Physics II Laboratory -Faculty of Engineering SIIE

Lab 5: Transient response of a RC circuit

Purpose

In this Lab, we will discuss the electrical transient responses seen in RC circuits. We will start by constructing charging and discharging circuits and analyze the rise and decay of voltage and current along with finding the time constants associated with each circuit. In cases where a large enough time constant is not possible to obtain, we will turn to our function generator and oscilloscope and look at the response of these circuits to a rectangular input pulse.

Transient Response

In electrical engineering and mechanical engineering, a transient response is the response of a system to a change from an equilibrium or a steady state.

For an example in the below circuit, **Voltage and the Current attained the steady state condition.** However at first we connect the voltage source to the circuit, it takes time to reach the steady state value for this voltage and current. In other words after a while we connect the voltage source to the circuit, it reaches its steady state values which are **Voltage =** V_1 and **Current =** I_1

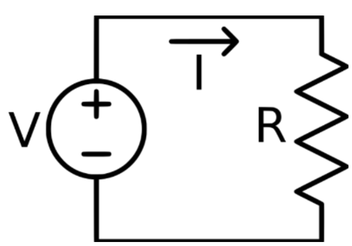


Figure 1: Simple Circuit

Similarly if we suddenly disconnect the power supply, the circuit will attain the new values of **Voltage** V_2 (theoretically 0v) and **Current** I_2 after certain amount of time. This time in the both cases known as **Transient time**. ***This transient period is quite a short amount of time which is in the range of Microseconds to Milliseconds***.

What is the importance of finding the transient response of Circuit Elements ? 😉

Transient response analysis of a capacitor

Case 1:

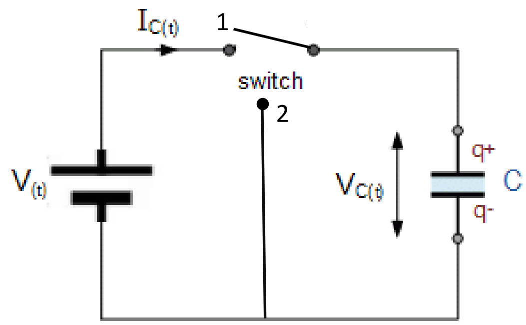


Figure 2: Simple Capacitor Open Circuit

- At time t = 0s, there is no voltage across the capacitor.
- At t = 0^{+} s, Voltage that appear across the Capacitor is V_0 , as soon as we close the switch.
- Therefore the Charge of a Capacitor is given as **Q = CV**
- Hence $\frac{dy}{dx} = C\frac{dv}{dt}$ confirming the capacitance C is not changing with the time
- Therefore the **current I through the capacitor** = $C\frac{dv}{dt}$
- If we suddenly turn on the switch, **dt** $\rightarrow$ **0**, there for the $\frac{dv}{dt} \rightarrow \infty$
- **Then the current I_c** $\rightarrow \infty$ which is an impossible scenario in practical
- Therefore this capacitor oppose to the instanteneous change of voltage
- Therefore **t = 0^{-}** $\longrightarrow$ **t = 0^{+} both limits of time t = 0,** the voltage across the capacitor also 0V
- After a long time when t = ∞ or in the steady state condition, $\frac{dv}{dt} \rightarrow \infty$ and I_c = 0
  Therefore the capacitor act as an open circuit

Case 2 :

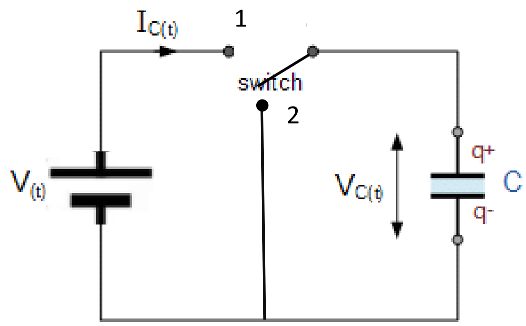


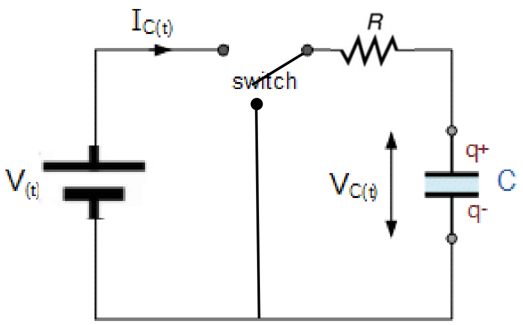
Figure 3: Simple Capacitor Close Circuit

- At time t = 0, and t = 0^+ the switch position has changed and V_0 will appear across the capacitor
- t = 0^- , Voltage across capacitor also V_0
- Therefore capacitor act as a Voltage source in this transient period
- Once this transient period is over, the $\frac{dv}{dt} = 0$, hence the current through capacitor I_C will be zero
- Therefore t = ∞ (Very long time or steady state condition), There will be no change in voltage, therefore the current through the capacitor is 0
- Hence the capacitor act as an open circuit

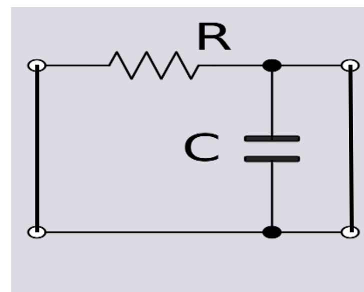
t =0	t =0^+	t =∞
	Short Circuit	Open Circuit
+ V0 -	+ V0 -	+ V0 -

Transient Analysis: First order(contain only one energy storage) RC

<u>Source free response</u>



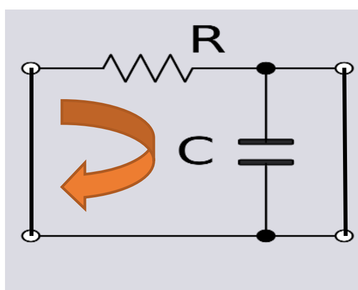
$t = 0^{-}$s



Equivalent circuit at $t = 0^{+}$s

- Both cases voltage across the capacitor = V_c

Apply KVL to the circuit,



$V_R + V_C = 0$, $i_R + V_C$

$$R\left[C\frac{dVc}{dt}\right] + V_c = 0$$

$$\frac{dVc}{dt} + \frac{1}{RC}V_C = 0$$ (first order differential eqn)

Solution for the first order differential eqn = $V_c(t) = e^{-t/RC}$

Therefore the discharging Voltage and the discharging current of a Capacitor

$$V_C = -V_0\, e^{\frac{-t}{RC}} \quad ; \quad I = -I_0\, e^{\frac{-t}{RC}}$$

RC = Ʈ = time Constant describes number of seconds required for the charge on a discharging capacitor to fall to 36.8% ($e^{-1} = 0.368$) of its initial value when **t = RC**

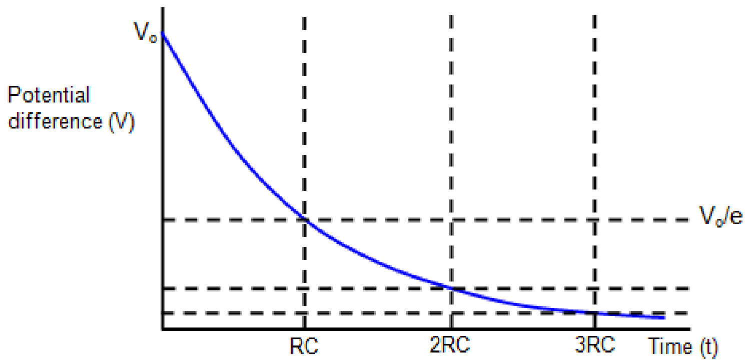
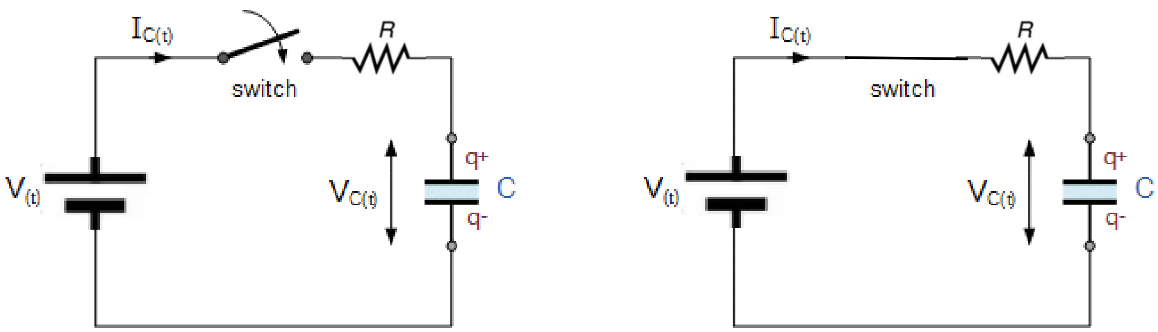


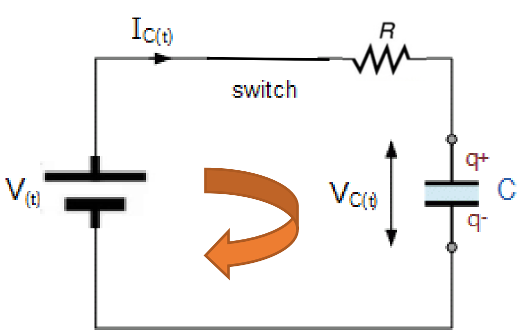
Figure 4: R-C discharging circuit and the transient response of the capacitor voltage

Forced response



$t = 0^-$s $t = 0^+$s

- Both cases voltage across the capacitor = $V_c = 0$



Apply KVL to the circuit,

$V_R + V_C = 0$, $i_R + V_C$

$$R\left[C\frac{dVc}{dt}\right] + V_c = 0$$

$$\frac{dVc}{dt} + \frac{1}{RC}V_C = \frac{V}{RC}$$ (first order differential eqn)

Solution for the first order differential eqn = $V_c(t) = \boldsymbol{V} - \boldsymbol{e}^{-t/RC}$

Therefore the discharging Voltage and the discharging current of a Capacitor

$$V_C = \text{v}(1 - e^{\frac{-t}{RC}}) \qquad I = I_0\, e^{\frac{-t}{RC}}$$

The product RC (unit is seconds) called the time constant of the circuit. The time constant is the amount of time required for the charge on a charging capacitor to rise to 63.2% ($1 - e^{-1} = 0.632$) of its final value when **t = RC**

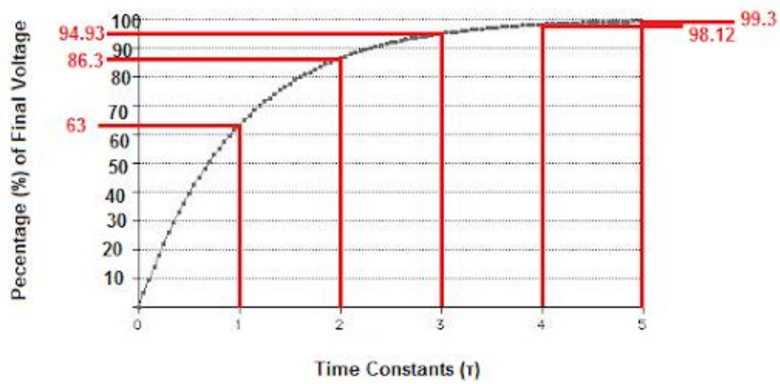
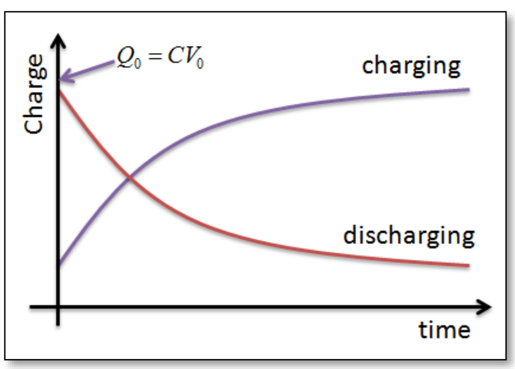
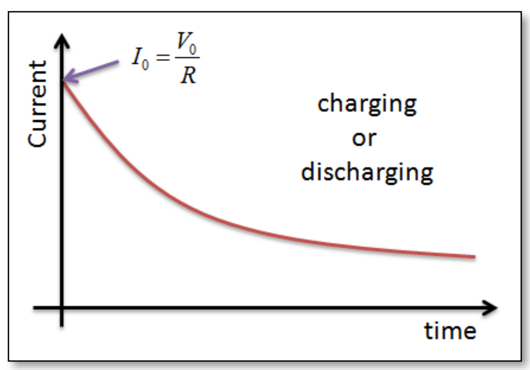
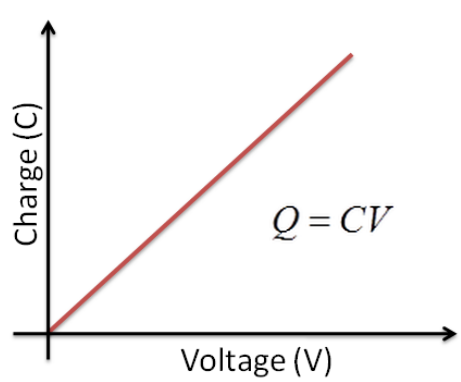


Figure 5: R-C charging circuit and the transient response of the capacitor voltage

<u>Summary of Charging and Discharging graphical characteristics of a capacitor</u>



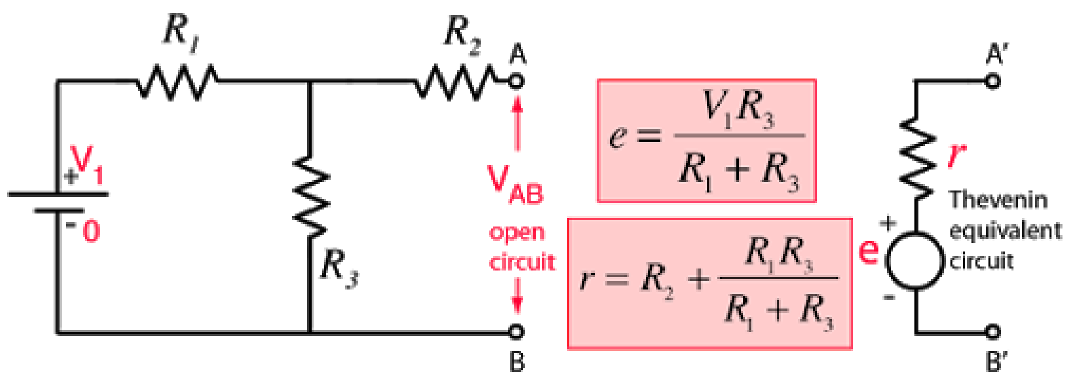




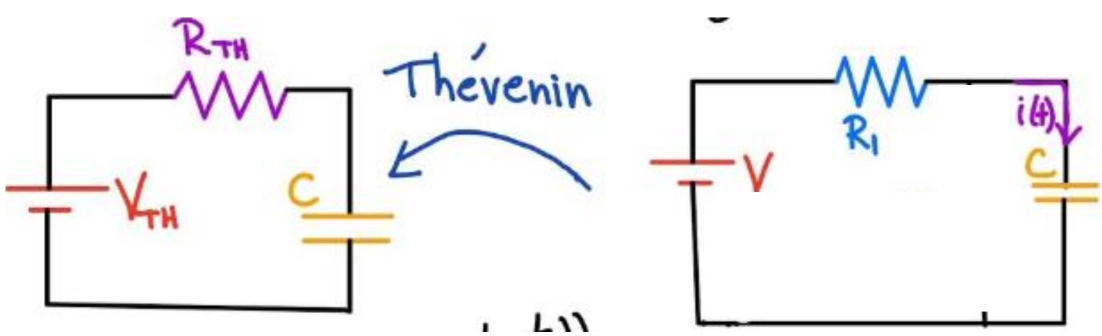
Thevenin's Theorem

Any combination of Power supplies or batteries and resistances with two terminals can be replaced by a single voltage source V and a single series resistor R.

- The value of V is the open circuit voltage at the terminals
- The value of R is the resistance between two terminals with all voltage sources replaced by short circuit.

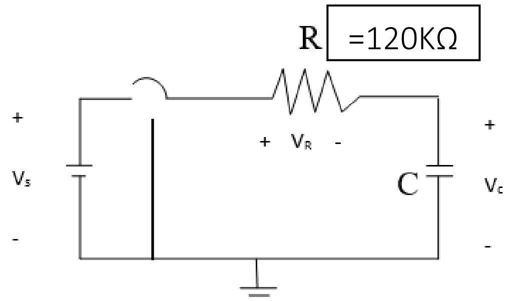


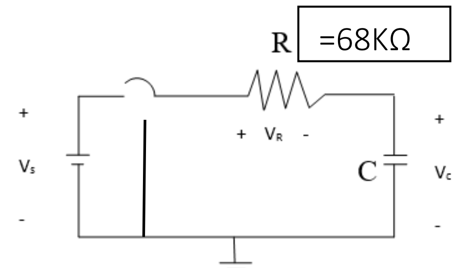
Thevenin's Theorem for a Capacitor



Task 1:

Set up the following simple circuits given below with the given Resistor R = 120KΩ, 68KΩ and capacitor 330µF capacitor





Set the power supply voltage to 15V. use a jumper wire as a switch

1. Measure the supply Voltage and record the values
2. Measure the given resistors and record the values
3. Start charging the capacitor and record the voltage across the capacitor every 5 seconds until 125 seconds using a multimeter with the help of a timer.
4. Record the values in a table and plot the charging curves for both circuits using Microsoft Excel
5. Now calculate the charging time constant Thevanin Voltage of the circuit.
6. And plot the values in the previous graphs
7. And find the corresponding time for each circuit for each Thevanin voltage.
8. How does this time compare with your calculations of the time constants? Why might this time be slightly different from your calculation
9. Similarly plot the graphs for discharging voltages.
10. And compare the graphs similar to the capacitor charging curves
11. What happened to the response time and the time constant with the smaller resistance?