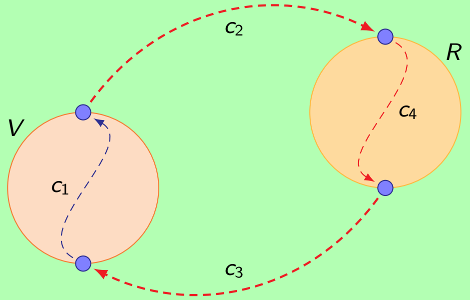


$$c = c_1 + c_2 + c_3$$

The Strategy for TSP



$c = c_1 + c_2 + c_3 + c_4$

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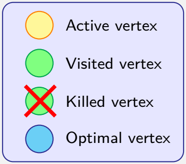

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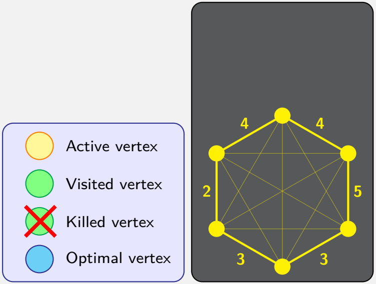

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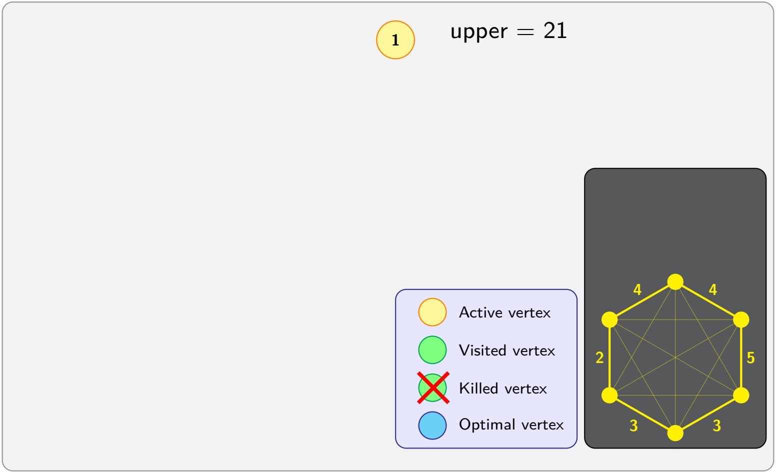

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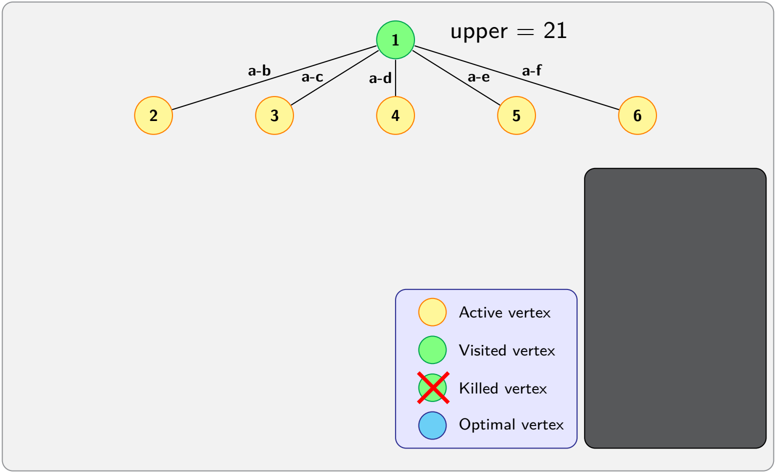

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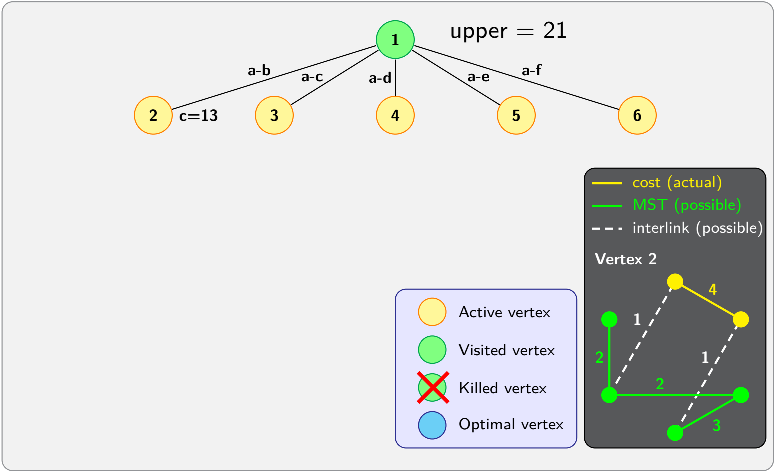

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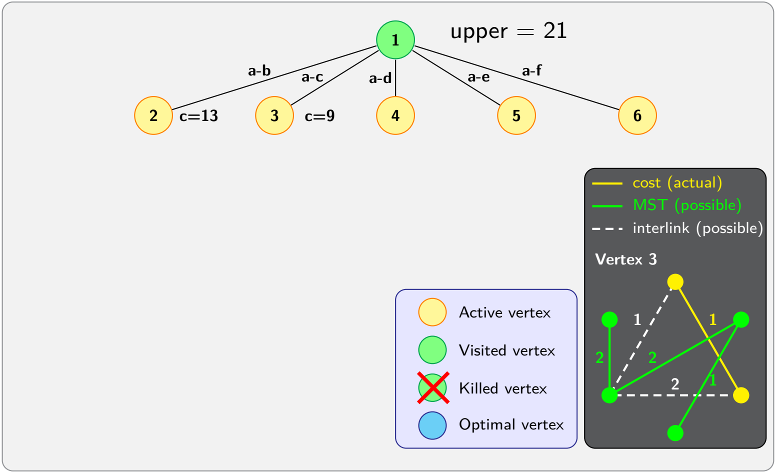

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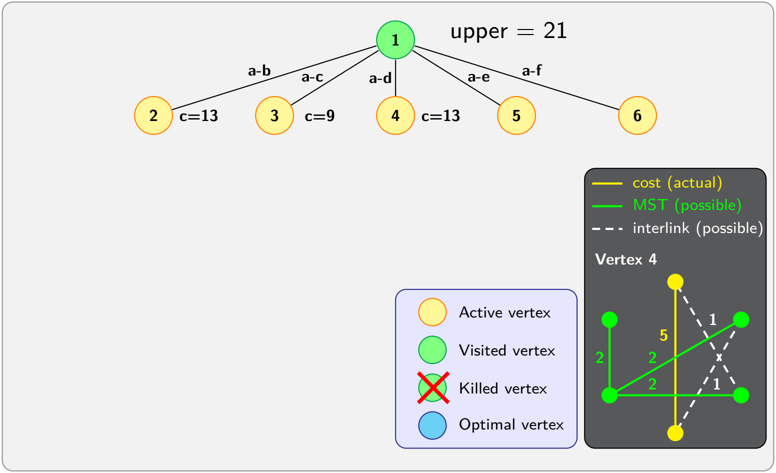

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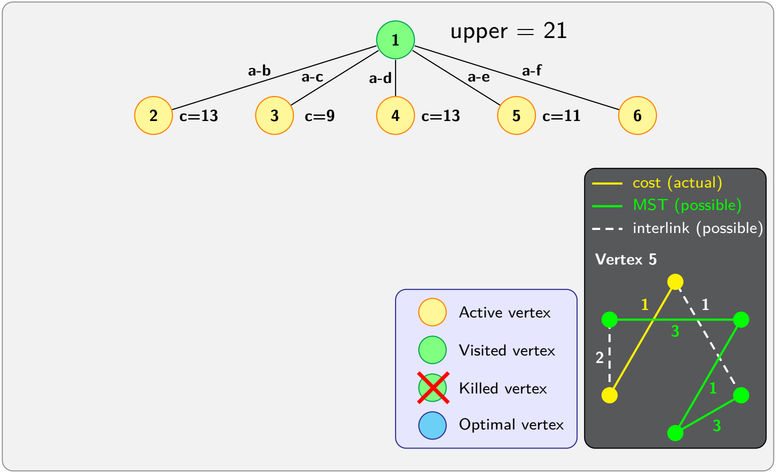

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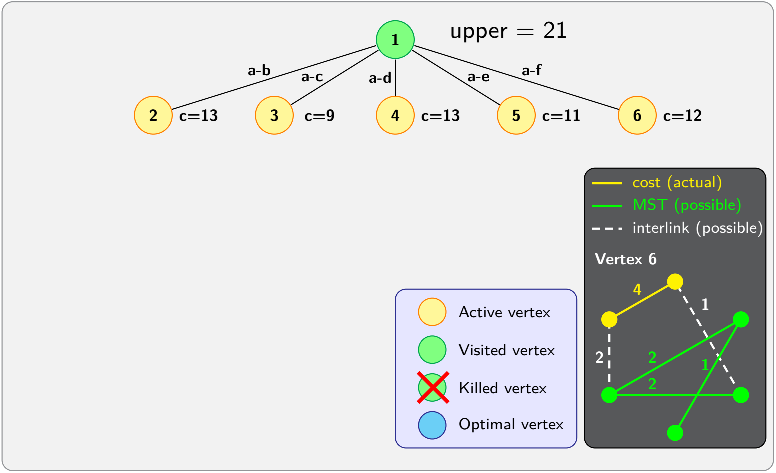

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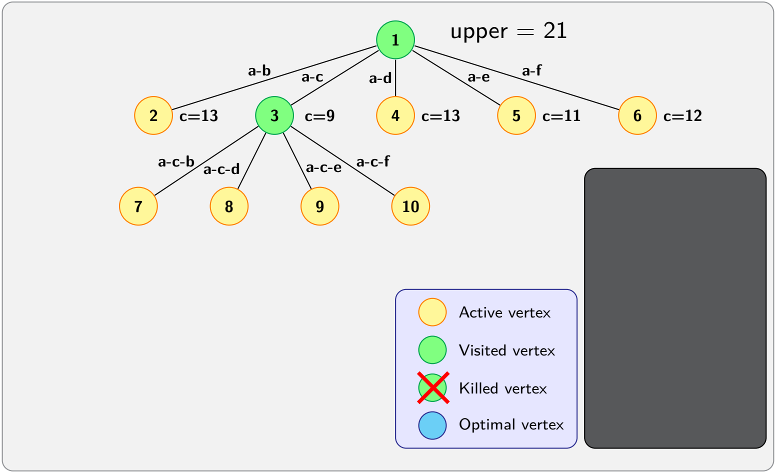

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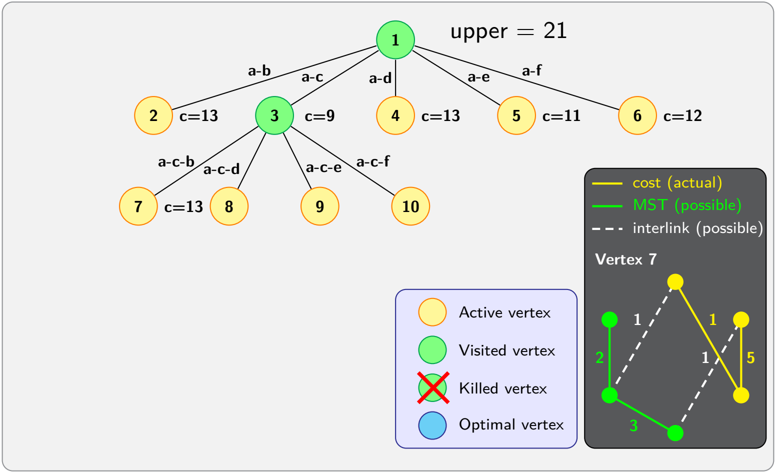

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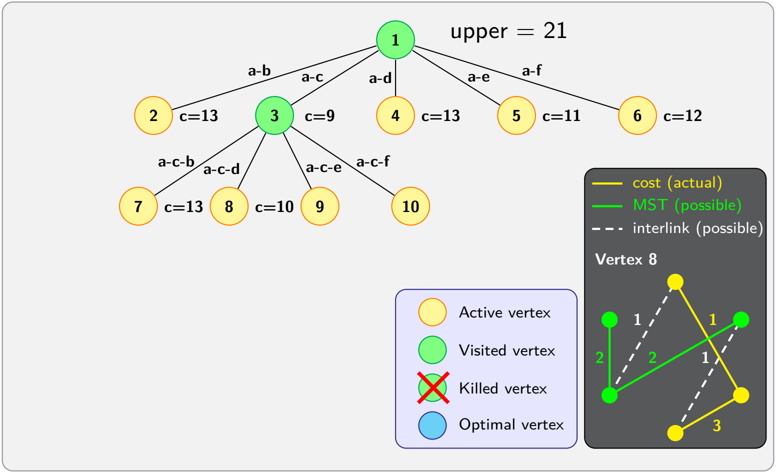

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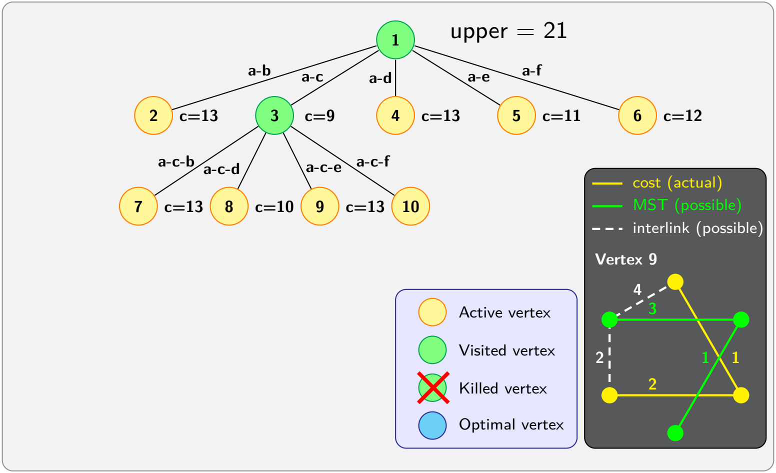

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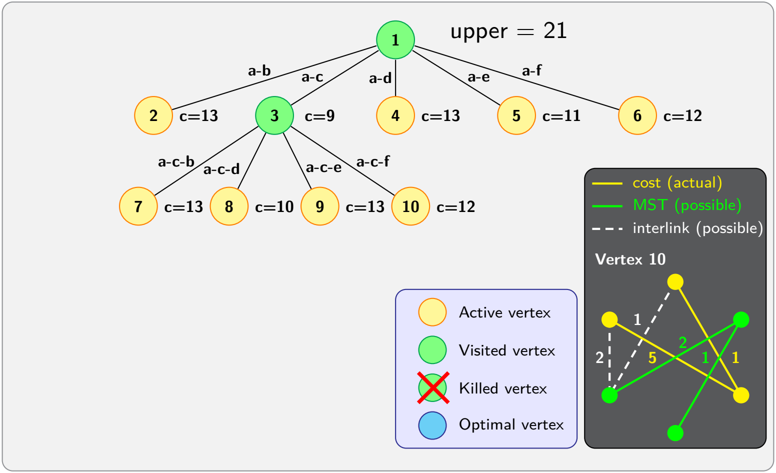

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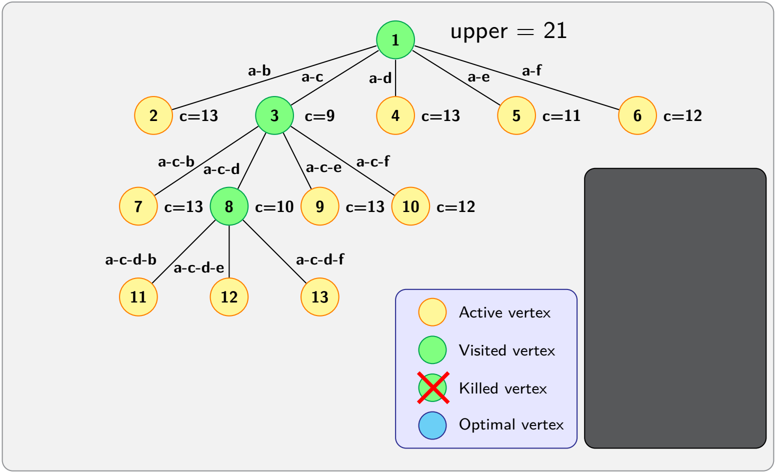

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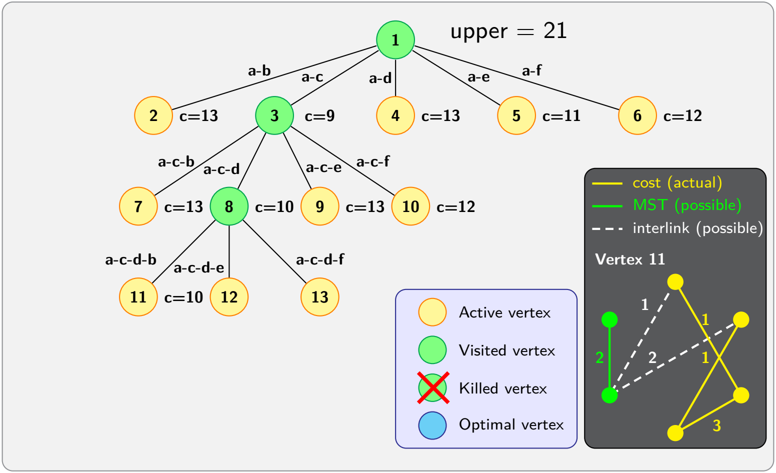

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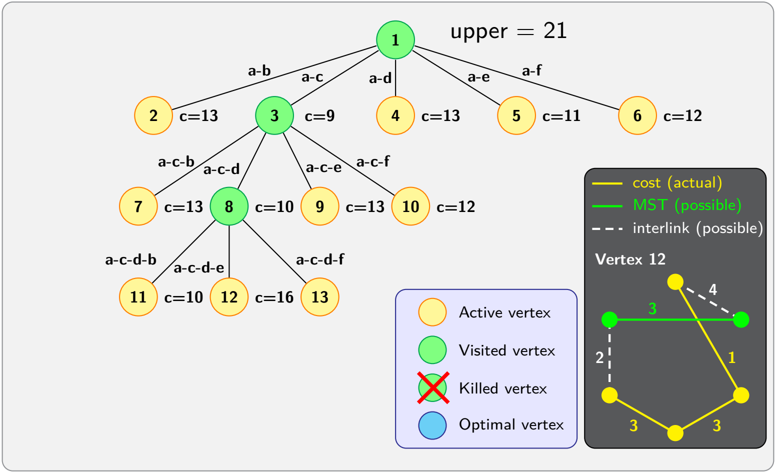

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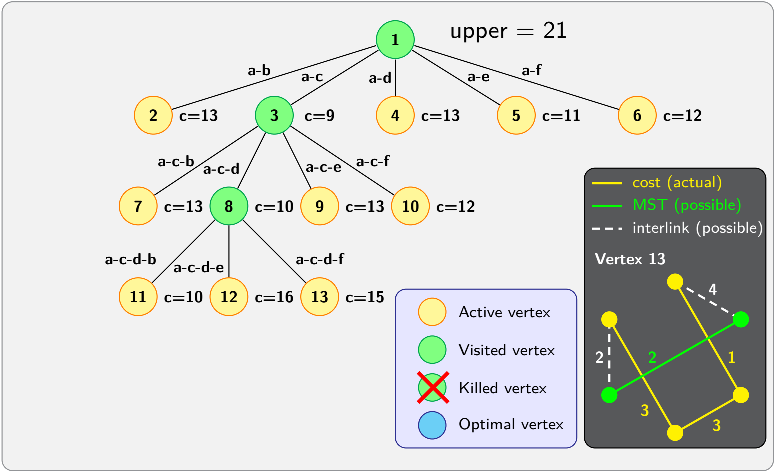

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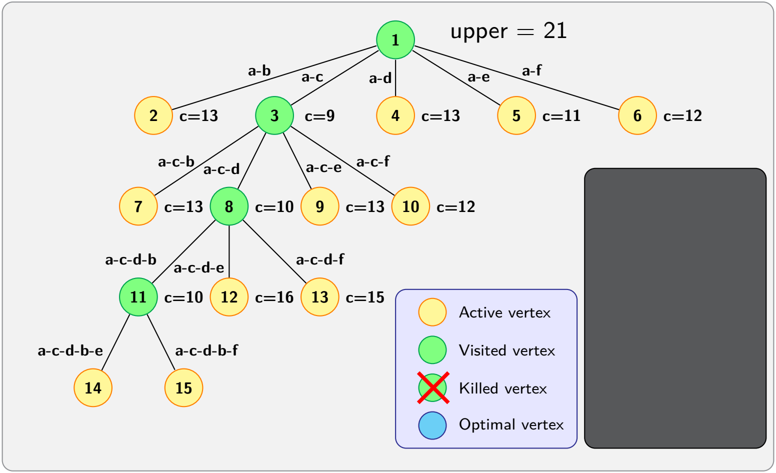

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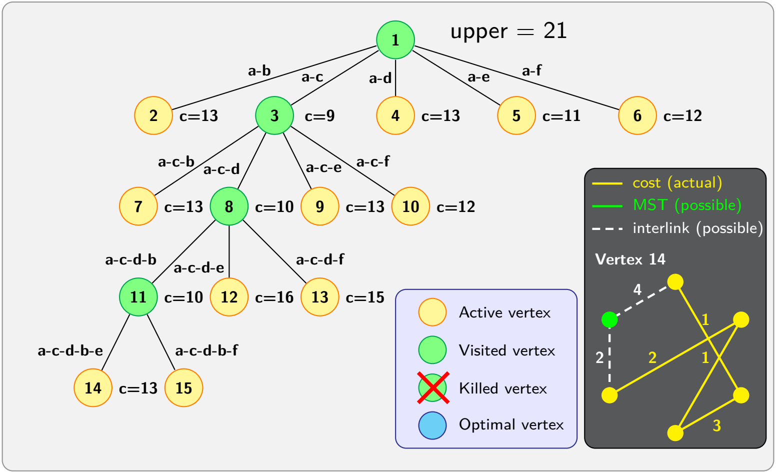

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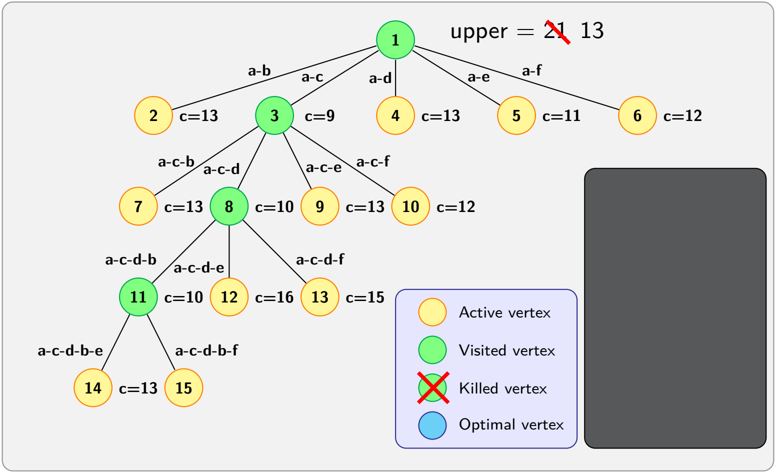

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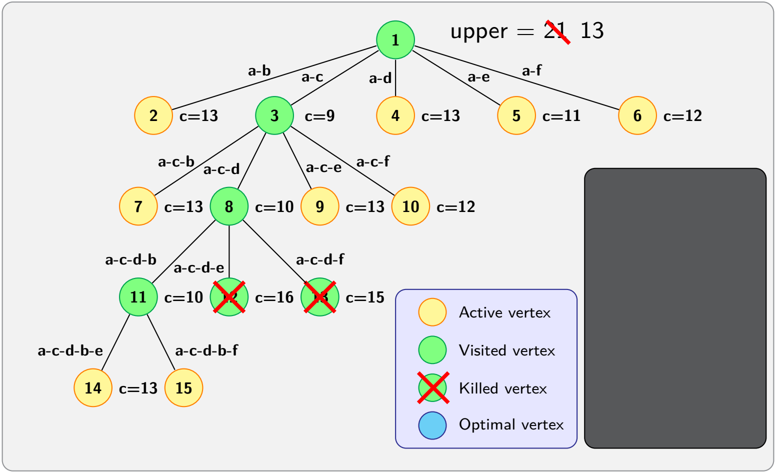

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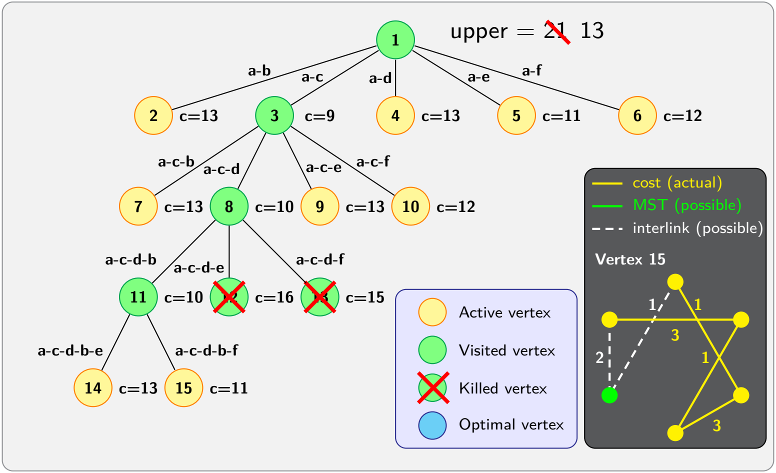

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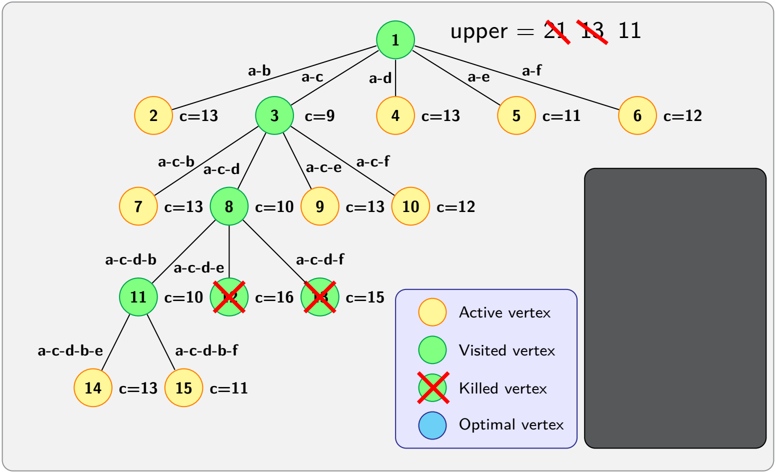

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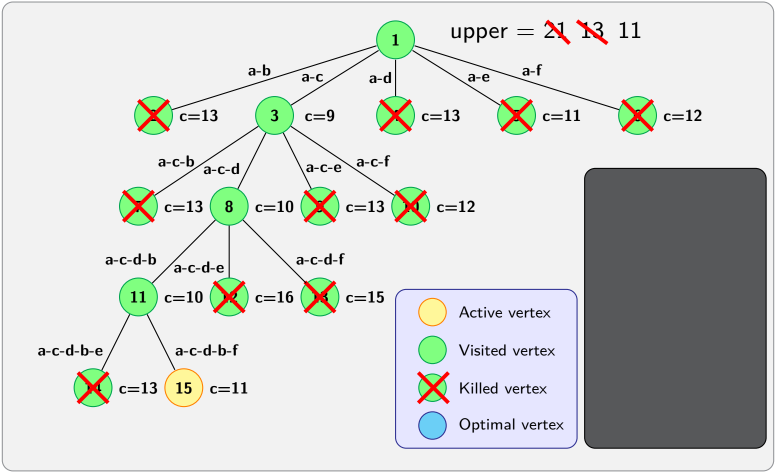

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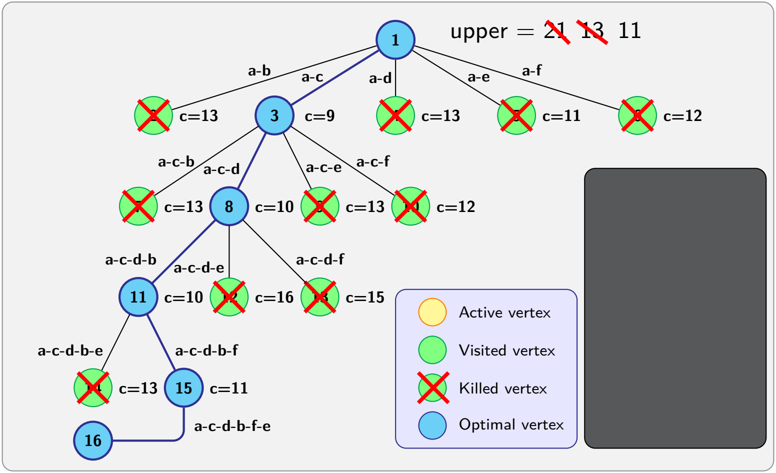

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