1 Problem 1

> **Assignment 1**
>
> A Grade 60, A706 reinforcing bar is pulled in tension from zero strain to the yield point in 0.2 seconds. What is the expected yield stress? The strain rate continues beyond yield. What is the expected ultimate stress?

The yield strain for Grade 60 steel is:

$$\epsilon_y = \frac{f_y}{E_{steel}} = \frac{60\,ksi}{29000\,ksi} \approx 2\text{‰}$$

Therefore the strain rate is:

$$\frac{\Delta\epsilon}{\Delta t} = \frac{0.002}{0.20s} \approx 1\%/s$$

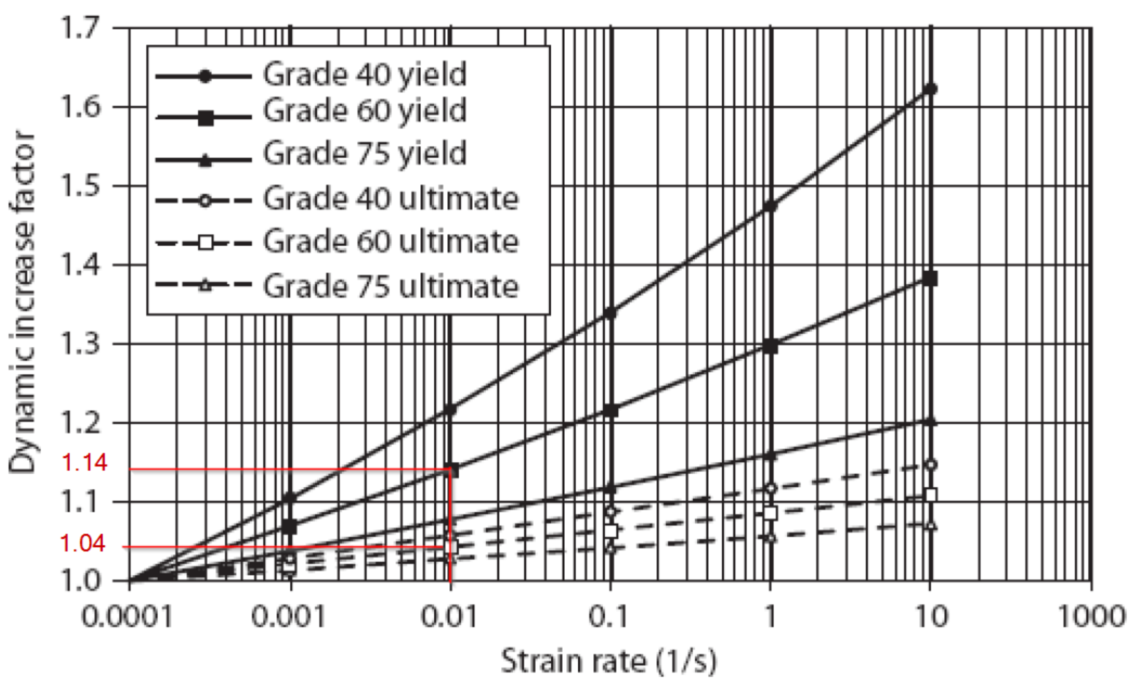

Figure 1: Expected yield and ultimate stresses.

Thus:

$$f_{y-expected} = 1.14 f_y = 79.344\,ksi\,(437\,MPa)$$

$$f_{u-expected} = 1.04 f_u \approx 100\,ksi\,(MPa)$$

2 Problem 2

Assignment 2

A Grade 60 reinforcing bar is subjected to the cyclic loading history shown in Figure 2. Using the Coffin-Manson relationship with coefficients M and m from Brown and Kunnath as presented in class, estimate (by calculations) the number of full cycles to failure.

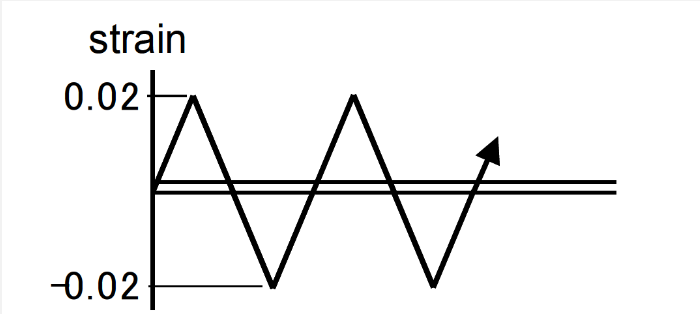

Figure 2: Cyclic behavior

The Coffin–Manson relation [1][2] provides useful information to estimate the number of subcycles $(2N_f)_u$ that lead to failure of the reinforcement steel. Henceforth, the upper script is used to denote the half strain amplitude considered in each case.

$$(2N_f)_u^{\epsilon_a=0.02} = \left(\frac{\epsilon_a}{M}\right)^{-1/m} = \left(\frac{0.02}{0.11}\right)^{-1/0.44} = 48.15 \tag{1}$$

Here, ϵ_a denotes half the strain amplitude, and the values $M = 0.11$ and $m = -0.44$ are adopted, as reported by Brown and Kunnath [3] to provide the best fit for No. 7 (25) bars.

It is thus concluded that 25 cycles are needed to produce bar failure at a half strain rate of $\epsilon_a = 0.02$.

3 Problem 3

Assignment 3

A reinforcing bar is subjected to the three cycles at strain amplitude 0.02 as shown in Figure 3. Subsequently it is subjected to cycles at twice that amplitude. Estimate by calculations the number of half cycles at the larger amplitude before fracture.

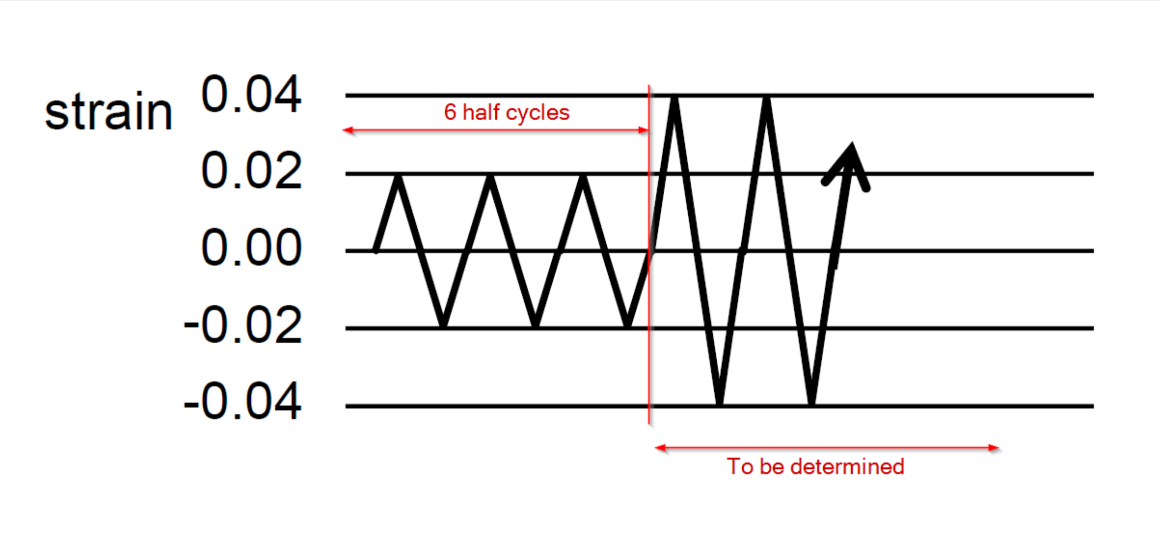

Figure 3: Low-cycle strain time behavior of analyzed reinforcement.

In this context $(2N_f) = 6$ is the number of half cycles that the material is subjected at half strain rate 0.02. Since from Equation 1 we have that $2Nf < (2N_f)_u^{\epsilon_a=0.02}$, it can be concluded that the steel reinforcement has not reached fracture in this first loading stage.

If $D = 1$ is the damage necessary to achieve failure, the damage accumulated after the 6 half cycles at half-strain amplitude 0.02 is:

$$D_{6\,half\,cycles}^{\epsilon_a=0.02} = \frac{2Nf}{(2N_f)_u^{\epsilon_a=0.02}} = \frac{6}{48.15} = 0.125\,(12.5\%\,damage)$$

Analogously to the derivation followed in **Eq. (1)**, the necessary number of half cycles at half strain amplitude of $\epsilon_a = 0.04$ is:

$$(2N_f)_{u-total}^{\epsilon_a=0.04} = \left(\frac{\epsilon_a}{M}\right)^{-1/m} = \left(\frac{0.04}{0.11}\right)^{-1/0.44} \approx 10$$

Having already accumulated a total damage of 12.5%, the number of half cycles at the larger amplitude before fracture is:

$$(2N_f)_{u-partial}^{\epsilon_a=0.04} = [1 - 0.125](2N_f)_{u-total}^{\epsilon_a=0.04} = 8.75$$

Precisely, it can be concluded that –after the bar was subjected to 6 half cycles at half strain rate 0.02– **8 half cycles will occur at half strain rate 0.04 before failure**.

4 Problem 4

Assignment 4

A normalweight concrete made with Type III cement has specified compressive strength of 5000 psi at 28 days. Suppose that it is ordered from a plant that has average quality control. At age of 1 year, a 6 x 12 in. cylinder containing that concrete is subjected to a monotonic uniaxial compression test, with load from zero to peak value in 0.2 seconds. Assume the cylinder has been stored in a fog chamber (similar to the requirement for the standard compression test) and is tested wet. What is the expected compressive strength? (You may need to do some extra reading in Chapter 3, Section 3.3.5 of the text to solve this problem.)

4.1 Charasteristic vs probable value

According to ACI 318 [4], the compressive strength of concrete follows a normal distribution that can be modelled as follows:

$$f'_{cr} \geq f'_c + 2.33\frac{\sigma}{\sqrt{3}} \tag{2}$$

$$f'_{cr} \geq f'_c + 2.33\sigma - max(500\text{psi}, 0.10f'_c) \tag{3}$$

Here, f_c is the specified compressive strength at 28 days by the manufacturer, f'_{cr} the average expected compressive strength at 28 days, and σ represents the standard deviation of the adopted normal distribution. **Eq. (2)** and **Eq. (3)** ensure that:

1. The probability that the average of three consecutive test results falls below f'_c is no greater than 0.01.
2. The probability that an individual test result falls more than 500 psi (3.5 MPa) below f'_c is no greater than 0.01 if $f'_c \leq 5000$ psi (34 MPa).

For average quality control plant, the Covariance of the normal distribution can be assumed to be $COV = 0.15 = \sigma/f'_{cr}$ [5]. Therefore, **Eq. (2)** and **Eq. (3)** can be re-expressed as:

$$f'_{cr} = \frac{f'_c}{\left(1 - \frac{0.15\cdot 2.33}{\sqrt{3}}\right)} = 6263.96\,psi \tag{4}$$

$$f'_{cr} = \frac{f'_c - max(500\text{psi}, 0.10f'_c)}{(1 - 0.15\cdot 2.33)} = 6917\,psi \tag{5}$$

Henceforth $f'_{cr} = 6917\,psi$ should be considered.

4.2 Cement Effect Over Time

Cement fineness influences the rate of strength gain over time[5]. Recent portland cements generally achieve higher early strengths than older ones; **Fig. 4** shows results for early-age and long-term strengths under continuous moist curing. Accordingly, the cylinder is assumed to have been stored in a fog chamber, and the strength at one year can be estimated.

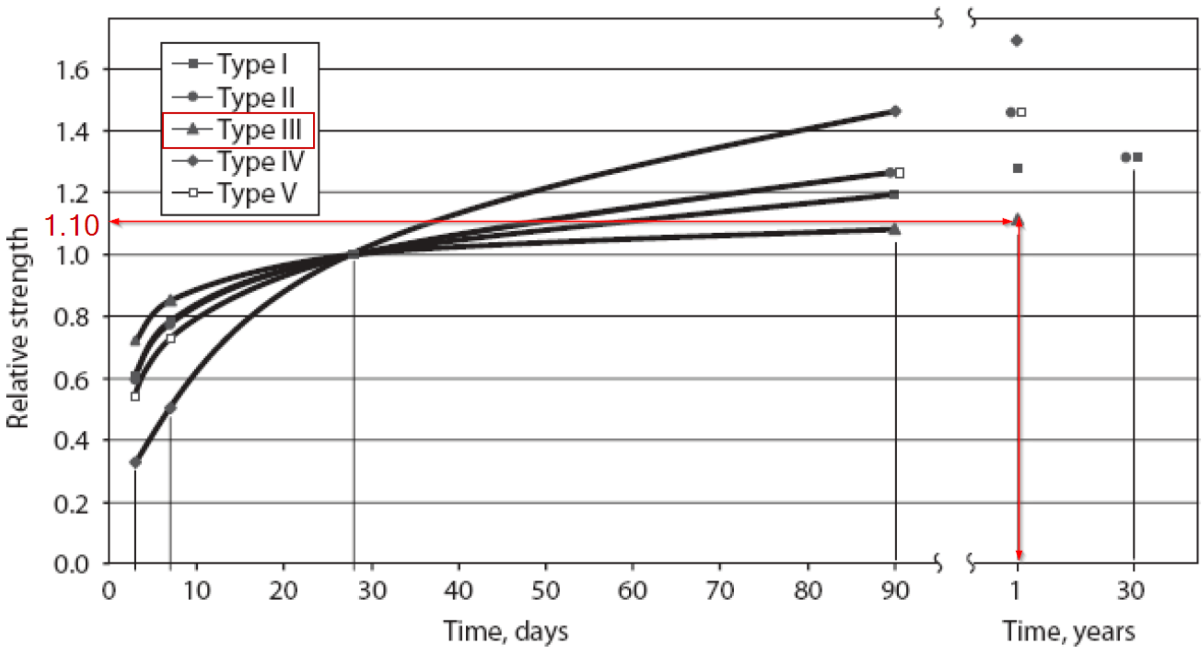

Figure 4: Concrete strength gain with time under moist curing for different types of portland cements.

The factor Δ_{cement} is defined to account for this effect such that.

$$\Delta_{cement} = 1.10 \tag{6}$$

4.3 Strain Rate Effects

Compressive strengths reported in **4.1 Charasteristic vs probable value** are based on standard test specimens loaded at a relatively slow rate [5]. Higher loading rates can increase the apparent strength, as captured by the dynamic increase factor:

$$\Delta_{\text{dynamic increase factor}} = \Delta_d = 1 + \frac{\sqrt[4]{\dot{\epsilon}}}{2}$$

The strain rate $\dot{\epsilon}$ is estimated by dividing the ultimate concrete strain at failure ($\epsilon_c = 0.002$ for plain concrete [5]) by the corresponding loading duration ($t = 0.2\,\text{s}$):

$$\dot{\epsilon} = \frac{\epsilon_c}{t} = \frac{0.002}{0.2} = 0.01\,\text{s}^{-1}.$$

Thus:

$$\Delta_{\text{d}} = 1 + \frac{\sqrt[4]{0.010}}{2} \approx 1 + \frac{0.32}{2} = 1.16. \tag{7}$$

4.4 Conclusion

In summary, the expected compressive strength of the cylinder at one year is obtained by combining the base value of f'_{cr} with the correction factors associated with cement fineness, strain rate, and curing conditions. Accordingly:

$$f'_{c,1\mathrm{yr}} = f'_{cr} \cdot \Delta_{cement} \cdot \Delta_d$$

Substituting the previously determined values from **Eq. (6)**, and **Eq. (7)**:

$$f'_{c,1\mathrm{yr}} = 6917 \times 1.10 \times 1.16 \approx 8\,826\,\mathrm{psi}.$$

Thus, the most probable compressive strength of the tested cylinder at one year is approximately:

$$f'_{c,1\mathrm{yr}} \approx 8.83\,\mathrm{ksi}.$$

5 References

[1] S. S. Manson, “Behavior of materials under conditions of thermal stress,” *NACA Technical Note 2933*, 1953.

[2] L. F. J. Coffin, “A study of the effects of cyclic thermal stresses on a ductile metal,” *Transactions of the ASME*, vol. 76, pp. 931–950, 1954.

[3] J. Brown and S. Kunnath, “Low-cycle fatigue failure of reinforcing steel bars,” *ACI Materials Journal*, vol. 101, pp. 457–466, 11 2004.

[4] ACI Committee 318, *Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary (ACI 318R-19).* Farmington Hills, MI: American Concrete Institute, 2019. Inch-Pound Units.

[5] J. Moehle, *Seismic Design of Reinforced Concrete Buildings.* New York: McGraw-Hill Education, 1st edition ed., 2015.