

A new method to estimate Weibull parameters for wind energy applications

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ABSTRACT

In recent years, Weibull distribution has been commonly used, accepted and recommended distribution in literature to express the wind speed frequency distribution. In this study, a new method is developed to estimate Weibull distribution parameters for wind energy applications. This new method is called power density (PD) method. In literature most frequently used methods, that are graphic, maximum likelihood and moment methods, are revisited and a comparison between these methods and PD method is carried out. Suitability of these methods is judged based on different goodness of fit tests for different geographical locations. Also to demonstrate the accuracy of PD method, comparisons are carried out based on power density and mean wind estimation results of previous studies. Results of this study indicate that PD method is an adequate method to estimate Weibull parameters and it might have better suitability than other methods. Some superiority of the new PD method are that, it has simple formulation, it does not require binning and solving linear least square problem or iterative procedure. If power density and mean wind speed are available it is very simple to estimate Weibull parameters.

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1. Introduction

Energy is one of the essential inputs for economic development and industrialization. Fossil fuels are the main resources and play a crucial role to supply world energy demand. However fossil fuel reserves are limited and usage of fossil fuel sources have negative environmental impacts. Therefore, management of energy sources, rational utilization of energy, and renewable energy source usage are vital.

Since the first oil crisis, renewable energy sources have gained a great importance due to their inexhaustibility, sustainability, ecological awareness and supply of energy security. So, renewable energy sources are expected to play an important role especially in electrical energy generation.

Among the renewable energy sources wind energy is currently viewed as one of the most significant, fastest growing, commonly used and commercially attractive source to generate electrical energy because of the mature and cost effective energy conversion system technology. So, electricity generation cost from wind energy system has become competitive with fossil fuel systems. Installed total wind power capacity has reached over 93 GW and installed wind power capacity generates more than 1% of the global electricity consumption [1].

In order to accurate assessment of wind energy potential and characteristics, it is necessary to carry out long term meteorological observations. Wind speed is a random variable and variation of wind speed over a period of time is represented by probability density functions. Detailed knowledge of wind characteristics and distribution are crucial parameters to select optimum wind energy conversion system to optimize energy output and minimization of electricity generation cost [2]. Wind speed frequency distribution has been represented by various probability density functions such as gamma, lognormal, three parameter beta, Rayleigh and Weibull distributions. However in recent years Weibull distribution has been one of the most commonly used, accepted, recommended distribution to determine wind energy potential and it is also used as a reference distribution for commercial wind energy softwares such as Wind Atlas Analysis and Application Program (WAsP) [3,4]. So, estimation of wind speed distribution parameters correctly is important in terms of selecting wind energy conversion system and obtains correct results about wind energy potential and the economic viability of project. But it is important to note that, Weibull distribution is not suitable to represent wind distribution for all geographical location in the world [5,6]. Therefore, new distributions have been proposed to represent wind speed frequency distribution and wind energy potential assessment of these locations.

Several methods have been proposed to estimate Weibull parameters [7–14]. Graphic method, maximum likelihood method and moment methods are commonly used to estimate Weibull parameters. In literature about wind energy, these methods are

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Nomenclature

| | | | |
|-------------|--------------------------------------|-------------|--|
| PDM | power density method | Pd_{ts} | power density of time series wind data |
| R^2 | coefficient of determination | Vm_{ts} | time series mean wind speed |
| $RMSE$ | root mean square error | k_{ref} | Weibull shape parameter of reference article |
| $f(v)$ | Weibull probability density function | c_{ref} | Weibull scale parameter of reference article |
| CDF | cumulative distribution function | Pw_{ref} | Weibull power density of reference article |
| k | Weibull shape parameter | Vm_{wref} | Weibull mean wind speed of reference article |
| c | Weibull scale parameter | k_{pdm} | Weibull shape parameter of PD method |
| \bar{V} | mean wind speed | c_{pdm} | Weibull scale parameter of PD method |
| v_i | ith wind speed | Pw_{pdm} | power density of PD method |
| \bar{V}^3 | mean of cube of wind speeds | Vm_{pdm} | mean wind speed of PD method |
| E_{pf} | energy pattern factor | | |

compared several times [12–20] however results and recommendations of the previous studies are different. For this reason, according to the results of the studies, it might be concluded that suitability of the method may vary with the sample data size, sample data distribution, sample data format and goodness of fit tests.

In order to assess the suitability of proposed distributions or methods for wind speed distribution of a region, goodness of fit tests are used. Several goodness of fit tests are used in literature. Celik indicates that judgment of suitability of distribution should be carried out based on power density estimation capability [21].

The main objective of the present study is to propose a new method to estimate Weibull parameters for wind energy applications. The rest of the study is organized as follows. In Section 2, graphic, maximum likelihood and moment methods are revisited and PD method is introduced. In Section 3, suitability of these methods and accuracy of PD method are judged from R^2 (coefficient of determination) and $RMSE$ analyses. Also power density and mean wind speed estimation capabilities are compared for Maden, Gökçeada, Çanakkale and Bozcaada regions in Turkey. In Section 4, to prove the accuracy of PD method, the results obtained by this method are compared with the results of previous studies in literature. Section 5 concludes the study.

2. Methods for estimating Weibull parameters

Weibull distribution has been commonly used, accepted and recommended distribution in literature to express the wind speed frequency distribution and estimate wind energy potential. The probability density function of Weibull distribution is given by,

$$f(v) = \frac{k}{c} \left(\frac{v}{c}\right)^{k-1} e^{-\left(\frac{v}{c}\right)^k} \quad (1)$$

where $f(v)$ is the probability of observing wind speed v , k the dimensionless shape parameter and c is scale parameter in units of wind speed. One of the important properties of Weibull distribution which makes this distribution more useful for wind applications is that once these parameters are estimated at one height, it is possible to adjust these parameters to different heights [7]. A literature survey for this study shows that, shape parameter of Weibull distribution range from 1.2 to 2.75 for most wind condition in the world. Cumulative distribution function (CDF) of Weibull distribution is given by

$$F(v) = 1 - e^{-\left(\frac{v}{c}\right)^k} \quad (2)$$

There are several methods to estimate Weibull parameters. We will discuss four different methods commonly used and one of them is derived by us and firstly represented in this study.

2.1. Graphic method

Graphic method is derived by using cumulative distribution function. Taking twice logarithm of CDF Eq. (2)

$$\ln[-\ln[1 - F(v)]] = k \ln v - k \ln c \quad (3)$$

we obtain $y = ax + b$ form respect to $\ln[-\ln[1 - F(v)]]$ and $\ln v$ and also k is the slope of the straight line. Using cumulative distribution function, we evaluate $(\ln v_i, \ln[-\ln[1 - F(v_i)]])$ pairs and then we solve linear least squares problem to obtain coefficients of straight line a, b . Hence

$$k = a \quad (4)$$

$$c = \exp(-b/a) \quad (5)$$

Implementation of this method consists of three stages such that: (i) using wind speed data, calculate cumulative frequency distribution or first evaluate frequency distribution, which requires sorting wind speed data into bins, and then using frequency distribution, obtain cumulative frequency distribution, (ii) calculate $(\ln v_i, \ln[-\ln[1 - F(v_i)]])$ pairs and (iii) solve linear least squares problem and find scale and shape parameters using Eqs. (4) and (5).

2.2. Maximum likelihood method

Maximum likelihood method is a method suggested by Stevens and Smulders [8], details on the development of this method can be found in [18]. Maximum likelihood method requires extensive iterative calculations. Shape and scale parameters of Weibull distribution are estimated by these two equations

$$k = \left(\frac{\sum_{i=1}^n v_i^k \ln(v_i)}{\sum_{i=1}^n v_i^k} - \frac{\sum_{i=1}^n \ln(v_i)}{n} \right)^{-1} \quad (6)$$

$$c = \left(\frac{\sum_{i=1}^n (v_i)^k}{n} \right)^{\frac{1}{k}} \quad (7)$$

where v_i is the wind speed and n is the number of nonzero wind speeds.

This method is implemented in two stages such that: (i) using wind speed data, calculate summations in Eqs. (6) and (7) with taking care of zero wind speeds which make logarithm indefinite and then calculate shape parameter with Eq. (7) and (ii) find scale parameter using a numerical technique in order to find the root of Eq. (6) around $k = 2$.

2.3. Moment method

This method is suggested by Justus et al. [7]. When the mean wind speed \bar{V} and standard deviation σ are available, shape and scale parameters can be estimated with this method using

$$k = \left(\frac{\sigma}{\bar{V}} \right)^{-1.086} \quad 1 \leq k \leq 10 \quad (8)$$

$$c = \frac{\bar{V}}{\Gamma(1 + \frac{1}{k})} \quad (9)$$

Since k and c are related to \bar{V} and σ by

$$\bar{V} = c\Gamma\left(1 + \frac{1}{k}\right) \quad (10)$$

$$\left(\frac{\sigma}{\bar{V}} \right)^2 = \left[\frac{\Gamma(1 + \frac{2}{k})}{\Gamma(1 + \frac{1}{k})^2} \right] - 1 \quad (11)$$

where Γ is gamma function.

Also mean wind speed is calculated by

$$\bar{V} = \left(\frac{1}{n} \sum_{i=1}^n v_i \right) \quad (12)$$

and standard deviation is calculated by

$$\sigma = \left[\left(\frac{1}{n-1} \sum_{i=1}^n (v_i - \bar{V})^2 \right) \right]^{0.5} \quad (13)$$

This method also requires two stages such that: (i) using wind speed data, calculate summations in Eqs. (12) and (13), calculate mean wind speed \bar{V} and standard deviation σ and (ii) find scale parameter k and shape parameter c using Eqs. (8) and (9). Also Eq. (11) can be solved numerically to obtain Weibull parameters [12].

2.4. Power density method

This is a new method that we suggest and recommend to be used estimating scale and shape parameters since it has simpler formulation, easier implementation and also requires less computation. Power density according to the Weibull distribution can be expressed as

$$P_w = \frac{1}{2} \rho \int_0^\infty v^3 f(v) dv \quad (14)$$

where ρ is air density of the region.

Using Eqs. (9) and (14)

$$\frac{\bar{V}^3}{(\bar{V})^3} = \frac{\Gamma(1 + 3/k)}{\Gamma(1 + 1/k)^3} \quad (15)$$

where \bar{V}^3 is mean of wind speed cubes and $\bar{V}^3/(\bar{V})^3$ is known as energy pattern factor (E_{pf}) and according to the literature survey conducted for this study shows that E_{pf} is between 1.45 and 4.4 for most wind distribution in the world. Weibull parameters can be estimated with solving energy pattern factor Eq. (15) numerically [22] or approximately by PD method using this simple formula

$$k = 1 + \frac{3.69}{(E_{pf})^2} \quad (16)$$

Also scale parameter is estimated by Eq. (10). In short PD method needs just mean of cube of wind speeds and mean wind speed. If time series mean wind speed and wind power density

$$Pd_{ts} = \frac{1}{2} \rho \bar{V}^3 \quad (17)$$

are available, then using energy pattern factor $\bar{V}^3/(\bar{V})^3$ Weibull parameters can be obtained even without whole time series wind speed data.

Also energy pattern factor for time series wind data can be calculated using Eqs. (12) and (18)

$$\bar{V}^3 = \frac{1}{n} \sum_{i=1}^n v_i^3 \quad (18)$$

Superiority of the new PD method are that: (i) it has a simple formulation, (ii) it does not require binning and solving linear least square problem or iterative procedure, (iii) If power density and mean wind speed are available it is very simple to estimate Weibull parameters and (iv) also it is more suitable to estimate power density for wind energy applications as shown in Section 3.

As stated earlier, k is generally between 1.20 and 2.75 for most wind conditions in the world and PD method estimates Weibull parameters significantly very well in this interval. In view of the fact that in many studies which deal with many regions k parameter is between these values.

Particularly, we will compare graphic, maximum likelihood, moment and power density methods in Section 3 using three different analyses; first one is R^2 ,

$$R^2 = 1 - \frac{\sum_{i=1}^N (y_i - x_i)^2}{\sum_{i=1}^N (y_i - \bar{y})^2} \quad (19)$$

and second is $RMSE$

$$RMSE = \left[\frac{1}{N} \sum_{i=1}^N (y_i - x_i)^2 \right]^{0.5} \quad (20)$$

where N is the total number of intervals, y_i the frequencies of observed wind speed data, x_i the frequency distribution value calculated with Weibull distribution, \bar{y} the average of y_i values. It is concluded as “better method” if R^2 magnitude is bigger or $RMSE$ value is smaller.

Power density error analysis was carried out by

$$Error (\%) = \left| \frac{P_w - Pd_{ts}}{Pd_{ts}} \right| \quad (21)$$

where Pd_{ts} is time series power density and P_w is Weibull distribution power density.

3. Comparison of methods

In order to compare the methods hourly mean wind data used for Maden, Gökçeada, Çanakkale and Bozcaada regions are obtained from Turkish State Meteorological Service cover the period of 1997–2006, 1997–2006, 1997–2000, 2002–2003, respectively. Wind speed measurements were carried out at 10 m above ground level. Information about the regions and mean wind and speeds energy densities are shown in Table 1. In this part of this study air density is assumed to be equal to 1.225 kg/m³ and constant for Maden, Gökçeada, Çanakkale and Bozcaada regions.

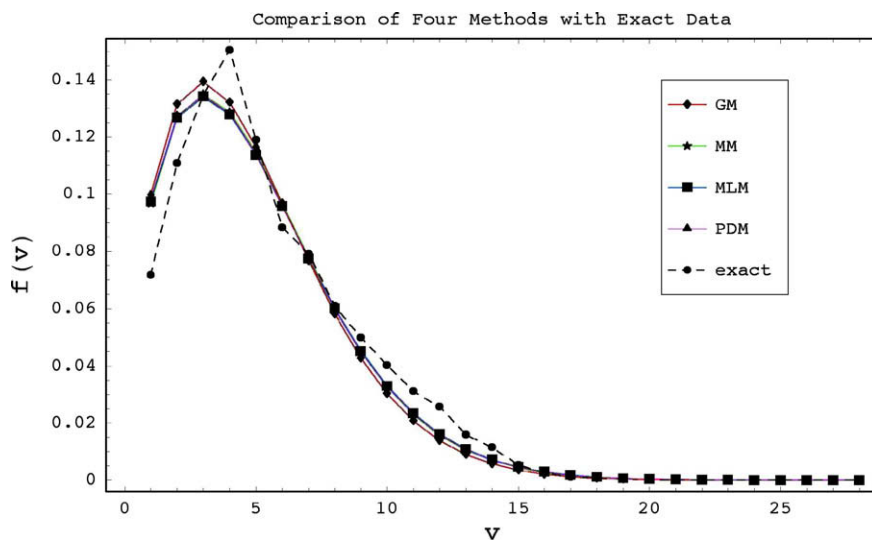
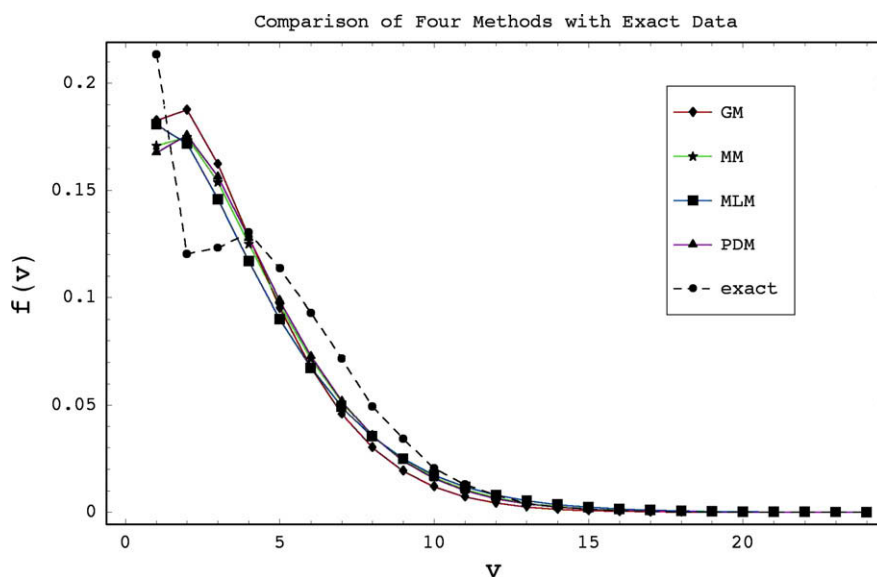
Table 1
Information about regions.

| Station | Longitude | Latitude | Mean speed (m/s) | Power density (W/m ²) | Years of wind data | Number of wind data |
|-----------|-----------|----------|------------------|-----------------------------------|--------------------|---------------------|
| Maden | 29°25' | 39°30' | 5.04422 | 199.632 | 1997–2006 | 87144 |
| Gökçeada | 40°11' | 25°54' | 3.77985 | 101.53 | 1997–2006 | 87384 |
| Çanakkale | 40°08' | 26°24' | 3.59473 | 75.6431 | 1997–2000 | 35064 |
| Bozcaada | 41°58' | 34°02' | 5.9532 | 299.558 | 2002–2003 | 17520 |

Table 2

Comparison of methods for different regions.

| Regions | Parameters | Graphic method | MLH method | Moment method | Numerical solution of Eq. (15) | PD method |
|-----------|------------|----------------|------------|---------------|--------------------------------|-----------|
| Maden | k (–) | 1.59536 | 1.57016 | 1.58143 | 1.57289 | 1.5722 |
| | c (m/s) | 5.45798 | 5.63616 | 5.6199 | 5.61691 | 5.61666 |
| | R^2 | 0.98512 | 0.9862 | 0.98581 | 0.98625 | 0.98628 |
| | RMSE | 0.00567 | 0.00546 | 0.00554 | 0.00545 | 0.00545 |
| Gökçeada | k (–) | 1.39653 | 1.27034 | 1.36157 | 1.38776 | 1.39165 |
| | c (m/s) | 3.84854 | 4.0657 | 4.1282 | 4.14139 | 4.14325 |
| | R^2 | 0.84609 | 0.90361 | 0.87524 | 0.8652 | 0.86365 |
| | RMSE | 0.02287 | 0.0181 | 0.02059 | 0.0214 | 0.02153 |
| Çanakkale | k (–) | 1.46907 | 1.57803 | 1.56044 | 1.52294 | 1.52203 |
| | c (m/s) | 4.00644 | 4.01898 | 3.99964 | 3.98911 | 3.98884 |
| | R^2 | 0.92183 | 0.92013 | 0.92155 | 0.92299 | 0.923 |
| | RMSE | 0.01885 | 0.01906 | 0.01889 | 0.01871 | 0.01871 |
| Bozcaada | k (–) | 1.69875 | 1.68755 | 1.70331 | 1.68471 | 1.68672 |
| | c (m/s) | 6.5515 | 6.6759 | 6.67304 | 6.66795 | 6.66851 |
| | R^2 | 0.98795 | 0.98972 | 0.98874 | 0.98984 | 0.989741 |
| | RMSE | 0.0046 | 0.00425 | 0.00445 | 0.00422 | 0.00424 |

**Fig. 1.** Comparison of probability density distributions for Maden. GM: graphic method, MM: moment method, MLM: maximum likelihood method, PDM: power density method.**Fig. 2.** Comparison of probability density distributions for Gökçeada. GM: graphic method, MM: moment method, MLM: maximum likelihood method, PDM: power density method.

Weibull parameters according to the four methods have been calculated and shown Table 2 with R^2 and $RMSE$ analyses results. Figs. 1–4 show comparison of the probability density distributions.

According to the results, PD method gives satisfactory according to R^2 and $RMSE$ analyses, consistently with numerical solution of energy pattern factor equation for three regions except Gökçeada. Result of analyses shows that, all methods have a better than 0.92 R^2 magnitude and the differences occur after third decimal point except Gökçeada. Estimated scale parameters are close, however graphic method generally estimates smaller value for scale parameter.

4. Comparison with previous studies

In order to show the accuracy of the PD method, the results obtained by PD method were compared with the results of previous studies in literature [23–26], to carry out further and extensive

comparison. Firstly we show that PD method has more satisfactory results than other methods according to estimations of power density and mean wind speed for monthly base. Table 3 shows the result of this comparison. To evaluate the accuracy of the method for yearly base is shown in Table 4 for different locations. According to the results in Tables 3 and 4 this method is more suitable than others for calculating power density and mean wind speed. Air density used in previous studies is calculated as follows:

$$\rho = \frac{P_w}{\frac{1}{2} C^3 \Gamma \left(1 + \frac{3}{k}\right)} \quad (22)$$

According to the Tables 3 and 4, introduced method gives more accurate results based on power density and mean wind speed estimation. As shown in Table 4, PD method makes smaller error in calculating wind power density except April and August, so it is obvious that this method calculates mean power density sensitively than other methods.

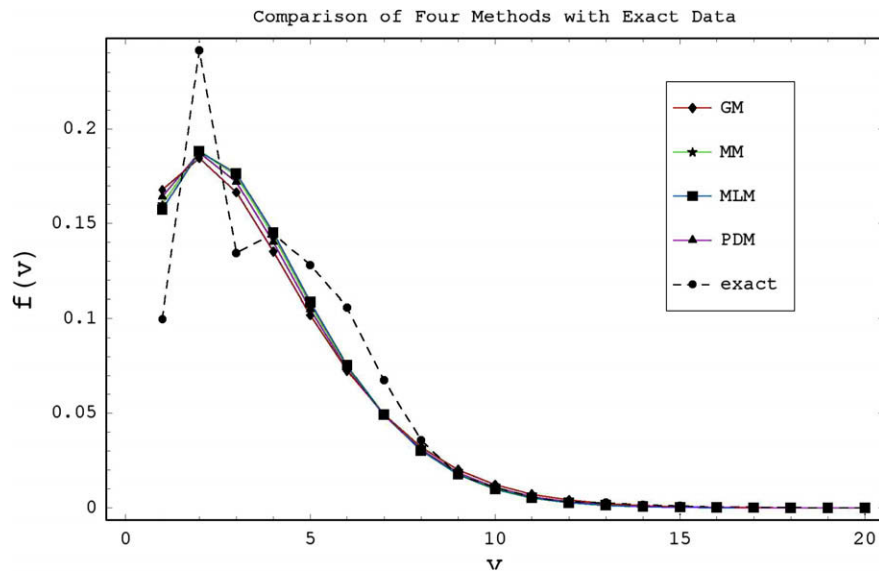


Fig. 3. Comparison of probability density distributions for Çanakkale. GM: graphic method, MM: moment method, MLM: maximum likelihood method, PDM: power density method.

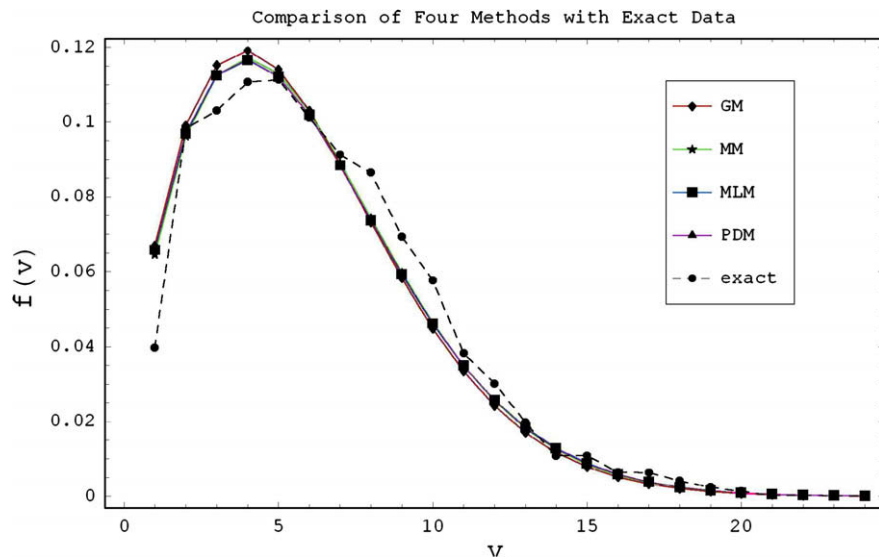


Fig. 4. Comparison of probability density distributions for Bozcaada. GM: graphic method, MM: moment method, MLM: maximum likelihood method, PDM: power density method.

Table 3

Comparison of different regions according to the power density and mean speed [23–25].

| Sites | $V_{m_{ts}}$ | $P_{d_{ts}}$ | k_{ref} | c_{ref} | Pw_{ref} | Power density error (%) | $V_{m_{wref}}$ | Mean speed error (%) | E_{pf} | k_{pdm} | c_{pdm} | Pw_{pdm} | Power density error (%) | $V_{m_{pdm}}$ |
|----------------|--------------|--------------|-----------|-----------|------------|-------------------------|----------------|----------------------|----------|-----------|-----------|------------|-------------------------|---------------|
| Keban-Elazig | 2.258 | 15.603 | 1.518 | 2.522 | 17.731 | 13.640 | 2.274 | 0.687 | 2.399 | 1.641 | 2.481 | 15.610 | 0.042 | 2.258 |
| Kirklareli | 4.68 | 142.75 | 1.75 | 5.25 | 138.850 | 2.732 | 4.680 | 0.000 | 2.274 | 1.714 | 5.248 | 142.450 | 0.210 | 4.68 |
| Kist | 4.07 | 84.03 | 2.25 | 5.22 | 80.060 | 4.725 | 4.624 | 13.600 | 2.636 | 1.531 | 4.519 | 84.104 | 0.088 | 4.07 |
| Al-Taweel | 5.33 | 105.99 | 2.64 | 6.79 | 150.500 | 41.995 | 6.034 | 13.204 | 1.551 | 2.533 | 6.005 | 106.751 | 0.718 | 5.33 |
| Rawdation | 3.87 | 45.02 | 2.21 | 5.26 | 80.930 | 79.765 | 4.658 | 20.374 | 1.688 | 2.295 | 4.368 | 44.949 | 0.157 | 3.87 |
| Ras As-Subiyah | 4.76 | 98.9 | 2.08 | 5.36 | 103.940 | 5.096 | 4.748 | 0.259 | 1.736 | 2.225 | 5.374 | 98.574 | 0.329 | 4.76 |
| Umm Omara | 4.88 | 97.94 | 2.19 | 5.83 | 123.420 | 26.016 | 5.163 | 5.802 | 1.648 | 2.358 | 5.507 | 97.973 | 0.034 | 4.88 |
| Al-Wafra | 5.52 | 122.71 | 2.7 | 7.18 | 166.680 | 35.832 | 6.385 | 15.671 | 1.705 | 2.270 | 6.232 | 122.436 | 0.223 | 5.52 |

Table 4

Comparison of different regions according to the power density [26].

| Months | V_{mts} | P_{mts} | k_{ref} | c_{ref} | Pw_{ref} | Power density error (%) | E_{pf} | k_{pdm} | c_{pdm} | Pw_{pdm} | Power density error (%) |
|-----------|-----------|-----------|-----------|-----------|------------|-------------------------|----------|-----------|-----------|------------|-------------------------|
| January | 2.71 | 28.46 | 1.66 | 3.04 | 29.58 | 3.935 | 2.288 | 1.705 | 3.038 | 28.406 | 0.191 |
| February | 1.62 | 7.54 | 1.49 | 1.8 | 7.34 | 2.653 | 2.838 | 1.458 | 1.785 | 7.538 | 0.026 |
| March | 2.14 | 18.79 | 1.37 | 2.35 | 19.35 | 2.980 | 3.068 | 1.392 | 2.346 | 18.705 | 0.452 |
| April | 2.28 | 31.02 | 1.17 | 2.41 | 31 | 0.065 | 4.188 | 1.210 | 2.429 | 29.077 | 6.264 |
| May | 2.22 | 25.38 | 1.21 | 2.37 | 26.89 | 5.950 | 3.712 | 1.268 | 2.391 | 24.572 | 3.183 |
| June | 3.38 | 63.69 | 1.46 | 3.73 | 68.39 | 7.379 | 2.639 | 1.530 | 3.753 | 63.746 | 0.088 |
| July | 3.35 | 62.84 | 1.42 | 3.69 | 70 | 11.394 | 2.674 | 1.516 | 3.716 | 62.893 | 0.084 |
| August | 2.74 | 56.74 | 1.1 | 2.84 | 61.22 | 7.896 | 4.413 | 1.189 | 2.906 | 52.248 | 7.918 |
| September | 1.87 | 16.35 | 1.23 | 2 | 15.43 | 5.627 | 4.000 | 1.231 | 2.000 | 15.537 | 4.976 |
| October | 2.43 | 30.23 | 1.36 | 2.66 | 28.52 | 5.657 | 3.371 | 1.325 | 2.641 | 29.782 | 1.482 |
| November | 1.91 | 9.77 | 1.79 | 2.15 | 9.45 | 3.275 | 2.243 | 1.733 | 2.143 | 9.745 | 0.252 |
| December | 2.07 | 11.63 | 1.89 | 2.34 | 11.37 | 2.236 | 2.098 | 1.838 | 2.330 | 11.578 | 0.450 |

5. Conclusions

In this study, PD method is developed and represented. Its superior features are shown over other methods. This new method is compared with three most common methods, graphic, maximum likelihood and moment methods and numerical solution of energy pattern factor. Suitability of these methods and accuracy of PD method is judged based on different goodness of fit tests for different geographical locations. Furthermore, PD method is compared with previous studies considering power density and mean wind speed estimation capability. It is worth to indicate that superiority of PD method over other methods can be obviously seen with estimation capability of power density. Then it is concluded that PD method is very suitable and efficient in order to estimate Weibull parameters for wind energy applications.

References

- [1] World Wind Energy Association. Press Release, February 2008. Wind turbines generate more than 1% of the global electricity; 2008. <<http://www.wwindea.org>> [accessed 22.02.08].
- [2] Pallabazzer R. Parametric analysis of wind siting efficiency. *J Wind Eng Indus Aerod* 2003;91:1329–52.
- [3] Carta JA, Ramirez P, Velázquez S. A review of wind speed probability distributions used in wind energy analysis. *Renew Sust Energy Rev*, in press, doi:10.1016/j.rser.2008.05.005.
- [4] Petersen LE et al. Wind power meteorology. Roskilde (Denmark): Risoe National Laboratory Press; 1997.
- [5] Jaramillo OA, Borja MA. Wind speed analysis in La Ventosa, Mexico: a bimodal probability distribution case. *Renew Energy* 2004;29:1613–30.
- [6] Ramirez P, Carta JA. The use of wind probability distributions derived from the maximum entropy principle in the analysis of wind energy. A case study. *Energy Convers Manage* 2005;47:2564–77.
- [7] Justus CG, Hargraves WR, Mikhail A, Graber D. Methods for estimating wind speed frequency distributions. *J Appl Meteorol* 1978;17:350–3.
- [8] Stevens MJM, Smulders PT. The estimation of the parameters of the Weibull wind speed distribution for wind energy utilization purposes. *Wind Eng* 1979;3:132–45.
- [9] Christofferson RD, Gillette DA. A simple estimator of the two-parameter Weibull distribution. *J Clim Appl Meteorol* 1987;26:323–5.
- [10] Al Hasan M, Nigmatullin RR. Identification of the generalized Weibull distribution in wind speed data by the Eigen-coordinates method. *Renew Energy* 2003;28:93–110.
- [11] Tar K. Some statistical characteristics of monthly average wind speed at various heights. *Renew Sust Energy Rev* 2008;12:1712–24.
- [12] Dörvlo ASS. Estimating wind speed distribution. *Energy Convers Manage* 2002;43:2311–8.
- [13] Seguro JV, Lambert TW. Modern estimation of the parameters of the Weibull wind speed distribution for wind energy analysis. *J Wind Eng Indus Aerod* 2000;85:75–84.
- [14] Mathew S. Wind energy. Fundamentals, resource analysis and economics. 1st ed. New York: Springer; 2006.
- [15] Cook JN. Discussion on modern estimation of the parameters of the Weibull wind speed distribution for wind speed energy analysis by Seguro JV, Lambert TW. *J Wind Eng Indus Aerod* 2001;89:867–9.
- [16] Shata ASA, Hanitsch R. The potential of electricity generation on the east coast of Red Sea in Egypt. *Renew Energy* 2006;31:1597–615.
- [17] Basumatary H, Sreevalsan E, Sasi KK. Weibull parameter estimation – a comparison of different methods. *Wind Eng* 2005;29:309–16.
- [18] Genc A et al. Estimation of wind power potential using Weibull distribution. *Energy Sources, Part A: Recov, Utiliz Environ Eff* 2005;27:809–22.
- [19] Pérez IA, García MA, Sánchez ML, Torre B. Analysis and parameterisation of wind profiles in the low atmosphere. *Sol Energy* 2005;78:809–21.
- [20] Bagioras HS, Mihalakakou G, Matthopoulos D. A statistical analysis of and speed distributions in the area of Western Greece. *Int J Green Energy* 2008;5:120–37.
- [21] Celik AN. On the distributional parameters used in assessment of the suitability of wind speed probability density functions. *Energy Convers Manage* 2004;45:1735–47.
- [22] Raichle BW, Carson WR. Wind resource assessment of the Southern Appalachian Ridges in the southeastern United States. *Renew Sust Energy Rev*, in press, doi:10.1016/j.rser.2007.12.005.
- [23] Gökçek M, Bayülken A, Bekdemir S. Investigation of wind characteristics and wind energy potential in Kirklareli, Turkey. *Renew Energy* 2007;32:1739–52.
- [24] Al-Nassar W, Alhajraf S, Al-Enizi A, Al-Awadhi L. Potential wind power generation in the State of Kuwait. *Renew Energy* 2005;30:2149–61.
- [25] Akpinar EK, Akpinar S. A statistical analysis of wind speed data used in installation of wind energy conversion systems. *Energy Convers Manage* 2005;46:515–32.
- [26] Celik AN. A techno-economic analysis of wind energy in southern Turkey. *Int J Green Energy* 2007;4:233–47.