Solution to analysis in Home Assignment 3

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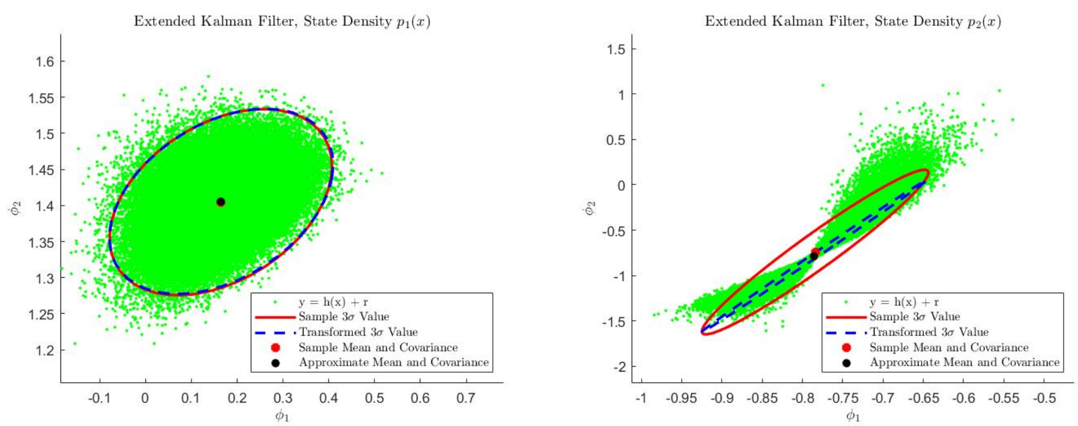
Analysis

In this report I will present my independent analysis of the questions related to home assignment 3. I have discussed the solution with Dongxu Guo but I swear that the analysis written here are my own.

1 Approximations of mean and covariance

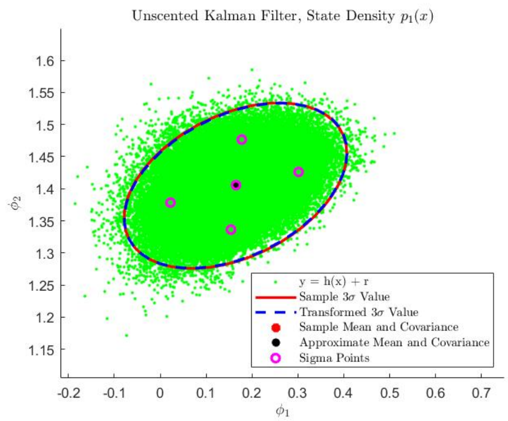
a,b,c,d

The plot of samples of y along with the sample mean and covariance and approximated mean and covariance for Extended Kalman Filter, Unscented Kalman Filter and Cubature Kalman Filter are represented as the figures 1.1, 1.2 and 1.3 respectively.

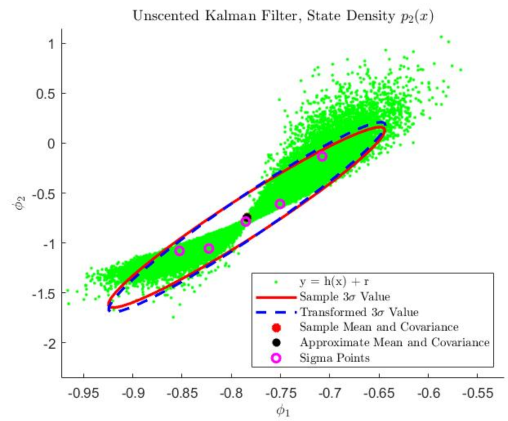


(a) p(y) = h(p_1(x))   (b) p(y) = h(p_2(x))

Figure 1.1: Mean and Covariance Approximation Using Extended Kalman Filter

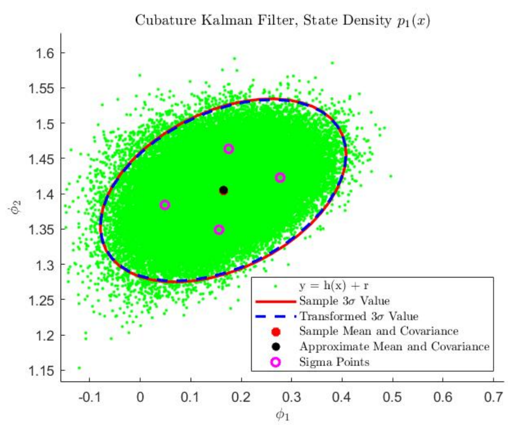


(a) p(y) = h(p_1(x))

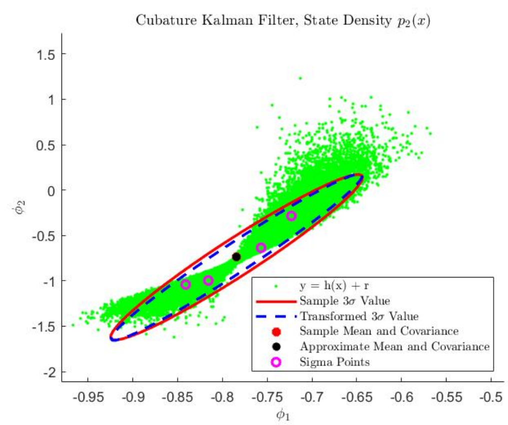


(b) p(y) = h(p_2(x))

Figure 1.2: Mean and Covariance Approximation Using Unscented Kalman Filter



(a) p(y) = h(p_1(x))



(b) p(y) = h(p_2(x))

Figure 1.3: Mean and Covariance Approximation Using Cubature Kalman Filter

It can be observed from the figures 1.1, 1.2 and 1.3 that the sample mean and sample covariances are a very good approximation of the actual mean and covariance since a very high number of samples ($N = 10^5$) are used. Therefore, considering the law of large numbers, the sample approximations can be considered equal to the "true" values. With this assumption, the performance of different Kalman filters are evaluated below.

On evaluating the state density $p_1(x)$, it can be concluded that all the three Kalman filters are approximating the mean and the covariance of the distribution equally well. This is because of the fact that the transformation h(x) is fairly linear around the point $\hat{x}_1$ and the transformed samples are very close to being a Gaussian distribution (Although in reality the transformed state density is not a Gaussian distribution). Due to this, it can be observed that even Extended Kalman Filter is able to approximate the mean and covariance fairly well.

However, on evaluating the state density $p_2(x)$, it can be concluded that there is a big difference in the performance of all the three Kalman filters as the transformation is highly non-linear around the point $\hat{x}_2$. Due to this, the transformed samples are highly non-Gaussian. Therefore, it can be observed that the Extended Kalman Filter performs very poorly in estimating the approximate mean and covariance of the transformed distribution. On the other hand, the performance of the Unscented Kalman Filter and Cubature Kalman Filter, which are based sigma point methods are fairly good and there is a negligible difference in performance between them.

e

It is a good idea to approximate the density $p(y)$ as a Gaussian density only if the transformation $y = h(x)$ is linear around the large mass of $p(x)$. As a consequence, the transformed density would be either Gaussian or close to Gaussian. The same phenomenon can be observed while evaluating the approximation of mean and covariance for the state density $p_1(x)$ in figures 1.1, 1.2 and 1.3. Although $h(x)$ is non-Gaussian, the transformation behaves well and the transformed densities are fairly close to Gaussian.

On the other hand, it is a bad idea to approximate the density $p(y)$ as a Gaussian density if the transformation $y = h(x)$ is highly non-linear around the large mass of $p(x)$. As a consequence, the transformed density is non-Gaussian and this can be confirmed by evaluating the transformed densities for the state density $p_2(x)$ in figures 1.1, 1.2 and 1.3. It can be observed that a high number of the samples are outside the 3σ region and there is a curve in the distribution. Therefore, in scenarios where the transformation is highly non-linear, it is a a bad idea to approximate the density $p(y)$ as a Gaussian density.

2 Non-linear Kalman filtering

a,b

The plot of sensor positions, measurements, true position sequence, estimated position sequence and 3σ contours at every 5^{th} estimate for different Kalman filters are represented as the figures 2.1 and 2.2 respectively.

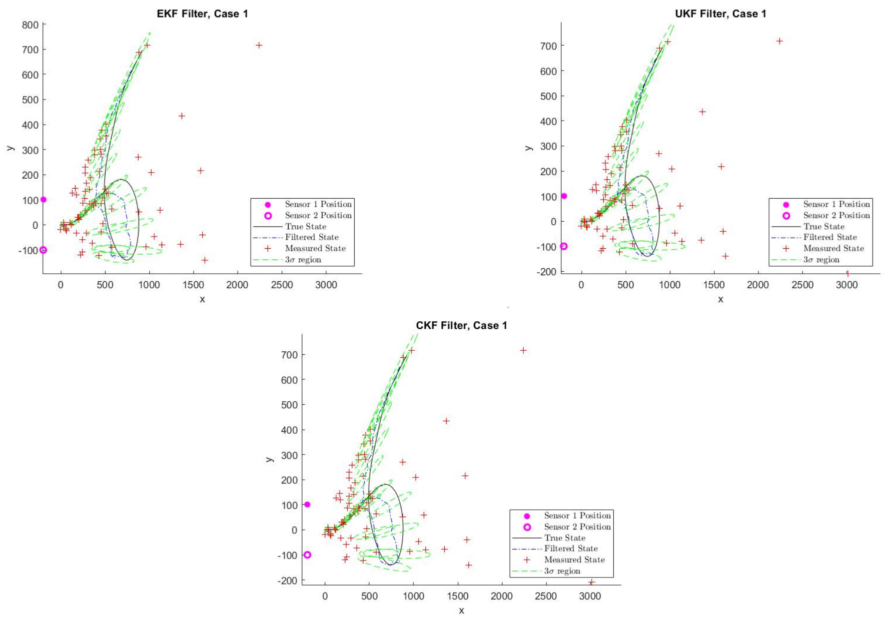


Figure 2.1: Sensor Position, Measurements and True Position for Noise Model 1

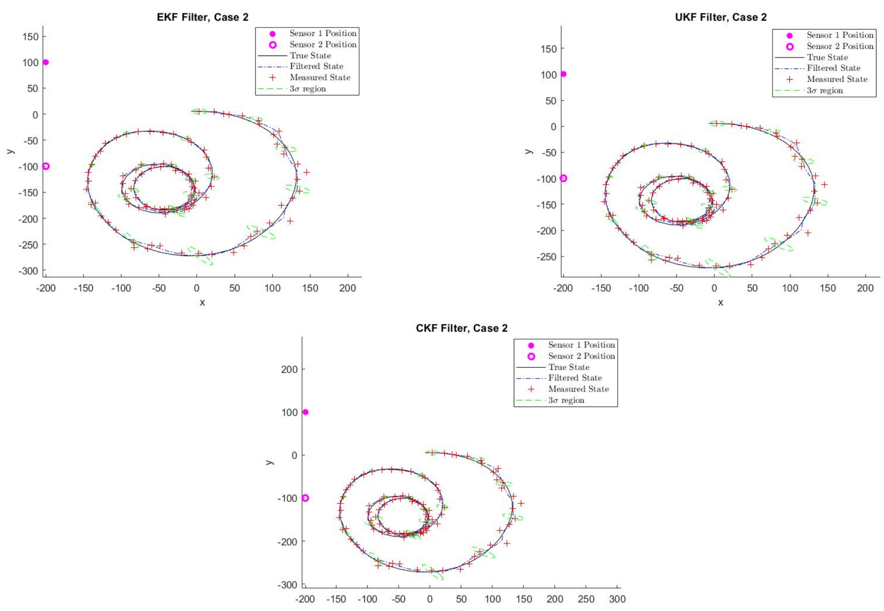


Figure 2.2: Sensor Position, Measurements and True Position for Noise Model 2

On observing the figures 2.1 and 2.2, it can be observed that the Extended Kalman Filter has a much worse performance in comparison to the Unscented Kalman Filter and the Cubature Kalman Filter. It can also be observed that the measurement noises are very high in the figure 2.1, but the Kalman filter is very good in estimating the position. In the figures 2.1 and 2.2, the 3σ ellipses are describing the uncertainity of the measurement.

On closely observing the figure 2.1, it can be concluded that the sensor 2 is giving better measurements as the major axis of the ellipses are aligned with sensor 2. Since the sensor 2 is giving better measurement, it can be assumed that the true position is aligned with sensor 2. Furthermore, it can be observed that the measurements from sensor 1 are highly uncertain. Despite the uncertainty, it can also be observed that the true position is aligned with both sensor 1 and sensor 2 as the position changes as soon as the angles from the sensors change. It can also be observed that the uncertainty of measurements in the figure 2.1 is much higher than in the figure 2.2.

On evaluating the performance of different Kalman filters, it can be clearly observed that the performance of Extended Kalman Filter is much worse performance in comparison to the Unscented Kalman Filter and the Cubature Kalman Filter. This is because of the way the Extended Kalman Filter performs the prediction and update steps. In case of an Extended Kalman Filter, the filter linearizes the non-linearities of the models around the prior mean. In case the models are highly non-linear, the performance of the Extended Kalman Filter is very poor. On the other hand, in case the models are fairly linear and most of the mass is concentrated around a certain point, the Extended Kalman Filter performs well. In case of a highly non-linear model, sigma point methods perform much better in estimating the posterior densities.

c

The histogram of the estimation errors for the position states in case 1 and 2 are represented as the figures 2.3 and 2.4 respectively.

As it can be observed in the figures 2.3 and 2.4, the Extended Kalman Filter performs very poorly in comparision to the Unscented Kalman Filter and the Cubature Kalman Filter. This is confirmed by the histogram plots present in the figures 2.3 and 2.4. Not only is the mean way off from zero, the standard deviations are also very high. On comparing the Unscented Kalman Filter and the Cubature Kalman Filter, there is not much difference between the two. However, the Cubature Kalman Filter performs better in terms of the mean and the covariance of the estimated error.

At the first sight, the histogram looks Gaussian. However, on a closer look, it can be concluded that the histogram is not Gaussian as the histogram generated from the first two moments is not similar to the normalized histogram present in the figures 2.3 and 2.4. This is because a high number of samples are placed outside the 3σ region of the Gaussian distribution.

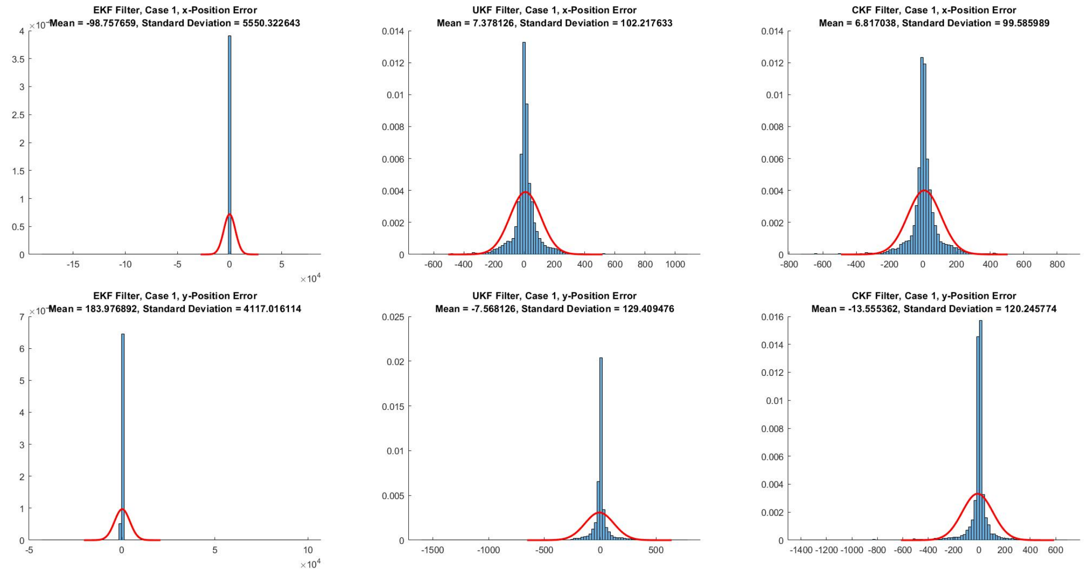


Figure 2.3: Histogram of the estimation errors for the position states in case 1

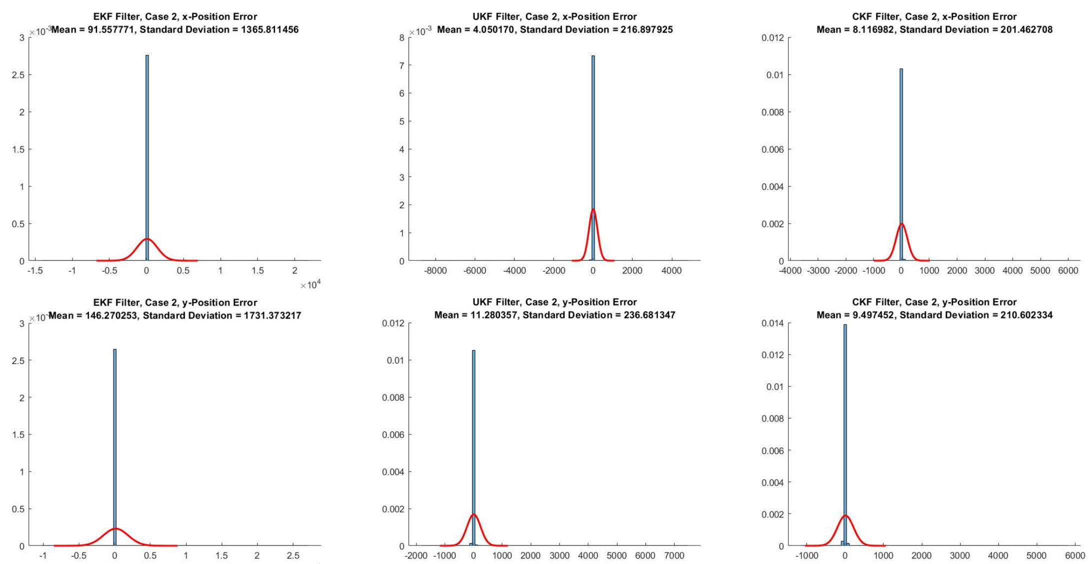


Figure 2.4: Histogram of the estimation errors for the position states in case 2

3 Tuning non-linear filters

In this exercise, the Cubature Kalman Filter is chosen due to its better performance in comparison to the other filters.

a

On varying the model noise covariance, the behaviours represented in the table 1 were observed. To evaluate the limits of the filter, the noise levels were modified by orders of 4.

Effect of changing process noise covariances	
Change	Effect
$\sigma_{\mathrm{v}} \uparrow$	Filtered position becomes very noisy. Speeds oscillates too much which is an unrealistic scenario for a vehicle driving on the road.
$\sigma_{\mathrm{v}} \downarrow$	The lower the velocity noise, the better the filter output. However, this is scenario is only valid if we have a good prior knowledge.
$\sigma_{\mathrm{w}} \uparrow$	Yaw becomes very noisy. Yaw rate oscillates too much which is an unrealistic scenario for a vehicle driving on the road.
$\sigma_{\mathrm{w}} \downarrow$	The lower the yaw noise, the more the vehicle tends to continue driving straight despite a turn. Due to this, the turning radius is fairly high.
$\sigma_{\mathrm{v}} \uparrow, \sigma_{\mathrm{w}} \uparrow$	The filter tends to trust the measurements more, which leads to a noisy output and unrealistic trajectory for a vehicle driving on the road.
$\sigma_{\mathrm{v}} \downarrow, \sigma_{\mathrm{w}} \downarrow$	The filter is not able to adapt to abrupt changes in velocity and yaw which leads to a very high turning radius.

Table 1: Effect of changing process noise covariances

b

Taking into account of the trajectory, it was found that the speed would be fairly constant over the complete trajectory. Since the prior information is quite good, it was concluded that a low velocity noise would be good for the Kalman filter since a higher velocity noise is only required during acceleration periods which is usually at the beginning of the simulation when we are not sure of the true velocity.

In case of the yaw noise, the yaw covariance was tuned in a way that the output was not too noisy and the output follows the trajectory as much as possible. Therefore, the values $\sigma_{\mathrm{v}} = 0.01$ and $\sigma_{\mathrm{w}} = \pi/200$ were found to be the best for the given trajectory. Furthermore, it is also important to ensure that the model noise was discretized before applying to the Kalman filter.

c

The plot of sensor positions, the positions corresponding to the measurements, the true position sequence, and, the estimated position sequence with corresponding covariance contours for high process noise, low process noise and well tuned process noise are represented as the figures 3.1, 3.2 and 3.3 respectively.

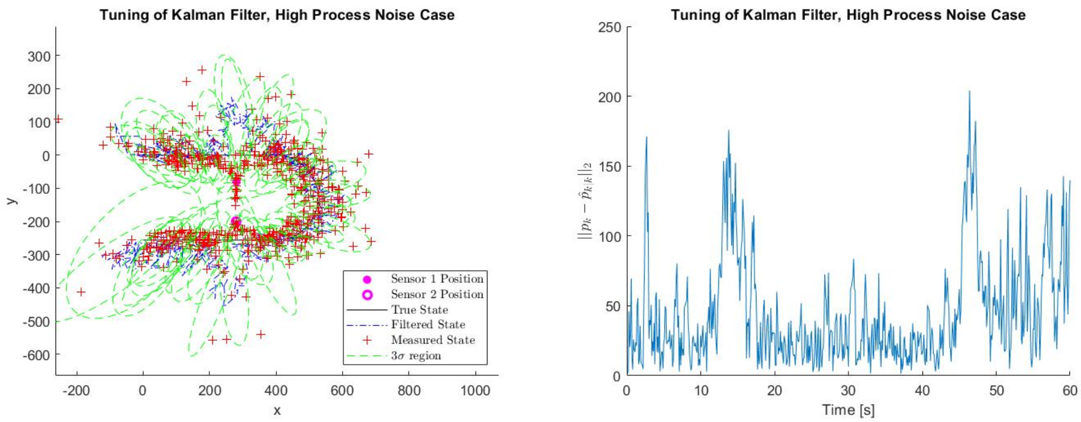


Figure 3.1: High Process Noise

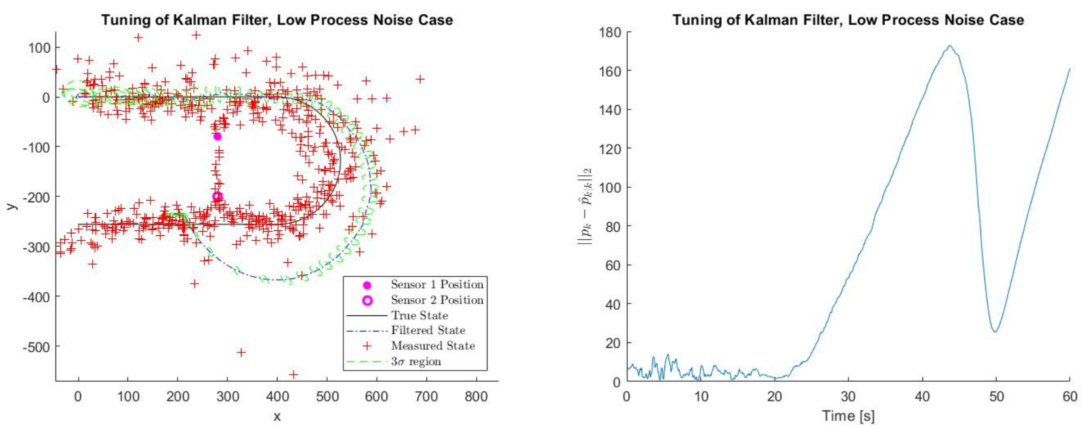


Figure 3.2: Low Process Noise

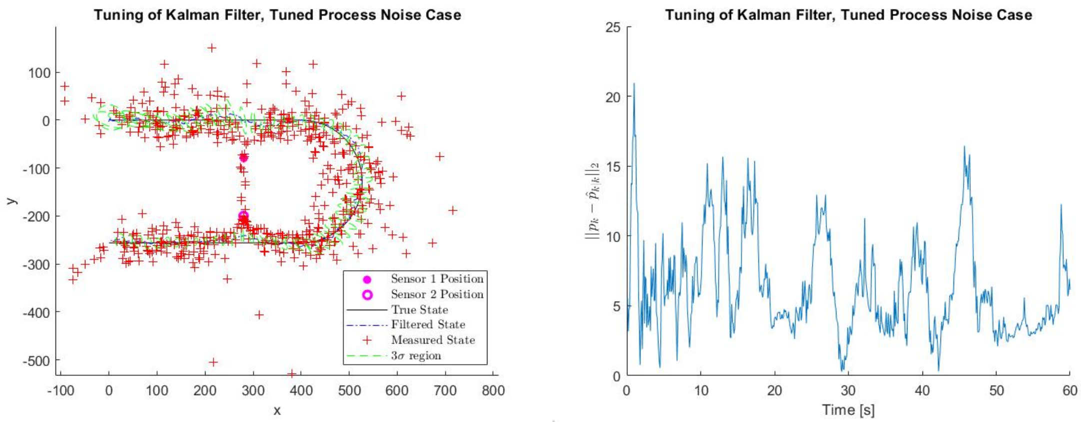


Figure 3.3: Tuned Process Noise

As it can be observed in the figure 3.1, when the noise covariances are higher, the filter tends to trust the measurements more than the model. This leads to a very noisy and unrealistic outputs. Since a small change in angle from the sensors can cause a big change in the position when the positions are aligned with the sensors, it can be observed that the error in the position is much higher during the straight trajectory.

On observing the figure 3.2, it can be concluded that when the noise covariances are low, the model is not able to accelerate in terms of velocity and yaw rate when changing from straight trajectory to the curved trajectory and vice-versa. This can be observed when the error increases abruptly when the vehicle leaves the straight trajectory to enter the curve and when the error increases abruptly when the vehicle leaves the curve to enter the straight trajectory.

On observing the 3.3, it can be concluded that when the noise covariance is well tuned, the model is able to maintain fairly constant and low errors throughout the entire trajectory. Therefore, the filter performs well in both the straight trajectory and the curve.

d

It is not possible to tune the filter to have accurate estimates of the velocity, heading and turn-rate for the whole trajectory. This is because both straight line and curve requires two very different values of process noise covariances.

During the straight trajectory, it is preferable to keep the velocity and the yaw covariance to be as low as possible since there is no change in velocity and yaw-rate. Although this would require a good prior, a low covariance ensures a low output error. However, as observed in the previous question, a low process noise covariance will also make it difficult to accelerate in terms of velocity and yaw rate when changing from straight trajectory to the curved trajectory and vice-versa. This is precisely observed in the figure 3.2.

In order to avoid high errors during transition from a straight trajectory to the curved trajectory and vice-versa, the process noise covariance must be set high. However, this creates an issue in the straight trajectory as a small error in angle from the sensors can cause a big error in the position when the positions are aligned with the sensors. This is precisely observed in the figure 3.1.

Therefore, the above issue makes it difficult to tune the filter for the whole trajectory. However, the value of process noise chosen in the figure 3.3 provides a fairly good estimates of the whole trajectory.