وَمَا أُوتِيتُمْ مِنَ الْعِلْمِ إِلَّا قَلِيلًا

Analog IC Design

Lecture 05
MOSFET Small Signal Model

Dr. Hesham A. Omran

Integrated Circuits Lab (ICL)
Electronics and Communications Eng. Dept.
Faculty of Engineering
Ain Shams University

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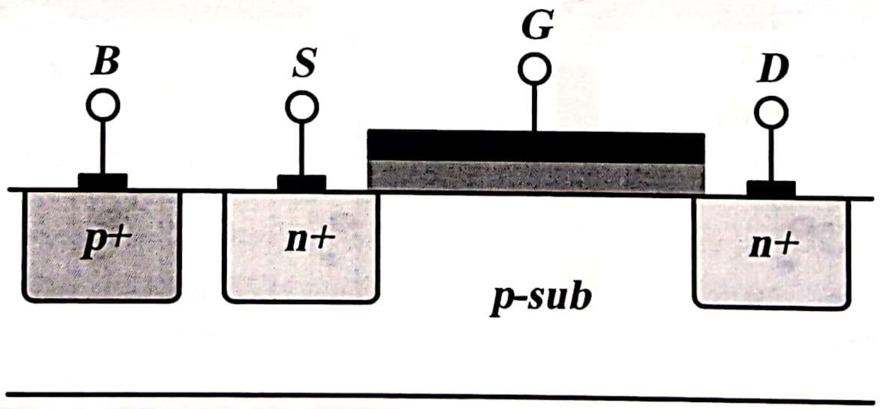
Outline

- Recapping previous key results
- The small signal approximation
- The transconductance (g_m)
- Body effect and body transconductance (g_{mb})
- Channel length modulation and output resistance ($r_o = \frac{1}{g_{ds}}$)
- Small signal model
- Short channel effects

N-Channel MOSFET Structure

- MOSFET: Metal-oxide-semiconductor field-effect transistor
- Three-terminal device: Gate (G), Source (S), and Drain (D)
- Substrate/Bulk/Body (S/B) can be treated as a fourth terminal



Regions of Operation Summary

OFF (Subthreshold)

$V_{GS} < V_{TH}$

ON

$V_{GS} > V_{TH}$

Triode

$V_{DS} < V_{ov}$

Or

$V_{GD} > V_{TH}$

$$I_D = \mu C_{ox} \frac{W}{L}\left(V_{ov}V_{DS} - \frac{V_{DS}^2}{2}\right)$$

Pinch-Off (Saturation)

$V_{DS} \geq V_{ov}$

Or

$V_{GD} \leq V_{TH}$

$$I_D = \frac{\mu C_{ox}}{2}\frac{W}{L}V_{ov}^2(1 + \lambda V_{DS})$$

★ We want to bias the device to work @ Saturation region.

MOSFET in Saturation

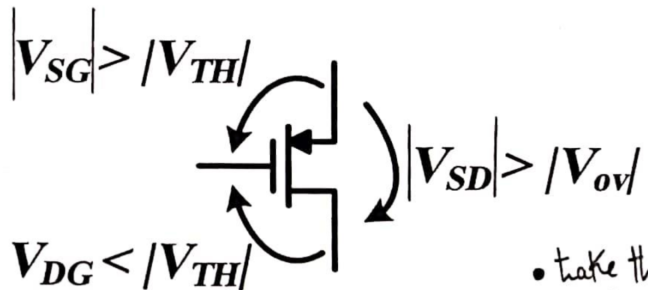
- The channel is pinched off if the difference between the gate and drain voltages is not sufficient to create an inversion layer

$$V_{GD} \le V_{TH} \quad OR \quad V_{DS} \ge V_{ov}$$

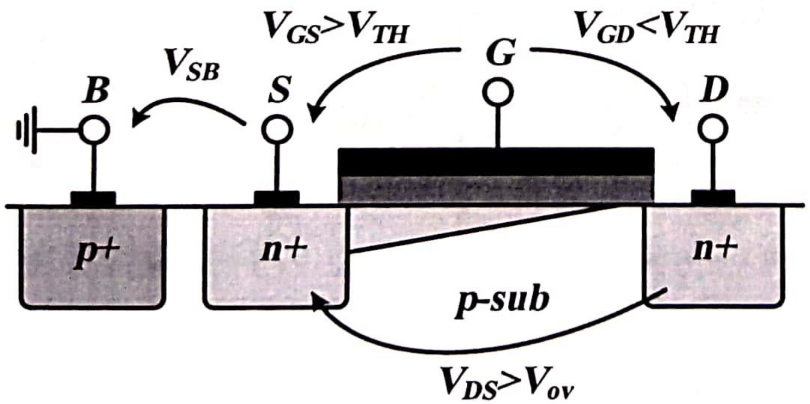
- Square-law (long channel MOS)

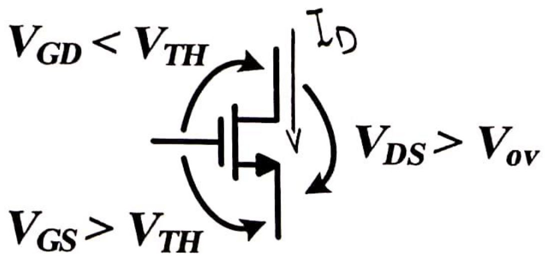
$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L} \cdot V_{ov}^2 (1 + \lambda V_{DS})$$

$$V_{SB} \uparrow \; \Rightarrow \; V_{TH} \uparrow \text{ (Body Effect)}$$



• take the Voltages with the arrow (Source).





• We want channel @ the Source
So, $V_{GS} > V_{TH}$
• we don't want channel @ the drain
So, $V_{GD} < V_{TH}$

Large Signal Model

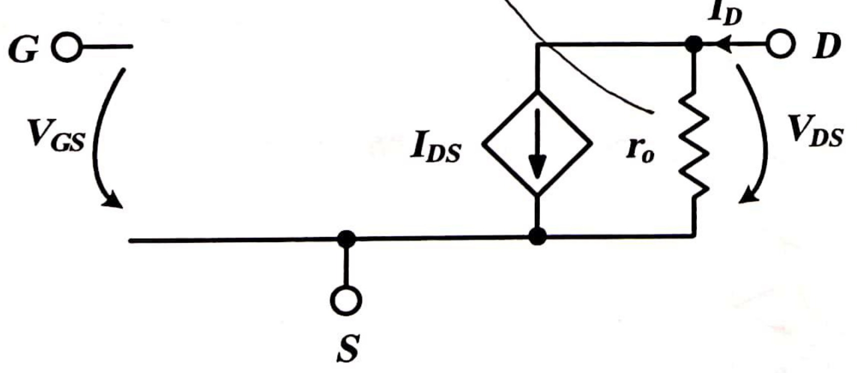
- The channel is pinched off if the difference between the gate and drain voltages is not sufficient to create an inversion layer

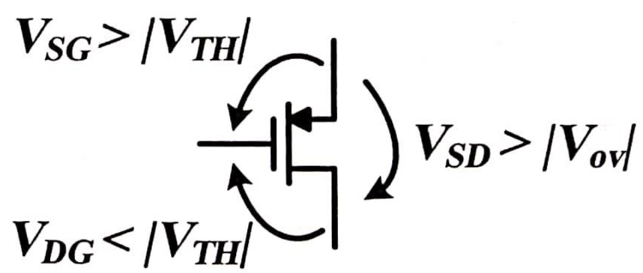
$$V_{GD} \le V_{TH} \quad OR \quad V_{DS} \ge V_{ov}$$

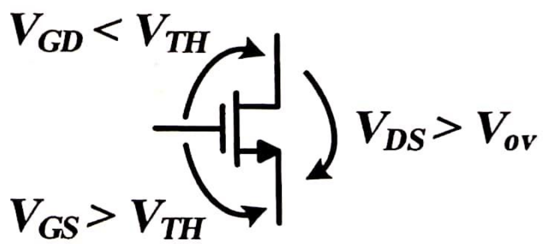
- Square-law (long channel MOS)

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} \cdot V_{ov}^2 (1 + \underline{\lambda} V_{DS})$$

$$V_{SB} \uparrow \;\Rightarrow\; V_{TH} \uparrow$$







"non linear VCCS"

Small Signal Approximation

- The transistor is a VCCS
- Transconductance: how well it converts the voltage to a current

→ Current/Voltage

i.e. it transfer the change from the input (control : volt) to the output (current)

$$g_m = \frac{\Delta I_D}{\Delta V_{GS}} = \frac{\partial I_D}{\partial V_{GS}}$$

Q: Quiescent operating Point Bias Point

1- in the long channel model ($I_D - V_{GS}$) is quadratic characteristics.

2- In Designing amplifiers we have two main steps:

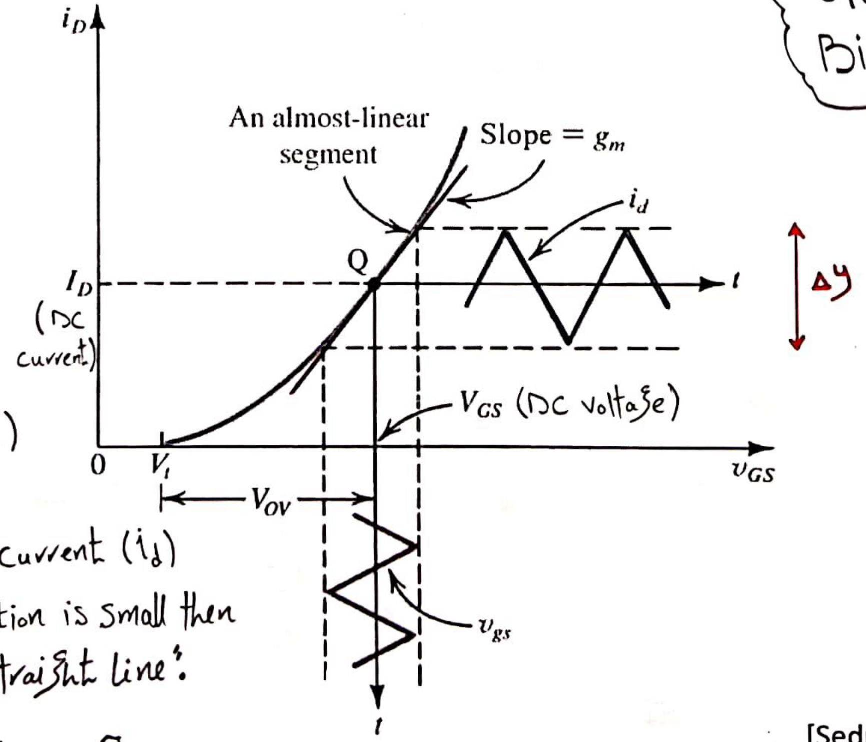
1- Bias Point (Biasing the amplifier) @ Q (DC Current "I_D", DC Voltage "V_{GS}")

2- We superimposed a small signal (v_{gs}) on the DC Component (V_{GS}).

- The small signal or the Perturbation (ΔX) will results a Perturbation (Δy)

∴ Perturbation in volt (v_{gs}) → Perturbation in current (i_d)

★ Small Signal approximation: "If the Perturbation is small then you can approximate the nonlinear curve into straight line".



★ g_m: is the most important Parameter in the transistor as it tells us: How can the transistor converts the input voltage on it into output current.

$$\therefore \frac{\Delta y}{\Delta X} = \text{Slope} = \text{derivative} = g_m$$

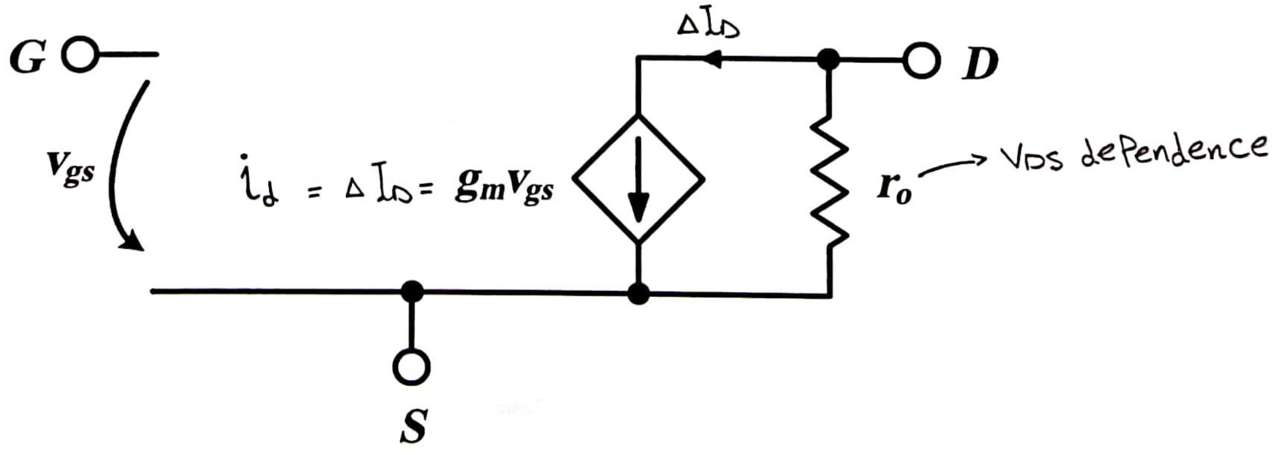
- The region of the slope on the curve is so small but this graph is an exaggeration.

[Sedra/Smith, 2015]

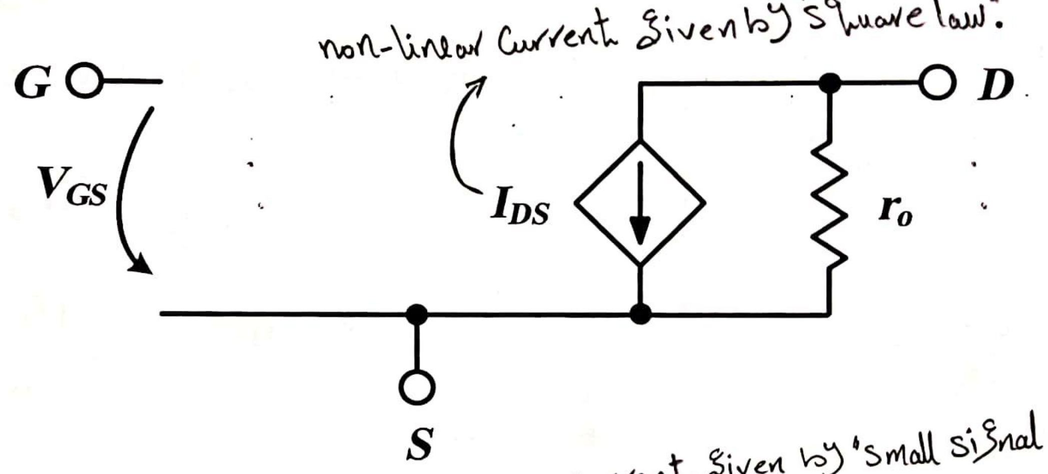
Small Signal Model

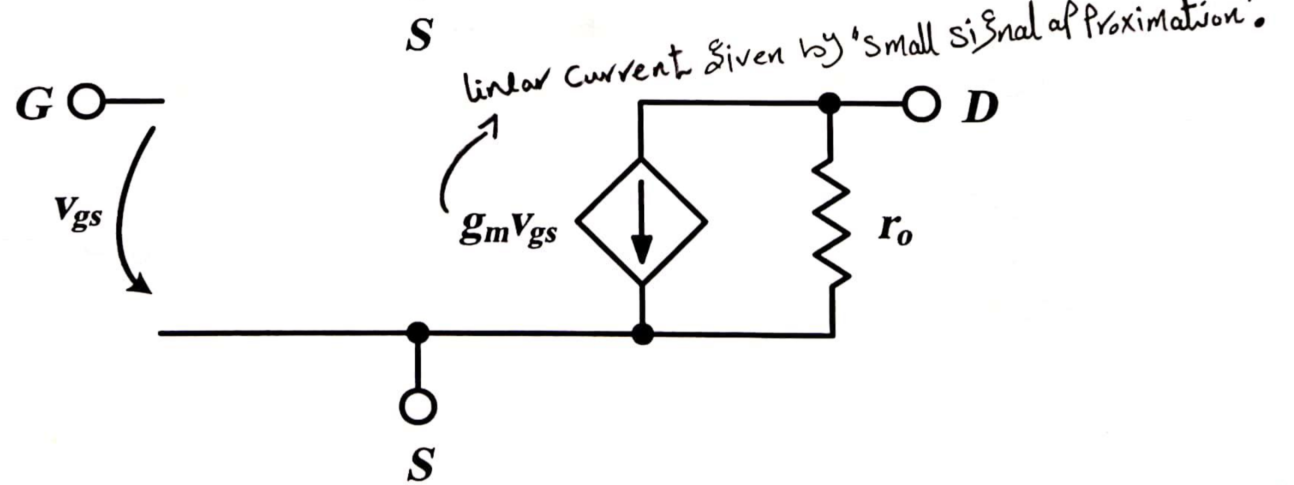
$$g_m = \frac{\Delta I_D}{\Delta V_{GS}} = \frac{\partial I_D}{\partial V_{GS}}$$

$$r_o = \frac{\Delta V_{DS}}{\Delta I_D} = \frac{1}{\frac{\partial I_D}{\partial V_{DS}}}$$



Large Signal vs Small Signal Model





★ Small Signal Approximation enables us to do the analysis for the amplifiers in an easy way. as we linearize the circuit so, it seams like we are solving linear circuit with only linear elements.

Transconductance

- The transistor is a VCCS
- Transconductance: how well it converts the voltage to a current

$$I_D \approx \frac{\mu_n C_{ox}}{2}\frac{W}{L}\cdot V_{ov}^2 \longrightarrow (1)$$

$$g_m = \frac{\Delta I_D}{\Delta V_{GS}} = \frac{\partial I_D}{\partial V_{GS}} = \frac{\partial I_D}{\partial V_{ov}}$$

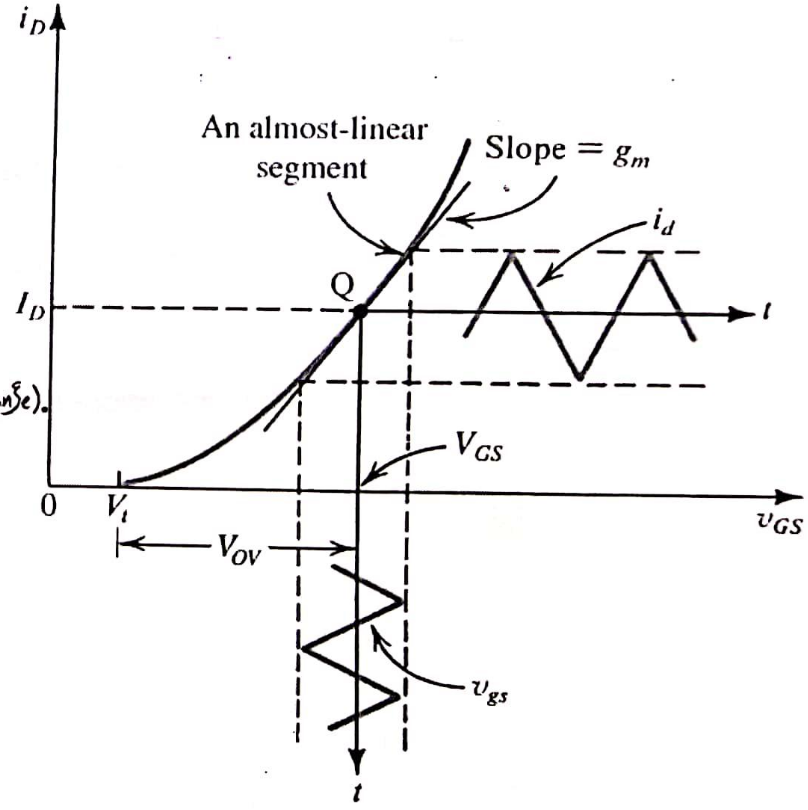
• to Derivate w.r.t V_{GS} equals to derivate w.r.t V_{ov} as the difference between them is V_{th} and we suppose that V_{th} is const. (won't change).

$$= \boxed{\mu C_{ox}\frac{W}{L}V_{ov}}$$

Sub. by V_{ov} from (1)

$$= \sqrt{\mu C_{ox}\frac{W}{L}\cdot 2I_D}$$

$$= \frac{2I_D}{V_{ov}}$$



[Sedra/Smith, 2015]

Transconductance

$$I_D \approx \frac{\mu_n C_{ox}}{2} \frac{W}{L} \cdot V_{ov}^2$$

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu C_{ox} \frac{W}{L} V_{ov} = \sqrt{\mu C_{ox} \frac{W}{L} \cdot 2 I_D} = \frac{2 I_D}{V_{ov}}$$

Limited with current budget

measuring device in the lab	Design with specific V_{ov}	Design with specific I_D
W/L constant	**V_{ov} constant**	**I_D constant**
$g_m \propto V_{ov}$	$g_m \propto W/L$	$g_m \propto \sqrt{W/L}$
$g_m \propto \sqrt{I_D}$	$g_m \propto I_D$	$g_m \propto 1/V_{ov}$

Measuring Designing

Body Effect

- V_{SB} affects the charge required to invert the channel
  - Increasing V_S or decreasing V_B increases V_{TH}

$$V_{TH} = V_{TH0} + \gamma\left(\sqrt{2\Phi_F + V_{SB}} - \sqrt{|2\Phi_F|}\right)$$

  - Φ_F = surface potential at threshold
    - Depends on doping level and intrinsic carrier concentration n_i

• $V_{SB} \uparrow$

$\therefore V_{Th} \uparrow$

$I_D \downarrow$

$\therefore V_{SB}$ as well as the gate changes the current

• $\Delta V_{GS} \xrightarrow{g_m} \Delta I_D$

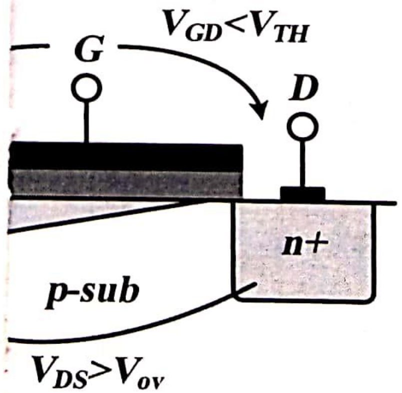
$\Delta V_{BS} \xrightarrow{g_{mb}} \Delta I_D$

★ Note: Prof. Ali Hajimiri (lecture 116N)

• MOSFT has a fourth terminal which is influencing the drain current 'I_D', But how?

• like the top gate changes current, the body can be as the bottom gate which changes the current.

★ $I_D = \frac{\mu_n C_{ox}}{2} \cdot \frac{W}{L}(V_{GS} - V_T)^2 (1 + \lambda V_{DS})$



Here, • $V_{TH} = V_{TH_0} + \gamma\left(\sqrt{2\phi_F + V_{SB}} - \sqrt{|2\phi_F|}\right)$

• $g_{mb} = \frac{\partial I_D}{\partial V_{BS}} = -\mu_n C_{ox} \frac{W}{L}(V_{GS} - V_T) \frac{\partial V_T}{\partial V_{BS}} = \chi g_m$

$-\frac{\partial V_T}{\partial V_{BS}} = \frac{\gamma}{2\sqrt{2\phi_F + V_{SB}}} = \chi \text{ (chi)} = \frac{C_{js}}{C_{ox}} = \frac{g_{mb}}{g_m}$

χ: 0.1 - 0.3

• C_{js} : is the capacitance per unit area of the depletion region under the channel.

• χ : is the rate of change of threshold voltage with body bias voltage. (Grey & Meyer)

★ It's more of a 'nuisance' most of the time, there are some topologies where (g_{mb}) is useful but in more scenarios it is nuisance.

Body Effect

- V_{SB} affects the charge required to invert the channel
  - Increasing V_S or decreasing V_B increases V_{TH}

$$V_{TH} = V_{TH0} + \gamma\left(\sqrt{2\Phi_F + V_{SB}} - \sqrt{|2\Phi_F|}\right)$$

  - Φ_F = surface potential at threshold
    - Depends on doping level and intrinsic carrier concentration n_i
  - γ = body effect coefficient
    - Depends on C_{ox} and doping

• $V_{SB} \uparrow$

$\therefore V_{Th} \uparrow$

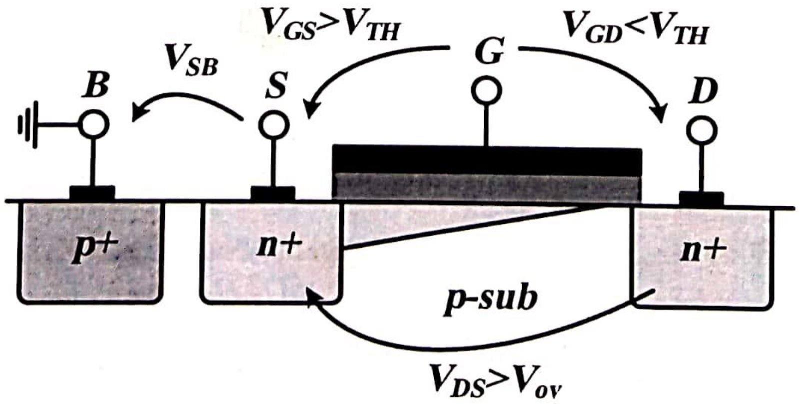
$I_D \downarrow$

$\therefore V_{SB}$ as well as the gate changes the current

• like the top gate changes the current, the body can be treated as the bottom gate which also changes the current.

• $\Delta V_{GS} \xrightarrow{g_m} \Delta I_D$

$\Delta V_{BS} \xrightarrow{g_{mb}} \Delta I_D$



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★ $I_D = \frac{\mu_n C_{ox}}{2} \cdot \frac{W}{L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS})$

Here, • $V_{TH} = V_{TH_0} + \gamma \left(\sqrt{2\phi_F + V_{SB}} - \sqrt{|2\phi_F|} \right)$

• $g_{mb} = \frac{\partial I_D}{\partial V_{BS}} = \underbrace{-\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T)}_{g_m} \frac{\partial V_T}{\partial V_{BS}} = \chi g_m$

$-\frac{\partial V_T}{\partial V_{BS}} = \frac{\gamma}{2\sqrt{2\phi_F + V_{SB}}} = \underset{0.1 - 0.3}{\chi} \text{(chi)} = \frac{C_{js}}{C_{ox}} = \frac{g_{mb}}{g_m}$

• C_{js}: is the capacitance per unit area of the depletion region under the channel.

• χ: is the rate of change of threshold voltage with body bias voltage. (Grey & Meyer)

★ It's more of a "nuisance" most of the time, there are some topologies where (g_{mb}) is useful but in more scenarios it is nuisance.

Bulk Transconductance

- The bulk behaves as a second gate that changes the output current

$$g_{mb} = \frac{\partial I_D}{\partial V_{BS}} = \eta g_m$$

η is typically $0.1 \rightarrow 0.25$

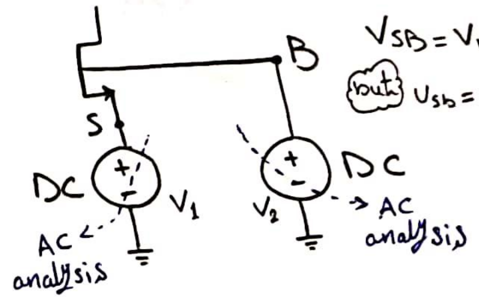
η : eta

1
- $g_m = \frac{\Delta I_D}{\Delta V_{GS}}$
- $g_{mb} < g_m$
  10% - 25%

2
- We only take into consideration the Body effect if : V_{BS} has a value.
  If Body & Source are "AC Short circuit" we don't care of g_{mb}

$V_{SB} = V_1 - V_2$ (DC)

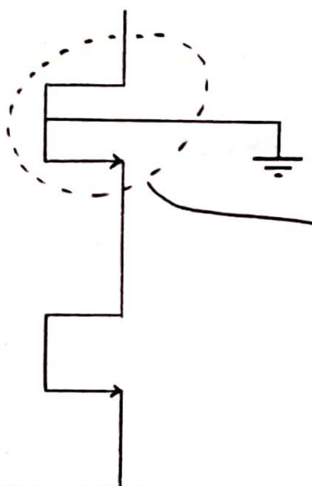
but $V_{sb} = 0$ (AC)



3
★ $\Delta I_D = g_{mb} \cdot \Delta V_{BS}$

- If g_{mb} has a value (like in cadence) doesn't mean ΔI_D have a value unless $\Delta V_{BS} \neq 0$.

★ Doesn't mean there is body effect.



4
We should take into consideration the Body Transconductance while dealing with this MOSFET.
↳ as V_s & V_b have different values.

Channel Length Modulation (CLM)

- The VCCS is not ideal: There is some dependence on V_{DS}

→ Ohm's Law

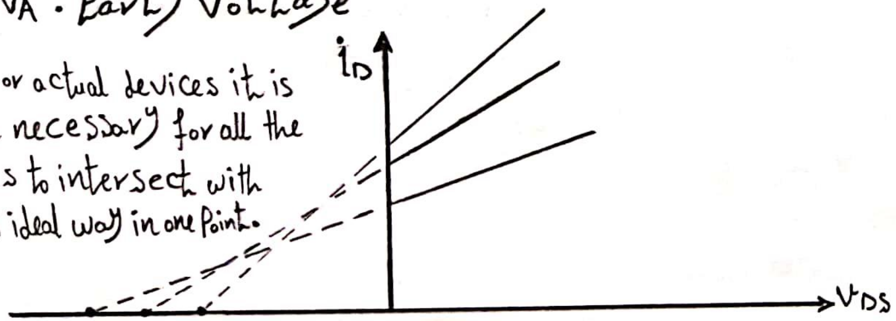
$$r_o = \frac{\Delta V_{DS}}{\Delta I_D} = \frac{1}{\partial I_D/\partial V_{DS}} = \frac{1}{g_{ds}} = \frac{V_A}{I_{DS}} = \frac{1}{\lambda I_{DS}}$$

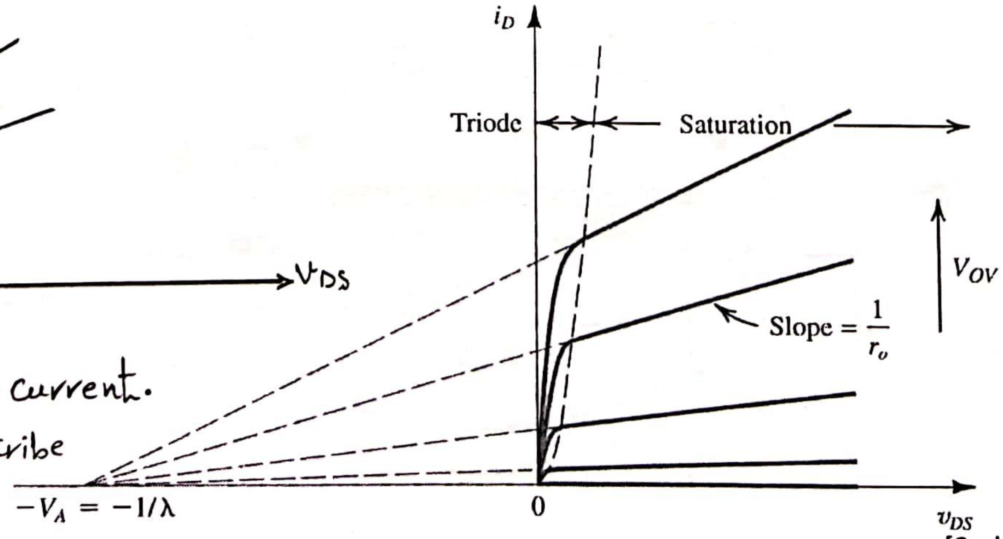
V_A: Early voltage ($V_A \propto L$) $\leftrightarrow$ λ: Channel length modulation coefficient ($\lambda \propto 1/L$)

$$I_D = I_{DS} + \frac{V_{DS}}{r_o} = I_{DS}\left(1 + \frac{V_{DS}/I_{DS}}{r_o}\right) = \frac{\mu C_{ox}}{2}\frac{W}{L}V_{ov}^2(1+\lambda V_{DS})$$

1- ★ V_A : Early voltage

- for actual devices it is not necessary for all the lines to intersect with this ideal way in one point.





2- • Early voltage can be a function of current.

3- • V_A is a complex parameter used to describe how the current depends on the drain voltage

[Sedra/Smith, 2015]

3- Due to many physical effect like: channel length modulation & DIBL

4- • we can also describe this effect with λ which equals $1/V_A$

★ Remember that: $V_A \propto L$ so, By increasing L $\therefore$ $V_A \uparrow$ & $r_o \uparrow$

5/ You don't have to understand the physical reasons which explain the I-V char. in all cases as it will be hard in some cases and confusing and will consumes time.
★ Briefly the contact between the device engineer & the circuit designer is the 'device model'.
• If you have accurate device model and you have I-V char. then you can proceed in the design process without understanding the physical reason.

6/ ∴ for you as a circuit designer the slope of (I_D-V_{DS}) $\partial I_D/\partial V_{DS} = 1/r_o$, also the slope of ($I_D$-$V_{GS}$) is g_m, it doesn't matter whether pinch-off, velocity sat., mobility degradation.

4/ Actually: as an Analog IC designer you are concerned with the I-V characteristics of the device
• If you have the I-V ch. then you can use it to design a specific circuit with specific specs.

Channel Length Modulation (CLM)

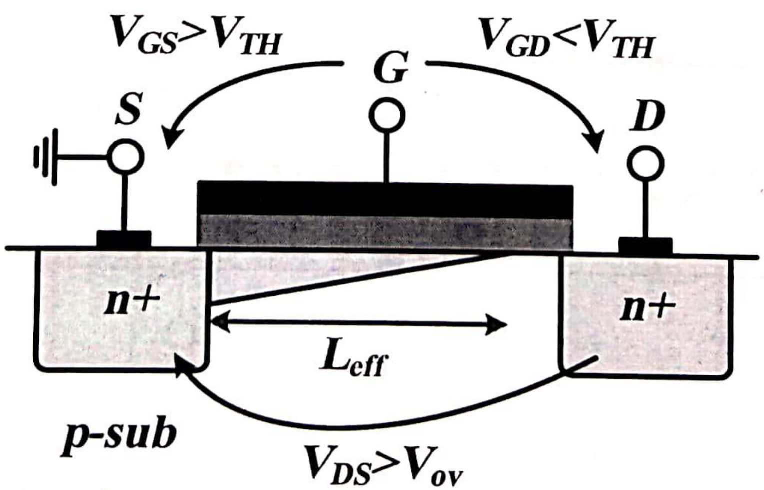
- L_{eff} decreases with V_{DS} → Shorter L gives more current
- V_A: Early voltage ($V_A \propto L$)
- λ: Channel length modulation coefficient ($\lambda \propto 1/L$)

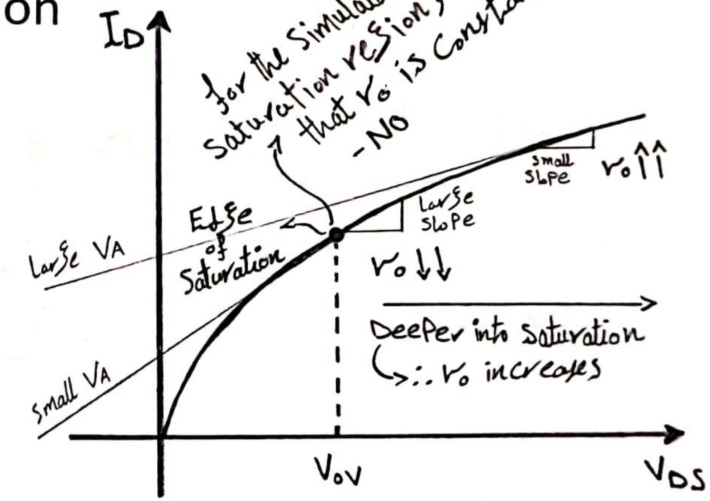
1/ ★ One of the important notes that you will see while designing the circuit practically
• is that the Early Voltage (V_A) is a strong function of V_{DS}.

$$I_D = \frac{\mu C_{ox}}{2}\frac{W}{L}V_{ov}^2(1+\lambda V_{DS}) \qquad r_o = \frac{V_A}{I_{DS}} = \frac{1}{\lambda I_{DS}}$$

- V_A increases with V_{DS}: higher r_o as we go deeper into saturation

2/ as we model 'r_o' by the parameter which is called the early voltage 'V_A'
∴ V_A is $f(V_{DS})$
V_{DS} ↑ ∴ V_A ↑ ∴ r_o ↑





3/ Physicists explain this with:
- when pinch-off occurs the tendency of movement of the pinch-off point (when we increase the drain voltage) is more than after while
★ after the pinch-off has moved the rate of the movement decreases (i.e. the amount of its movement with the increase of V_{DS}) as most of V_{DS} lies on the high resistance depletion region, That means you weaken the CLM. (weaken CLM means the change in L_{eff} is less, means the change in I_D is less, means high r_o)

• increasing r_o is a good thing for analog design but increasing V_{DS} on this device costs me in signal swing as we will see

We will, when we study frequency response.

We didn't put into consideration the effect of the capacitors yet.

Low-Frequency Small-Signal Model

1. • g_m ($V_{gs} \rightarrow V_{bs}$)

the two currents will be the same direction.

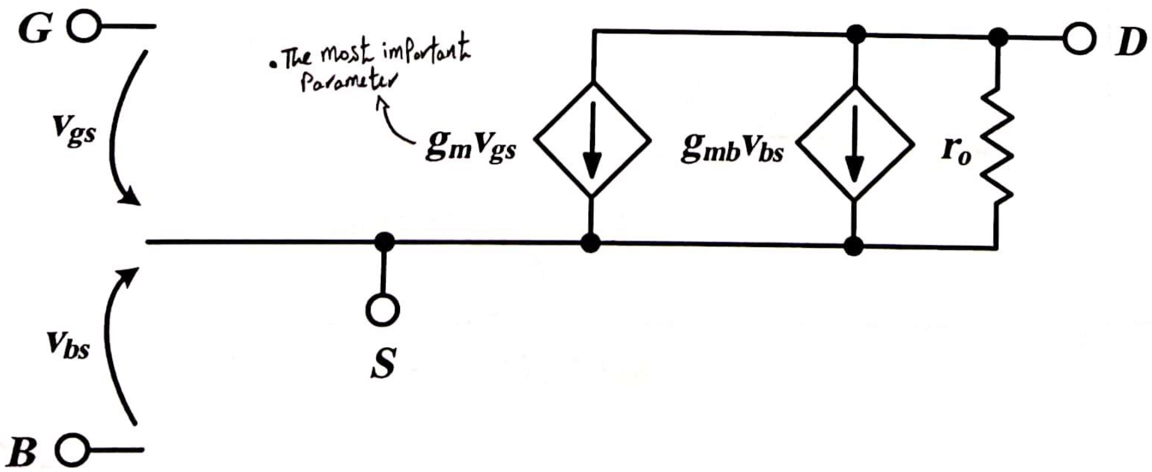
2. • But you say before that V_{SB} is +ve so, V_{BS} should be -ve?
- yes, but this is the large signal
• if $V_{SB} = 0.5$ V and a perturbation occurs with ± 10 mV is the sign of the AC signal important? - No
↳ it is a perturbation on the large signal.

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu C_{ox}\frac{W}{L}V_{ov} = \sqrt{\mu C_{ox}\frac{W}{L}\cdot 2I_D} = \frac{2I_D}{V_{ov}}$$

$$g_{mb} = \eta g_m \qquad \eta \approx 0.1 - 0.25$$

3. If you bias @ the edge of saturation ∴ you'll have small V_A ∴ small r_o
∴ it's not good to bias the device @ the edge of saturation.

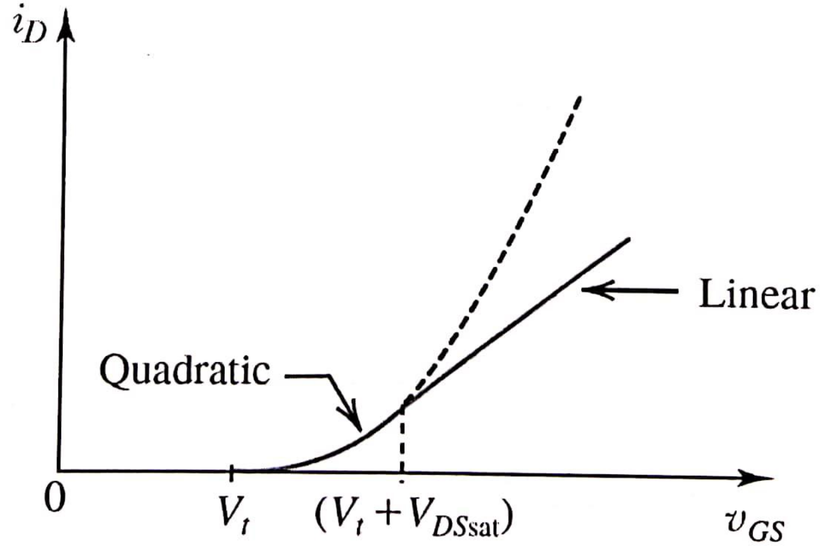
$$r_o = \frac{1}{\partial I_D/\partial V_{DS}} = \frac{V_A}{I_D} = \frac{1}{\lambda I_D} \qquad V_A \propto L \leftrightarrow \lambda \propto \frac{1}{L} \qquad V_{DS}\uparrow V_A\uparrow$$

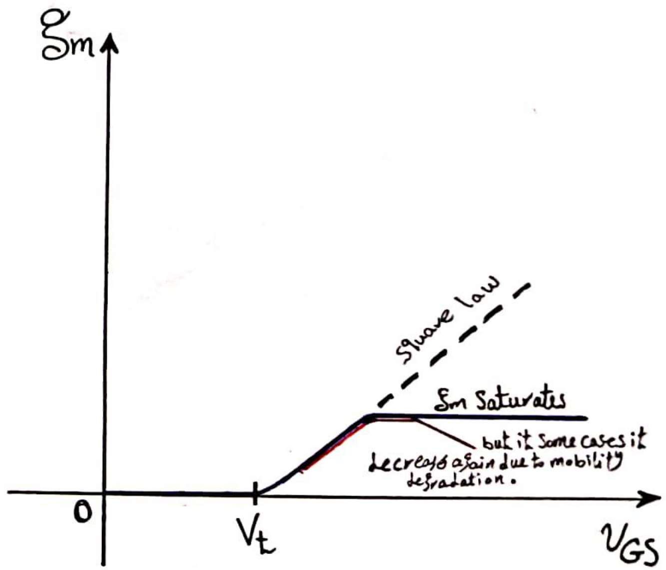


★ $V_{GS}\uparrow$ - $I_D\uparrow$
• take both in the same direction
$V_{SB}\uparrow$ - $I_D\downarrow$ (as $V_{th}\uparrow$)
$V_{BS}\uparrow$ - $I_D\uparrow$

Short Channel Effects: Velocity Saturation

- ❑ ID-VGS quadratic: $g_m = \frac{\partial I_D}{\partial V_{GS}} = \text{linear} \rightarrow g_m$ increases with V_{GS} (Long channel Model)
- ❑ ID-VGS linear: $g_m = \frac{\partial I_D}{\partial V_{GS}} = \text{constant} \rightarrow g_m$ saturates (Velocity Saturation)

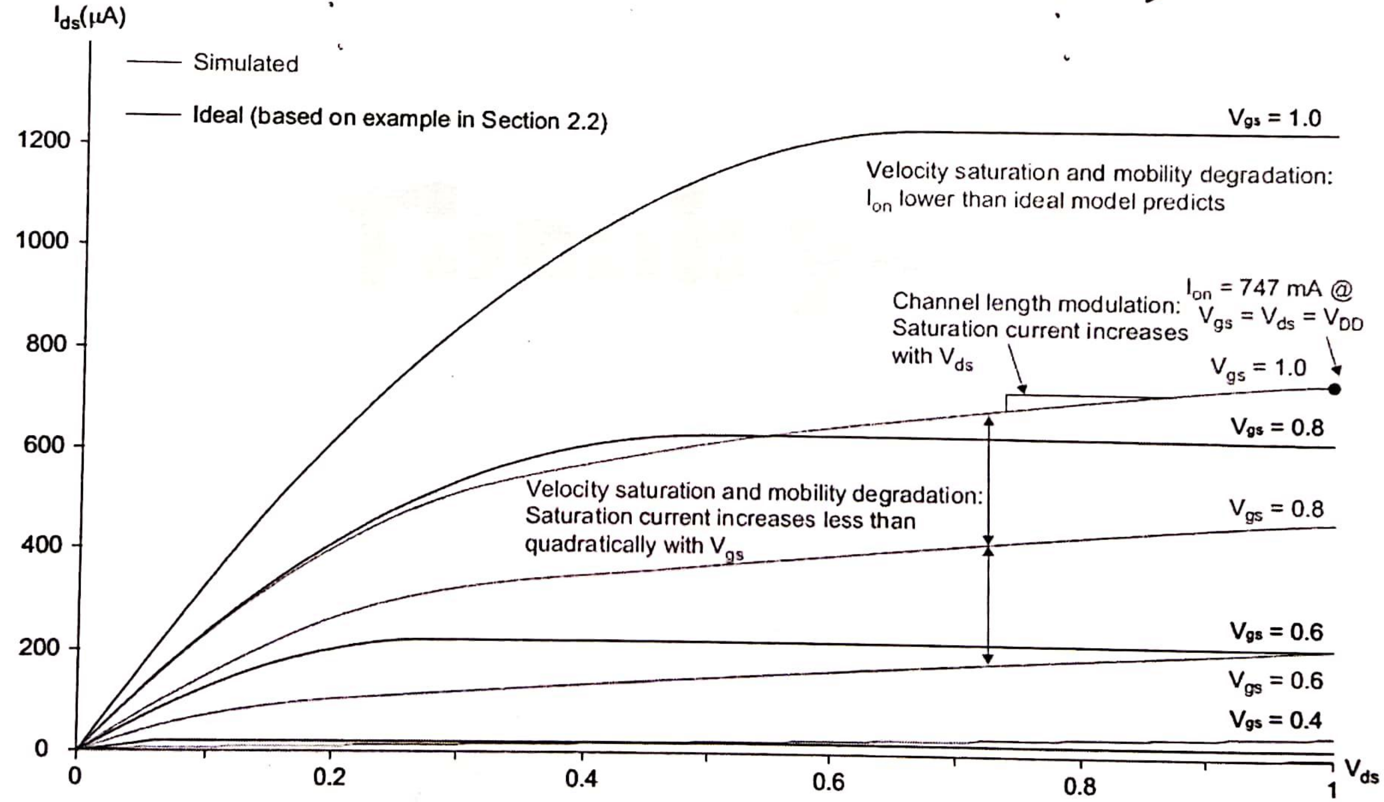




[Sedra/Smith, 2015]

Short Channel Effects: CLM and DIBL

- ID-VDS horizontal: no V_{DS} dependence → ideal current source
- More slope → more V_{DS} dependence → smaller r_o → smaller gain (short channel devices) → we will see
  - for higher output resistance (r_o) use long channel devices.



[Weste & Harris]

Thank you!

Dr. Hesham Omran's Lectures

Fady Sabry Negm's Notes

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وَمَا أُوتِيتُمْ مِنَ الْعِلْمِ إِلَّا قَلِيلًا

Allah almighty said in the Qur'an :
“and you ⌜O humanity⌝ have been given but little knowledge.”

لو أن الناس كلما استصعبوا أمرًا تركوه ما قام للناس دنيا ولا دين

The Muslim Caliph Umar ibn Abdulaziz, May Allah have mercy on him, Said:
“If every time people found something difficult, they abandoned it, then neither worldly affairs nor religion would have ever been established for people.”

- All Credits for these lectures go to **Dr. Hesham Omran**, Associate Professor at Ain Shams University and CTO at Master Micro, May Allah bless Dr. Hesham for these Lectures.

  https://www.master-micro.com/professional-courses/analog-ic-design

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