Solution to analysis in Home Assignment 2

Sourab Bapu Sridhar (sourab)

Analysis

In this report, I will present my independent analysis of the questions related to home assignment 2. I have discussed the solution with Varun Ganapati Hegde but I swear that the analysis written here are my own.

1 A first Kalman filter and its properties

a

The state sequence and the measurement sequence from the motion model and the measurement model is represented as the figure 1.1.

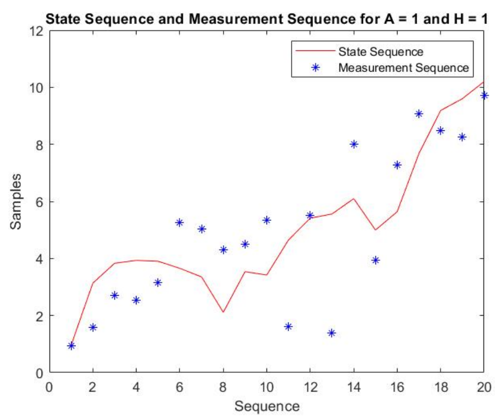


Figure 1.1: State Sequence $\mathrm{x_k}$ and Measurement Sequence $\mathrm{y_k}$

It can be concluded from the figure 1.1 that the measurements behave according to the model as 99.7% of the measurements are within the 3σ region of the state sequence.

b

The measurement sequence filtered using a Kalman filter is represented as the figure 1.2.

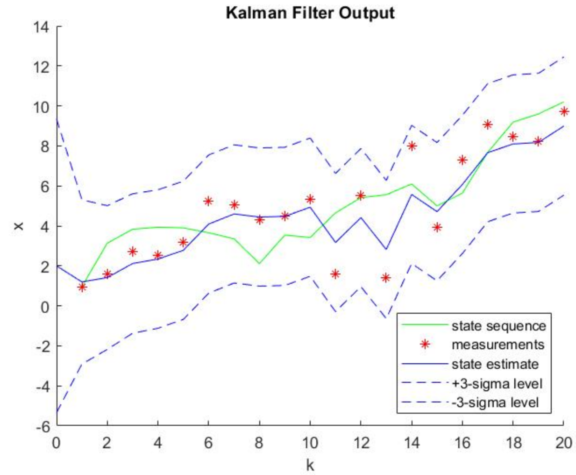
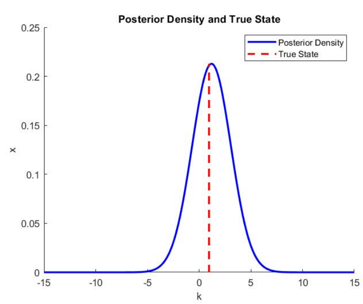
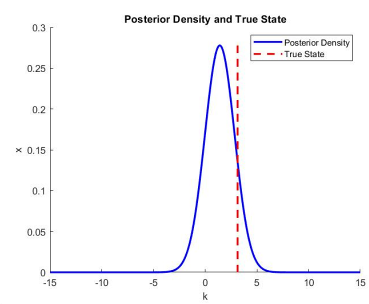


Figure 1.2: Kalman Filter Estimate of the Measurement Sequence

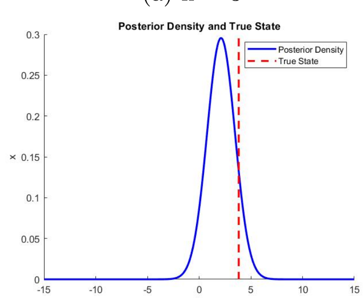
Furthermore, the posterior density along with the true state at various time instances is represented as the figure 1.3.



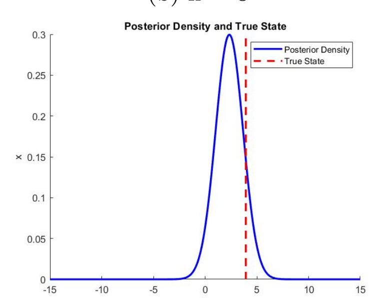
(a) k = 3



(b) k = 8



(c) k = 13



(d) k = 18

Figure 1.3: Posterior Density and True Value for Various Time Instances

From the figure 1.2, it can be concluded that the Kalman filter outputs are reasonable estimates as the filter uses information from the motion model along with the information from the measurement model to predict the estimate. If the Kalman filter had only used information only from the measurement model, the estimate would have been far worse. Additionally, the error co-variance represent the uncertainty in the estimates well as all the posterior densities lie within the 3σ region of the state sequence. This can be further observed in the figure 1.3 where the likelihood of the true value is quite high in the posterior density which indicates that the estimates are quite reasonable.

c

The plots $\mathrm{p(x_{k-1}|y_{1:k-1})}$, $\mathrm{p(x_k|y_{1:k-1})}$, $\mathrm{y_k}$ and $\mathrm{p(x_k|y_{1:k})}$ for the time instance $\mathrm{k} = 10$ is represented as the figure 1.4.

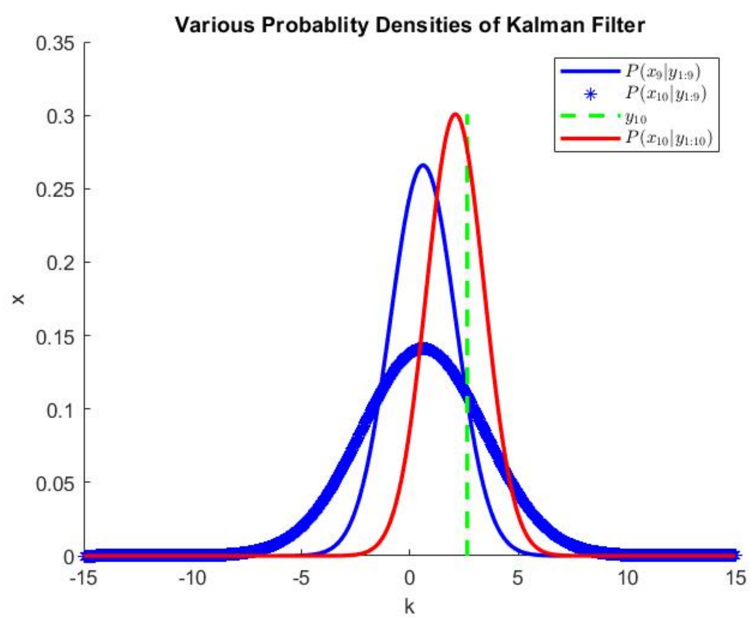


Figure 1.4: Prior, Prediction and Posterior Density for time instance $\mathrm{k} = 10$

On observing the figure 1.4, it can be concluded that the mean of the prior $\mathrm{p(x_{k-1}|y_{1:k-1})}$ and the mean of the prediction $\mathrm{p(x_k|y_{1:k-1})}$ is the same, which is reasonable. However, it can be also observed that the co-variance after the prediction step increases in comparison to the prior as the filter predicts the future state. Once the new measurement $\mathrm{y_k}$ is recorded, the posterior density $\mathrm{p(x_k|y_{1:k})}$ is calculated based on the likelihood and the prior. It can be observed that the mean of the posterior density is not equal to the measurement as the Kalman filter uses information from both the motion model and the measurement model. Furthermore, the co-variance of the posterior density is lesser in comparison with the prediction step as the new measurement is considered for calculating the posterior using the update step. Therefore, it can be concluded that the behaviour of the prediction and the update step is reasonable.

d

The histogram of the estimation error $(x_k - \hat{x_k})$ and the probability density function $\mathcal{N}(x; 0, P_{N|N})$ is represented as the figure 1.5.

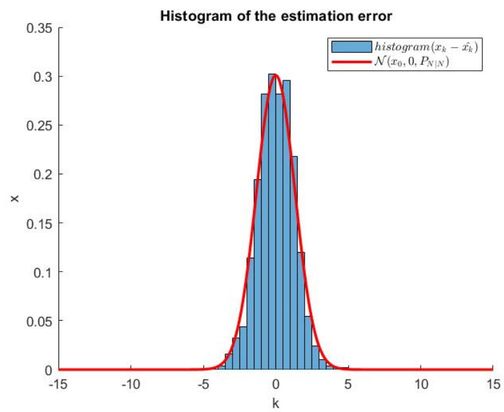


Figure 1.5: Histogram of estimation error $(x_k - \hat{x_k})$ and the probability density function $\mathcal{N}(x; 0, P_{N|N})$

On observing the figure 1.5, it can be noticed that the probability density function fits very closely to the histogram of the estimation error. This is because the estimation error must be zero mean if the filter is performing well and over time the co-variance $P_{N|N} = \mathrm{Cov}(x_k|y_{1:k})$ will converge to a value which is equal to the co-variance of the estimation error.
Another important consistency condition for Linear Kalman Filter is the auto-correlation of the innovation. For a linear Kalman Filter, the $\mathrm{cov}(v_k, v_{k-l}) = 0$, for $l \neq 0$. This can be observed in the figure 1.6 which represents the auto-correlation of the innovation function.

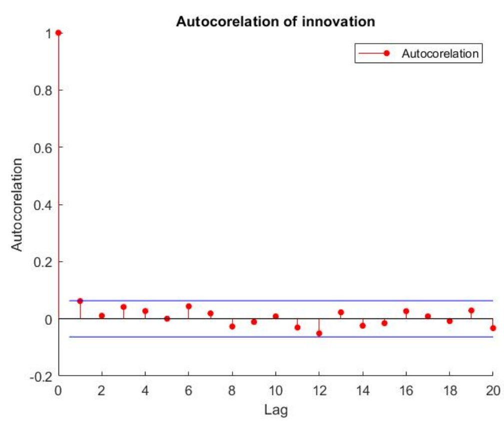


Figure 1.6: Auto-correlation of Innovation

On observing the figure 1.6, it can be noticed that the auto-correlation is very high for no lag (i.e. $l = 0$) and is close to zero for various values of lag (i.e. $l \neq 0$) which means that the filter is performing well.

f

The Kalman filter estimate for correct and incorrect values of prior is represented as the figure 1.7.

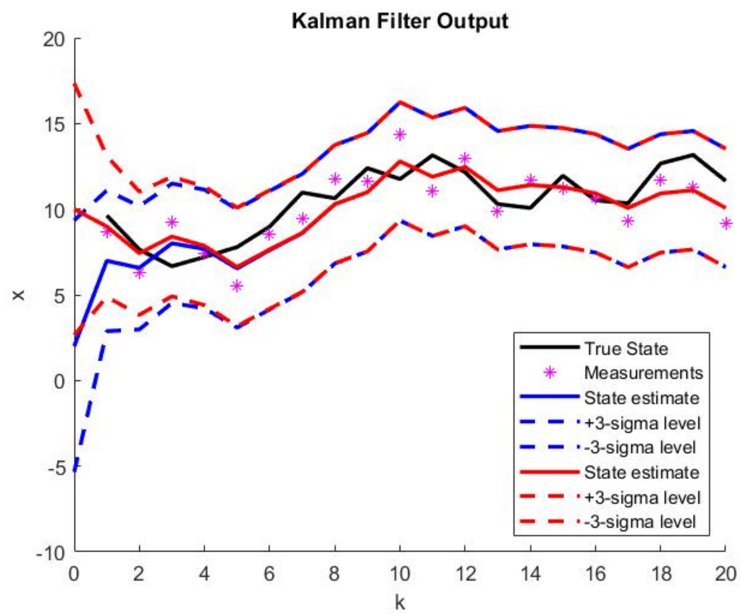


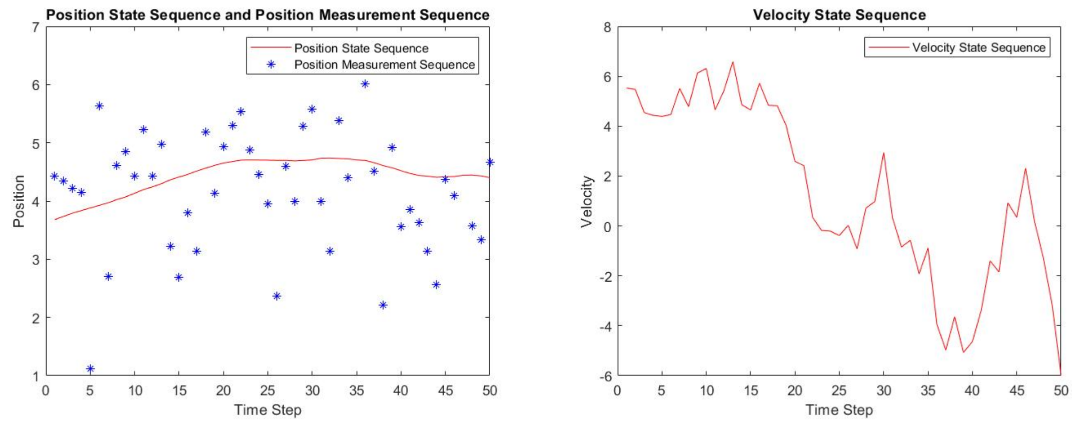
Figure 1.7: Kalman Filter Estimate for Correct and Incorrect Values of Prior

On observing the figure 1.7, it can be noticed that the Kalman filter estimate with the incorrect prior values rapidly converges to the Kalman filter estimate with the correct prior values. It can also be observed that the 3σ values for both the estimates also converges rapidly over time. This is possible because the influence of the first prior reduces over time on the posterior density $p(x_k|y_{1:k})$. Furthermore, the history of the measurements ($y_{1:k}$) will improve the posterior density over time.

2 Kalman filter and its tuning

a

The state sequence and the measurement sequence from the Constant Velocity (CV) motion model is represented as the figure 2.1.



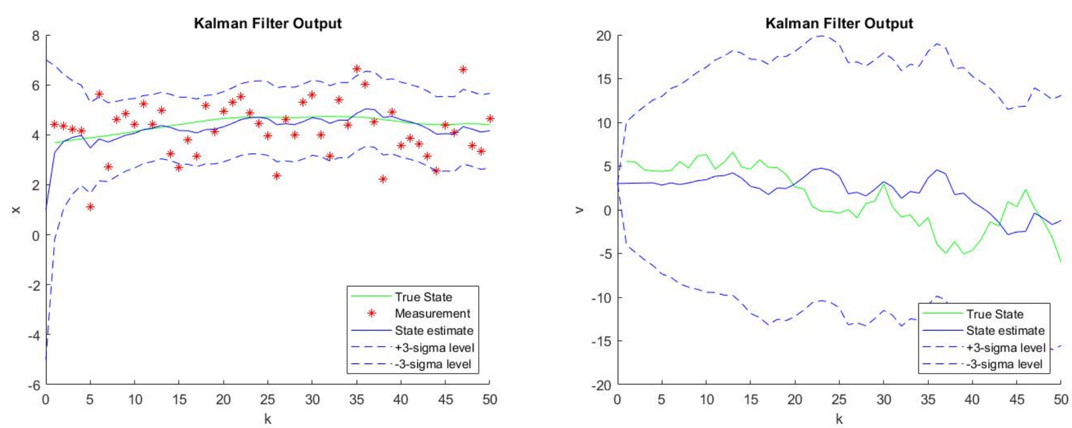
(a) Position (b) Velocity

Figure 2.1: State sequence and Measurement sequence for the Constant Velocity (CV) motion model

As observed from the figure 2.1, the position state and the measurement sequence is based on the Constant Velocity (CV) Model. Furthermore, the change in the velocity is represented as the zero-mean Gaussian white noise. Since the velocity can be represented as piece-wise constant, it can be concluded that the result is reasonable.

b

The filtered value of the generated measurements for the Constant Velocity (CV) model is represented as the figure 2.2.



(a) Position Estimate (b) Velocity Estimate

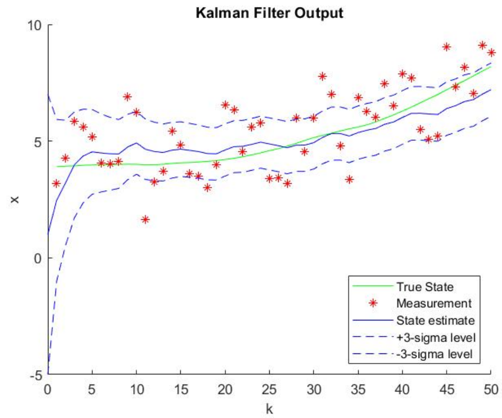
Figure 2.2: True State, Measurement Sequence and Estimate State for the Constant Velocity (CV) Model

On observing the figure 2.2, it can be concluded that the Kalman filter estimates accurate values of position and velocity despite the high amount of noise. This is because the motion model selected is a very good representation of the observed system. Since the assumed Constant Velocity (CV) model represents the observed system precisely, the prediction is accurate and hence the estimates are good. It can also be observed that the velocity is fairly constant, although the velocity estimate is influenced by the measured position.

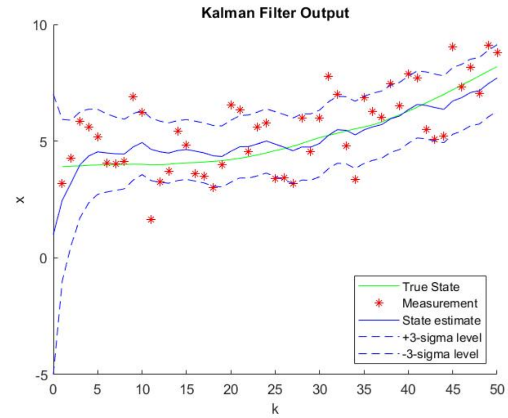
c

The Kalman filter estimate for various values of process noise co-variance is represented as the figure 2.3. On observing the figure, it can be noticed that for smaller values of motion noise co-variance (i.e. Q), the Kalman Filter estimate is quite stable. This observation can be also confirmed by assuming an extreme case of Q = 0 where the estimate would not change. When the motion noise co-variance is smaller compared to the measurement noise co-variance, the Kalman filter trusts the model more than the measurements and hence the estimates are closer to the true value. As a consequence, as long as the true state is stable, the estimated states are also stable. However, it can also be observed in the first figure that as the true state varies due to acceleration (i.e. change in velocity), the estimate deviates from the true state over time as the low motion noise co-variance does not allow the system dynamics to vary a lot. This causes a bias in the estimate.

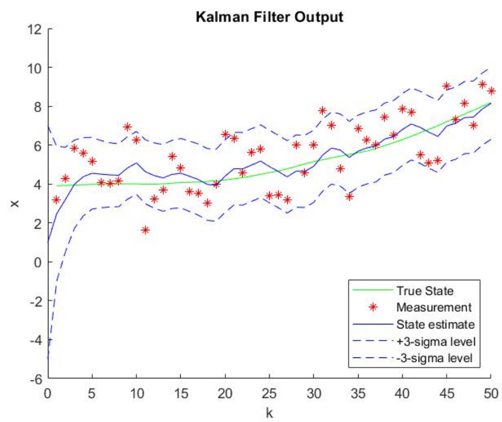
On the other hand, it can be observed that for higher values of motion noise co-variance (i.e. Q), the Kalman Filter estimate can keep track with the changes in the true state due to acceleration (i.e. change in velocity). This is caused because the Kalman filter starts to trust the measurements more. In case the motion noise co-variance is higher than the measurement noise, the Kalman filter starts to trust the measurements more than the model. As a consequence of this, the co-variance of estimate is higher for higher values of motion noise co-variance. This can be observed by comparing the 3σ regions for various values of motion noise co-variance.



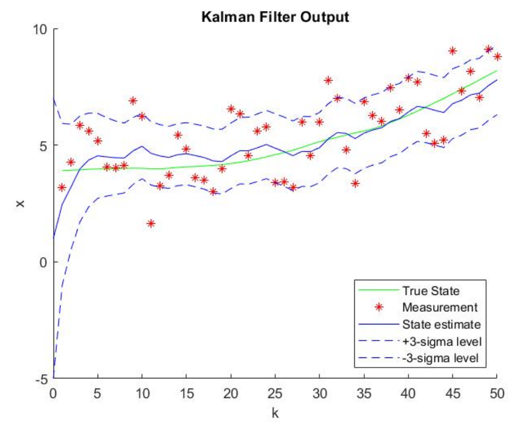
(a) Q = 0.1



(b) Q = 1



(c) Q = 10



(d) Q = 1.5

Figure 2.3: Position Estimate for Various Values of Motion Noise Co-variance