1. Diket : $2n+1 < 2^n$ untuk $n \geq 3$

- Metode 1 - Induksi:

$n=3 \rightarrow 2(3)+1 = 7$, $2^3 = 8$ $\rightarrow 7 < 8$

asumsi: $2k+1 < 2^k$

Induksi: $2(k+1)+1 = 2k+3$

$= (2k+1)+2 < 2^k + 2 \leq 2^k + 2^k = 2^{k+1}$

- Metode 2 - Komparasi: $2^n \geq n^2 > 2n+1$

$n=3 \rightarrow 7 < 8$

$n=4 \rightarrow 2^4 = 16$

$= 4^2 > 2 \cdot 4 + 1 = 9$

$n \geq 4$

$2^n \geq n^2$

$2^{k+1} = 2 \cdot 2^k \geq 2k^2 \geq (k+1)^2$

karena $n^2 > 2n+1$ untuk $n \geq 4$

maka: $2^n \geq n^2 > 2n+1$

2. $2^{3n}-1$ habis dibagi 7

- Metode 1 - Modulo

$$2^3 = 8 \equiv 1 \pmod 7$$

$$2^{3n} = (2^3)^n \equiv 1^n \equiv 1 \pmod 7$$

$$2^{3n} - 1 \equiv 0 \pmod 7$$

- Metode 2 - Induksi

$n=1 \rightarrow 2^{3 \cdot 1} - 1 = 7$

asumsi: $7 \mid (2^{3k}-1)$

induksi: $2^{3(k+1)} - 1 = 8 \cdot 2^{3k} - 1$

$= 7 \cdot 2^{3k} + (2^{3k} - 1)$

3. $n^3 - 7n + 3$ habis dibagi 3

- Metode 1 - modulo per kasus
  - $n \equiv 0 \pmod 3 \rightarrow n^3 - 7n + 3 \equiv 0$
  - $n \equiv 1 \pmod 3 \rightarrow 1 - 7 + 3 = -3 \equiv 0$
  - $n \equiv 2 \pmod 3 \rightarrow 8 - 14 + 3 = -3 \equiv 0$

- Metode 2. - Induksi

$n = 0 \rightarrow 3$

asumsi : Induksi :

$3 \mid (k^3 - 7k + 3)$ $\quad (k+1)^3 - 7(k+1) + 3 = (k^3 - 7k + 3) + 3(k^2 + k - 2)$