Online Calculus and Pre-Calculus Modules

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This paper is a report on the development of online modules to be used as self-directed learning support for mathematics skill development. The learning support modules can be used in multiple learning support contexts. It can be used as self-directed learning support based on the needs of students in courses which include a calculus related-curriculum, as well as to be incorporated in various course designs.

Introduction

We have developed 12 online modules to be used as self-directed learning support for mathematics skill development. Modules 1-8 cover foundational concepts, and modules 9-12 cover advanced concepts. The learning support modules can be used in multiple learning support contexts. It can be used as self-directed learning support based on the needs of students in courses which include a calculus related-curriculum, as well as to be incorporated in various course designs. For example, learners in a Calculus for Life Sciences course might focus on applications of trigonometry, while math major students might focus on proof techniques. These learning support modules have been designed so that the course instructor may select, from a broader menu, those modules relevant to the scope of a particular course. The pace at which a learner advances through the modules would also be dependent on the specific context and structure of the course. The learning support modules could also be used as a short preparatory sequence in the summer before students enter university. Some instructors might assign the foundational modules as a self-diagnostic at the commencement of a course, and include other modules as necessary for appropriate learning support for the remainder of the course. The content and activities are supplemented with a flexible assessment framework, allowing course instructors to adopt assessments as iterative formative assessment, or as part of their overall summative grading scheme according to their needs. Accessibility purposes where taken into consideration throughout the entire design process in order to maximize the usefulness of these modules to all learners.

In summary, the design of the learning support modules are such that instructors have flexibility to use them any place, at any pace, just-in-time, and just-enough for their specific goals. In order to achieve this objective, the online materials are provided as a set of sequenced resources, illustrative of a sequence of scaffolded lesson units. These may be used for self-directed learning, or integrated by the instructor into structured course design, which may include instructor guided or peer-based collaborative activities.

Goals and Objectives

Relevant learning objectives promote deep learning (Becoming Outstanding – Secondary Instructional Video Series, 2010.) In the design stage of this project, the general goals and specific learning objectives for each module were identified carefully.

The general goals for the learning support modules are:

- Provide students with a strong support for basic learning calculus concepts: limits, derivatives, and integration.
- Help students communicate mathematical ideas through the practice of proper mathematical notations.

- Help students to verify mathematical ideas through the practice of proper mathematical proof techniques.
- Developing mathematical thinking and understanding in students by guiding them towards deep thinking rather than "memorizing all the rules".
- Increase students awareness of alternate means of learning such as group study, as well as strategies that will enhance the learning of mathematics.

The **specific learning outcomes** for each module are as follows:

Module 1: Algebraic Manipulations
At the end of this unit students are able to

- recognize different type of numbers (natural, integer, rational and irrational numbers) based on a criteria or a simple justification
- use proper interval and set notations and operations for expressing a given set of numbers
- apply proper algebraic manipulations and operations for simplifying an expression

Module 2: Equations and Inequalities
At the end of this unit students are able to

- solve quadratic, radical and absolute value equations
- solve linear and nonlinear inequalities

Module 3: Analytic Geometry
At the end of this unit students are able to

- find distance formula between two given points
- write the equation of a line if two points or a point and the slope is known
- find the intersection of two given curves
- classify equation of curves such as circles, ellipses, hyperbolas and parabolas

Module 4: Functions
At the end of this unit students are able to

- classify a function as a power function, root function, polynomial function, rational function, and algebraic function
- find the domain of a given function
- sketch the curve of a function after shifting or stretching a given curve
- identify odd and even functions and their related symmetries
- understand the relation between a function and its inverse

Module 5: Exponential & Logarithmic Functions
At the end of this unit students are able to

- solve exponential and logarithmic equations
- sketch the curve of simple exponential and logarithmic functions
- sketch a logarithmic function as the inverse of the related exponential function and vise versa

Module 6: Trigonometry
At the end of this unit students are able to

- find the measure of an angle in radians if it is given in degrees and vice versa
- write basic trigonometry relations using the unit circle and right triangles
- use addition and subtraction formulas and derive various trigonometric identities
- solve real world problems using trigonometry

Module 7: Trigonometry Functions
At the end of this unit students are able to
- define various trigonometry functions
- identify basic trigonometric graphs
- describe inverse trigonometric functions and sketch their graphs

Module 8: Limits
At the end of this unit students are able to
- provide intuitive and precise definition of right-limit, left-limit, and limit of a given function
- compute infinite limits and find vertical asymptote of functions
- use limit laws to compute limits

Module 9: Continuity
At the end of this unit students are able to
- define the concept of continuity of a given function both intuitively and by using the precise definition
- distinguish among various types of discontinuity points including removable discontinuity, infinite discontinuity, and jump discontinuity
- apply continuity laws to determine behavior of the function at a given point
- understand Intermediate Value Theorem and apply it for discussing the roots of a function
- understanding the Pinching Theorem and use it to find certain limits

Module 10: Derivative (Part I & Part II)
At the end of this module students are able to
- use definition of derivative to determine whether a function is differentiable or not at certain points
- use Power Rule, Product Rule, Quotient Rule, Chain Rule, Implicit Differentiation to calculate derivative
- calculate derivatives of Inverse Trigonometric Functions
- calculate derivatives of Logarithmic and exponential Functions
- compute critical values
- sketch curves using first and second derivatives
- evaluating different types of indeterminate forms using L'Hospital's Rule

Module 11: Integration (Part I & Part II)
At the end of this module students are able to
- use definition of integration to determine whether a function is integrable or not on a closed interval
- apply properties of definite integral to derive information about a given integral
- define antiderivatives or indefinite integrals of a given function
- compute integrals using substitution, integration by parts, trigonometric substitution, and integration by partial fractions

Module 12: Proof Techniques
At the end of this module students are able to
- understand the justification of calculus facts and relations by applying methods of direct proof, proof by contradiction, proof by contrapositive and proof by induction

Organization, Structure and Approach

The module structure reflects a student centric design. Key components include an orientation stage, with links to additional guides and supporting resources. Materials and assignments are organized into "chunked" units to support ease of access and progression of the student through sequenced and scaffolded learning activities. Screencast segments present content, but also model approaches to problem solving. To maximize students' engagement in the learning the designed activities reinforce individual learning, but may also be adapted by instructors to support peer-to-peer activities using synchronous learning environments or discussion boards, or for use in the context of programming within academic success skills centres. Thus, in addition to self-directed activities, the modules include optional activities for individual or group assignments in online courses or in the classroom component of hybrid courses. An instructor's guide will accompany each of these activities.

It is known that students who have an active, inquiring mind are more successful in learning mathematics (Brown, S. 1981 & Hunsburger, W. F. 2008) In each module, we model the inquiry process and guide students in *asking* questions, as well as answering questions. The modules are targeted to strengthen the foundation skills for success in all standard first year calculus courses as well as provide thorough explanation of the calculus topics. Students are able to use these units independently or follow the guidance of their instructor to use the modules to support their learning. A scaffolded progression through activities will support development of the active level of mathematical thinking appropriate to the university undergraduate program context. The aim of the activities is to improve students' skills, enthusiasm, motivation and confidence.

Module Components

1) Diagnostic Assessment

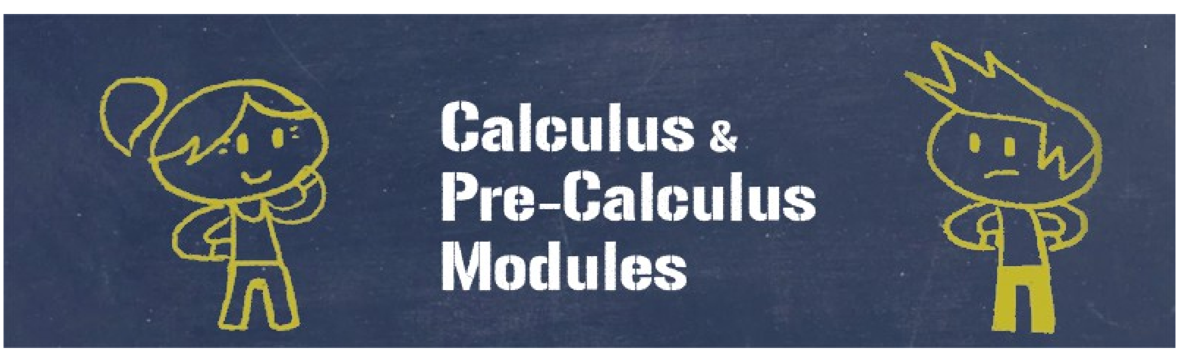
There is an initial diagnostic test that informs learners and their course instructors of the level of students' strength for basic skills. The test is in multiple choice format and includes questions related to Modules 1-7. The test includes 56 questions, 8 questions from each module 1-7. Further diagnostic assessments will be provided at the end of Module 7 which is a transition point between foundational and advanced concepts (Hay, I., 2011 & Picone-Zocchia, J., Martin-Kniep, G. O., 2008.)

2) Animations

We have developed a 3-4 minutes animation for each of the 12 modules (for a total of 39 minutes). The main purpose of the animations is to introduce the module topics in an innovative way. The animations are also useful for visualization of difficult topics. Our goal was to create a learning tool that would help students to change their negative attitudes toward learning mathematics. Our Lead Educational Developer, Lead Instructor, and Creative Advisory Team worked hard together for writing appropriate content and creative elements for the animation scripts, communicating with the animation company Flikli,Video Content Studios, contributing to several rounds of revisions of the storyboards, providing feedback to the animation company, and sending the animations to collaborators and other reviewers for further comments and improvements.

The reviewers were impressed by the final product; they remarked that the videos were not only educational, but were also fun to watch.

For accessibility purposes, the animations are provided with audio transcripts and descriptive video scripts.



Sample Animation Video Script (Module 6: Trigonometry)

General production comments
Fun, upbeat music is playing in the background with one narrator.
Learning outcomes for modules
At the end of this unit students are able to:

- ✓ Find the measure of an angle in radians if it is given in degrees and vice versa
- ✓ Write basic trigonometry relations using the unit circle and right triangles
- ✓ Use addition and subtraction formulas and derive various trigonometric identities
- ✓ Solve real world problems using trigonometry

Scene 1
Narrator: An angle AOB consists of two rays AO and BO with a common vertex O.
A 45 degree angle AOB is formed.
Narrator: We can interpret an angle as a rotation of the ray AO onto BO, with the measure of its angle being the amount of rotation necessary to move AO onto BO.
The 45 degree angle pivots to a 30 degree angle as the rays AO acutely moves closer to ray BO. The AO ray pivots again to a straight horizontal line, this time completely replacing the ray BO and representing a zero degree angle.
Narrator: You've probably seen angles measured in degrees. Degrees and radians are two different units that can be used to measure angles. For trigonometric functions we usually use the radian measure.
A new 45 degree angle is formed. The angle pivots obtusely into a 135 degree angle. The angle continues to pivot counter-clockwise into a complete circle representing 360 degrees.
Narrator: For degrees we define a complete revolution to be 360 degrees. For radians we define a complete revolution to be 2 π radians. So 180 degrees would be π radians. 90 degrees is π/2 radians.
Using trigonometric functions we can describe the relations between angles and the distances between them. For instance, if we take a triangle with an angle of one radian, here the angle is equal to one radian when the length of the arc "s" which subtends the angle is also a length of 1.

Scene 2
Narrator: Let's take a circle with a radius of 1, and plot it on the Cartesian plane.
A circle is formed on the Cartesian plane. The axis is in units of one, labeled from minus one to plus one. In Quadrant I there is a right angle triangle with sides x, y, and z which is the hypotenuse is equal to one. The angle of is theta, and the radius is one. The point where the triangle meets the arc of the circle is labelled P (x, y) = (Cosine theta, Sine theta).
Narrator: If we look at a point P = (x, y) corresponding to angle theta , on the unit circle, we can define many Trigonometric functions. We'll explain these functions in more detail in the next module. As a shortcut in the meantime, with a good calculator you'll be able to calculate these values now. Sine theta will give you the length of the line segment y, and calculating co-sine theta will give you the length of the line segment x. But there are other functions that you'll need too, like tangent, secant, co-secant, and co-tangent.
Sine theta = y
Cosine theta = x
Tangent theta = y/x
Cosecant theta = 1/y
Secant theta = 1/x
Cotangent theta = x/y
Narrator: You can use these trigonometric functions to, for instance, solve a triangle. This simply means figuring out the lengths of each side, and determining the angles within the triangle. There are many other applications you may encounter.

Scene 3

Narrator: Recall, the Pythagorean Theorem deals with right triangles like we have here. Knowing that the longest side has a length of 1, if we also know the angle theta.

After applying Pythagorean, we get sine theta squared + co-sine theta squared equals 1. This is the most important identity in trigonometry. But there are many more. Be sure to check the provided cheat sheet for all the common (and uncommon) identities you're likely to encounter.

A right angle triangle labeled a, b, and hypotenuse c with the formula: 'a' squared plus 'b' squared equals 'c' squared. The screen shift back to the triangle from the previous Scene, where the hypotenuse is equal to one. The formula changes to x-squared plus y-squared equals one squared. To imply applying Pythagorean, the triangle sides change to Sine theta and Cosine theta and the formula also changes to Sine theta squared plus Cosine theta squared equals 1.

Scene 4

Narrator: You may think trigonometry is not relevant to your everyday life. Like calculating the height of a mountain, the distance to shore from a point in the ocean, or the distance between the stars and planet we see in the sky!

The examples are illustrated on screen, with 'h' representing the height of the mountain, and 'd' representing the distance between a star and the planet that appears to be Saturn, and the distance from shore to a boat on the ocean

Narrator: Or even to determine the angle escape from a tight situation.

Susie is on screen, blinking and doing her thing, unsuspectingly. A dome with an opening facing the moon forms around her.

Narrator: Luckily for Susie, she understands her trigonometric functions well.

A light bulb appears on top of Susie's head!

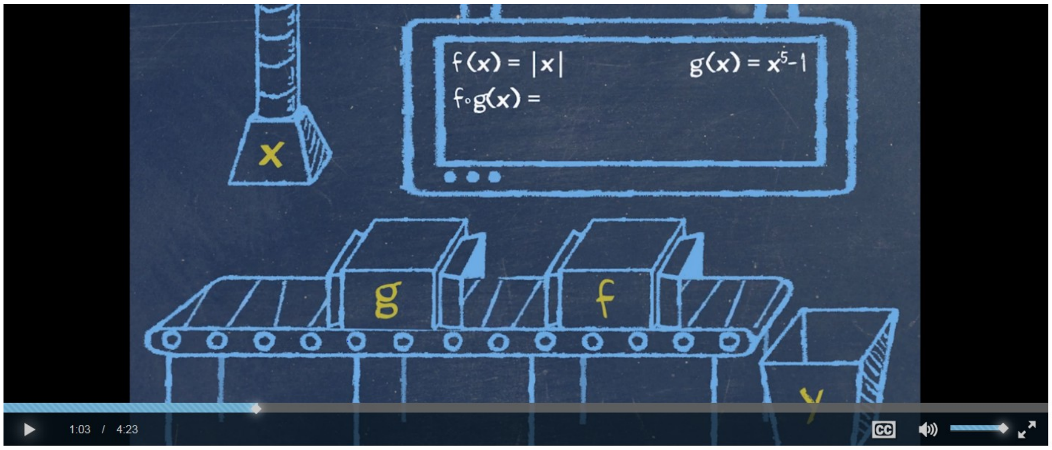
Narrator: Susie knows a right triangle has an internal angle of 45 degrees only when the two sides, opposite the hypotenuse, are of equal length.

Susie has a laser pointer, and shifts over to be right under the dome opening. Looking up, she points the laser pointer towards the moon. This forms the side 'y', she then shifts back to create the hypotenuse side of the triangle 'z'. With the floor representing 'x', Susie successfully completes the right angle triangle xyz with an angle of 45 degrees! She knows it is a 45 degree angle when x and y are of equal lengths.

Susie happens to have a jet-pack, and positioning herself at a 45 degree angle from the opening, she blasts off through the opening in the dome above (following the angle of the triangle that was traced), and disappears off into the atmosphere towards the moon.

Scene 5

Susie and Tommy are on the moon wearing space suits. Using their jet-pacts, they fly up and give each other a high five.



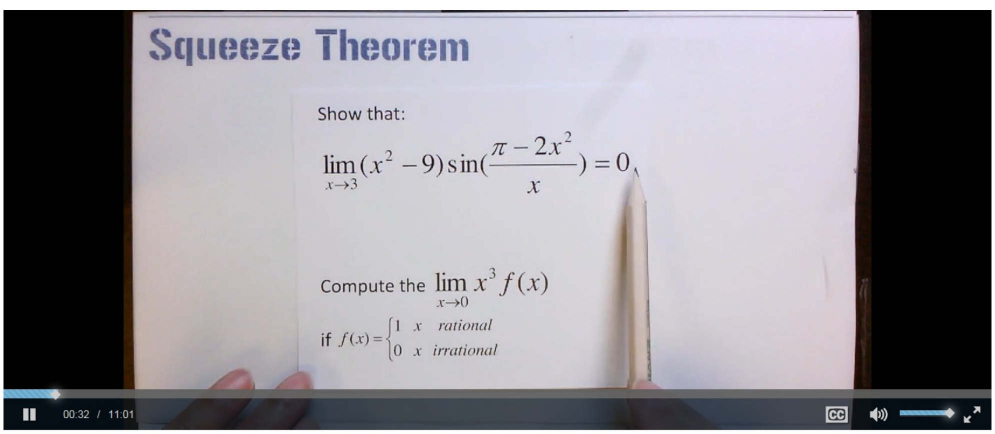
Math Instruction: 05-Animated intro to Composition and Inverse

3) Instructional Videos

Each module includes 2-5 videos which model approaches to problem solving in mathematics related to each module's content. The length of instructional videos in total is around 10 hours. The main purpose of the videos is to engage students in the learning of mathematics concepts in a simple, natural, and constructive manner. Significant amount of work has been done by the Lead Instructor and the Lead Educational Developer to design the content and presentation style to allow students to be able to follow the lessons without difficulty. The lessons have been organized into "chunked" units in order to support ease of access and the students' natural progression of skill through sequenced and scaffolded learning activities.

The produced videos were sent to University of Toronto Mississauga (UTM), Ontario Institute for Studies in Education (OISE), and McMaster collaborators and reviewers for further improvements.

For accessibility purposes, the lessons are provided with audio transcripts.



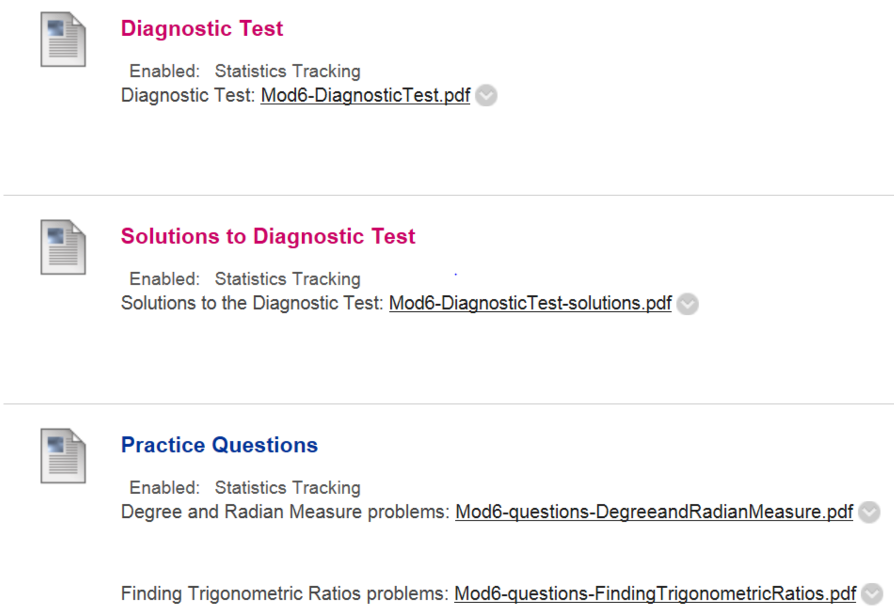
Math Instruction: 09-Continuity, Squeeze Theorem lesson

4) Additional Resources

For modules 1-8, additional problem sets, diagnostic tests, and assessment tests, all together with their detailed solutions (a total of about 80 pages) were prepared. A team consisting of collaborators from UTM, U of T St. George, and the Lead Instructor were responsible for the creation of these questions and tests.

For modules 9-12, which focus on more advanced calculus concepts, the Lead Instructor and a collaborator from Western University have developed a 40-page handbook explaining selected topics through carefully chosen examples, and including a set of additional practice questions and answers. This is expected to help students gather a better understanding of the concepts introduced and to improve their capability of solving related problems.

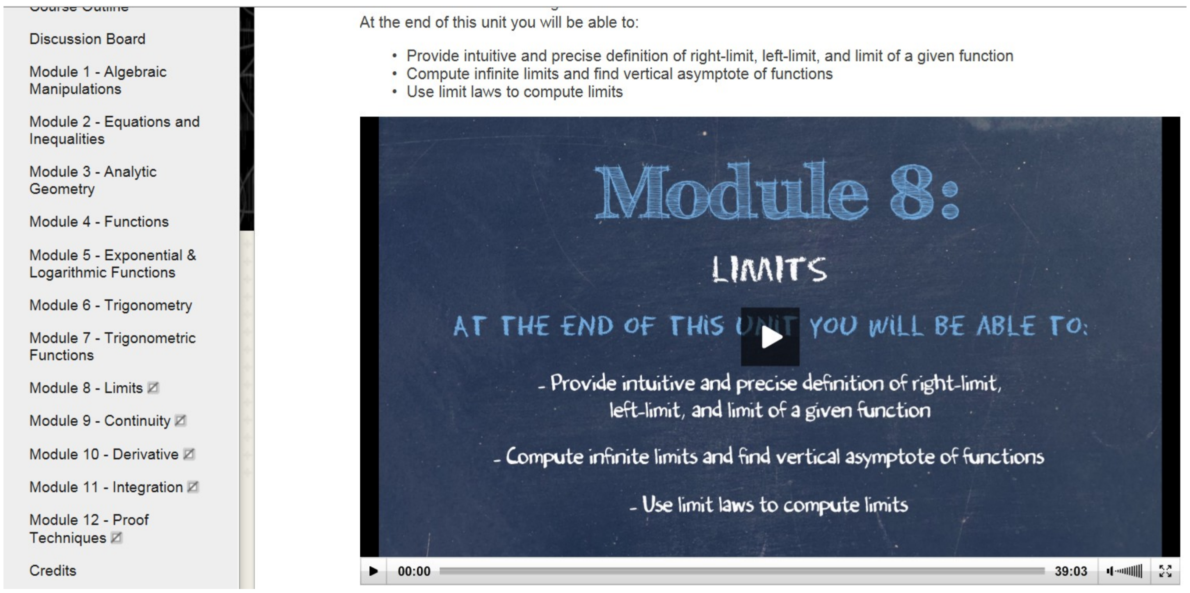
LaTeX files are available for accessibility accommodation purposes.



Implementation

In fall 2015, the modules were used extensively in MATA31F (Calculus for Computer and Mathematical Sciences offered at University of Toronto Scarborough (UTSC)) as an additional resource. In addition, modules 1-7 will be the main resources for the New Online Mathematics Preparedness Course offered at UTSC, a course that is directed for students lacking a solid background in high school mathematics.

In different stages of the project, our Student Advisor and collaborators from UTM, Western University, OISE, and U of T St. George were involved to help prepare the material and to provide feedback about the developed resources. The collaborators plan to use the modules as a resource in their first year courses.



Licensing Information

The project website offers materials available for download in *SCORM-compliant Common Cartridge v1.2 and Blackboard Archive formats* for instructors to use in developing their own course on this topic. The materials have been published as shared educational resources and licensed under a Creative Commons Attribution-NonCommercial 4.0. Educators are welcome to re-use the entire set of modules, or to use an appropriate selection of modules to best support course goals and/or student learning requirements. The URL for the project is: math.onlinelearning.utoronto.ca .

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