ANEXO 3

CONT...

Undergraduate Texts in Mathematics

Apostol: Introduction to Analytic Number Theory.
1976. xii, 338 pages. 24 illus.

Armstrong: Basic Topology.
1983. xii, 260 pages. 132 illus.

Bak/Newman: Complex Analysis.
1982. x, 224 pages. 69 illus.

Banchoff/Wermer: Linear Algebra Through Geometry.
1983. x, 257 pages. 81 illus.

Childs: A Concrete Introduction to Higher Algebra.
1979. xiv, 338 pages. 8 illus.

Chung: Elementary Probability Theory with Stochastic Processes
1975. xvi, 325 pages. 36 illus.

Croom: Basic Concepts of Algebraic Topology.
1978. x, 177 pages. 46 illus.

Fischer: Intermediate Real Analysis.
1983. xiv, 770 pages. 100 illus.

Fleming: Functions of Several Variables. Second edition.
1977. xi, 411 pages. 96 illus.

Foulds: Optimization Techniques: An Introduction.
1981. xii, 502 pages. 72 illus.

Franklin: Methods of Mathematical Economics. Linear and Nonlinear Programming. Fixed-Point Theorems.
1980. x, 297 pages. 38 illus.

Halmos: Finite-Dimensional Vector Spaces. Second edition.
1974. viii, 200 pages.

Halmos: Naive Set Theory.
1974, vii, 104 pages.

Iooss/Joseph: Elementary Stability and Bifurcation Theory.
1980. xv, 286 pages. 47 illus.

Jänich: Topology
1984. ix, 180 pages (approx.). 180 illus.

Kemeny/Snell: Finite Markov Chains.
1976. ix, 224 pages. 11 illus.

Lang: Undergraduate Analysis
1983. xiii, 545 pages. 52 illus.

Lax/Burstein/Lax: Calculus with Applications and Computing, Volume I. Corrected Second Printing.
1984. xi, 513 pages. 170 illus.

LeCuyer: College Mathematics with A Programming Language.
1978. xii, 420 pages. 144 illus.

Macki/Strauss: Introduction to Optimal Control Theory.
1981. xiii, 168 pages. 68 illus.

Malitz: Introduction to Mathematical Logic: Set Theory - Computable Functions - Model Theory.
1979. xii, 198 pages. 2 illus.

Martin: The Foundations of Geometry and the Non-Euclidean Plane.
1975. xvi, 509 pages. 263 illus.

Martin: Transformation Geometry: An Introduction to Symmetry.
1982. xii, 237 pages. 209 illus.

Millman/Parker: Geometry: A Metric Approach with Models.
1981. viii, 355 pages. 259 illus.

continued after Index

Peter Lax
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Volume I

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Preface

1 Mathematics is vigorously and brilliantly pursued in our time on a very broad front; yet the authors of this text feel that not enough mathematical talent is devoted to furthering the interaction of mathematics with other sciences and disciplines. This imbalance is harmful to both mathematics and its users; to redress this imbalance is an educational task which must start at the beginning of the college curriculum. No course is more suited for this than the calculus; there students can learn at first hand that mathematics is the language in which scientific ideas can be precisely formulated, that science is a source of mathematical ideas which profoundly shape the development of mathematics, and last but not least that mathematics can furnish brilliant answers to important scientific problems.

2 Our purpose in writing this text has been to emphasize this relation of calculus to science. We hope to accomplish this by devoting whole connected chapters to single—or several related—scientific topics, letting the reader observe how the notions of calculus are used to formulate the basic laws of science and how the methods of calculus are used to deduce consequences of those basic laws. Thus the student sees calculus at work on worthwhile tasks. The traditional course too often resembles the inventory of a workshop; here we have hammers of different sizes, there saws, yonder planes; the student is instructed in the use of each instrument, but seldom are they all put together in the building of a truly worthwhile object.

3 Finding numerical answers is a very important part of an application of mathematics. Even when qualitative rather than quantitative understanding is the aim, the calculation of a well-chosen special case can take the role of a crucial experiment in confirming an old speculation, or pointing to a new one. Also, the design of effective numerical methods is one of the finest applications of the ideas of calculus. Numerical methods are presented in

this text as organic parts of calculus, not as a mere list of recipes appended as an afterthought.

Almost all numerical examples and exercises presented in this text can be calculated with the aid of programmable hand calculators. The use of a computer is advocated for those problems whose output is more than a few numbers and must be displayed in tabulated or graphical form to extract the essential features.

Review

The educational value of numerical examples worked out by the students themselves, individually or as members of small teams, cannot be overestimated. A good student is likely to be a better and more enterprising computer programmer than his instructor, and this will enable him to experiment on his own, instead of being bound to his text or the instructor's apronstrings. This active participation is a welcome alternative to merely sitting back and absorbing knowledge.

Our fairly radical outlook on what purpose calculus serves is echoed by a critical revision of some purely mathematical notions of calculus. Our treatment is rigorous without being pedantic; we do not hesitate to break with tradition where we feel that a change is called for. We pinpoint these changes in the following chapter-by-chapter account of our treatment:

Chapter 1 deals with real numbers; we teach the student to think of them in three complementary ways: (i) as entities which can be added, multiplied, etc., subject to the usual rules of algebra; (ii) as points on the number line; and (iii) as infinite decimals.

Infinite decimals are Platonic ideals of which we mortals only see shadows that appear as finite digits on the register, or printout, of calculators. The enormous advantage of thinking of real numbers as infinite decimals is that we can recognize at a glance when two numbers are close to each other. The notion of a convergent sequence can be explained without resorting to Greek letters, merely by noting that a sequence converges if more and more of the digits of its members are identical.

verbos en infinitivo

In Chapter 2 on functions we emphasize the role of functions in describing the relation of two quantities. We explain that complicated functions can be built out of very simple ones by composing, adding, multiplying, inverting, etc., functions repeatedly. Our definition of continuity is uniform continuity on a given interval; this is far more appropriate than the notion of continuity at each point. We define uniform convergence of a sequence of functions, a natural, useful, and elementary concept that Victorian prudishness usually reserves for mature audiences. We explain the notion of an algorithm for calculating a function, and give examples of distinct algorithms which calculate the same function, where one algorithm is faster and more accurate than the other.

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In Chapter 3 on differentiation, the derivative is defined as the uniform limit of difference quotients. This makes it evident that a function whose derivative is positive on an interval is an increasing function. This observation is used as a workhorse throughout the book; in this chapter it is used to

prove the mean value theorem, Taylor's theorem, and the characterization of maxima and minima. We give many illustrations of the notion of derivative and devote a section to one-dimensional mechanics.

The definite integral is defined in Chapter 4 in terms of two basic properties which are amply illustrated and motivated. We show that all properties of the integral follow from these two, including its relation to the derivative, the approximation of integrals by sums, and the rules of integration such as changing variables or integration by parts. These techniques change a given integral into another integral; we explain to the student that although in a few spectacular instances the application of these techniques leads to the explicit evaluation of an integral in terms of known functions, this is not so in most cases. Nevertheless an intelligently chosen change of variables or integration by parts can change an integral into another one that is far easier to approximate numerically than the original integral. Simpson's rule is introduced and applied lovingly.

In Chapter 5 the exponential function is defined as modeling growth; the functional equation of the exponential function is derived from this model, and from it the differential equation. We emphasize that all properties of the exponential function, including our ability to find accurate approximations to it, follow from this differential equation. The logarithmic function is defined as the inverse of the exponential function, and its usual properties are explored.

Chapter 6 is an introduction to probability theory, both discrete and continuous. The information content of a probability distribution is defined. Gauss' law of error is derived and applied to the diffusion process.

In Chapter 7 we derive the addition formulas for sine and cosine from the arithmetic of complex numbers. The differential equations for sine and cosine are derived; we emphasize that all properties of sine and cosine follow from these differential equations. There is a brief discussion of two-dimensional mechanics in terms of complex numbers. The basic facts of gravitational motion are derived.

In Chapter 8 on vibration the emphasis is placed on the law of conservation of energy. There is an elementary but nontrivial discussion of nonlinear vibrations which is a mixture of theory and numerical experimentation.

The last chapter on populations dynamics discusses the differential equations governing the growth of populations and the differential equations of chemical reactions. We emphasize that properties of solutions, qualitative and quantitative, can be deduced directly from the differential equations themselves without relying on explicit formulas for their solution.

Our experience in teaching this material convinces us that more is included in this text than can be covered in two semesters. The following has been taught successfully to an average freshman class at Washington Square College of New York University.

First semester: All of Chapter 1, including infinite sums; Chapter 2, excluding functions of several variables and partial fractions; Chapter 3, excluding Taylor's theorem; Chapter 4, omitting the section on the existence of the integral but including improper integrals.

19 Second semester: Chapter 5, including numerical methods for calculating exponential and logarithmic functions; all of Chapter 6 including the section on diffusion; Chapter 7, except for the section on isometry, and merely touching on complex valued functions; all of Chapter 8 minus nonlinear vibrations. In Chapter 9 we suggest covering either population dynamics or chemical kinetics, but not necessarily both. We would like to emphasize that Chapters 6, 8 and 9 are independent applications of the rest of the text.

20 About a week was spent in the first semester on a crash course in computing.

21 Volume II is in preparation; it will deal with functions of several variables in the same spirit as Volume I treated functions of a single variable. Vectors will be used from the outset.

22 It is a pleasure to thank friends and colleagues, within the Courant Institute and elsewhere, for critical review of parts of the manuscript and for general good advice. We particularly thank Robert Walker for an overall review of the first version of our manuscript, which appeared in 1972 in the Courant Institute Lecture Notes series. We also thank Paul Gans for a critical review of the section on chemical kinetics. One of the authors (SB) would like to thank his wife, Elaine, for her patience and understanding over many months of preparation and preoccupation. Finally, a bouquet of thanks to Gloria Lee for expert typing.

23 Each section was submitted to trial by fire, i.e., in the classroom. The students' reactions were taken into account in shaping the final version. We thank them for their cooperation and enthusiasm.

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