Solution to analysis in Home Assignment 1

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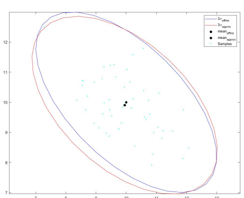
Analysis

In this report, I will present my independent analysis of the questions related to home assignment 1. I have discussed the solution with Varun Ganapati Hegde and Neel Kachhawah but I swear that the analysis written here are my own.

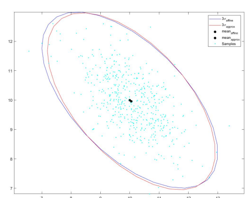
1 Transformation of Gaussian random variables

a

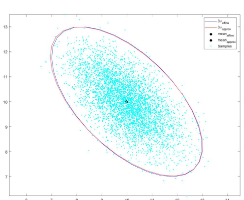
The plot of the transformed Gaussian random variables from **affineGaussianTransform()** and **approxGaussianTransform()** for different number of samples are represented as the figure 1.1.



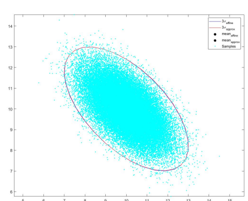
(a) Number of samples = 50



(b) Number of samples = 500



(c) Number of samples = 5000



(d) Number of samples = 50000

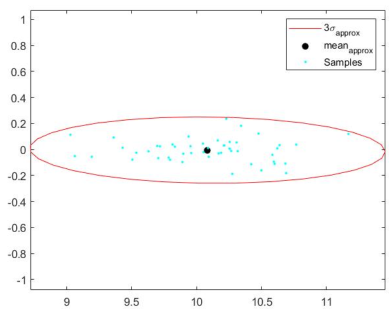
Figure 1.1: Transformed Gaussian random variables for various samples

As observed in the figure 1.1, the approximated ellipse's fit matches with the analytically calculated ellipse if the number of samples increases. Furthermore, it can also be observed that as the total number of samples increases, the number of samples outside the 3σ value increases which is expected.

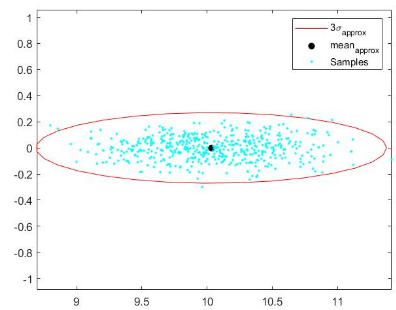
Since most of the transformed samples fit within the ellipses, it can be concluded that the ellipses match the sample points well.

b

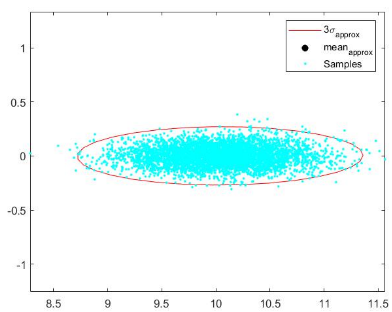
The plot of the transformed Gaussian random variables from **approxGaussianTransform()** for different number of samples are represented as the figure 1.2.



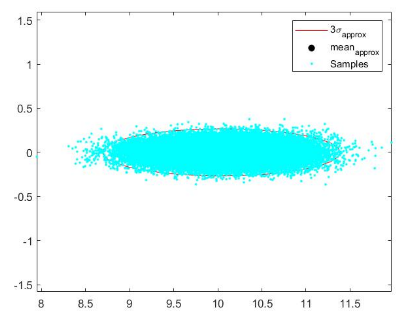
(a) Number of samples = 50



(b) Number of samples = 500



(c) Number of samples = 5000



(d) Number of samples = 50000

Figure 1.2: Transformed Gaussian random variables for various samples

As observed in the figure 1.2, the number of samples outside the 3σ value increases as the total number of samples increases which is expected. Since most of the transformed samples fit within the ellipses, it can be concluded that the ellipse match the sample points well.

2 Snow depth in Norway

a

The plot of the 3σ-ellipsoid of $\mathrm{p}(\mathrm{x}, \mathrm{y})$ for the snow depth at Hafjell and Kvitfjell is represented as the figure 2.1.

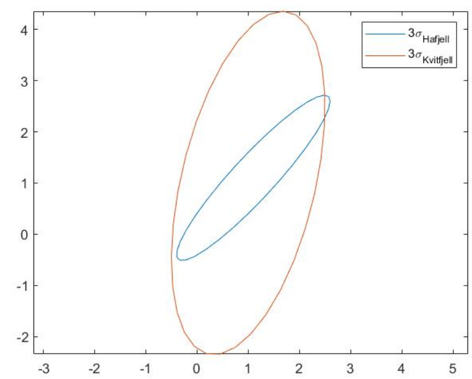


Figure 2.1: 3σ-ellipsoid of $\mathrm{p(x, y)}$ for the snow depth at Hafjell and Kvitfjell

As observed in the figure 2.1, the actual depth of snow (i.e. x) is more dependent on the measurement (i.e. y) in case of the snow depth at Hafjell whereas the actual depth of snow (i.e. x) is less dependent on the measurement (i.e. y) in case of the snow depth at Kvitfjell.

b

The plot of posterior densities $\mathrm{p(x_H|y_A)}$ and $\mathrm{p(x_K|y_E)}$ is represented as the figure 2.2

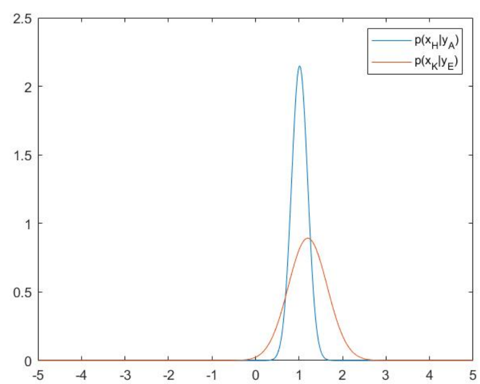


Figure 2.2: Posterior densities $\mathrm{p(x_H|y_A)}$ and $\mathrm{p(x_K|y_E)}$

As observed in the figure 2.2, the 1-dimensional posterior density calculated using the function **posteriorGaussian()** is proportional to the joint density $\mathrm{p(x, y)}$ calculated using the function **jointGaussian()**. This proportionality is a feature of the Bayes rule.

c

Anders would go to ski in Hafjell if he bases his decision on maximising the expected snow depth. However, if Anders bases his decision on the mean of the conditional densities, he would ski in Kvitfjell.

3 MMSE and MAP estimates for Gaussian mixture posteriors

a

The posterior density along with the MMSE and MAP estimate is represented as the equation 1 and the figure 3.1.

$$\mathrm{p}(\theta|\mathrm{y}) \;=\; 0.1\mathcal{N}(\theta;\,1,\,0.5) \,+\, 0.9\mathcal{N}(\theta;\,1,\,9) \tag{1}$$

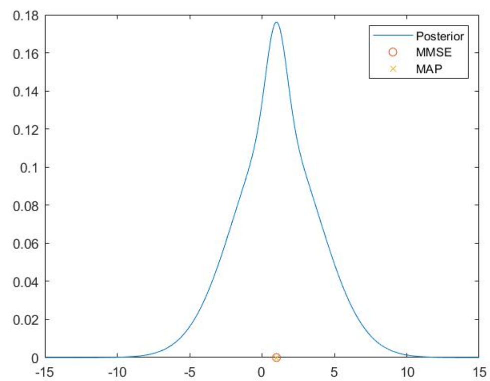


Figure 3.1: Posterior density along with MMSE and MAP estimates

As it can be observed from the figure 3.1, the MAP and MMSE estimator values are same.

b

The posterior density along with the MMSE and MAP estimate is represented as the equation 2 and the figure 3.2.

$$\mathrm{p}(\theta|\mathrm{y}) \;=\; 0.49\mathcal{N}(\theta;\,5,\,2) \,+\, 0.51\mathcal{N}(\theta;\,-5,\,2) \tag{2}$$

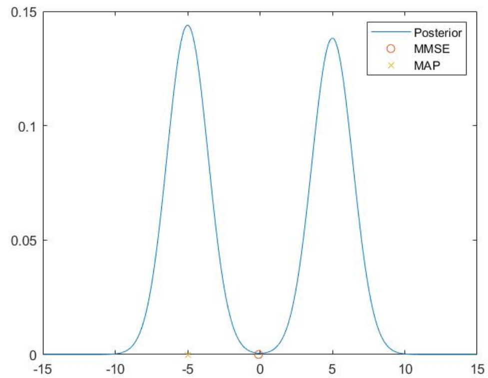


Figure 3.2: Posterior density along with MMSE and MAP estimates

As it can be observed from the figure 3.2, the MAP and MMSE estimator values are different. The MAP estimator provides the mode of the posterior density whereas the MMSE provides the mean of the posterior density.

c

The posterior density along with the MMSE and MAP estimate is represented as the equation 3 and the figure 3.3.

$$\mathrm{p}(\theta|\mathrm{y}) \;=\; 0.4\mathcal{N}(\theta;\,1,\,2) \,+\, 0.6\mathcal{N}(\theta;\,2,\,1) \tag{3}$$

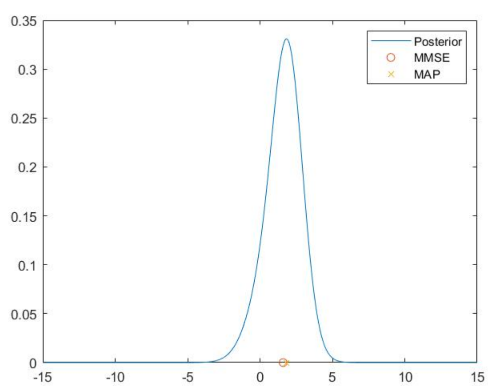


Figure 3.3: Posterior density along with MMSE and MAP estimates

As it can be observed from the figure 3.3, the MAP and MMSE estimator values are slightly different.

The MAP estimator provides the mode of the posterior density whereas the MMSE provides the mean of the posterior density.