Quantum Computing Formula Sheet

Fady Morris Ebeid

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Chapter 1 Preliminary Math

1 Classic Logic Gates

XOR gate:

$$Q = A \oplus B = \bar{A}B + A\bar{B}$$

A	B	Q
0	0	0
0	1	1
1	0	1
1	1	0

2 Linear Algebra

Tensor Product:

$$\begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a \cdot \begin{pmatrix} c \\ d \end{pmatrix} \\ b \cdot \begin{pmatrix} c \\ d \end{pmatrix} \end{pmatrix} = \begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix}.$$

3 Complex Numbers

$$i^2 = -1 \qquad \sqrt{-1} = i$$

$$e^{i\pi} = -1 \qquad e^{i\frac{\pi}{2}} = i$$

Euler identity:

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Complex Conjugate

$$z = a + bi \qquad z^* = a - bi$$

$$z + z^* = 2a$$

$$zz^* = a^2 + b^2$$

$$|z| = \sqrt{zz^*} = \sqrt{a^2 + b^2}$$

$$\frac{z_1}{z_2} = \frac{z_1 z_2^*}{z_2 z_2^*}$$

Conjugate Transpose

$$\boldsymbol{U}^\dagger \boldsymbol{U} = \boldsymbol{U}\boldsymbol{U}^\dagger = \mathbf{I}$$

Chapter 2 Quantum States

1 Dirac Notation (Bra-Ket Notation)

Qubits are represented by *bra–ket* notation, or *Dirac notation.*

Ket: represents a column vector, for example:

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Bra: represents a row vector, for example:

$$\langle 0| = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad \langle 1| = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

Inner product: (scalar product/dot product)

$$\langle 0|0\rangle = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1$$

Outer product:

$$|1\rangle\langle 0| = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

2 1 Qubit

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

A *superposition* of *zero* and *one*:

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$$\left\| \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \right\| = \sqrt{\alpha^2 + \beta^2} = 1$$

Where α and β are called *probability amplitueds*. They are *complex numbers* and $|\alpha|^2 + |\beta|^2 = 1$

In a balanced superposition of n qubits, $\alpha = \frac{1}{\sqrt{2^n}}$

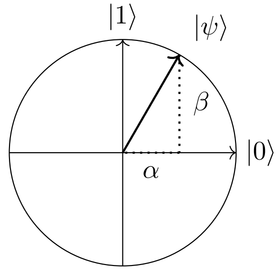
The *probability* of observing the system in state $|0\rangle$ is α^2.

The *probability* of observing the system in state $|1\rangle$ is β^2.

2.1 Visualization of a (Real-Valued) Qubit

Any real-valued quantum state ψ can be represented as a point in the unit circle, and it can be described by one angle θ.

If we assume that α and β are real numbers, the quantum state ψ can be represented using a unit circle:



$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix}$$

2.2 Visualization of a Complex-Valued Qubit

The Bloch Sphere

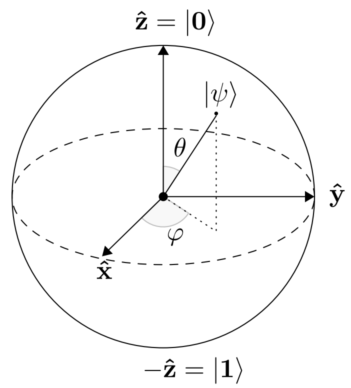


Figure 2.1: The bloch sphere

A single qubit can be written as:

$$|\psi\rangle = \cos\frac{\theta}{2}\,|0\rangle + e^{i\phi}\sin\frac{\theta}{2}\,|1\rangle$$

Where $0 \le \theta \le \pi$ and $0 \le \phi \le 2\pi$

3 2 Qubits

$$\begin{array}{cccc} |00\rangle & |01\rangle & |10\rangle & |11\rangle \\ \begin{bmatrix}1\\0\\0\\0\end{bmatrix} & \begin{bmatrix}0\\1\\0\\0\end{bmatrix} & \begin{bmatrix}0\\0\\1\\0\end{bmatrix} & \begin{bmatrix}0\\0\\0\\1\end{bmatrix} \end{array}$$

$$\begin{aligned} |10\rangle &= |1\rangle \otimes |0\rangle \\ &= \begin{bmatrix}0\\1\end{bmatrix} \otimes \begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}0\\0\\1\\0\end{bmatrix} \end{aligned}$$

$$|\psi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle = \begin{bmatrix}a\\b\\c\\d\end{bmatrix}$$

$$\left\| \begin{bmatrix}a\\b\\c\\d\end{bmatrix} \right\| = \sqrt{a^2+b^2+c^2+d^2} = 1$$

Note: n qubits give 2^n states

Chapter 3 Quantum Gates (Operators)

Quantum circuits are reversible. Number of inputs equals number of outputs.

1 1 Qubit Gates

1.1 Identity

Symbol: $-\boxed{I}-$

Leaves the basis states $|0\rangle$ and $|1\rangle$ unchanged.

$$\mathbf{I} = |0\rangle\langle 0| + |1\rangle\langle 1| = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$\mathbf{I}|0\rangle = |0\rangle \qquad \mathbf{I}|1\rangle = |1\rangle$$

1.2 Hadamard Gate

Symbol: $-\boxed{H}-$

Takes $|0\rangle$ or $|1\rangle$ and put it into *equal superposition*

$$\mathbf{H} = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix} = \begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{bmatrix}$$

$$\mathbf{H}|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix}$$

$$\mathbf{H}|1\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}\end{bmatrix}$$

$$\mathbf{H}|0\rangle = |+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$\mathbf{H}|1\rangle = |-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

Inverse operation:

$$\mathbf{H}\begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix} = |0\rangle$$

$$\mathbf{H}\begin{bmatrix}\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix} = |1\rangle$$

On a unit circle, Hadamard operator is a reflection across the line obtained by rotating x-axis with $\frac{\pi}{8}$ radians in counter-clockwise direction. This line is given by the equation:

$$y = \frac{\sin(\pi/8)}{\cos(\pi/8)}x$$

1.3 NOT Gate (Pauli-X Gate)

Symbol: $-\boxed{X}-$ or $-\oplus-$

Inverts the quantum state.

$$\alpha|0\rangle + \beta|1\rangle \;-\!\oplus\!-\; \alpha|1\rangle + \beta|0\rangle$$

$$\begin{aligned} \mathbf{X} &= |0\rangle\langle 1| + |1\rangle\langle 0| \\ &= \begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}0 & 1\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}1 & 0\end{bmatrix} \\ &= \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix} + \begin{bmatrix}0 & 0\\1 & 0\end{bmatrix} \\ &= \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} \end{aligned}$$

$$\mathbf{X}\begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}b\\a\end{bmatrix}$$

$$\mathbf{X}|0\rangle = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix} = |1\rangle$$

$$\mathbf{X}|1\rangle = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix} = |0\rangle$$

Representing the last two operations in dirac notation:

$$\begin{aligned} \mathbf{X}|0\rangle &= \Big(|0\rangle\langle 1| + |1\rangle\langle 0|\Big)|0\rangle \\ &= |0\rangle\langle 1|0\rangle + |1\rangle\langle 0|0\rangle \quad \text{by distribution} \\ &= 0\,|0\rangle + 1\,|1\rangle \\ &= \boxed{|1\rangle} \end{aligned}$$

$$\begin{aligned} \mathbf{X}|1\rangle &= \Big(|0\rangle\langle 1| + |1\rangle\langle 0|\Big)|1\rangle \\ &= |0\rangle\langle 1|1\rangle + |1\rangle\langle 0|1\rangle \quad \text{by distribution} \\ &= 1\,|0\rangle + 0\,|1\rangle \\ &= \boxed{|0\rangle} \end{aligned}$$

On a unit circle, $\mathbf{X}$ gate is a reflection across the line $y = x$.

1.4 Pauli-Y Gate

- Maps the $|0\rangle$ state to the $i\,|i\rangle$ state and the $|1\rangle$ state to the $-i\,|0\rangle$ state.
- It can be viewed on the bloch sphere as rotation over the y-axis.

Symbol: $-\boxed{Y}-$

$$\alpha|0\rangle + \beta|1\rangle \;-\boxed{Y}-\; \alpha i|1\rangle - \beta i|0\rangle$$

$$\mathbf{Y} = \begin{bmatrix}0 & -i\\i & 0\end{bmatrix}$$

1.5 Pauli-Z Gate

- Leaves the $|0\rangle$ state unchanged, and flips the sign of the $|1\rangle$ state changing it to $-|1\rangle$
- It can be viewed on the bloch sphere as rotation over the z-axis.
- For real values, on a unit circle it can be viewed as a reflection across the x-axis.

Symbol: $-\boxed{Z}-$

$$\alpha|0\rangle + \beta|1\rangle \;-\boxed{Z}-\; \alpha|0\rangle - \beta|1\rangle$$

$$\mathbf{Z} = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$$

$$\mathbf{Z}(\alpha|0\rangle + \beta|1\rangle) = \alpha|0\rangle - \beta|1\rangle$$

1.6 Rotation Operator

$$q : -\boxed{R_y(\theta)}-$$

$\mathbf{R}$ gate is a single-qubit rotation through angle θ (radians) around the y-axis.

$$\mathbf{R}(\theta) = \begin{bmatrix}\cos(\theta) & -\sin(\theta)\\ \sin(\theta) & \cos(\theta)\end{bmatrix}$$

1.7 Reflection Operator

Let θ be the angle of the line of reflection. Then, the martix form of reflection is represented as follows:

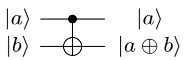
$$\mathbf{Ref}(\theta) = \begin{pmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{pmatrix}$$

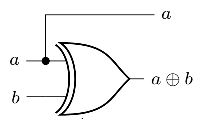
1.8 Composition of Gates

$$\mathbf{HZH} = \mathbf{X}$$
$$\mathbf{HXH} = \mathbf{Z}$$
$$\mathbf{Ref}(\theta) = \mathbf{R}(2\theta)\mathbf{Z}$$

2 2 Qubit Gate

2.1 CNOT(Controlled X or CX)





input		output	
$\lvert a\rangle$	$\lvert b\rangle$	$\lvert a\rangle$	$\lvert a\oplus b\rangle$
$\lvert 0\rangle$	$\lvert 0\rangle$	$\lvert 0\rangle$	$\lvert 0\rangle$
$\lvert 0\rangle$	$\lvert 1\rangle$	$\lvert 0\rangle$	$\lvert 1\rangle$
$\lvert 1\rangle$	$\lvert 0\rangle$	$\lvert 1\rangle$	$\lvert 1\rangle$
$\lvert 1\rangle$	$\lvert 1\rangle$	$\lvert 1\rangle$	$\lvert 0\rangle$

input		output	
a	b	a	$a\oplus b$
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

In the circuit diagram $\bullet$ denotes the *control or input* and $\oplus$ denotes the *target*

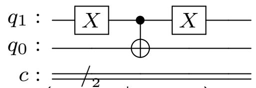
$$\text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\text{CNOT} = \left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right) = \left(\begin{array}{c|c} \mathbf{I} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{X} \end{array}\right)$$

$$\mathbf{CX} = |0\rangle\langle 0| \otimes \mathbf{I} + |1\rangle\langle 1| \otimes \mathbf{X}$$

$\text{CNOT}\,|00\rangle = |00\rangle$
$\text{CNOT}\,|01\rangle = |01\rangle$
$\text{CNOT}\,|10\rangle = |11\rangle$
$\text{CNOT}\,|11\rangle = |10\rangle$

CNOT operator activated when in state 0:



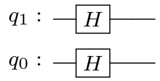
$$\text{C}_0\,\text{NOT} = \left(\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) = \left(\begin{array}{c|c} \mathbf{X} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{I} \end{array}\right),$$

$$\text{C}_0\,\text{NOT} = (\mathbf{X} \otimes \mathbf{I}) \cdot (\text{CNOT}) \cdot (\mathbf{X} \otimes \mathbf{I})$$

Make the second bit the control bit:
$\mathbf{H}^{\otimes 2} \cdot (\text{CNOT}) \cdot \mathbf{H}^{\otimes 2}$

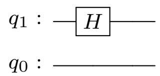
2.2 Operators on Two Qubits

Two Hadamard Gates:



$$\begin{aligned} \mathbf{H}^{\otimes 2} &= \mathbf{H} \otimes \mathbf{H} \\ &= \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \otimes \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \end{aligned}$$

One Hadamard on First Qubit:



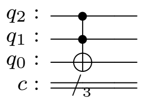
Operator:

$$\mathbf{H} \otimes \mathbf{I}$$

3 3 Qubit Gate

3.1 CCNOT(Controlled-Controlled X or CCX or Toffoli Gate)

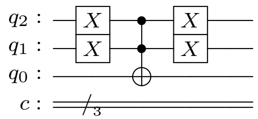
CCNOT :



$$\mathbf{CCNOT} = \left(\begin{array}{cc|cc|cc|cc} \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right)$$

$$= \left(\begin{array}{c|c|c|c} \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{X} \end{array}\right)$$

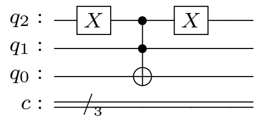
$\mathbf{C_0C_0NOT}$:



$$\mathbf{C_0C_0NOT} = (\mathbf{X} \otimes \mathbf{X} \otimes \mathbf{I}) \cdot (\mathbf{CCNOT}) \cdot (\mathbf{X} \otimes \mathbf{X} \otimes \mathbf{I})$$

$$= \left(\begin{array}{cc|cc|cc|cc} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} \end{array}\right)$$

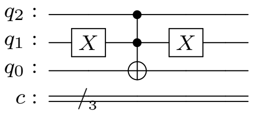
$\mathbf{C_0C_1NOT}$:



$$\mathbf{C_0C_1NOT} = (\mathbf{X} \otimes \mathbf{I} \otimes \mathbf{I}) \cdot (\mathbf{CCNOT}) \cdot (\mathbf{X} \otimes \mathbf{I} \otimes \mathbf{I})$$

$$= \left(\begin{array}{cc|cc|cc|cc} \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} \end{array}\right)$$

$\mathbf{C_1C_0NOT}$:

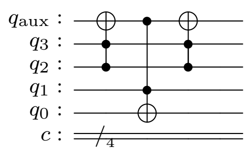


$$\mathbf{C_1C_0NOT} = (\mathbf{I} \otimes \mathbf{X} \otimes \mathbf{I}) \cdot (\mathbf{CCNOT}) \cdot (\mathbf{I} \otimes \mathbf{X} \otimes \mathbf{I})$$

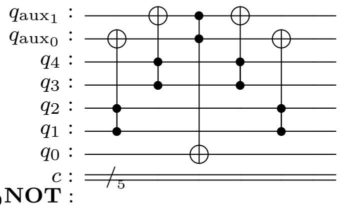
$$= \left(\begin{array}{cc|cc|cc|cc} \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} \end{array}\right).$$

4 Multiple Control Constructions

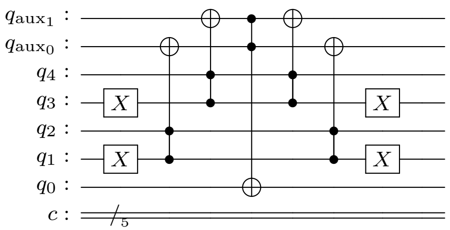
CCCNOT:



CCCCNOT:



$\mathbf{C_1C_0C_1C_0NOT}$:



Chapter 4
Quantum Algorithms

Bell states: are four maximally entangled quantum states of *two qubits*. The are *maximally entangled* and *perfectly correleated.*

$$\left|\mathbf{\Phi}^+\right\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

$$\left|\mathbf{\Phi}^-\right\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right)$$

$$\left|\mathbf{\Psi}^+\right\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)$$

$$\left|\mathbf{\Psi}^-\right\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$$

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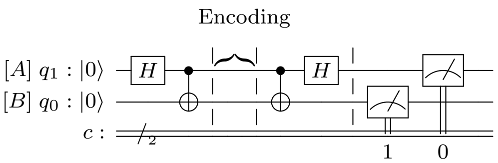
1 Superdense Coding

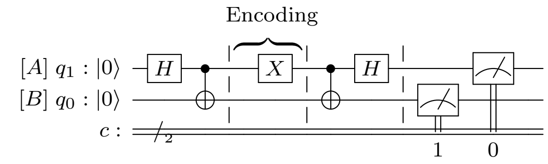
Superdense coding is a procedure that allows sending two classical bits to another party using just a single qubit of communication.

Circuit:

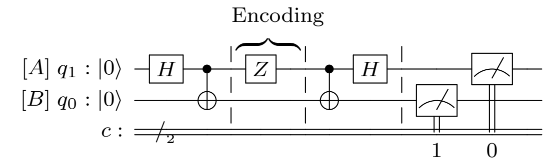
00:



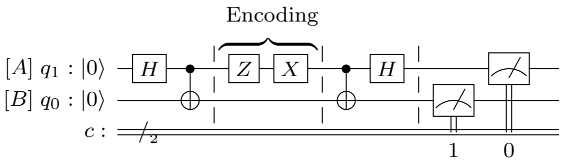
01:



10:



11:



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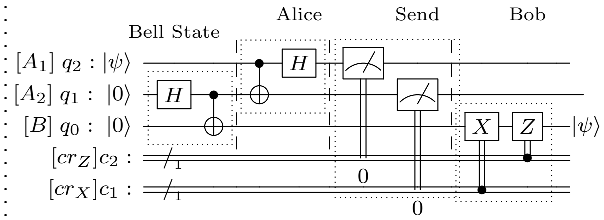
2 Quantum Teleportation

Alice wants to send the qubit state $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$ to Bob by passing information about α and β.

Using two classical bits (cr_X) and (cr_Z) and an entangled qubit pair (q_1 and q_0), Alice can transfer her state $|\psi\rangle$ of qubit q_2 to Bob's qubit q_0.

A third party creates the entangled Bell pair of qubits and gives one to Alice (q_1) and one to Bob (q_0)

This is called *teleportation* because Bob will have $|\psi\rangle$ and Alice won't anymore.



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3 Grover's Search Algorithm

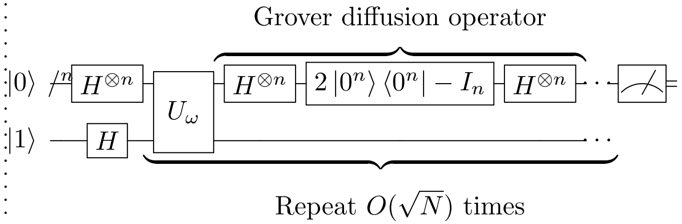

Figure 4.1: Grover's Algorithm
Source: Wikimedia Commons

Where U_ω is the oracle function.