1 Introduction

Consider the weightless frame shown. Theroller at c can resist both downward and upward forces. Beam depth is 30 in. and column depth is 24 in. Service loads on the beam include 1 klf dead load and 0.5 klf live load, both acting downward. Dead load D is along the entire span. Live load L, which is associated with distributed load of 50 psf and is not associated with a garage or area of public assembly, can be placed anywhere along the span. Earthquake load E, which can act either left or right, is 10 kips acting at the beam centerline. (Sections 1.5.3 and 1.5.4 of the textbook have useful information on load combinations and an example problem.)

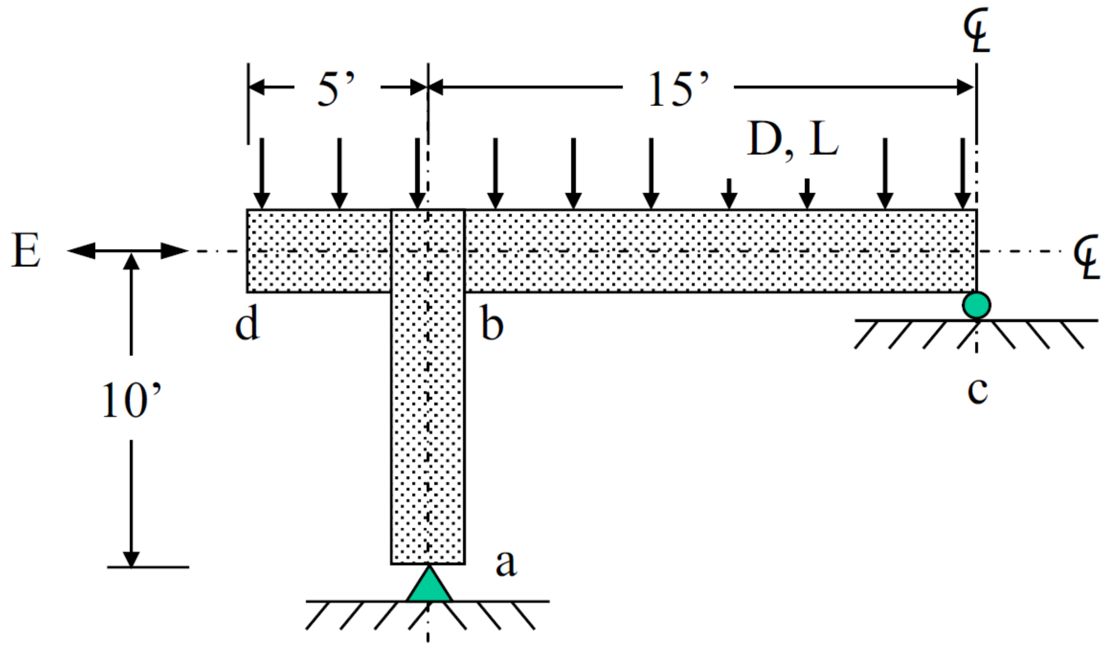


Figure 1: Weightless frame for problems 1-5.

To address Problems 1 through 4 and to better understand the structural response, section-force diagrams were constructed for the prescribed load patterns. Since the system is isostatic, no structural analysis software was required; all calculations were carried out manually. These diagrams constitute valuable tools, as load effects can be directly combined by applying the principle of superposition, thereby enabling a clear assessment of the structure's behavior under more complex loading and required scenarios.

The structural diagrams can be found from **Fig. 2** to **Fig. 7**, the values in orange indicate the section forces:

1. In the beam at the righ-hand face of the column.

2. In the column at the face of the beam (bottom).

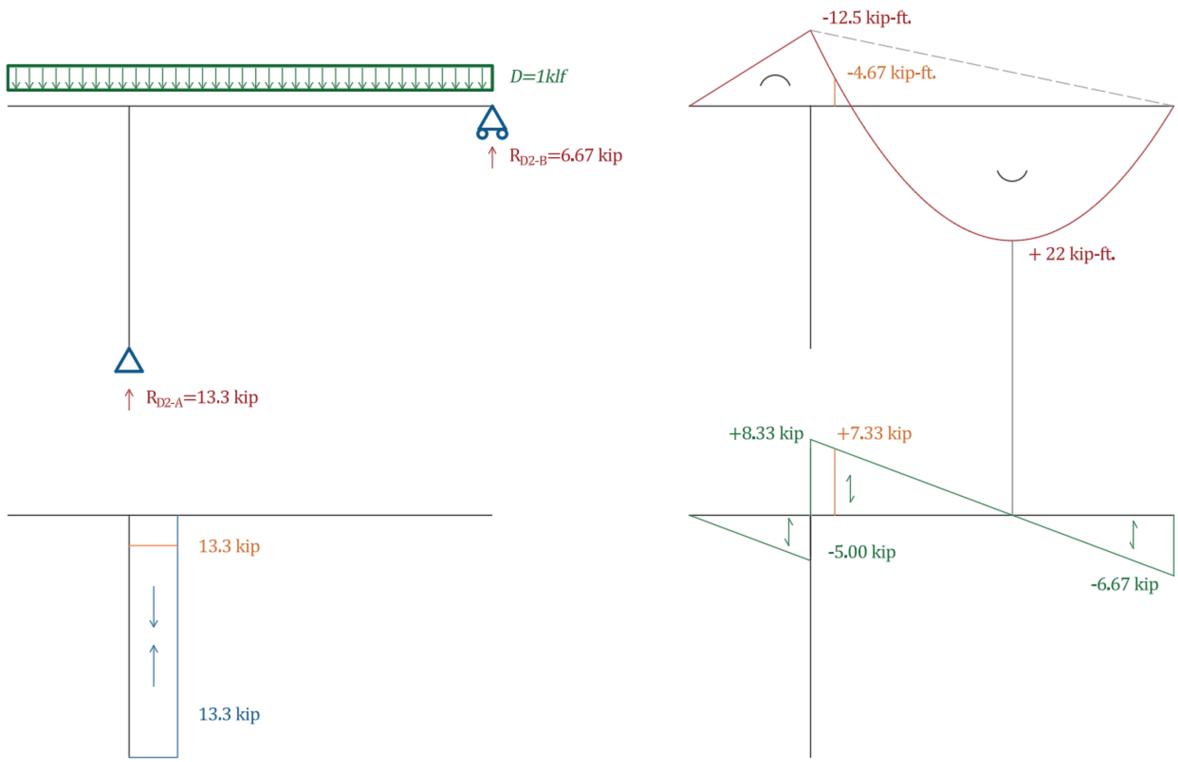


Figure 2: Section forces for dead load D.

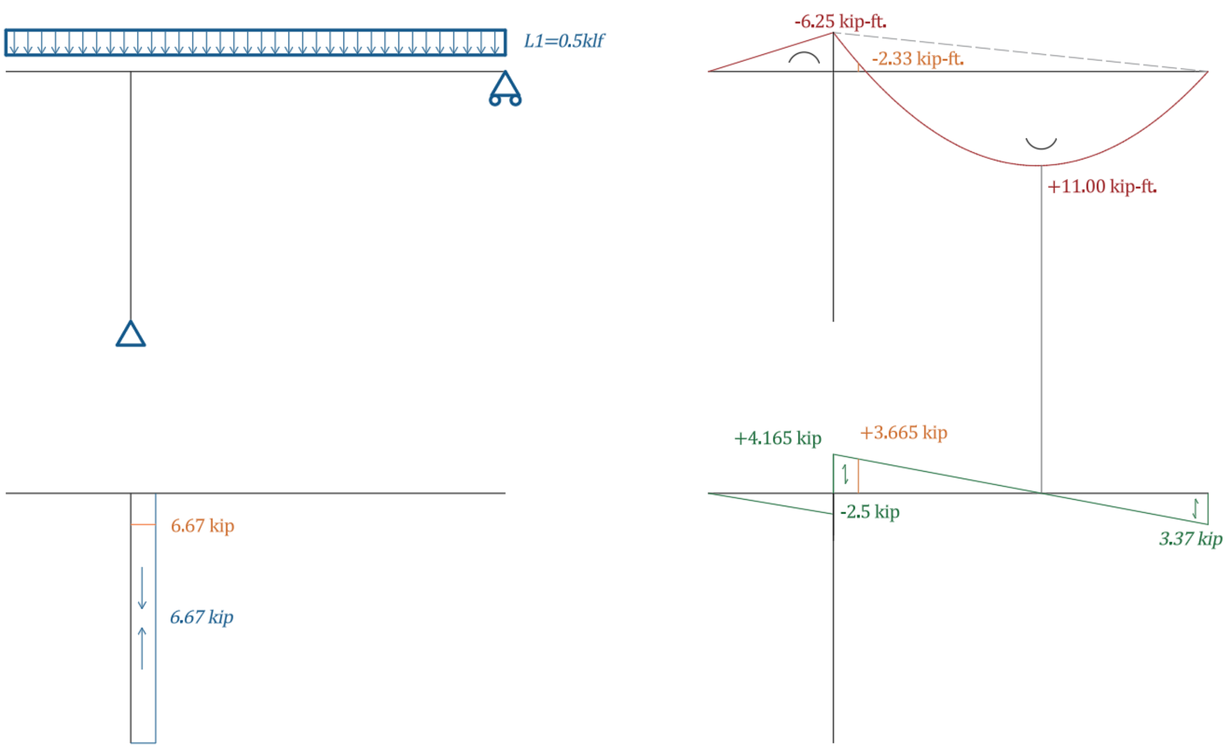


Figure 3: Section forces for live load L1.

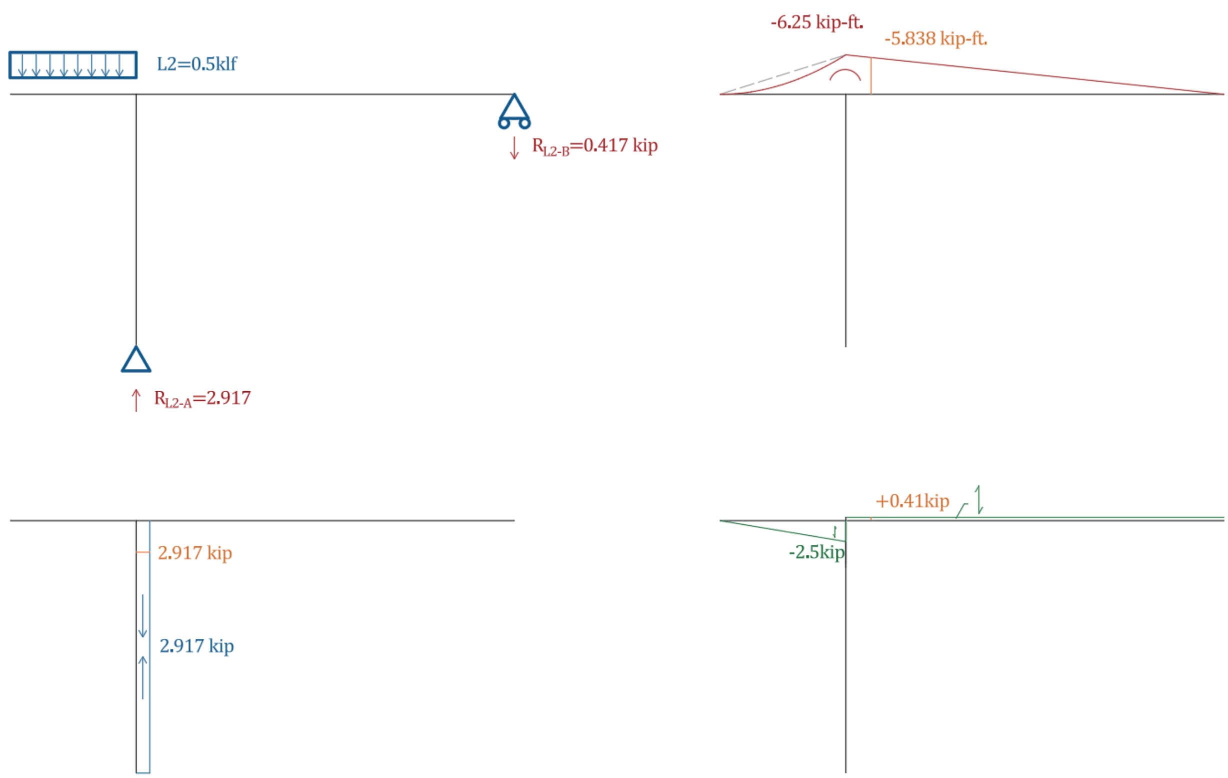


Figure 4: Section forces for live load L2.

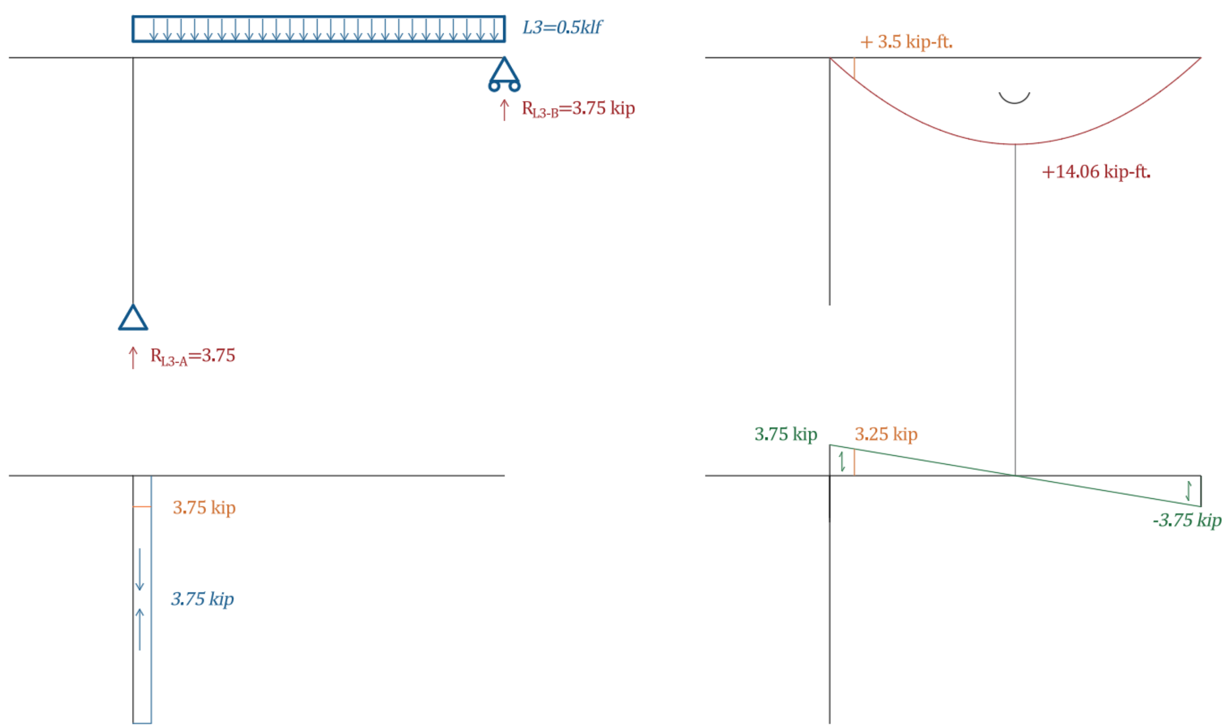


Figure 5: Section forces for live load L3.

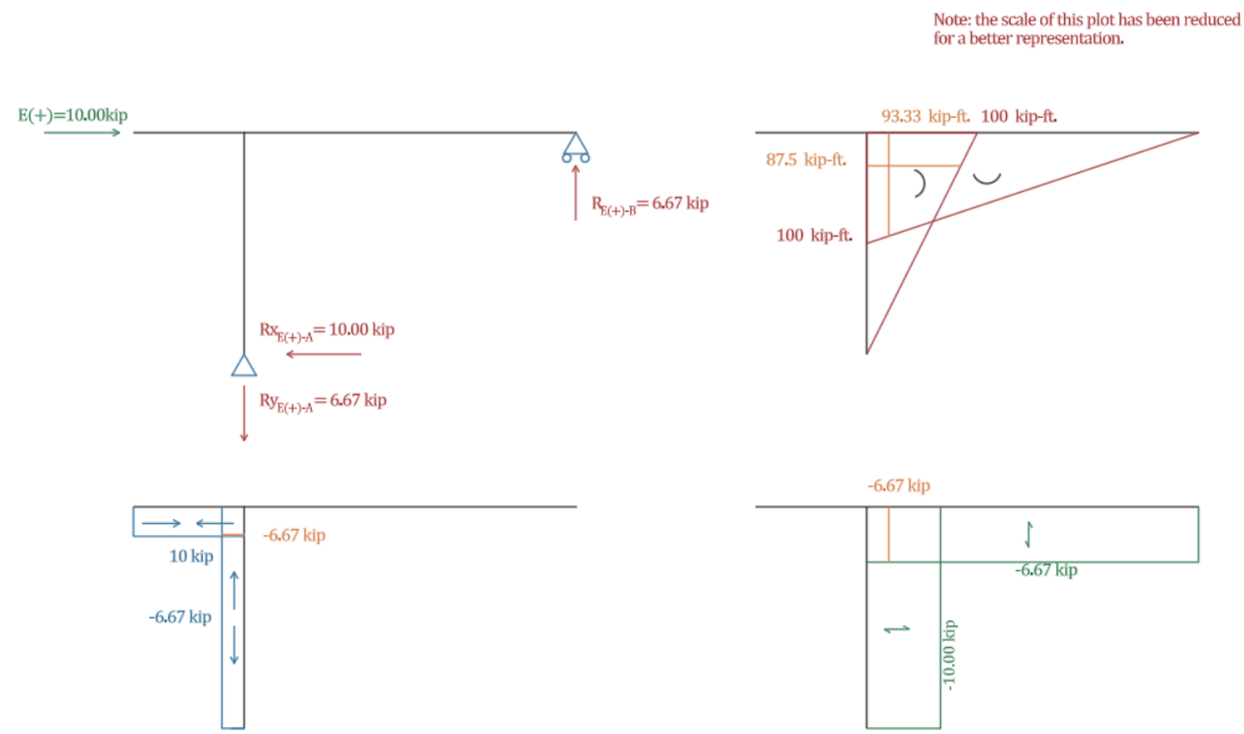


Figure 6: Section forces for earthquake load in positive X direction.

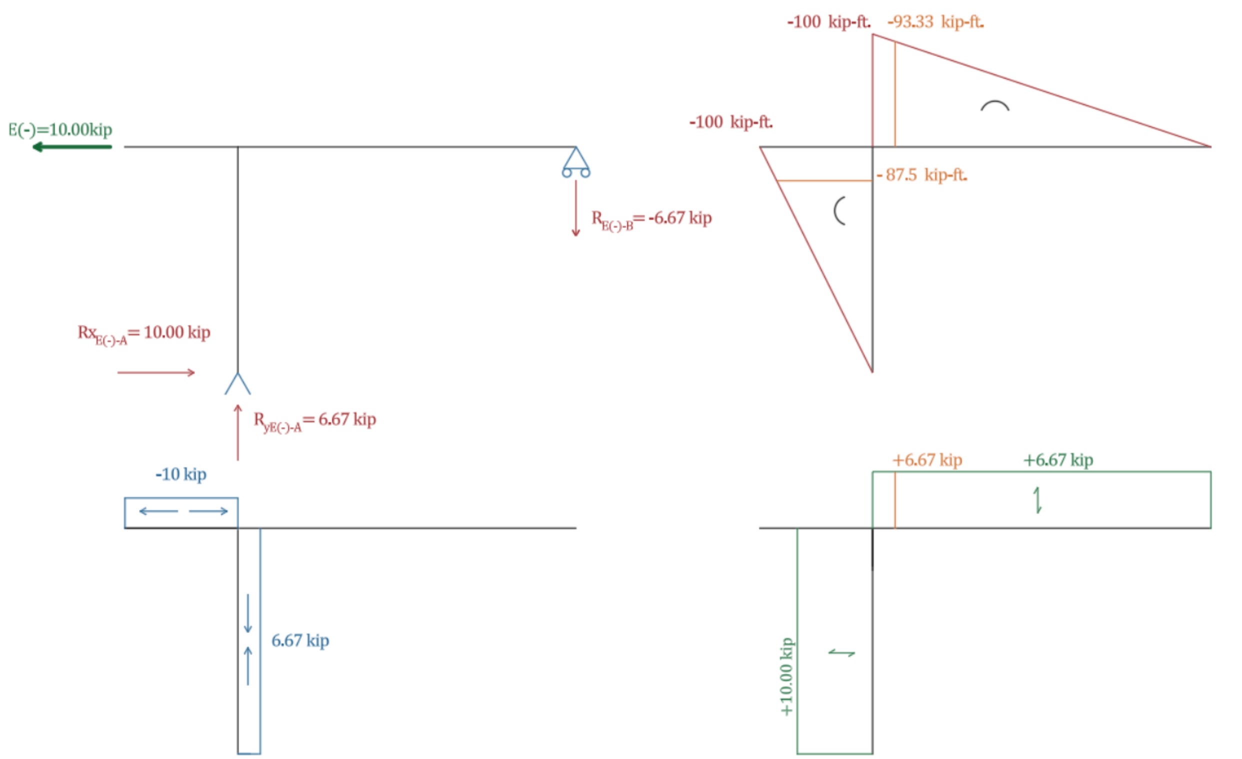


Figure 7: Section forces for earthquake load in negative X direction.

2 Problem 1

Assignment 1

Use the LRFD method to calculate design moments in the beam at the right-hand face of the column near point b. Report both the most positive moment (bottom in tension) and most negative moment (top in tension). Report positive moments as + and negative moments as –. If it turns out that there is no positive moment, report the least negative moment instead of the most positive moment. Present results in a table that summarizes the moments for each relevant load combination. See table format below. Hint: Don't guess the critical loadings. Instead, use a methodical approach of defining the load cases (D, L, E, each considered independently in various possible patterns). Then for each load case, solve for the reactions and the moment in the beam at point b. Finally, find the beam moments at point b for each load combination using algebraic combinations of the values already obtained and write the design values as most negative and most positive results.

In order to obtain the ***beams's design moments*** at the right-hand face of the column, the section forces at the wanted coordinate for the uncombined load patterns are computed. The results from **Fig. 2** to **Fig. 7** are summarized in **Table 1**:

Load Pattern	**Descrption**	**Beam section force at right-hand column face**	
		Bending Moment [kip-ft.]	Shear [kip]
D	Dead Load.	-4.667	7.333
L1	Live load applied on entire beam span.	-2.333	3.667
L2	Live load applied at left portion of beam.	-5.838	0.410
L3	Live load applied at right portion of beam.	3.500	3.250
E(+)	Earthquake load in +X direction.	93.333	-6.667
E(-)	Earthquake load in -X direction.	-93.333	6.667

Table 1: Beam section forces at the right-hand column face for load patterns.

The considered load combinations, according to ASCE 7-22 (1) are:

1. 1.4D
2. 1.2D + 1.6L1
3. 1.2D + 1.6L2
4. 1.2D + 1.6L3
5. 1.2D + 0.50[1]L1 + E
6. 1.2D + 0.50[1]L2 + E
7. 1.2D + 0.50[1]L3 + E

[1] L is a distributed load associated with 50psf100psf

8. 1.2D + 0.50^1L1 − E
9. 1.2D + 0.50^1L2 − E
10. 1.2D + 0.50^1L3 − E
11. 0.90D + E
12. 0.90D − E

Performing a linear combination on the values of **Table 1** we obtain the design moments in the beam at the right-hand face of the column near point b. See **Table 2**.

Id	Load Comb	Mu [kip-ft.]	Vu [kip]	Comment
1	1.4D	-6.53	10.27	
2	1.2D + 1.6L1	-9.33	14.67	
3	1.2D + 1.6L2	-14.94	9.46	
4	1.2D + 1.6L3	0.00	14.00	
5	1.2D + 0.50L1 +E	86.57	3.97	
6	1.2D + 0.50L2 +E	84.81	2.34	
7	1.2D + 0.50L3 +E	89.48	3.76	Most positive moment.
8	1.2D + 0.50L1 -E	-100.10	17.30	Most positive shear.
9	1.2D + 0.50L2 -E	-101.85	15.67	Most negative moment and most positive shear.
10	1.2D + 0.50L3 -E	-97.18	17.09	
11	0.90D+E	89.13	-0.07	Most negative shear.
12	0.90D-E	-97.53	13.27	

Table 2: Beam factored bending and shear at the right-hand column face for the combination of loads.

Thus the desing moments M_u are:

Id	Load Comb	Mu Design Moment [kip-ft.]
7	1.2D + 0.50L3 +E	89.48
9	1.2D + 0.50L2 -E	-100.10

Table 3: Beam design bending moment at the right-hand column face for load patterns. $\phi = 0.90$ due to tension-controlled section assumption.

3 Problem 2

Assignment 2

WRepeat Problem 1 except in this case solve for the combinations of Pu and Mu acting on the column at the face of the beam near point b. Plot the combinations of Pu and Mu for each load combination on a P-M interaction diagram. (You don't need to identify which combinations are most critical, just plot all the combinations. An engineer would need to ensure that the column design capacity was sufficient for all the combinations.)

Following a similar procedure, the section forces for the load patterns at the face of the beam are fist computed in **Table 4**:

Load Pattern	**Descrption**	**Column section force at beam face** P [kip] Comp (+)	M [kip-ft.]
D	Dead Load.	13.333	0.000
L1	Live load applied on entire beam span.	6.667	0.000
L2	Live load applied at left portion of beam.	2.917	0.000
L3	Live load applied at right portion of beam.	3.750	0.000
E(+)	Earthquake load in +X direction.	-6.667	87.500
E(-)	Earthquake load in -X direction.	6.667	-87.500

Table 4: Column section forces at the beam face for load patterns.

Based on **Table 4**, all factored Pu, Mu are computed in **Table 5**

Id	**Load Combo**	**Pu [kip]**	**Mu [kip-ft]**
1	1.4D	18.67	0.00
2	1.2D + 1.6L1	26.67	0.00
3	1.2D + 1.6L2	20.67	0.00
4	1.2D + 1.6L3	22.00	0.00
5	1.2D + 0.50L1 +E	12.67	87.50
6	1.2D + 0.50L2 +E	10.79	87.50
7	1.2D + 0.50L3 +E	11.21	87.50
8	1.2D + 0.50L1 -E	26.00	-87.50
9	1.2D + 0.50L2 -E	24.13	-87.50
10	1.2D + 0.50L3 -E	24.54	-87.50
11	0.90D+E	5.33	87.50
12	0.90D-E	18.67	-87.50

Table 5: Column factored section forces Pu, Mu at the beam face for load patterns.

Finally, the results are ploted in a P-M interaction diagram. See **Fig. 8**.

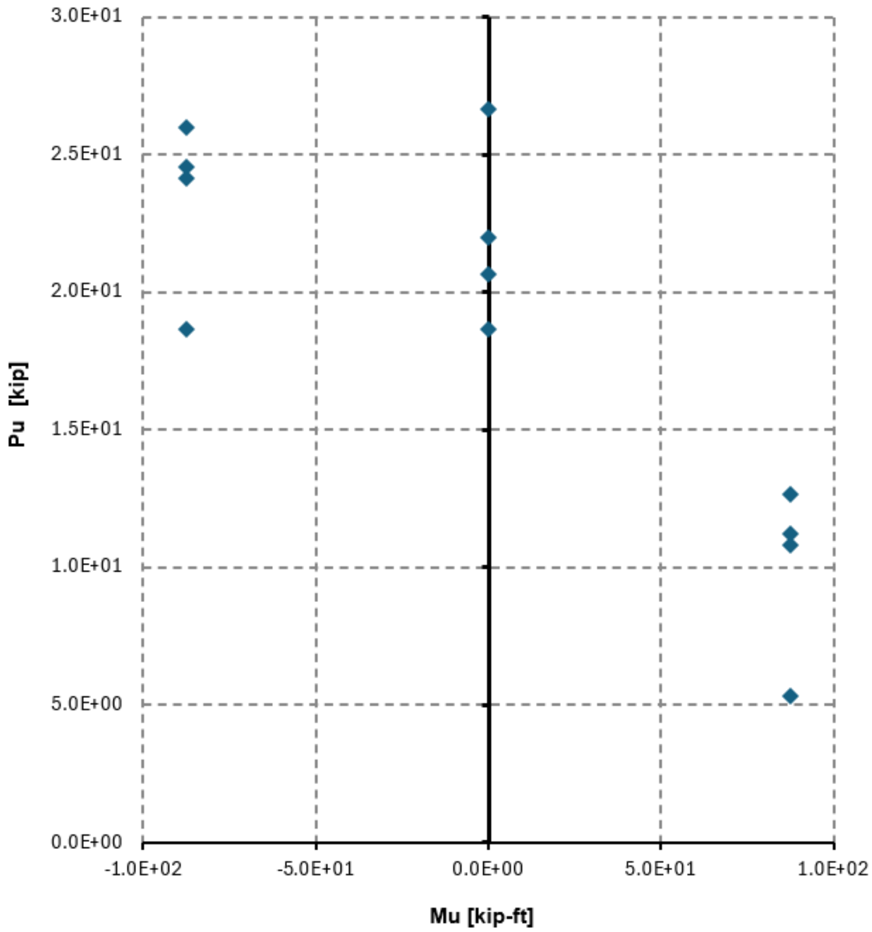


Figure 8: P-M interaction diagram for factored loads.

> **Assignment 3**
>
> Again using the load combinations of the LRFD method (not capacity design), determine the most positive and most negative values of Vu in the beam at the right-hand face of the column near point b. Present results in a table as you did for Problem 1.

The expected result is obtained by summarizing the values of **Table 2**:.

Id	Load Comb	Vu [kip]
9	1.2D + 0.50L1 -E	17.30
11	0.90D+E	-0.07

Table 6: Column section forces at the beam face for load patterns.

4 Problem 4

> **Assignment 4**
>
> What is the minimum required nominal shear strength for the beam given the shear determined in Problem 3?

According to **Table 6**, the minimum required nominal shear strength is given by V_u/ϕ, where ϕ is either 0.75 or 0.60 depending on the provisions of ACI 318-19, Article 21.2.4.1 (2):

> ***21.2.4.1*** *For any member designed to resist E, ϕ for shear shall be 0.60 if the nominal shear strength of the member is less than the shear corresponding to the development of the nominal moment strength of the member. The nominal moment strength shall be the maximum value calculated considering factored axial loads from load combinations that include E.*

Assuming 21.2.4.1 is not met, the minimum required nominal shear strength for the beam is

$$V_n = \frac{V_u}{\phi} = \frac{17.30kip}{0.75} = 23.06kip$$

5 Problem 5

Assignment 5

Suppose that the beam is designed such that it is capable of developing a probable moment strength Mpr in both positive and negative bending equal to 169.7 k-ft (note that, in this example, this is approximately 1.7 times the maximum required moment strength from Problem 1.) What is the minimum required nominal shear strength for the beam if it is required that the beam have a ductile failure mode under overloads?

In order for the beam to exhibit ductile failure, flexural plastic hinges must be developed before reaching its shear capacity. Thus, shear failure—which is characterized by brittle behavior—must be avoided.

As noted in **Table 3**, the critical load combination that produced the maximum moment (in absolute value) is *1.2D + 0.50L2 + E(-)*. Accordingly, the distributed load applied is calculated as

$$1.20 \cdot 1.00\,\text{klf} + 0.50 \cdot 0.50\,\text{klf} = 1.45\,\text{klf}.$$

In **Fig. 9**, the probable moment strength M_{pr} is applied at both ends of the beam in the direction that generates the maximum shear demand in the left end. However, it is noted that the sign of M_{pr} may be reversed, and the shear demand in the extremes should be switched .

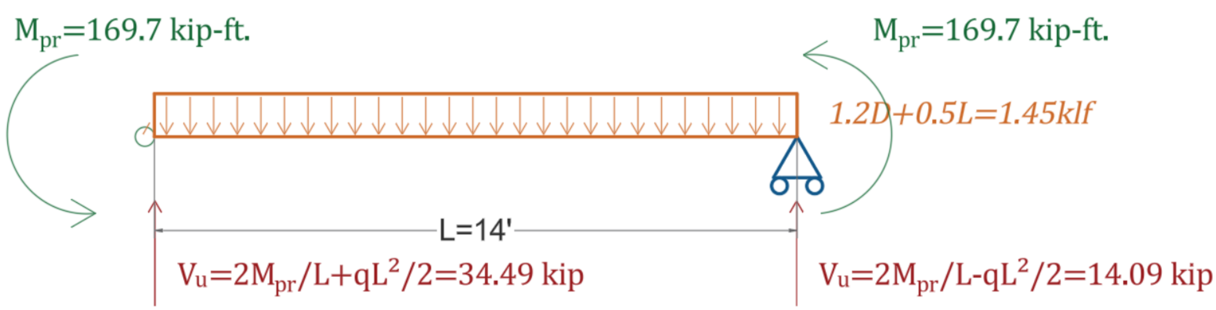


Figure 9: Capacity design for beam at its right-face from column.

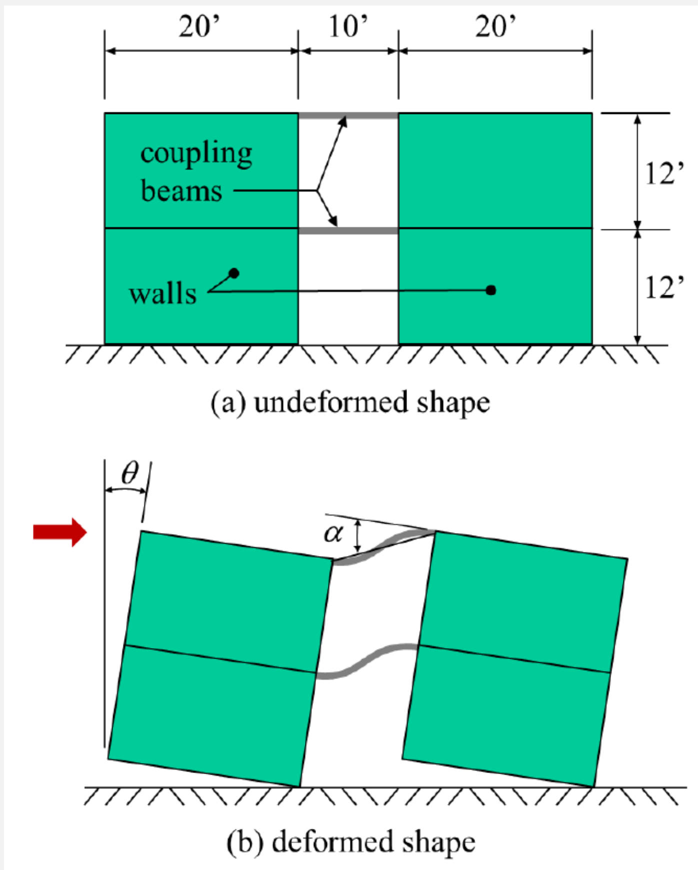
The required nominal shear strength for both ends is:

$$V_{n-required} = \frac{V_u}{\phi_{shear}} = \frac{34.49kip}{0.75} = 45.98kip$$

6 Problem 6

Assignment 6

Two rigid walls are coupled by two coupling beams. Under earthquake loading, structural analysis shows that the walls rock about their corners and undergo lateral displacement corresponding to a drift ratio $\theta = 0.02$. In a displacement-based design, it would be necessary to design the coupling beams to be capable of undergoing the rotation α due to the drift ratio 0.02. What is the value of the chord rotation angle α corresponding to the drift ratio 0.02?



To determine the chord rotation angle α, defined as the angle between the deformed coupling beams, 6 points defined in the structure as per **Fig. 10** are used.

Here, the angle α is defined as the angle between the vectors $\vec{EF}$ and $\vec{EC}$, which represent the orientation of the deformed positions of the coupling beams. The steps followed to compute this angle are as follows:

1. The initial configuration consists of two rigid walls defined by points $A(0', 0')$ and $B(30', 0')$, which represent the bottom corners of the left and right walls, respectively. These points remain stationary during the rotational motion.

2. After the imposed drift, the coordinates of the deformed corners are:

$$C = (0.48', 23.995'), \quad E = (10.484', 24.395'), \quad F = (30.48', 23.995').$$

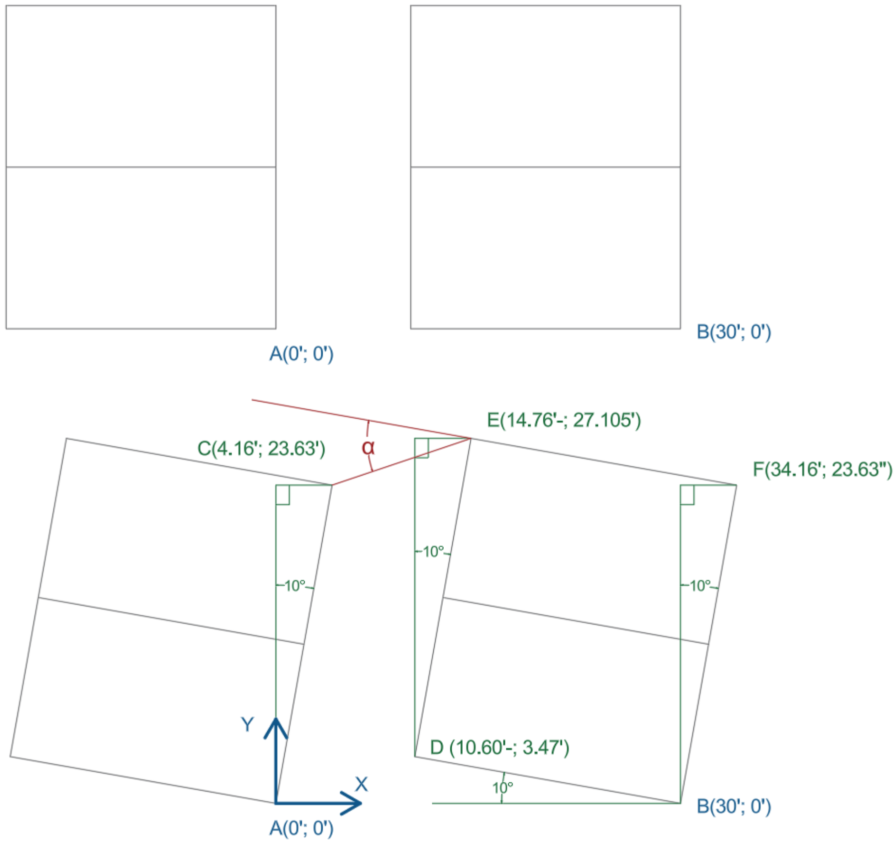


Figure 10: Points defined in the structure and their coordinates.

Note that $0.48' = 0.02 * 24'$ which correspond with a angle of $atan(0.02) = 1.145$

3. With points C, E, and F known, the following vectors are defined:

$$\vec{EC} = C - E = (-10.004, \ -0.400), \qquad \vec{EF} = F - E = (19.996, \ -0.400).$$

4. The dot product and magnitudes are:

$$\vec{EC} \cdot \vec{EF} = (-10.004)(19.996) + (-0.400)(-0.400) = -199.88,$$

$$\|\vec{EC}\| = \sqrt{(-10.004)^2 + (-0.400)^2} \approx 10.012, \qquad \|\vec{EF}\| = \sqrt{(19.996)^2 + (-0.400)^2} = 20.00.$$

5. The angle between the vectors is then:

$$\theta = \cos^{-1}\left(\frac{\vec{EC} \cdot \vec{EF}}{\|\vec{EC}\| \, \|\vec{EF}\|}\right) = \cos^{-1}(-0.9991) \approx 176.564^\circ.$$

6. Since this is the obtuse angle, the chord rotation is obtained as the supplement:

$$\alpha = 180^\circ - \theta \approx 3.436^\circ \ (\approx 0.060 \, \text{rad}).$$

This value of α represents the chord rotation experienced by the coupling beam due to the drift imposed by the motion of the walls under earthquake loading.

7 References

[1] American Society of Civil Engineers, *Minimum Design Loads and Associated Criteria for Buildings and Other Structures*. Reston, Virginia: American Society of Civil Engineers, 2022. ASCE Standard ASCE/SEI 7-22. Library of Congress Control Number: 2021951104.

[2] ACI Committee 318, *Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary (ACI 318R-19)*. Farmington Hills, MI: American Concrete Institute, 2019. Inch-Pound Units.