5 Thermal Systems

5.1 Heat and Temperature

The concept of heat arises from intuitive judgements of how objects "feel" when touched. The experience of our tactile sensations is that objects can be ordered in a sequence of "hotter" or "colder". Furthermore, this conceptual property "heat" exhibits flow properties. Two bodies, one hotter than the other, tend to retain this difference if isolated one from the other, but if they are brought into close contact they tend to lose any temperature difference. These notions are systematized by the introduction of "temperature" as a measure of the degree of hotness and a quantity "heat" which tends to flow from hotter to cooler bodies. A quantitive measure of temperature is obtained by fixing a scale of measurement against some controlled events which are characterized by specific degrees of hotness. Temperature scales in common use, Celsius and Fahrenheit, are now fixed with respect to the characteristic temperature of water. The amount of heat can be similarly quantified by an interaction experiment between two bodies of initially different temperatures T_1 and T_2. Then postulating that the amount of heat associated with a body is proportional to the product of mass and temperature, the conservation principle for energy can be envoked to determine the constant of proportionality which characterizes the substance. In this way the phenomenon of heat can be rationalized as follows: If a mass m_1 of substance is heated from temperature T_1 to T_2 the amount of heat H which it acquires is given by

$$H = m_1 C_p (T_2 - T_1), \tag{79}$$

where C_p is the specific heat of the substance.

Thus the quantities heat flow rate (q) and temperature (T) apparently qualify as an effort/flow pair. Heat is readily thought of as a flow variable, and temperature is an across variable since a temperature datum is required for thermometer calibration. This notion is valid, but should be used with care since heat (H) itself is an energy variable, so that generalizations concerning energy will not carry over.

5.2 Thermal Flow Store

The ability of a specific material to store heat is a measure of its thermal capacity. The constitutive relation (equation (79)) of a thermal flow store relates in a linear fashion the total heat transferred and the temperature change. The total heat is related to the heat flow rate q by the dynamic relation:

$$q = \mathrm{d}H/\mathrm{d}t. \tag{80}$$

The constitutive relation for a thermal flow store, together with its symbolic representation, is shown in Fig. 3.19. The total energy stored in the flow store is the time integral of the heat flow variable.

5.3 Thermal Effort Store

There is apparently no thermal element which displays an energy storage mechanism which is complementary to the flow store.

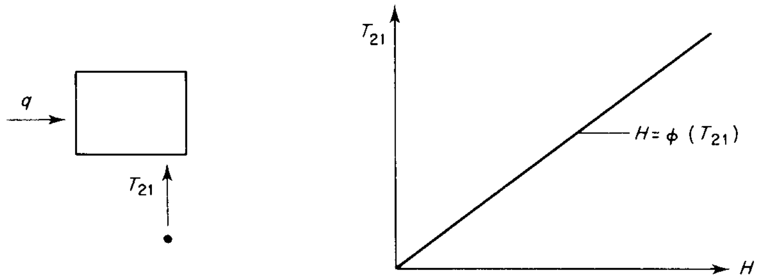

Figure 3.19. Symbol and constitutive relation for a thermal flow store.

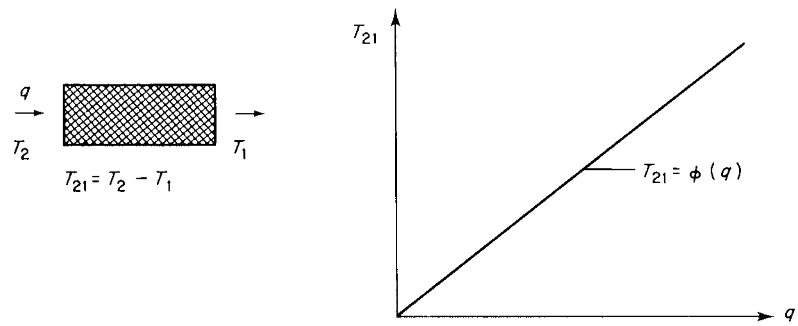

Figure 3.20. Symbol and constitutive relation for a thermal dissipator.

5.4 Thermal Dissipation

The flow of heat through a substance is accompanied by thermal resistance and a consequent temperature gradient. The material relation which governs the flow of heat by conduction is Fourier's law:

$$q = \frac{\sigma_c A}{l}(T_2 - T_1), \tag{81}$$

where σ_c is the thermal conductivity of the material: A is the cross-sectional area of the object; l is the length of the object; and $T_2 - T_1$ is the temperature difference across the ends of the object.

If the transport of heat is by convection, the resistance to heat flow is difficult to analyse. In this instance the notion of an overall heat transfer coefficient for a specific physical object is employed. The resultant constitutive relation is

$$q = C_h A(T_2 - T_1), \tag{82}$$

where the heat coefficient C_h is defined in terms of the constitutive relation of a particular object.

The third mechanism for heat transfer is radiation, this is described analytically by the Stefan-Boltzmann law:

$$q = C_r(T_2^4 - T_1^4), \tag{83}$$

where C_r is a constant determined by the geometry of the surfaces radiating and receiving heat.

The symbol for a thermal dissipator is depicted in Fig. 3.20, as is a typical constitutive relation. Like other types of dissipator elements, thermal dissipators do not lose energy. The heat flow is associated with a net increase in the entropy of the transporting medium and a net decrease in "useful" heat energy.

6 Notes and References

1. The discussion of basic system elements given here can be found in similar forms in a number of text books. A particularly clear exposition is given in:

   Shearer, J. L., Murphy, A. T. and Richardson, H. H. (1967). "Introduction to systems dynamics". Addison Wesley, Reading, Mass.

2. We have also found useful the perspective set in:

   Feather, N. (1959). "Mass, length and time". Pelican Books, London.

3. The representation of thermal energy in terms of a pair of variables is questionable on a number of levels. First, temperature and heat flow are not a true energy pair (entropy flow rate and temperature are more appropriate). Second, the class of thermal systems which can be modelled by a pair of variables is limited. Fortunately, the dynamics of heat conduction dominate most other aspects of thermal systems and in a practical sense justify the restrictions invoked here.
4. A good discussion of spring and dissipator constitutive relations encountered in practice is given in:

   Shigley, J. E. (1977). "Mechanical Engineering Design", 3rd edn. Series in mechanical engineering design. McGraw-Hill, New York.
   See especially chapters, 3,8, 10.

7 Problems

7.1. Determine the constitutive relation for a spherical fluid reservoir of radius r. Obtain the stored energy as a function of the volume of stored liquid, and the stored co-energy as a function of the pressure at the tank bottom.

7.2. The open top container shown in Fig. 3.21 is used as a reservoir for an incompressible fluid. Sketch the constitutive relation of the reservoir, and obtain equations for the stored co-energy and energy.

7.3. An electrical inductor has inductance which is a function of current:

$$L = 0.01(i)^{-1/2}$$

Sketch the constitutive relation of the device and evaluate the stored energy and co-energy when a current of 0.2 amperes passes through the inductor.

7.4. A mechanical dissipator consists of a paddle rotating in a viscous fluid. A torque of 0.1 newton metre is required before the paddle will rotate, the torque then increases linearly with angular velocity given by 0.2 newton metres per rad/second. Find the content and co-content of the device at a velocity of 0.5 rad/second.