Computer Communications Formula Sheet

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Chapter 1 Fundamentals of Network Communication

1 Delay

Propagation delay:

$t_{\text{prop}} = \frac{d}{v}$

where:

$t_{\text{prop}} \longrightarrow$ time for signal to propagate across the medium (in seconds).

$d \longrightarrow$ distance between two nodes (in meters).

$v \longrightarrow$ speed of light in the transmission medim $3 \times 10^8 m/s$ in vacuum

Transmission delay:

$t_{\text{trans}} = \frac{L}{R}$

where:

$t_{\text{trans}} \longrightarrow$ time to transmit a message (seconds).

$L \longrightarrow$ number of bits in the message.

$R \longrightarrow$ Transmission rate. Bandwidth of digital transmission system in bps(bits/second).

Overall delay:

$$\begin{aligned}\text{overall delay} &= t_{\text{prop}} + t_{\text{trans}} \\ &= \frac{d}{v} \quad + \frac{L}{R}\end{aligned}$$

2 Error Control - Parity Checks

Codeword:

$$n = m + k$$

where:

$n \longrightarrow$ codeword length.

$m \longrightarrow$ message data.

$k \longrightarrow$ check bits.

2.1 Single Parity Check

Information bits:	$\langle b_1, b_2, b_3, \ldots, b_k \rangle$
Check bit:	$b_{k+1} = b_1 + b_2 + b_3 + \ldots + b_k \mod 2$
Codeword:	$\langle b_1, b_2, b_3, \ldots, b_k, b_{k+1} \rangle$

Redundancy:

$$\text{overhead} = \frac{1}{m+1}$$

Error vector: $\langle e_1, e_2, \ldots, e_n \rangle$. Where $e_i = 1$ if an error occurs in the i^{th} transmitted bit and $e_i = 0$ otherwise.

2.2 Error probability:

For an n-bit frame, the probability of occurrence of exactly j-bit errors with an error probability p is given by the binomial probability distribution:

$$p(j) = \binom{n}{j} p^j (1-p)^{n-j}$$

3 Error Control – Polynomial Codes (CRC)

3.1 Binary Polynomial Arithmetic

Binary vectors map to polynomials(of degree $k-1$):

$$\begin{aligned}&\langle i_{k-1}, i_{k-2}, \ldots, i_2, i_1, i_0 \rangle \\ &\longrightarrow i_{k-1}x^{k-1} + i_{k-2}x^{k-2} + \ldots + i_2x^2 + i_1x + i_0\end{aligned}$$

Addition (mod 2):

$$\begin{aligned}(x^7 + x^6 + 1) + (x^6 + x^5) &= x^7 + x^6 + x^6 + x^5 + 1 \\ &= x^7 + (1+1)x^6 + x^5 + 1 \\ &= x^7 + x^5 + 1 \qquad \text{(since 1+1=0 mod 2)}\end{aligned}$$

Multiplication (mod 2):

$$\begin{aligned}(x+1)(x^2+x+1) &= x(x^2+x+1) + 1(x^2+x+1) \\ &= x^3 + x^2 + x + x^2 + x + 1 \\ &= x^3 + 1\end{aligned}$$

Division:

$$p(x) = q(x)g(x) + r(x)$$

$p(x) \longrightarrow$ encoding dividend polynomial.

$q(x) \longrightarrow$ quotient.

$g(x) \longrightarrow$ generator polynomial (divisor).

$r(x) \longrightarrow$ remainder polynomial (CRC checkbits).

3.2 CRC Encoding Procedure

Given an *information polynomial*(message) $i(x)$ and a *generator polynomial* $g(x)$:

- Information polynomial $i(x)$ has k information bits (degree $k-1$).

$$i(x) = i_{k-1}x^{k-1} + i_{k-2}x^{k-2} + \ldots + i_2x^2 + i_1x + i_0$$

- Generator polynomial $g(x)$ has degree $n-k$:

$$g(x) = x^{n-k} + g_{n-k-1}x^{n-k-1} + \ldots + g_2x^2 + g_1x + 1$$

1. Multiply $i(x)$ by x^{n-k} (puts $n-k$ zeros in $(n-k)$ low order positions).
2. Divide $x^{n-k}i(x)$ by g(x), and get a remainder polynomial $r(x)$ of at most degree $n-k-1$. The remainder is the *CRC checkbits*;
$$x^{n-k}i(x) = q(x)g(x) + r(x)$$
3. Add remainder $r(x)$ to $x^{n-k}i(x)$; (put check bits in the n-k lower- order positions). The resulted polynomial will be the transmitted codeword
$$b(x) = x^{n-k}i(x) + r(x)$$
4. The receiver verifies the transmitted message by dividing the transmitted message by the generator polynomial. The remainder should be zero.
$$b(x) \mod g(x) = 0$$

Example:

Given an information message length $k = 4$ bits, codeword length $n = 7$ bits, CRC checkbits length $n-k = 3$ bits:

Information	1100	$i(x) = x^3 + x^2$
Encoding dividend	1100000	$p(x) = x^3 i(x) = x^6 + x^5$
Generator polynomial	1011	$g(x) = x^3 + x + 1$
Remainder(CRC checkbits)	010	$r(x) = x$
Transmitted codeword	1100010	$b(x) = x^6 + x^5 + x$

3.3 Undetectable Error Patterns

Imagine a transmission error $e(x)$ occurs so that instead of $b(x)$ arriving, $R(x) = b(x) + e(x)$ arrives.

Blind spot: If $e(x)$ is a multiple of $g(x)$ (if $e(x)$ is divisible by $g(x)$), then :

$$R(x) = b(x) + e(x) = q_b(x)g(x) + q_e(x)g(x)$$

and

$$R(x) \mod g(x) = 0$$

so, the error will slip by.

4 Internet Checksum (IP Checksum) Algorithm

- Let IP header consists of L, 16-bit words, $\mathbf{b}_0, \mathbf{b}_1, \mathbf{b}_2, ..., \mathbf{b}_{L-1}$
- The algorithm appends a 16-bit checksum $\mathbf{b}_L$ to the header. The checksum $\mathbf{b}_L$ is calculated as follows:
  1. Treating each 16-bit word as an integer, find $\mathbf{x} = (\mathbf{b}_0 + \mathbf{b}_1 + \mathbf{b}_2 + ... + \mathbf{b}_{L-1}) \mod 2^{16} - 1$
  2. The checksum is then given by: $\mathbf{b}_L = -\mathbf{x}$
     Thus, the headers must satisfy the following pattern:
     $$(\mathbf{b}_0 + \mathbf{b}_1 + \mathbf{b}_2 + ... + \mathbf{b}_{L-1} + \mathbf{b}_L) \mod 2^{16} - 1 = 0$$
  3. The checksum calculation is carried out in software using one's complement arithmetic

Chapter 2 Peer-to-Peer Protocols and Local Area Networks

1 Peer-to-Peer Protocols

ARQ stands for *Automatic Repeat Request*.

1.1 Stop-and-Wait ARQ Protocol

Stop-and-Wait ARQ works well on channels that have low propagation delay. It becomes inefficient when the propagation delay is much greater than the time to transmit a frame (transmission delay).

Stop and wait delay model:

$$\begin{aligned} t_0 &= 2t_{\text{prop}} + 2t_{\text{proc}} + t_f + t_{\text{ack}} \\ &= 2t_{\text{prop}} + 2t_{\text{proc}} + \frac{n_f}{R} + \frac{n_a}{R} \end{aligned}$$

Where:
$t_0 \longrightarrow$ Total time to transmit one frame (seconds).
$t_{\text{prop}} \longrightarrow$ Propagation delay (seconds)
$t_{\text{proc}} \longrightarrow$ Processing time (seconds)
$t_f \longrightarrow$ Frame transmission time.
$R \longrightarrow$ Channel transmission rate (bandwidth) (bit/s).
$n_f \longrightarrow$ Number of bits in the information frame.
$n_a \longrightarrow$ Number of bits in the ack frame.

Effective transmission rate:

$$\begin{aligned} R_{\text{eff}}^0 &= \frac{\text{number of information bits delivered to destination}}{\text{total time required to deliver the information } n \text{ bits}} \\ &= \frac{n_f - n_o}{t_0} \end{aligned}$$

Where:
$n_o \longrightarrow$ Number of bits for header and CRC.

Transmission efficiency

$$\eta_0 = \frac{R_{\text{eff}}^0}{R} = \frac{\frac{n_f - n_o}{t_0}}{R} = \frac{1 - \frac{n_o}{n_f}}{1 + \frac{n_a}{n_f} + \frac{2(t_{\text{prop}} + t_{\text{proc}})R}{n_f}}$$

$\frac{n_o}{n_f} \longrightarrow$ Effect of frame overhead.
$\frac{n_a}{n_f} \longrightarrow$ Effect of ACK frame.
$\frac{2(t_{\text{prop}} + t_{\text{proc}})R}{n_f} \longrightarrow$ Effect of *Delay-Bandwidth Product*.

Delay-Bandwidth Product:

Delay-bandwidth product is a key element in performance evaluation of network protocols.

$$\begin{aligned} \text{Delay-Bandwidth Product} &= 2(t_{\text{prop}} + t_{\text{proc}}) \times R \\ &= \text{RTT} \times R \end{aligned}$$

Where:
RTT $\longrightarrow$ Round Trip Time.

Stop and Wait Efficiency in Channel with Errors

$$\eta_{\text{SW}} = \frac{R_{\text{eff}}^0}{R} = \frac{\frac{\frac{n_f - n_o}{t_0}}{1 - p_f}}{R} = \frac{1 - \frac{n_o}{n_f}}{1 + \frac{n_a}{n_f} + \frac{2(t_{\text{prop}} + t_{\text{proc}})R}{n_f}}(1 - p_f)$$

$p_f \longrightarrow$ Probability that the frame arrives with errors (frame loss probability).
$\frac{1}{1 - p_f}$ Average number of transmissions to first correct arrival (geometric distribution).
$\frac{t_0}{1 - p_f} \longrightarrow$ Average total time per frame.
$(1 - p_f) \longrightarrow$ Effect of frame loss.

1.2 Go-back-N ARQ

Maximum Window Size:

Given m-bit sequence numbers, the maximum allowable window size is:

$$W_s = 2^m - 1$$

Where:
$W_s \longrightarrow$ Sender's window size (outstanding frames).
Receiver's window size W_r is often 1.

Required timeout and window size:

Timeout value should allow for:

$$t_{\text{out}} > 2t_{\text{prop}} + 2t_f + t_{\text{proc}}$$

$t_{\text{out}} \longrightarrow$ Timeout.
Window size W_s should be large enough to keep the channel busy for t_{out}

$$D = 2(t_{\text{prop}} + t_{\text{proc}})$$

$$W_s = 1 + \frac{D \times R}{L}$$

Where:
$D \longrightarrow$ Delay (round-trip time).
$DR \longrightarrow$ Delay-Bandwidth Product.
$L \longrightarrow$ number of bits in the message.

1.3 Selective Repeat ARQ

Maximum Window Size

$$M = 2^m$$
$$W_s + W_r = 2^m$$

Where:
$M \longrightarrow$ maximum sequence number for m-bits.
$W_s \longrightarrow$ sending window size.
$W_r \longrightarrow$ receiving window size.

Efficiency of Selective Repeat

$$\eta_{\text{SR}} = \frac{\frac{\frac{n_f - n_o}{t_f}}{1 - p_f}}{R} = \left(1 - \frac{n_o}{n_f}\right)(1 - p_f)$$

$\frac{t_f}{1 - p_f} \longrightarrow$ Average transmission time.

2 TCP Flow Control

Adaptive RTT

TCP uses adaptive estimation of RTT, measures RTT each time ACK is received and calculates the RTT exponentially weighted average:

$$t_{\text{RTT}}(\text{new}) = \alpha \cdot t_{\text{RTT}}(\text{old}) + (1 - \alpha)\tau_n$$

where:
$t_{\text{RTT}} \longrightarrow$ Round trip time.
$\tau_n \longrightarrow$ New RTT time measured.
$\alpha \longrightarrow$ Momentum. Typically $\alpha = \frac{7}{8}$

RTT Variability

Approximate estimation of deviation σ_{RTT}:

$$d_{\text{RTT}}(\text{new}) = \beta d_{\text{RTT}}(\text{old}) + (1 - \beta)|\tau_n - t_{\text{RTT}}|$$

Approximate estimation of timeout:

$$t_{\text{out}} = t_{\text{RTT}} + k\sigma_{\text{RTT}}$$
$$t_{\text{out}} = t_{\text{RTT}} + 4d_{\text{RTT}}$$

3 Multiplexer Modeling

Parameters:

- Inter-arrival time: time between arrival of packets.
- Service time: How long does it take to transmit a packet.
- Service discipline: Order of transmission.
- Buffer discipline: If buffer is full, which pacet is dropped?

Performance measures:

- Delay distribution.
- Packet loss probability.
- Line utilization.

Delay(total time in the system) = Waiting time in queue + Service Time.
Waiting time in queue: The time from arrival instant to beginning of service.

3.1 Poisson Arrivals & Queuing

Number of arrivals in interval of time t is a Poisson random variable with mean λt

$$p(k \text{ arrivals in } t \text{ seconds}) = \frac{(\lambda t)^k}{k!} e^{-\lambda t} \tag{2.1}$$

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4 Two Station MAC

- Each frame transmission requires $2t_{\text{prop}}$ of quiet time.
- Station B needs to be quiet t_{prop} before and after when station A transmits.

$$\text{Efficiency} = \rho_{\max} = \frac{1}{L + 2t_{\text{prop}}R} = \frac{1}{1 + 2\frac{t_{\text{prop}}R}{L}} = \frac{1}{2a}$$

$$a = \frac{t_{\text{prop}}}{t_{\text{trans}}} = \frac{t_{\text{prop}}}{L/R}$$

Where
$R \longrightarrow$ Transmission bit rate (bandwidth).
$L \longrightarrow$ Frame Length (bits).
$a \longrightarrow$ Normalized delay-bandwidth product.
$t_{\text{prop}} \longrightarrow$ Propagation delay.
$t_{\text{trans}} \longrightarrow$ Transmission delay.

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5 MAC Random Access

5.1 ALOHA Model

$$S = GP_{\text{success}} \tag{2.2}$$

Where:
$X \longrightarrow$ Frame transmission time (assume constant).
$S \longrightarrow$ Throughput (average number of successful frame transmissions per X seconds)
$G \longrightarrow$ Load (average number of transmission attempts per X seconds)
$P_{\text{success}} \longrightarrow$ Probability that a frame transmission is successful.

Abramson's Assumption

Probability of arrivals in the vulnerable period:

Given G average number of arrivals per X seconds, and X is divided into n intervals of duration Δ. Probability of arrival in Δ interval is p.

$$\Delta = \frac{X}{n}$$
$$G = np$$

$$\begin{aligned}
P_{\text{success}} &= P(0 \text{ arrivals in } 2X \text{ second}) \\
&= P(0 \text{ arrivals in } 2n \text{ intervals}) \\
&= \lim_{n\to\infty} (1-p)^{2n} \\
&= \lim_{n\to\infty} \left(1 - \frac{G}{n}\right)^{2n} \\
&= e^{-2G}
\end{aligned}$$

$$S \overset{(2.2)}{=} Ge^{-2G} \tag{2.3}$$

Throughput of Slotted ALOHA

$$\begin{aligned}
S &\overset{(2.2)}{=} GP_{\text{success}} \\
&= GP(\text{no arrivals in } X \text{ seconds}) \\
&= GP(\text{no arrivals in } n \text{ intervals}) \\
&= \lim_{n\to\infty} G(1-p)^n \\
&= \lim_{n\to\infty} G\left(1 - \frac{G}{n}\right)^n \\
&= Ge^{-G}
\end{aligned}$$

ALOHA schemes are simple, but low maximum system throughput.

Maximum throughput for ALOHA (G=0.5) :

$$S \overset{(2.3)}{=} \frac{1}{2e} = 0.184$$

Maximum throughput for slotted ALOHA (G=1) :

$$S \overset{(2.3)}{=} \frac{1}{e} = 0.368$$

5.2 CSMA/CD Model

CSMA/CD stands for: Carrier Sense Multiple Access / Collision Detection

Contention Resolution

- Assume n busy stations, and each may transmit with a probability p in each contention time slot.

$$P_{\text{success}} = np(1-p)^{n-1}$$

- By taking the derivative of P_{success} we find the max occurs at $p = \dfrac{1}{n}$

$$\begin{aligned}
P_{\text{success}}^{\max} &= \lim_{n\to\infty} n\frac{1}{n}\left(1 - \frac{1}{n}\right)^{n-1} \\
&= \lim_{n\to\infty} \left(1 - \frac{1}{n}\right)^{n-1} \\
&= \frac{1}{e}
\end{aligned}$$

- On average it takes $\dfrac{1}{P_{\text{success}}^{\max}} = e = 2.718$ time slots to resolve contention.

  Average contention period $= 2t_{\text{prop}}e$ seconds.

CSMA/CD Throughput

At maximum throughput, systems alternates between contention periods and frame transmission times.

$$\rho_{\max} = \frac{X}{X + t_{\text{prop}} + 2et_{\text{prop}}} = \frac{1}{1 + (2e+1)a} = \frac{1}{1 + (2e+1)Rd/vL} \tag{2.4}$$

Where:
$R \longrightarrow$ Transmission rate (bits/sec).
$L \longrightarrow$ Frame length (bits/frame).
$X \longrightarrow$ Frame transmission time $X = L/R$ seconds/frame.
$a \longrightarrow$ Normalized propagation delay ($a = t_{\text{prop}}/X$).
$v \longrightarrow$ Signal speed (meters/sec).
$d \longrightarrow$ Diameter of the system (meters).
$2e + 1 \longrightarrow = 6.44$

5.3 Scheduling Approaches

Efficiency of Reservation Systems

Assume mini-slot duration $= vX$ ($v < 1$; negligible delay).

- For a single frame reservation for M stations with X frame transmission, a single frame transmission requires $(1+v)X$ seconds.

  Maximum efficiency: $\rho_{\max} = \dfrac{MX}{MvX + MX} = \dfrac{1}{1+v}$

- A k frame reservaiton scheme: k frame transmissions can be reserved with a single reservation message. If there are M stations, Mk frames can be transmitted in $XM(k+v)$ seconds.

  Maximum efficiency:

$$\rho_{\max} = \frac{MkX}{MvX + MkX} = \frac{1}{1 + \frac{v}{k}}$$

Random Access: Slotted ALOHA Reservation Scheme

Effective time required for the reservation is $evX = 2.71vX$

$$\rho_{\max} \frac{X}{X(1+ev)} = \frac{1}{1+2.71v}$$

Typical MAC Efficiencies

- Two-Station Example:

$$\text{Efficiency} = \frac{1}{1+2a}$$

- CSMA-CD protocol:

$$\text{Efficiency} \overset{(2.4)}{=} \frac{1}{1+6.44a}$$

- Token-ring network:

$$\text{Efficiency} = \frac{1}{1+a'}$$

  Where:
  $a' \longrightarrow$ latency of the ring (bits)/average frame length.

If $a << 1$ then efficiency is close to 100%
As a approaches 1, the efficiency becomes low.

Chapter 3 Packet Switching Networks and Algorithms

1 Message Switching vs. Packet Switching Minimum Delay

Notation:
$\tau \longrightarrow$ Propagation delay.
$T \longrightarrow$ Message transmission time(delay).
$L \longrightarrow$ Number of Hops.
$P \longrightarrow$ Packet transmission time.
$k \longrightarrow$ Number of Packets.

- Message switching:

$$\begin{aligned}\text{Delay} &= L\tau + LT \\ &= L\tau + (L-1)T + T\end{aligned}$$

- Packet switching with store-and-forward:

$$T = kP$$

$$\begin{aligned}\text{Delay} &= L\tau + LP + (k-1)P \\ &= L\tau + (L-1)P + T\end{aligned}$$

- Cut-Through packet switching (immediate forwarding after header):

$$\text{Delay} = L\tau + T$$