Numerical Analysis

Project 1: Numerical Methods

- **Pseudocode:**

1. Bisection Method:

```
def bisection(l, u, f, prec, iters, old_mid=None):
   mid ← (l + u) / 2
   fmid ← f(mid)
   ea ← None if old_mid is None else abs((mid - old_mid) / mid)
   if fmid == 0:
      return mid, ea
   if iters == 0:
      return None, None
   if old_mid is not None and ea <= prec:
      return mid, ea
   else:
      if f(u) * fmid > 0:
         return bisection(l, mid, f, prec, iters - 1, mid)
      else:
         return bisection(mid, u, f, prec, iters - 1, mid)
```

2. **False Position Method:**

```
def falseposition(l, u, f, prec, iters, old_mid=None):
  mid ← (l * f(u) - u * f(l)) / (f(u) - f(l))
  fmid ← f(mid)
  ea ←  None if old_mid is None else abs((mid - old_mid) / mid)
  if fmid == 0:
    return mid, ea
  if iters == 0:
    return None, None
  if old_mid is not None and ea <= prec:
    return mid, ea
  else:
    if fmid > 0:
      return falseposition(l, mid, f, prec, iters - 1, mid)
    else:
      return falseposition(mid, u, f, prec, iters - 1, mid)
```

3. Fixed Point Method:

```
def fixedPoint(xr, g, prec, iters):
   xr_old ← xr
   xr ← g(xr_old)
   ea ← abs((xr - xr_old) / xr)
   if ea > prec:
      iters ← iters - 1
      if iters == 0:
         return None, None
      return fixedPoint(xr, g, prec, iters)
   else:
      return xr, ea
```

4. Newton Raphson Method:

```
def newton_raphson(x_i_1, f, f_prime, prec, iters):
   if iters == 0:
      return None, None
   x_i ← x_i_1 - f(x_i_1) / f_prime(x_i_1)
   if abs(x_i - x_i_1) <= prec:
      return x_i, abs((x_i - x_i_1) / x_i)
   else:
      return newton_raphson(x_i, f, f_prime, prec, iters - 1)
```

5. Secant Method:

```
def secant_method(x_i_1, x_i, f, prec, iters):
    x_new ← x_i - f(x_i) * (x_i - x_i_1) / (f(x_i) - f(x_i_1))
    if iters == 0:
        return None, None
    if abs((x_new - x_i) / x_new) <= prec:
        return x_new, abs((x_new - x_i) / x_new)
    else:
        return secant_method(x_i, x_new, f, prec, iters - 1)
```

- **Data structure used:**

  No data structures used.

➢ Analysis:

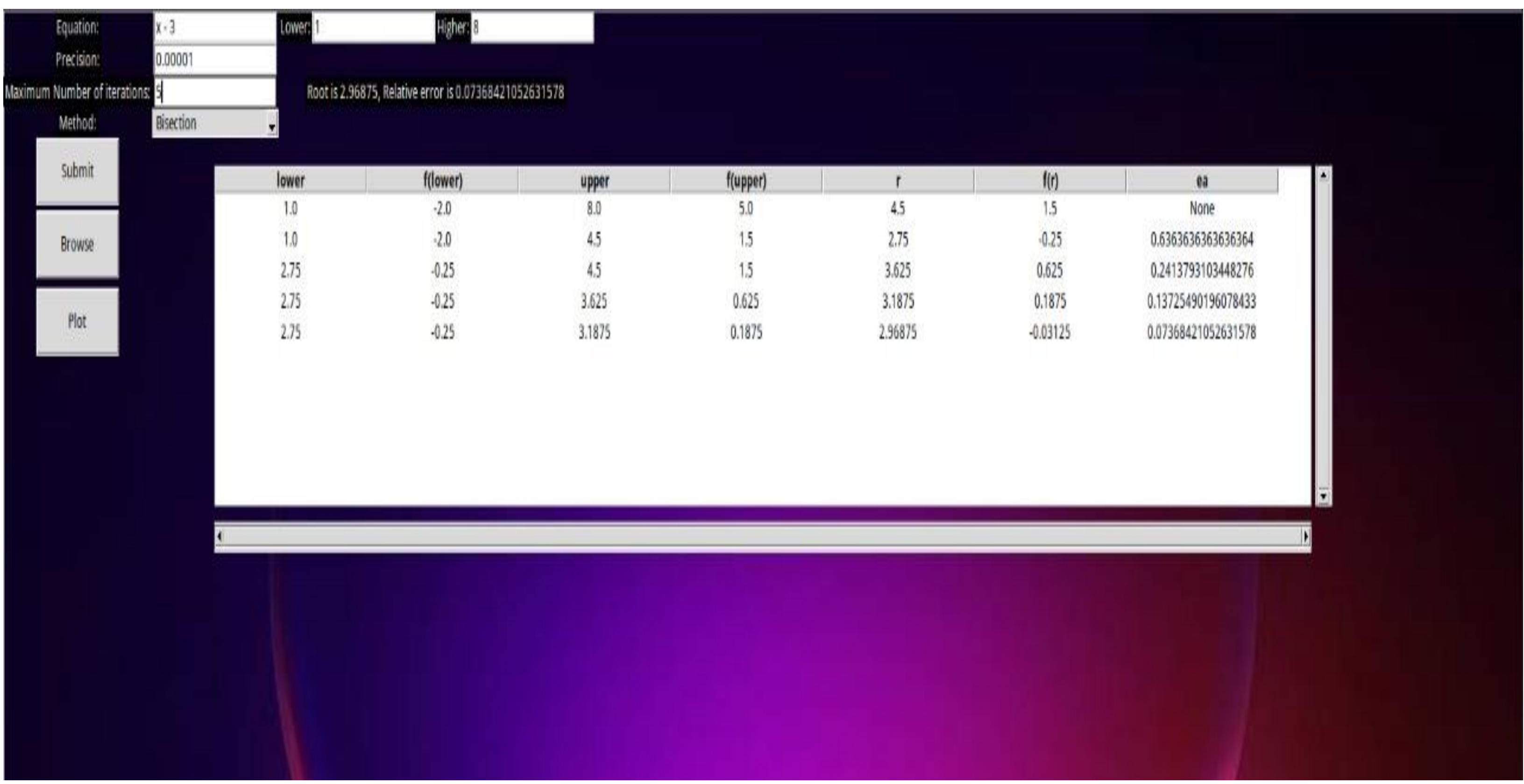
1. Bisection Method:

Explanation:

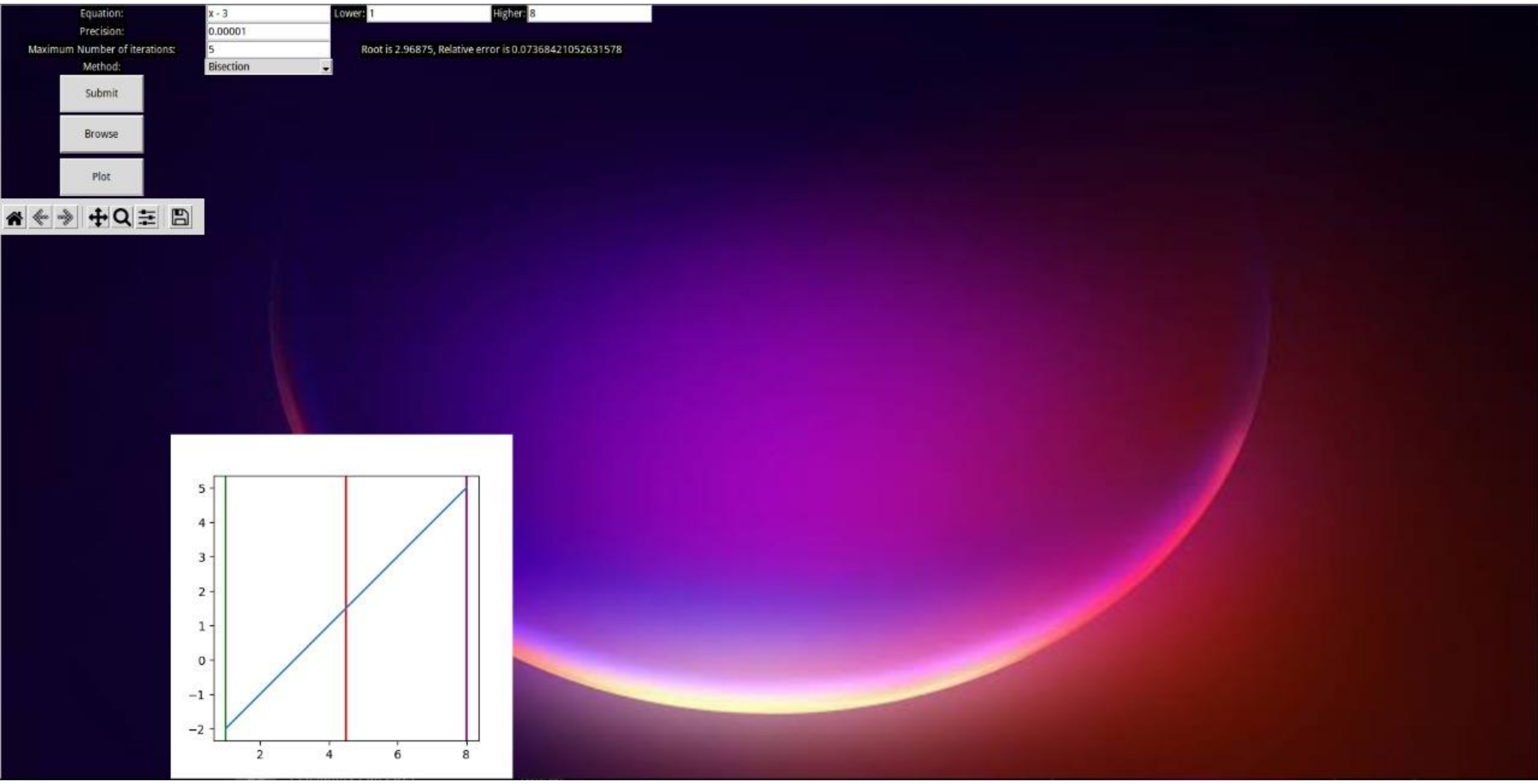
- Find two points, say “l” and “u“ such that l < u and f(l)* f(u) < 0
- Find the midpoint of l and u, say “mid”
- mid is the root of the given function if f(mid) = 0; else follow the next step
- Divide the interval [l, u]. If f(l)*f(mid) < 0, there exist a root between l and mid – else if f(mid) *f (u) < 0, there exist a root between mid and u
- Repeat above three steps until f(mid) = 0 or absolute error < tolerance.

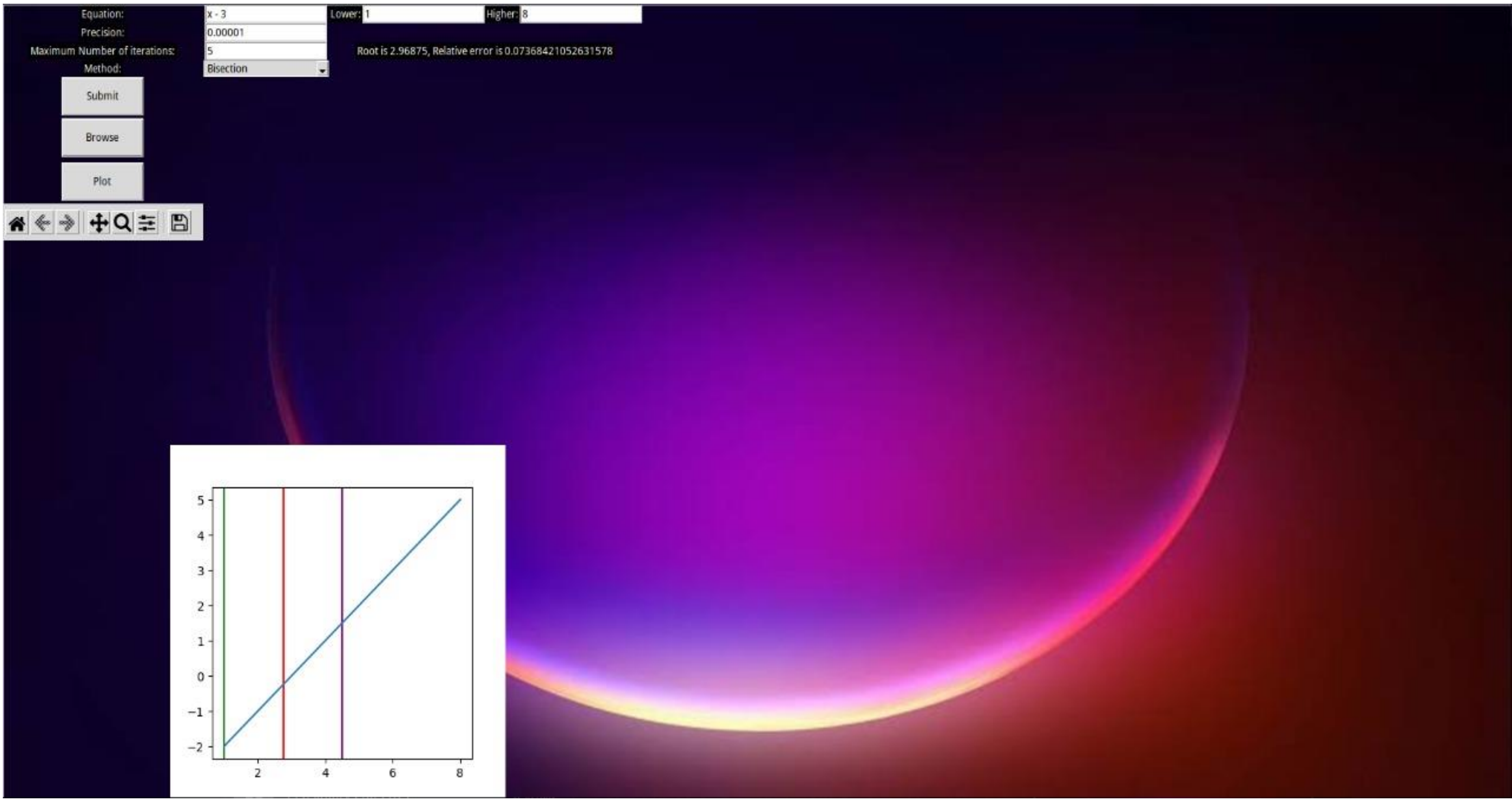
Examples:

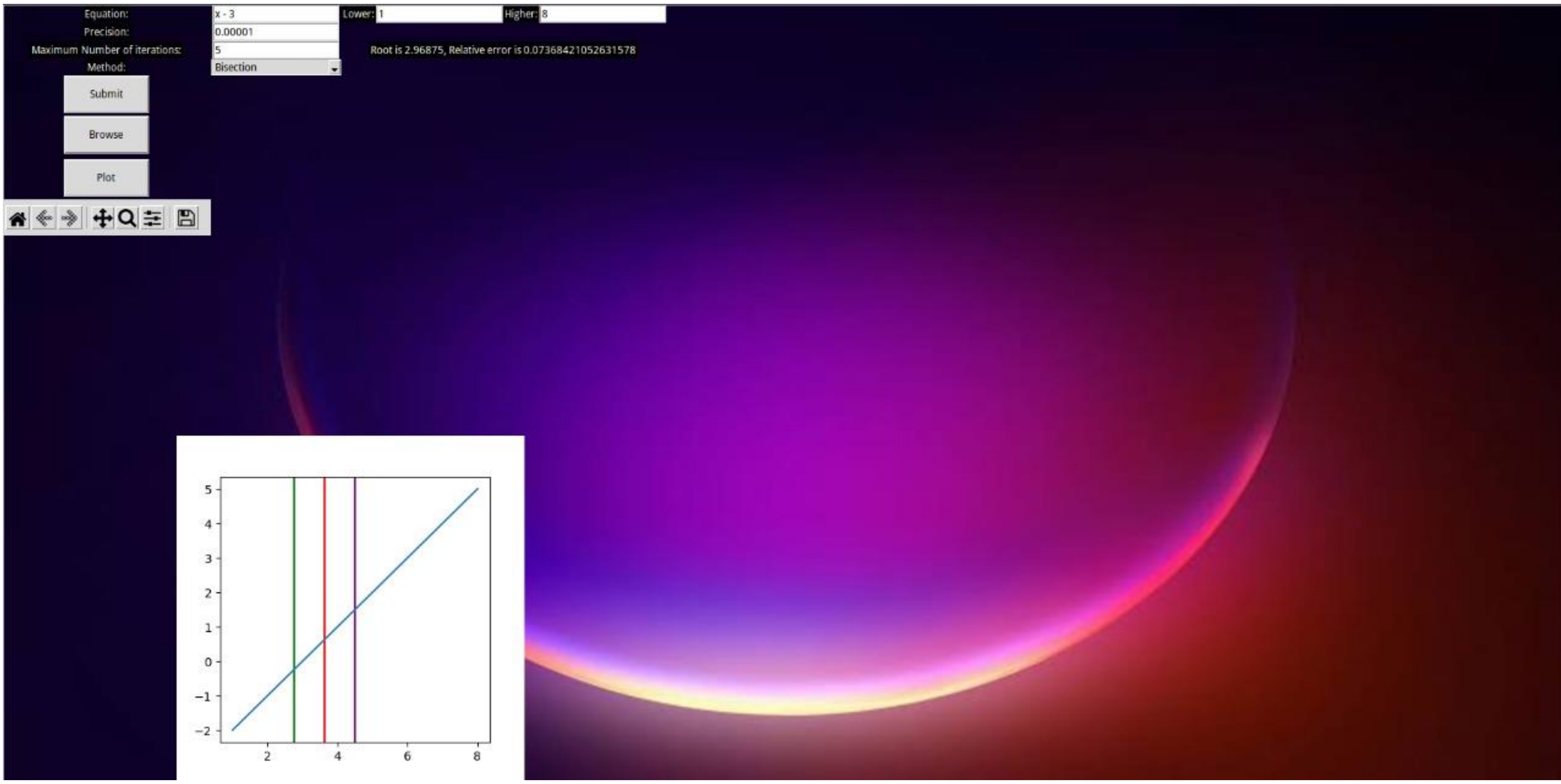
#Iterations = 5:

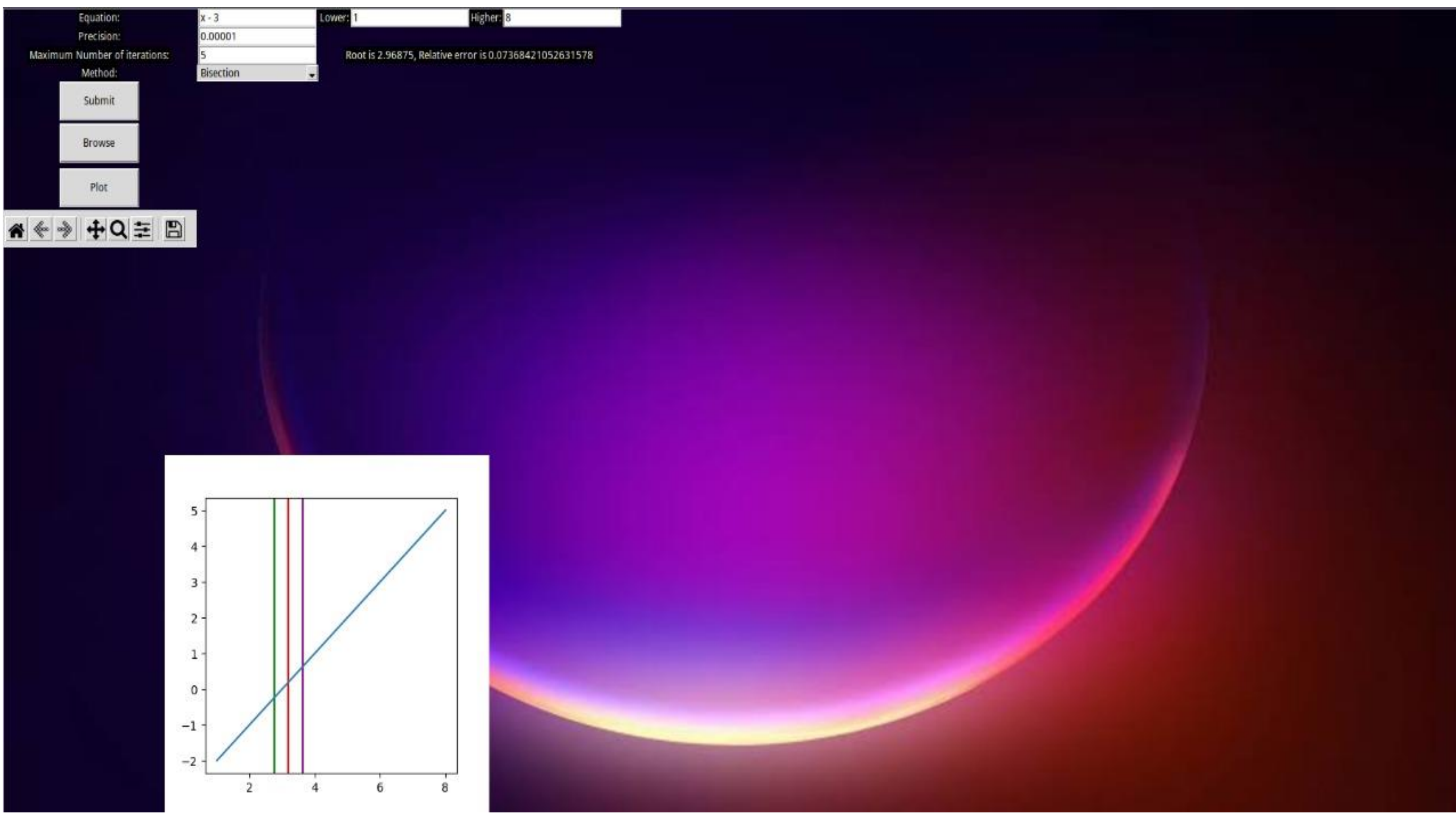


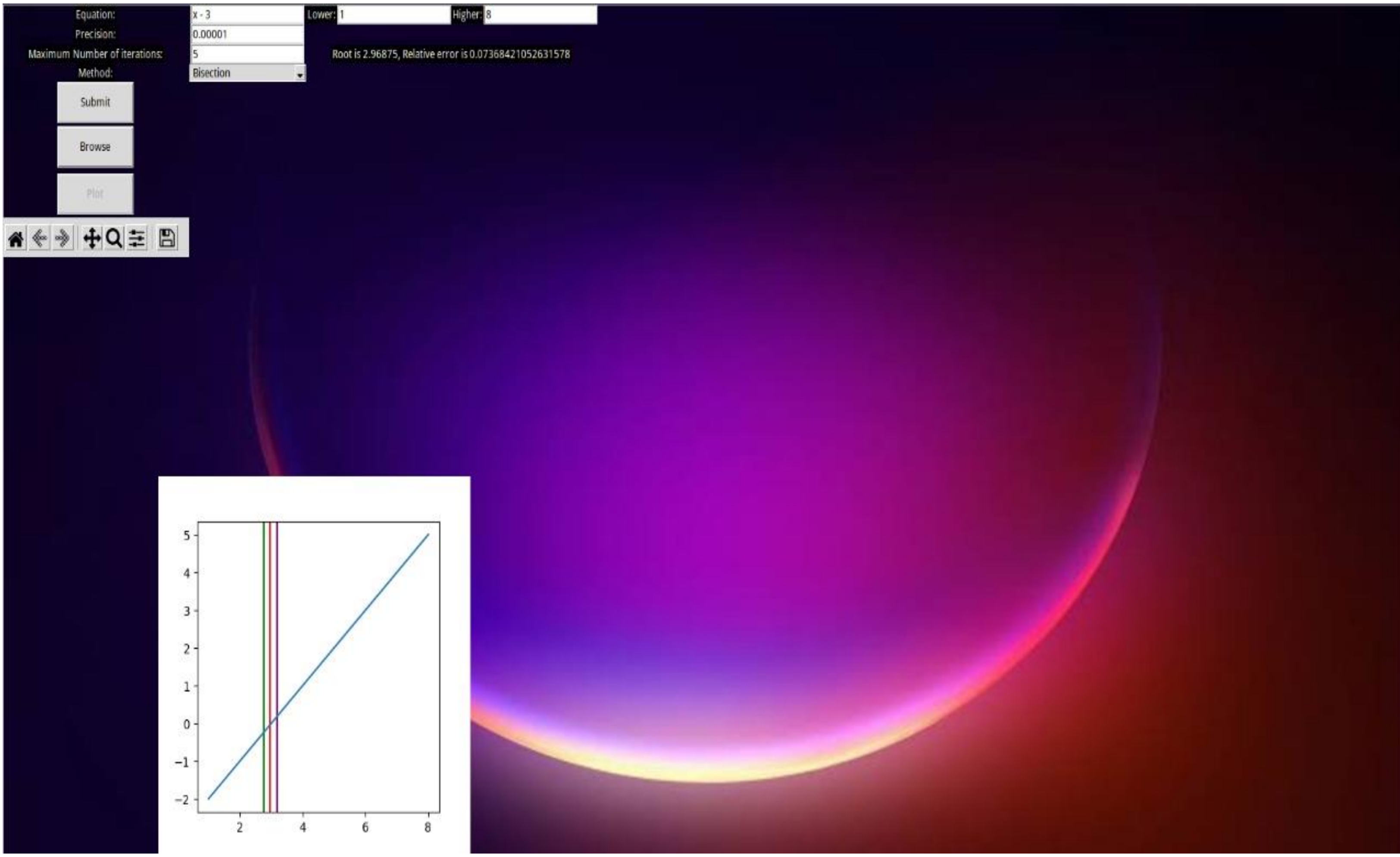
lower	f(lower)	upper	f(upper)	r	f(r)	ea
1.0	-2.0	8.0	5.0	4.5	1.5	None
1.0	-2.0	4.5	1.5	2.75	-0.25	0.6363636363636364
2.75	-0.25	4.5	1.5	3.625	0.625	0.2413793103448276
2.75	-0.25	3.625	0.625	3.1875	0.1875	0.13725490196078433
2.75	-0.25	3.1875	0.1875	2.96875	-0.03125	0.07368421052631578


Equation:
x - 3
Lower: 1
Higher: 8
Precision:
0.00001
Maximum Number of iterations:
5
Root is 2.96875, Relative error is 0.07368421052631578
Method:
Bisection
Submit
Browse
Plot

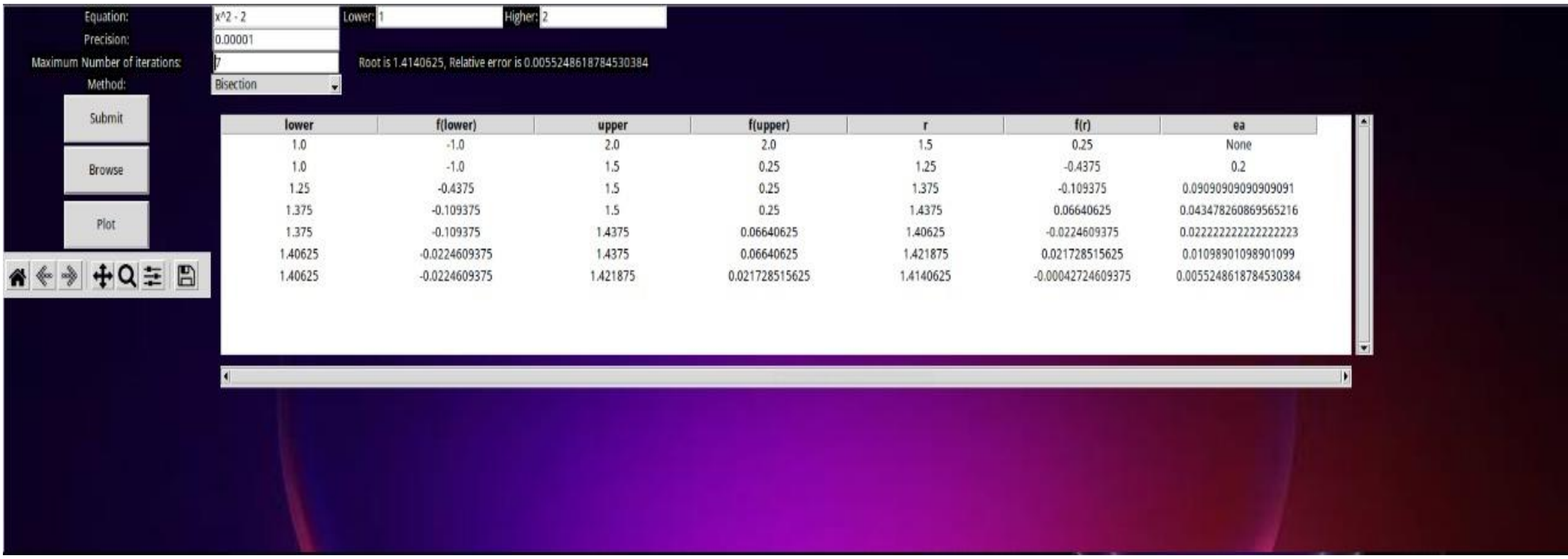

Equation:
x - 3
Lower: 1
Higher: 8
Precision:
0.00001
Maximum Number of iterations:
5
Root is 2.96875, Relative error is 0.07368421052631578
Method:
Bisection
Submit
Browse
Plot


Equation:
x - 3
Lower:
1
Higher:
8
Precision:
0.00001
Maximum Number of iterations:
5
Root is 2.96875, Relative error is 0.07368421052631578
Method:
Bisection
Submit
Browse
Plot
5
4
3
2
1
0
−1
−2
2
4
6
8

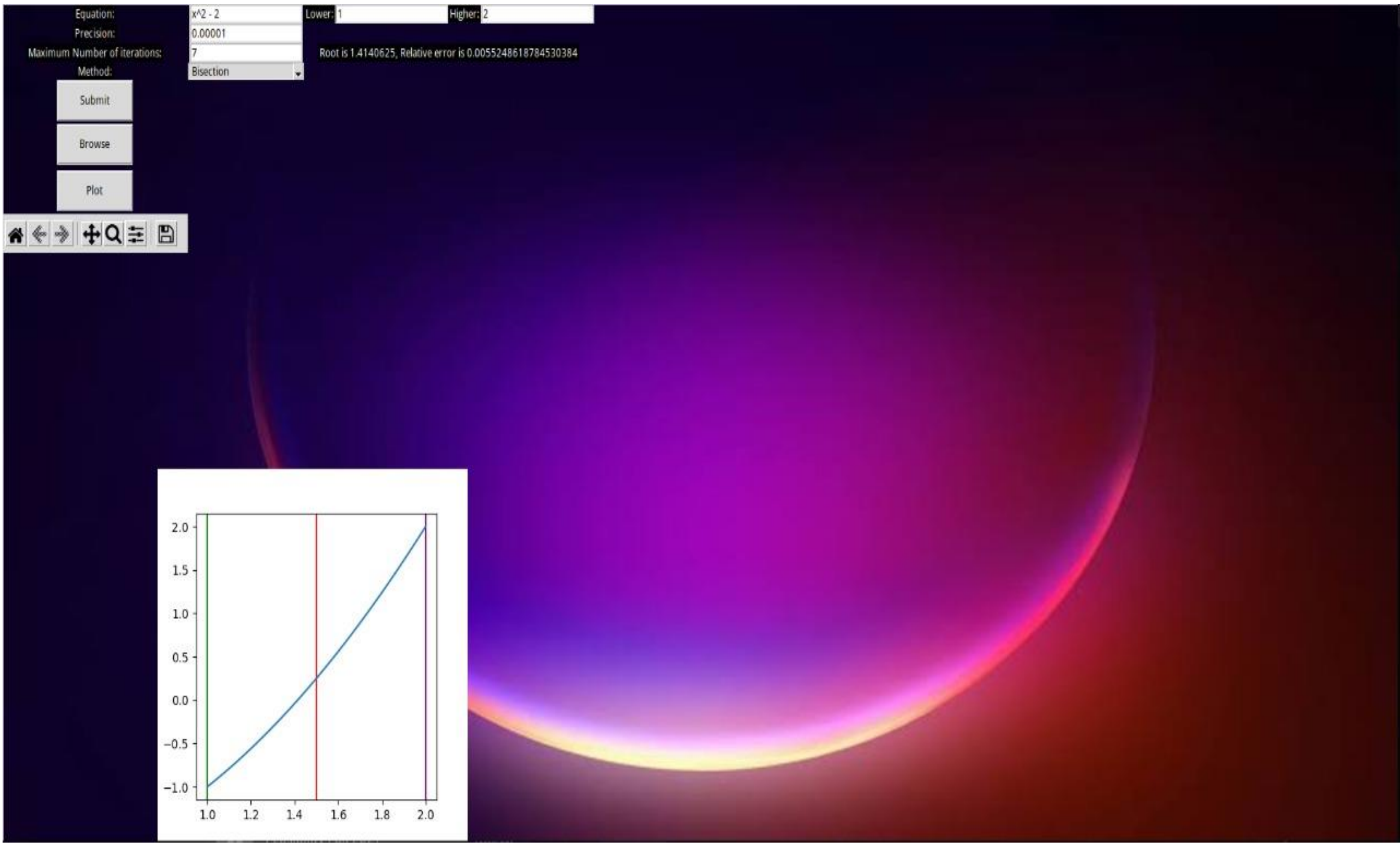

Equation:
x - 3
Lower:
1
Higher:
8
Precision:
0.00001
Maximum Number of iterations:
5
Root is 2.96875, Relative error is 0.07368421052631578
Method:
Bisection
Submit
Browse
Plot
5
4
3
2
1
0
−1
−2
2
4
6
8

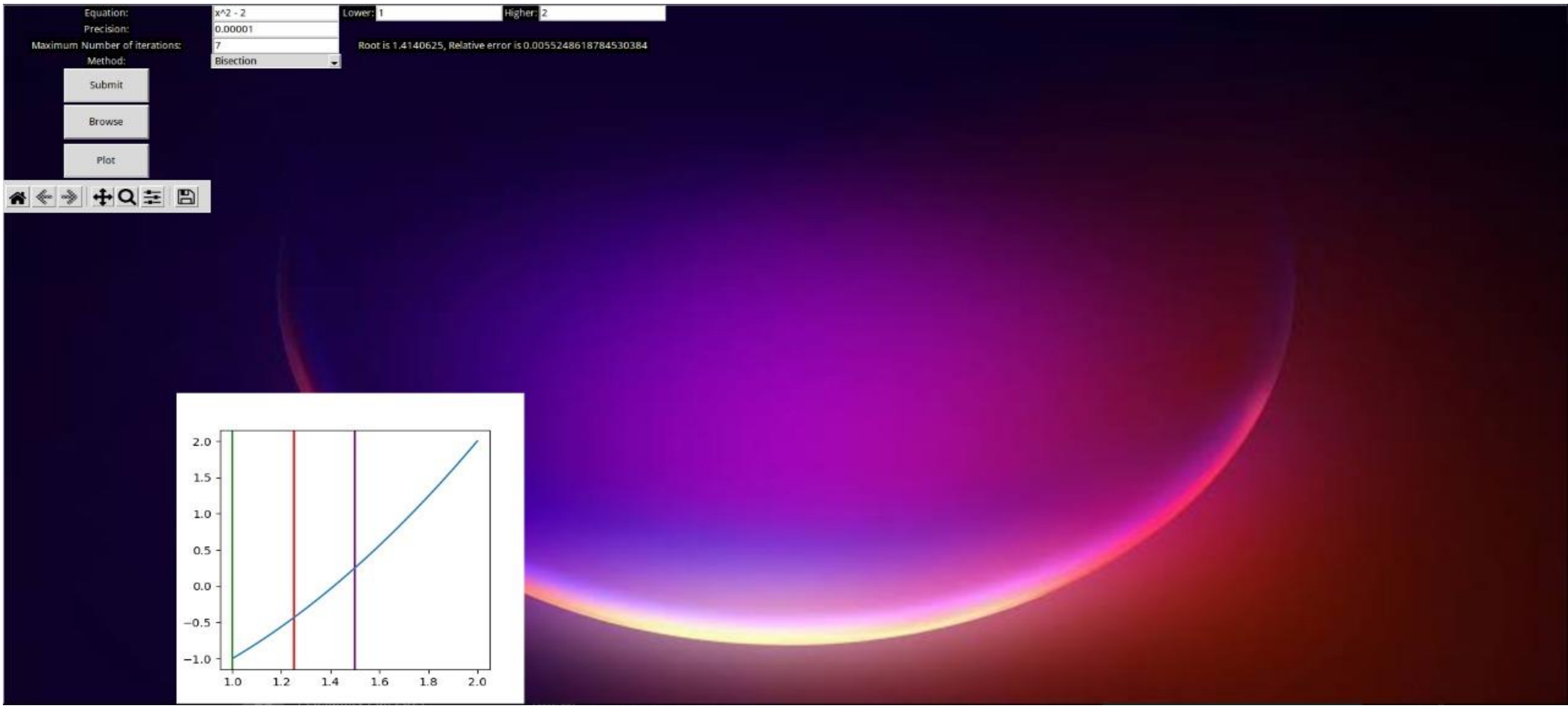

Equation:
x - 3
Lower:
1
Higher:
8
Precision:
0.00001
Maximum Number of iterations:
5
Root is 2.96875, Relative error is 0.07368421052631578
Method:
Bisection
Submit
Browse
Plot
5
4
3
2
1
0
−1
−2
2
4
6
8

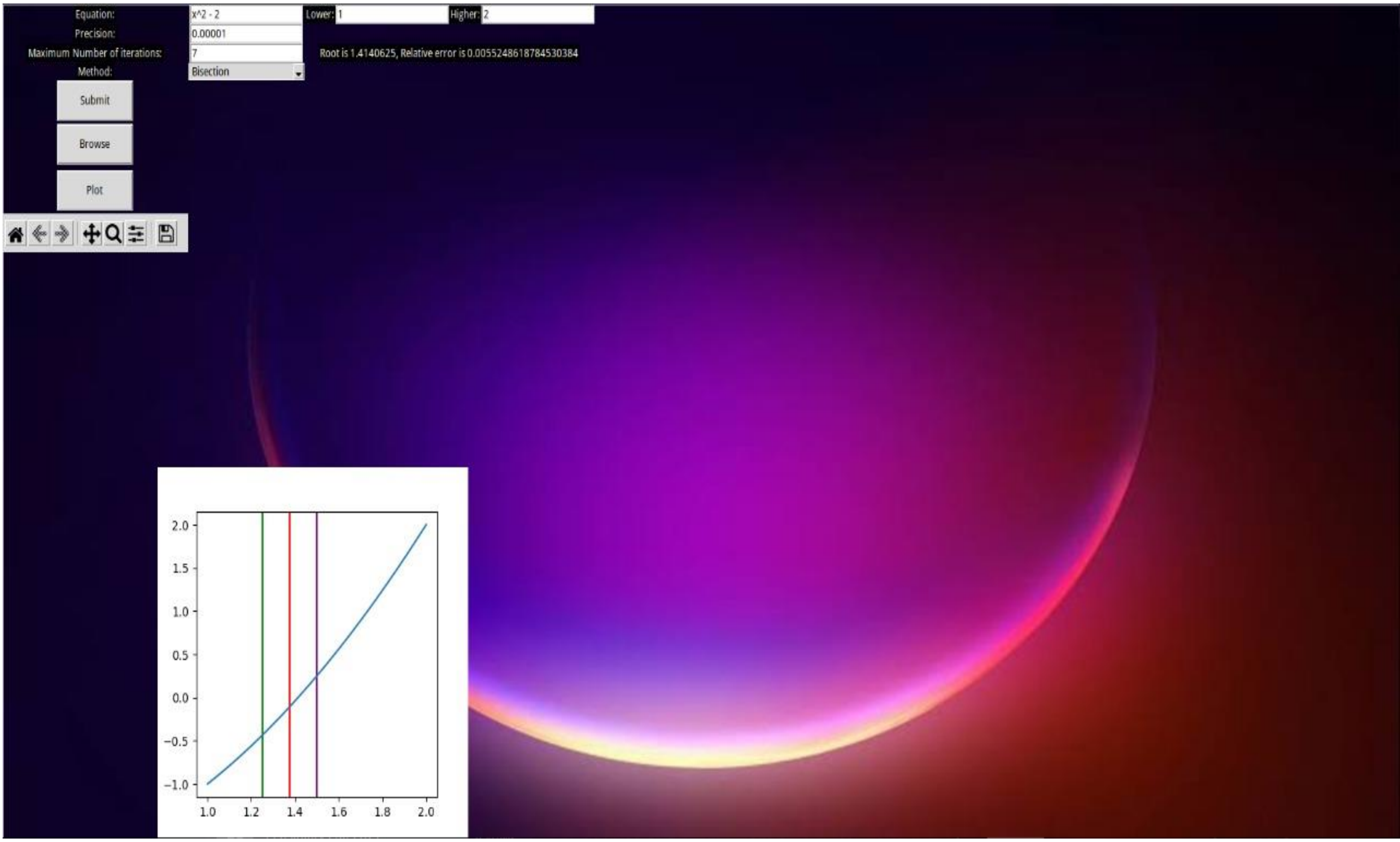
#Iterations = 7:

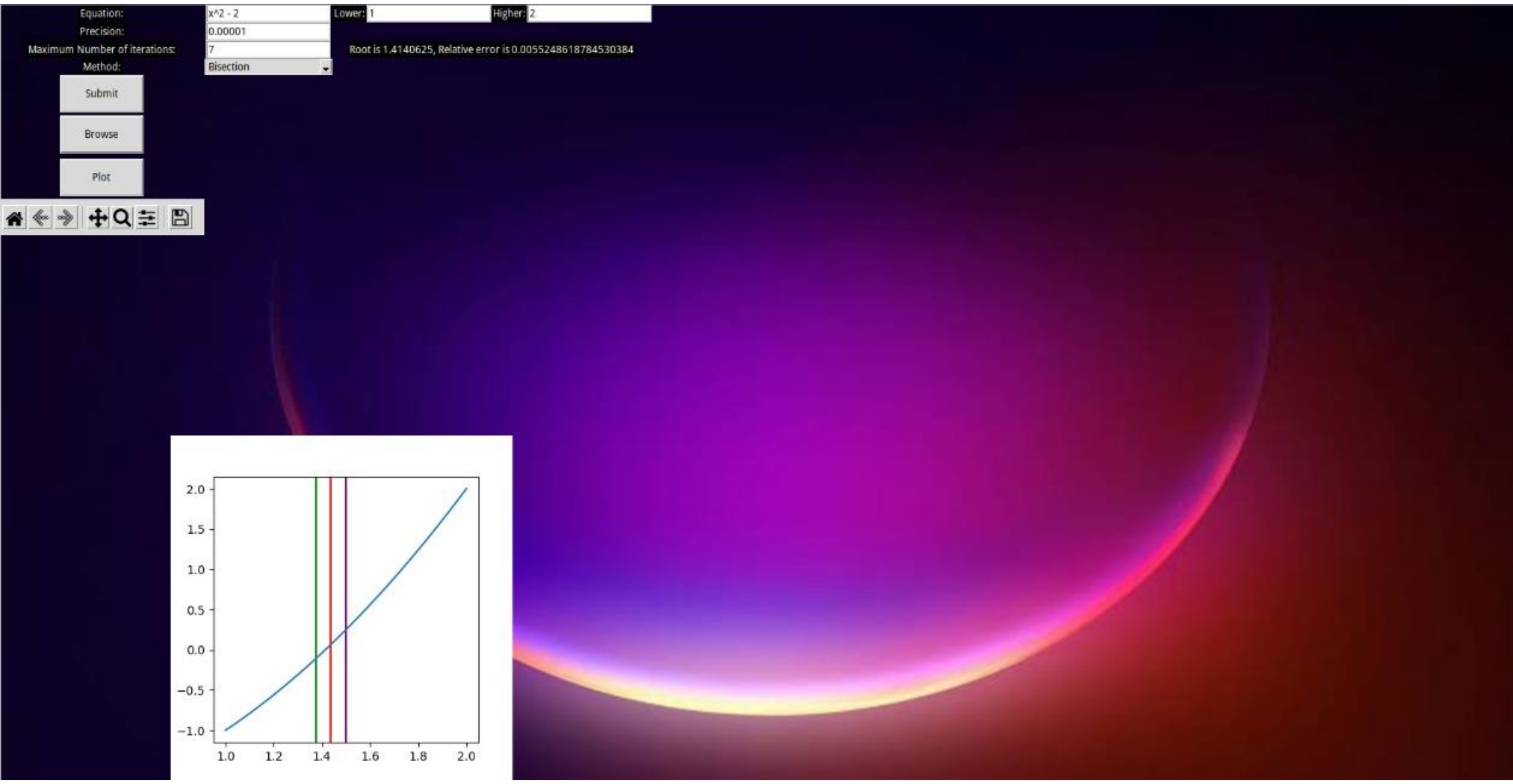


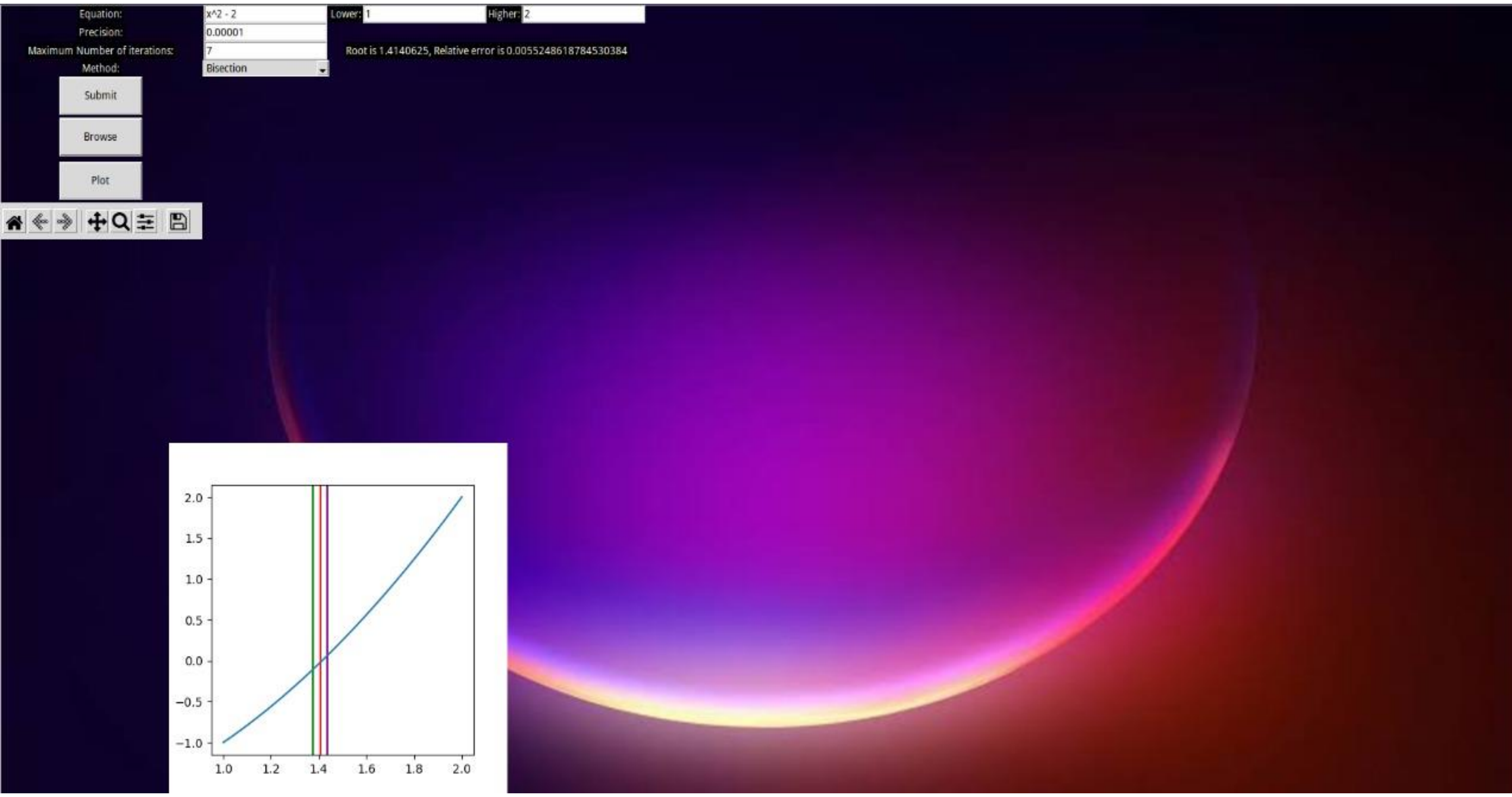
lower	f(lower)	upper	f(upper)	r	f(r)	ea
1.0	-1.0	2.0	2.0	1.5	0.25	None
1.0	-1.0	1.5	0.25	1.25	-0.4375	0.2
1.25	-0.4375	1.5	0.25	1.375	-0.109375	0.09090909090909091
1.375	-0.109375	1.5	0.25	1.4375	0.06640625	0.043478260869565216
1.375	-0.109375	1.4375	0.06640625	1.40625	-0.0224609375	0.022222222222222223
1.40625	-0.0224609375	1.4375	0.06640625	1.421875	0.021728515625	0.01098901098901099
1.40625	-0.0224609375	1.421875	0.021728515625	1.4140625	-0.00042724609375	0.0055248618784530384

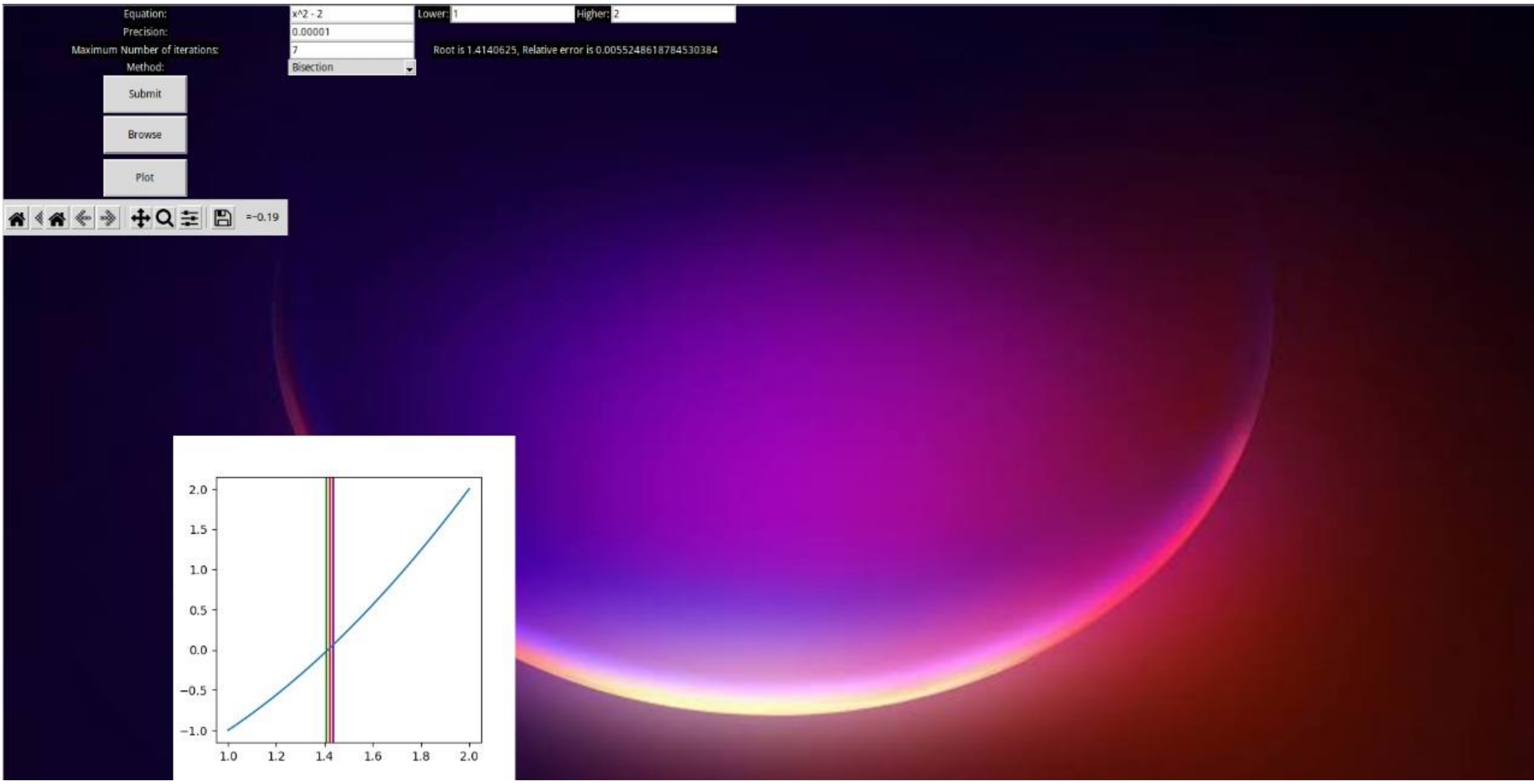



Equation:
x^2 - 2
Lower:
1
Higher:
2
Precision:
0.00001
Maximum Number of iterations:
7
Root is 1.4140625, Relative error is 0.0055248618784530384
Method:
Bisection
Submit
Browse
Plot
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
1.0
1.2
1.4
1.6
1.8
2.0


Equation:
x^2 - 2
Lower:
1
Higher:
2
Precision:
0.00001
Maximum Number of iterations:
7
Root is 1.4140625, Relative error is 0.0055248618784530384
Method:
Bisection
Submit
Browse
Plot
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
1.0
1.2
1.4
1.6
1.8
2.0


Equation:
x^2 - 2
Lower: 1
Higher: 2
Precision:
0.00001
Maximum Number of iterations:
7
Root is 1.4140625, Relative error is 0.0055248618784530384
Method:
Bisection
Submit
Browse
Plot


Equation:
x^2 - 2
Lower: 1
Higher: 2
Precision:
0.00001
Maximum Number of iterations:
7
Root is 1.4140625, Relative error is 0.0055248618784530384
Method:
Bisection
Submit
Browse
Plot


Equation:
x^2 - 2
Lower:
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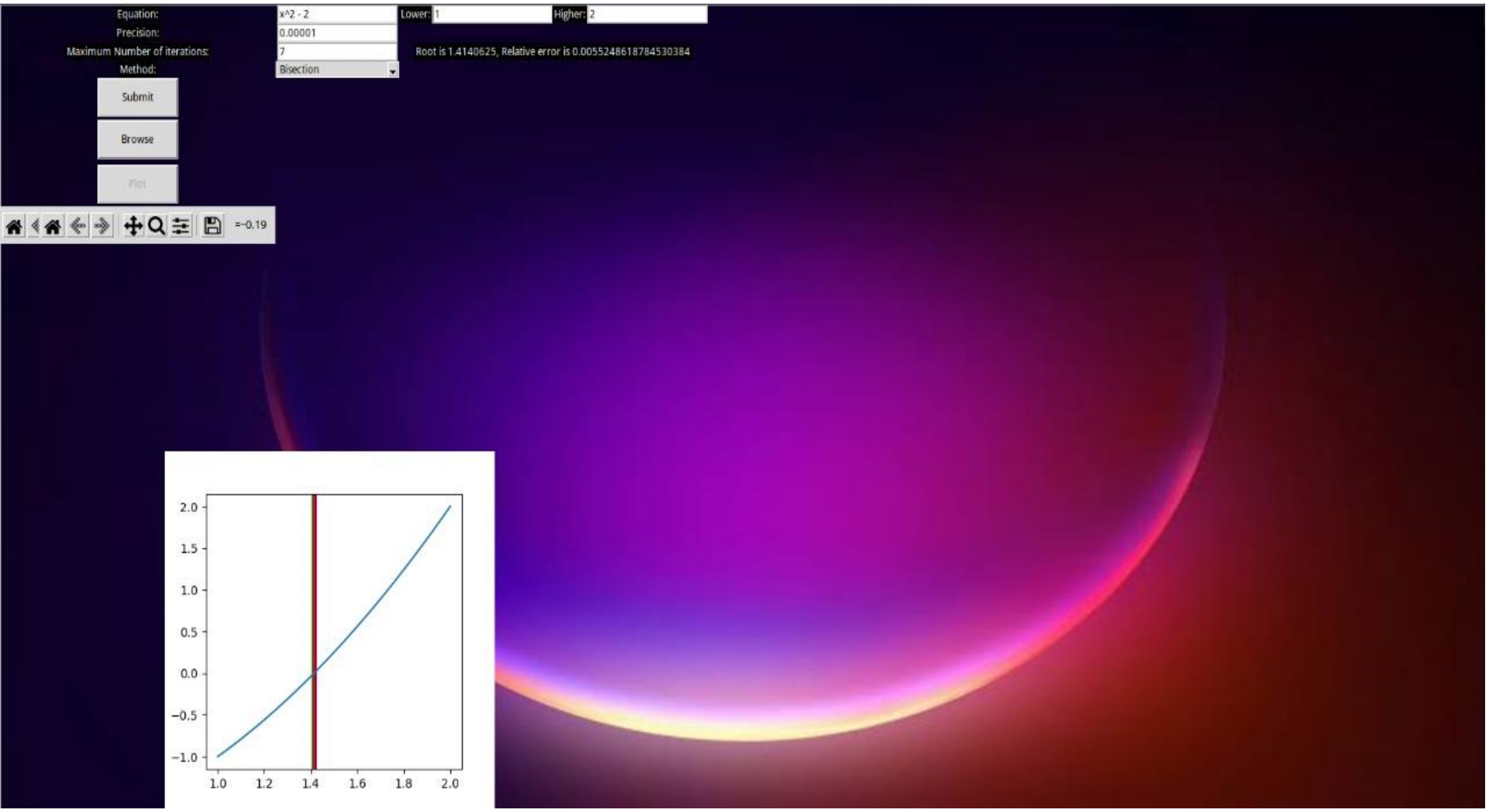
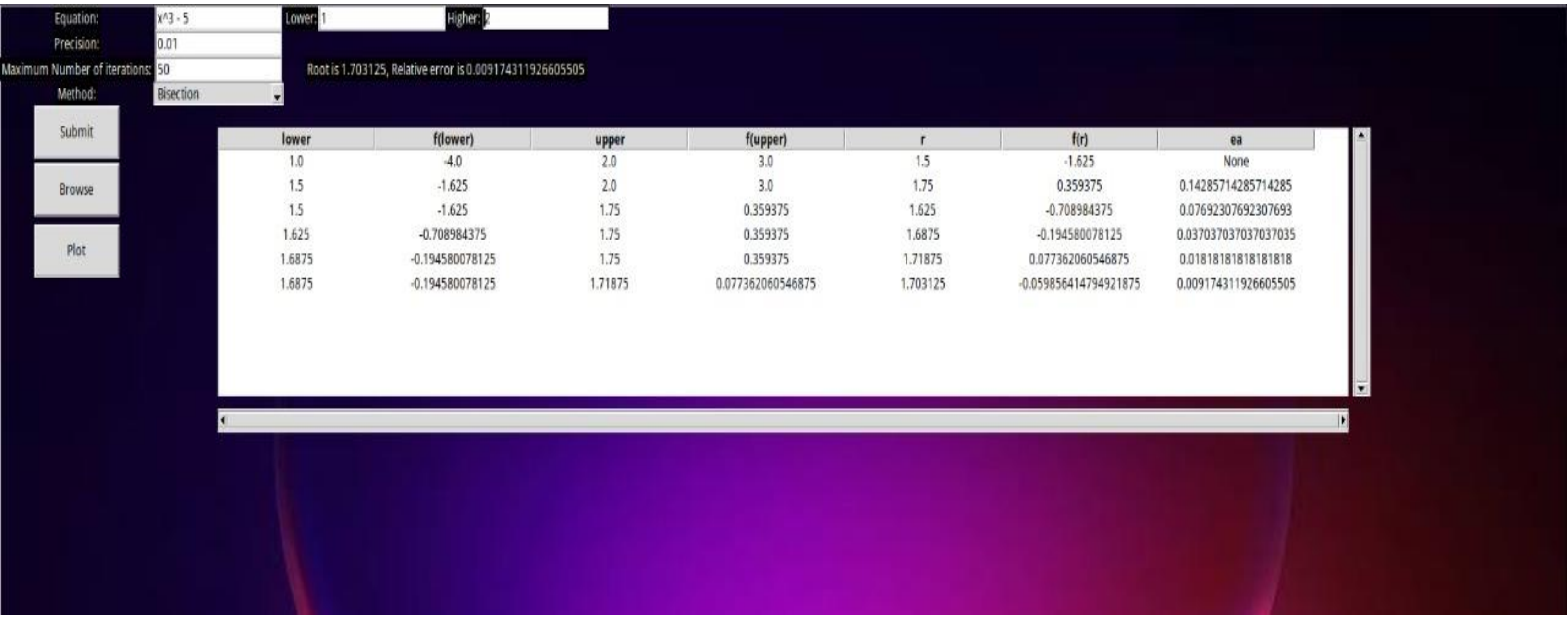
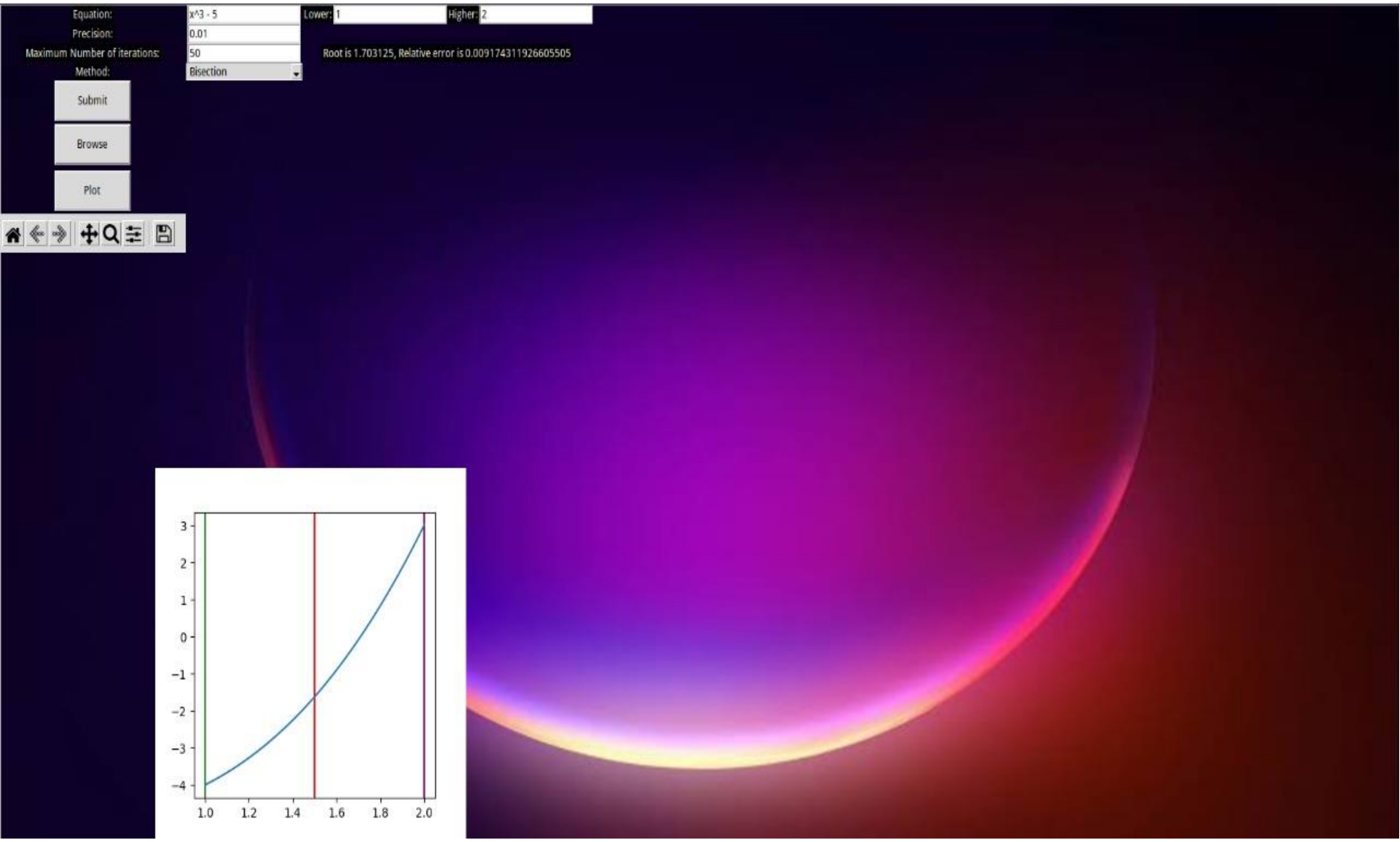
Higher:
2
Precision:
0.00001
Maximum Number of iterations:
7
Root is 1.4140625, Relative error is 0.0055248618784530384
Method:
Bisection
Submit
Browse
Plot
=-0.19
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
1.0
1.2
1.4
1.6
1.8
2.0

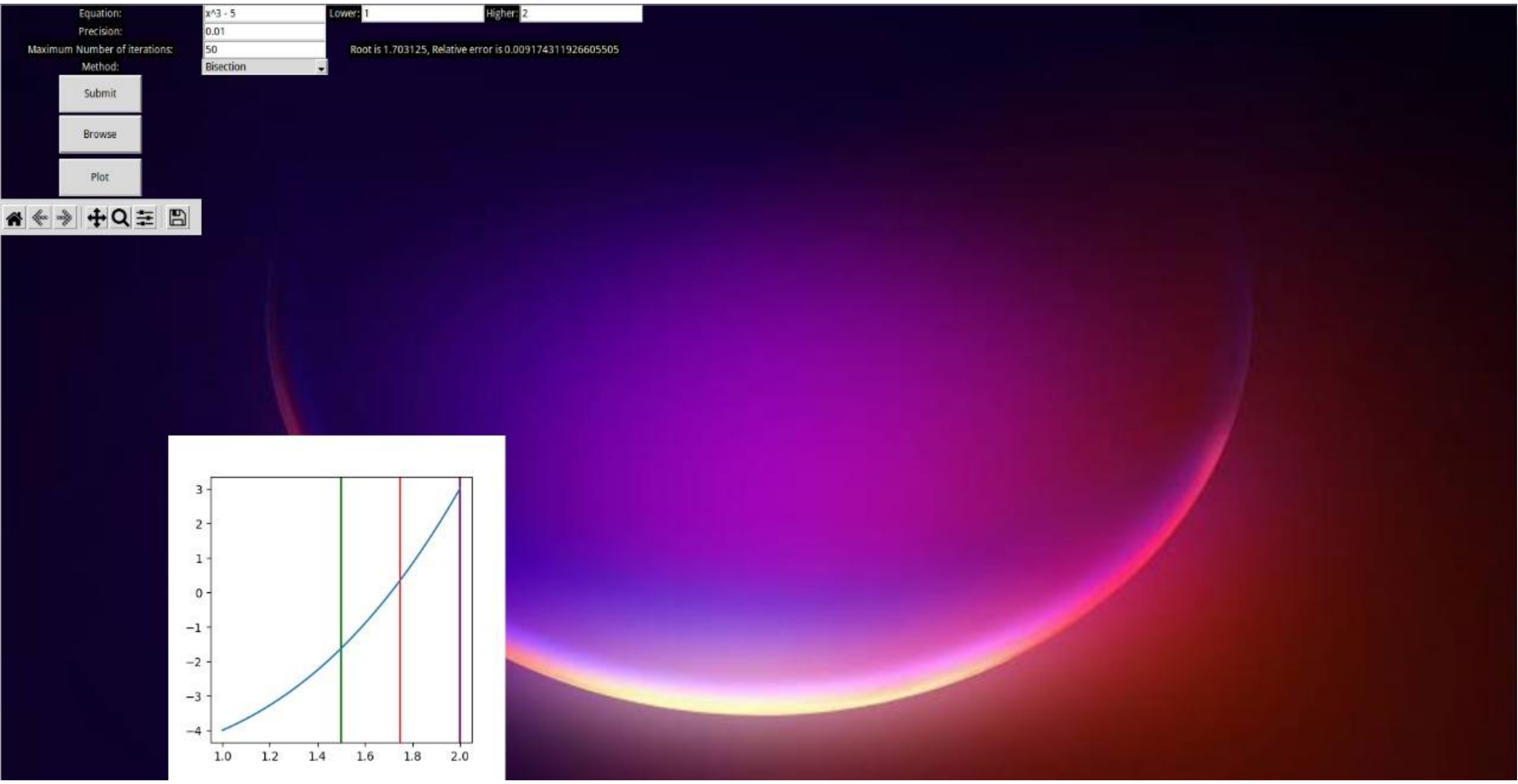

Equation:
x^2 - 2
Lower:
1
Higher:
2
Precision:
0.00001
Maximum Number of iterations:
7
Root is 1.4140625, Relative error is 0.0055248618784530384
Method:
Bisection
Submit
Browse
Plot
=-0.19
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
1.0
1.2
1.4
1.6
1.8
2.0

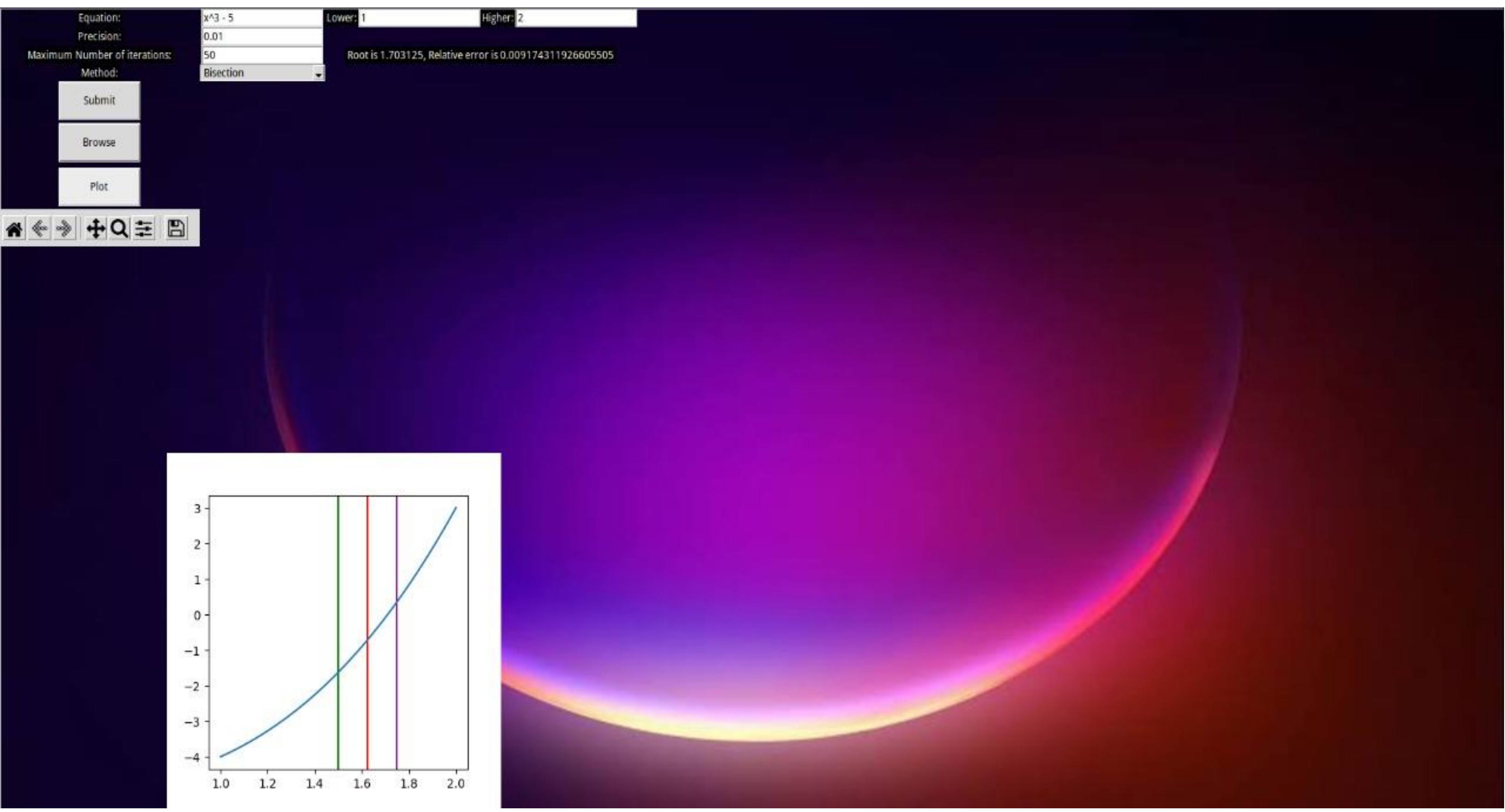
#Iterations = 50:

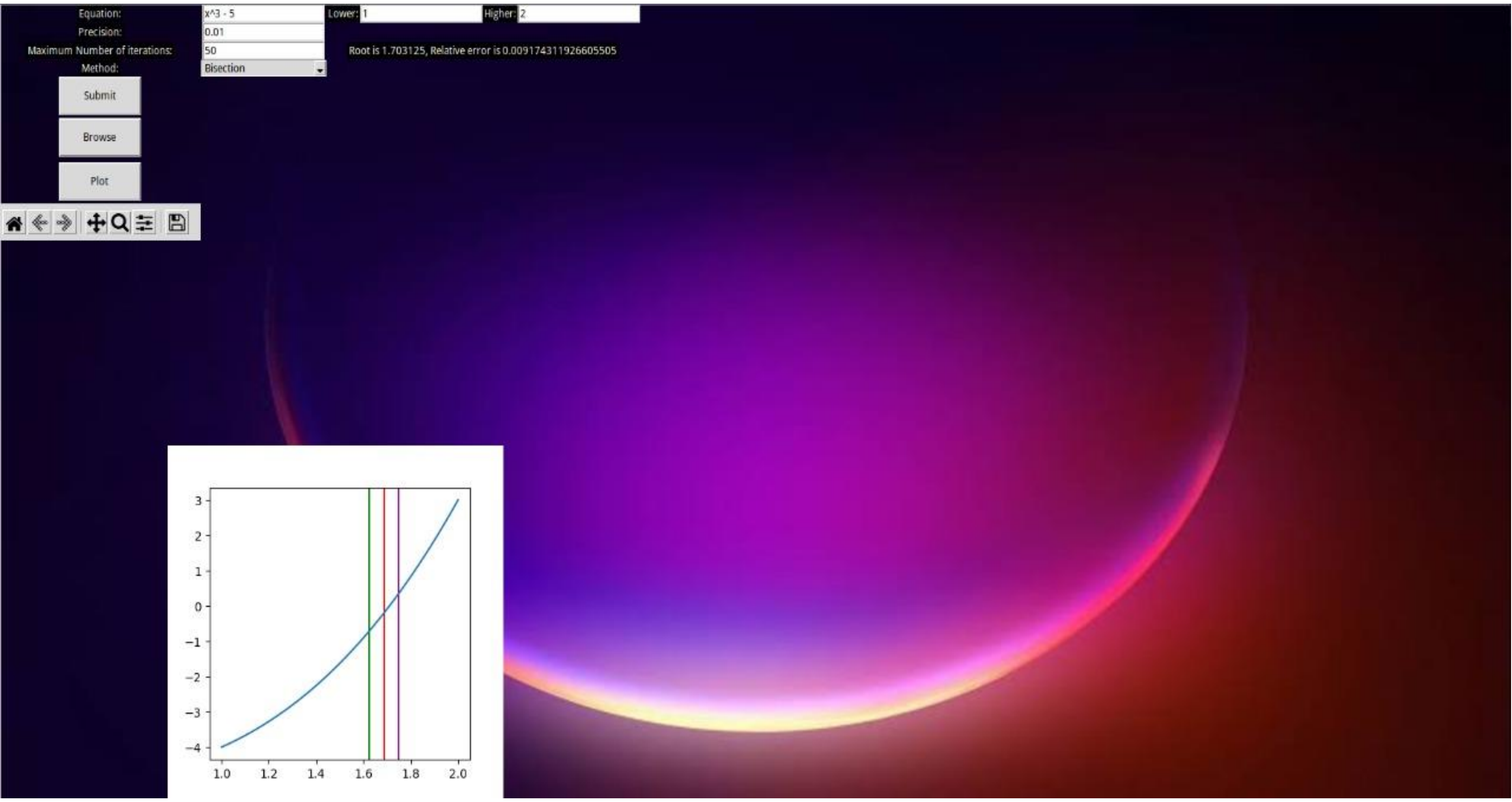


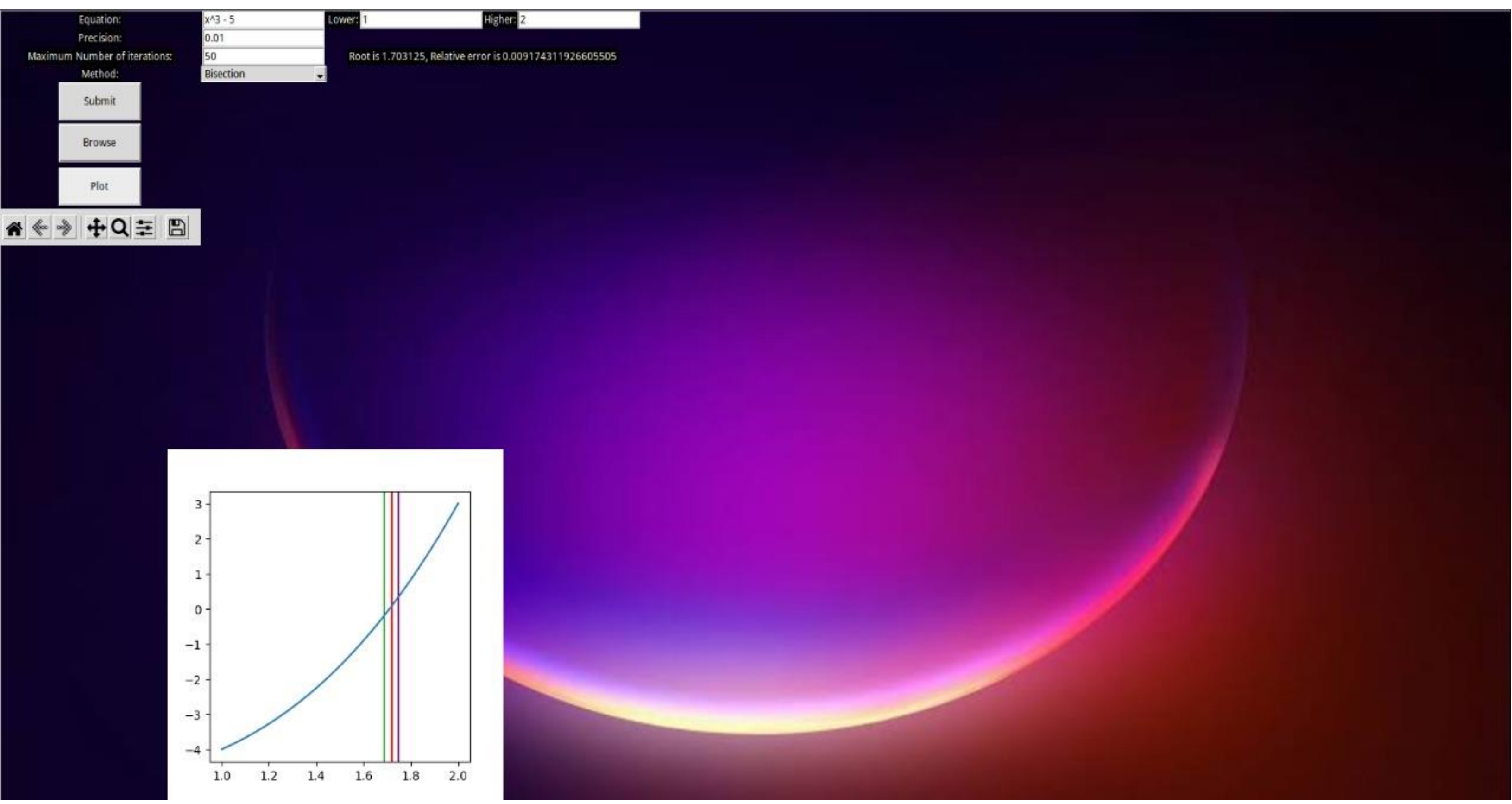
lower	f(lower)	upper	f(upper)	r	f(r)	ea
1.0	-4.0	2.0	3.0	1.5	-1.625	None
1.5	-1.625	2.0	3.0	1.75	0.359375	0.14285714285714285
1.5	-1.625	1.75	0.359375	1.625	-0.708984375	0.07692307692307693
1.625	-0.708984375	1.75	0.359375	1.6875	-0.194580078125	0.037037037037037035
1.6875	-0.194580078125	1.75	0.359375	1.71875	0.077362060546875	0.01818181818181818
1.6875	-0.194580078125	1.71875	0.077362060546875	1.703125	-0.059856414794921875	0.009174311926605505




Equation:
x^3 - 5
Lower:
1
Higher:
2
Precision:
0.01
Maximum Number of iterations:
50
Root is 1.703125, Relative error is 0.009174311926605505
Method:
Bisection
Submit
Browse
Plot
3
2
1
0
−1
−2
−3
−4
1.0
1.2
1.4
1.6
1.8
2.0


Equation:
x^3 - 5
Lower:
1
Higher:
2
Precision:
0.01
Maximum Number of iterations:
50
Root is 1.703125, Relative error is 0.009174311926605505
Method:
Bisection
Submit
Browse
Plot
3
2
1
0
−1
−2
−3
−4
1.0
1.2
1.4
1.6
1.8
2.0


Equation:
x^3 - 5
Lower:
1
Higher:
2
Precision:
0.01
Maximum Number of iterations:
50
Root is 1.703125, Relative error is 0.009174311926605505
Method:
Bisection
Submit
Browse
Plot
3
2
1
0
−1
−2
−3
−4
1.0
1.2
1.4
1.6
1.8
2.0


Equation:
x^3 - 5
Lower:
1
Higher:
2
Precision:
0.01
Maximum Number of iterations:
50
Root is 1.703125, Relative error is 0.009174311926605505
Method:
Bisection
Submit
Browse
Plot
3
2
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1.0
1.2
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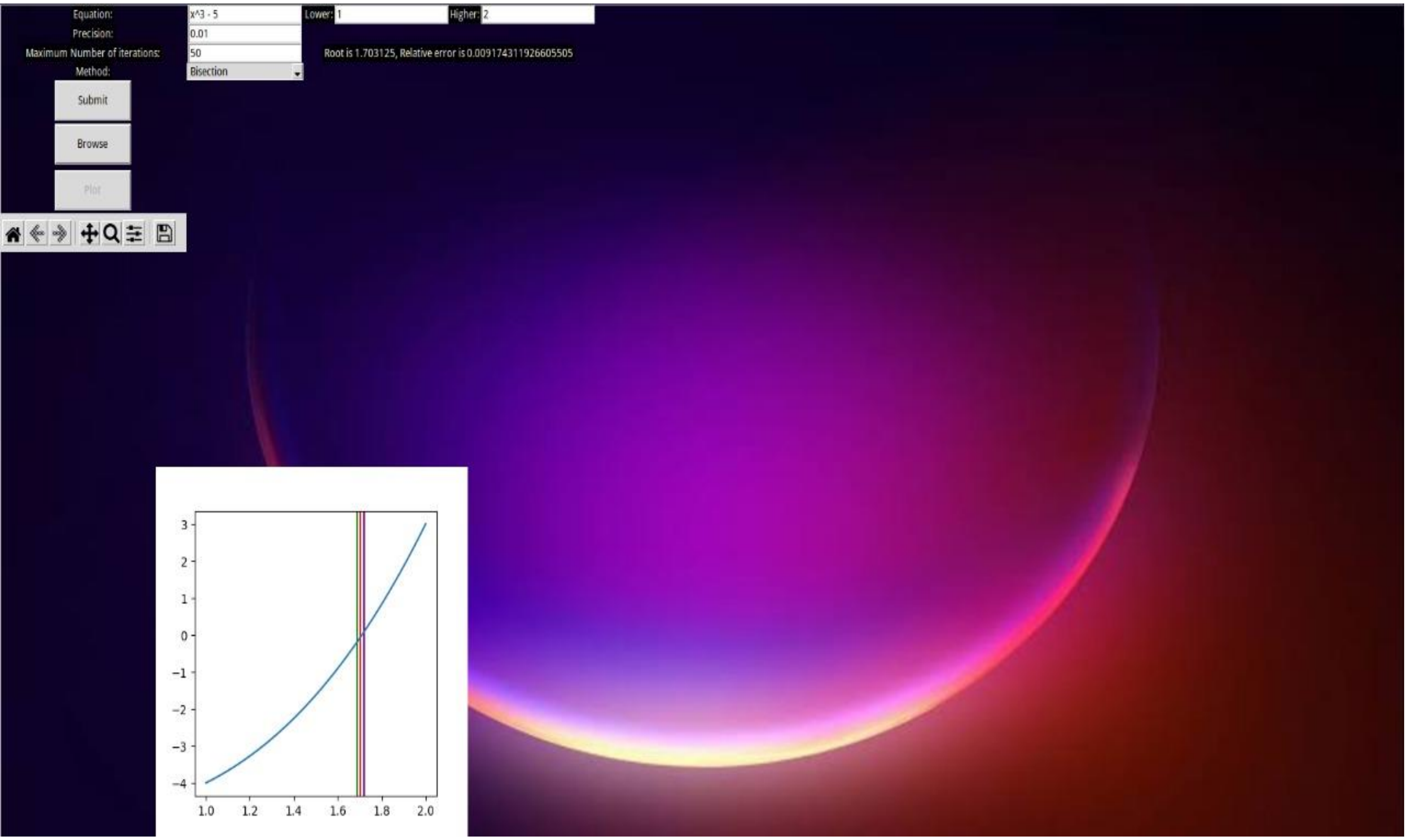
1.6
1.8
2.0



Observation:

- As the number of iterations increases, the more accurate result for the root we get (less error).
- Bisection method always converges to the root (slow convergence).

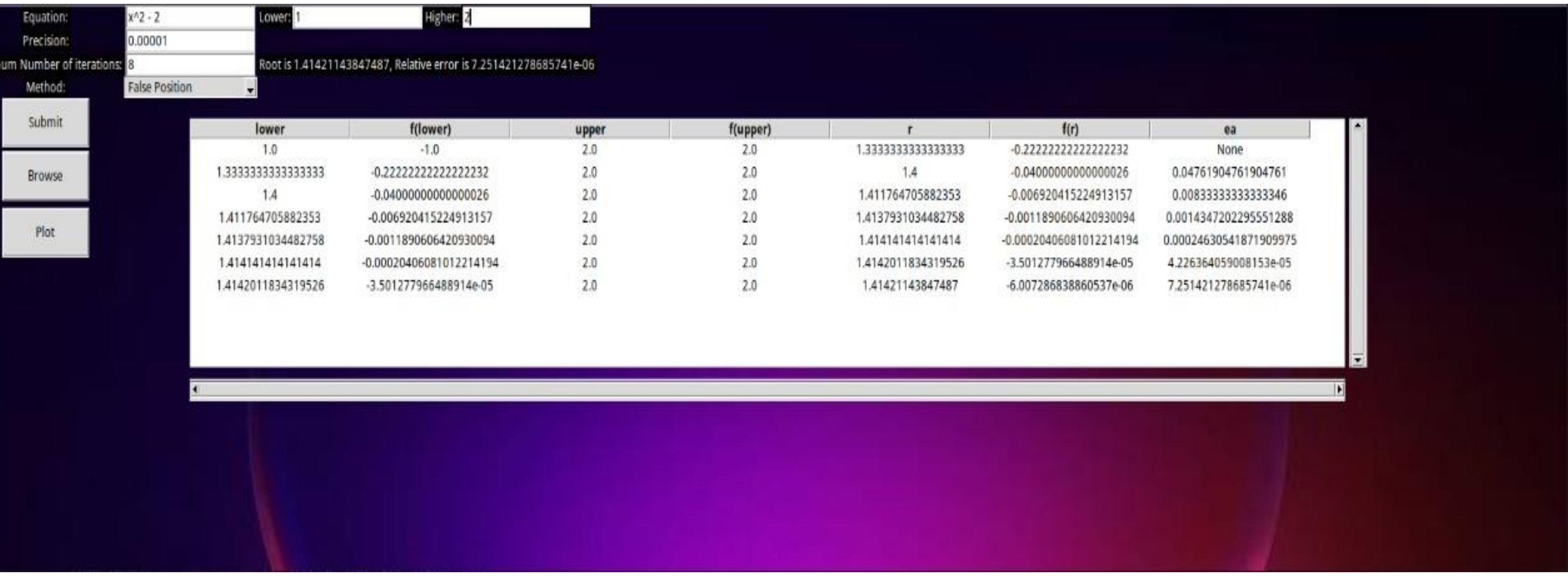
2. False Position Method:

Explanation:

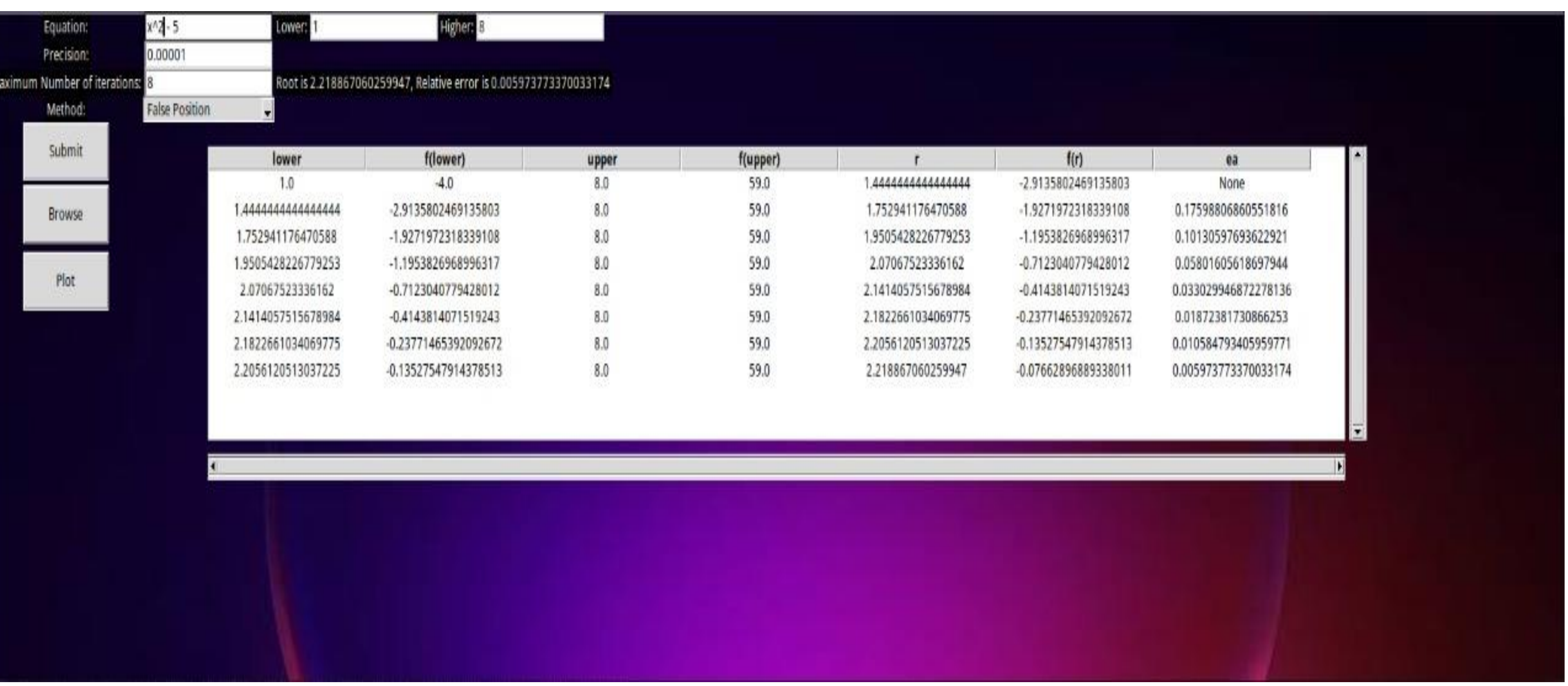
- Same Assumptions: This method also assumes that function is continuous in [a, b] and given two numbers ‘a’ and ‘b’ are such that f(a) * f(b) < 0.
- Always Converges: like Bisection, it always converges, usually considerably faster than Bisection–but sometimes very much more slowly than Bisection.
- It differs in the fact that we make a chord joining the two points [a, f(a)] and [b, f(b)]. We consider the point at which the chord touches the x axis and named it as c.

Examples:

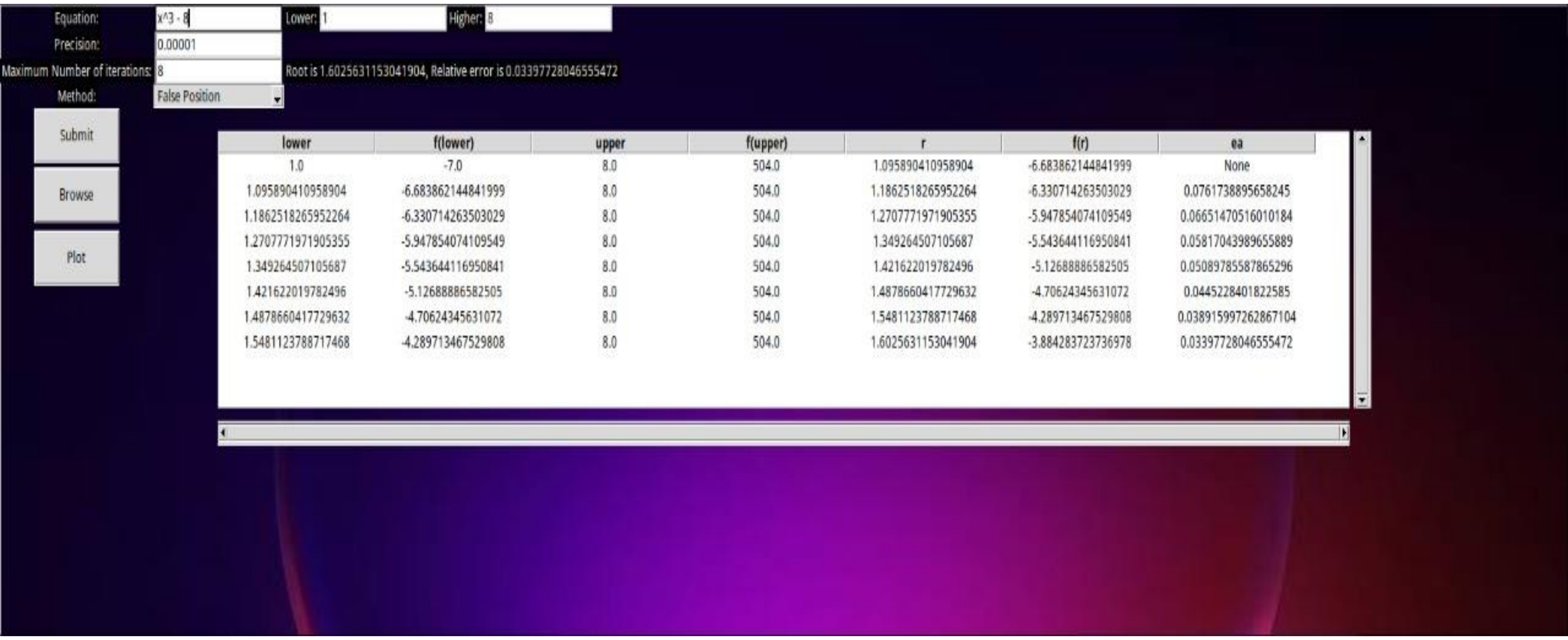
#Iterations = 8:



lower	f(lower)	upper	f(upper)	r	f(r)	ea
1.0	-1.0	2.0	2.0	1.3333333333333333	-0.2222222222222232	None
1.3333333333333333	-0.2222222222222232	2.0	2.0	1.4	-0.04000000000000026	0.04761904761904761
1.4	-0.04000000000000026	2.0	2.0	1.411764705882353	-0.006920415224913157	0.00833333333333346
1.411764705882353	-0.006920415224913157	2.0	2.0	1.4137931034482758	-0.0011890606420930094	0.001434720229555128
1.4137931034482758	-0.0011890606420930094	2.0	2.0	1.414141414141414	-0.00020406081012214194	0.00024630541871909975
1.414141414141414	-0.00020406081012214194	2.0	2.0	1.4142011834319526	-3.501277966488914e-05	4.226364059008153e-05
1.4142011834319526	-3.501277966488914e-05	2.0	2.0	1.41421143847487	-6.007286838860537e-06	7.251421278685741e-06



lower	f(lower)	upper	f(upper)	r	f(r)	ea
1.0	-4.0	8.0	59.0	1.4444444444444444	-2.9135802469135803	None
1.4444444444444444	-2.9135802469135803	8.0	59.0	1.752941176470588	-1.9271972318339108	0.17598806860551816
1.752941176470588	-1.9271972318339108	8.0	59.0	1.9505428226779253	-1.1953826968996317	0.10130597693622921
1.9505428226779253	-1.1953826968996317	8.0	59.0	2.07067523336162	-0.7123040779428012	0.05801605618697944
2.07067523336162	-0.7123040779428012	8.0	59.0	2.1414057515678984	-0.4143814071519243	0.033029946872278136
2.1414057515678984	-0.4143814071519243	8.0	59.0	2.1822661034069775	-0.23771465392092672	0.01872381730866253
2.1822661034069775	-0.23771465392092672	8.0	59.0	2.2056120513037225	-0.13527547914378513	0.010584793405959771
2.2056120513037225	-0.13527547914378513	8.0	59.0	2.218867060259947	-0.07662896889338011	0.005973773370033174



lower	f(lower)	upper	f(upper)	r	f(r)	ea
1.0	-7.0	8.0	504.0	1.095890410958904	-6.683862144841999	None
1.095890410958904	-6.683862144841999	8.0	504.0	1.1862518265952264	-6.330714263503029	0.0761738895658245
1.1862518265952264	-6.330714263503029	8.0	504.0	1.2707771971905355	-5.947854074109549	0.06651470516010184
1.2707771971905355	-5.947854074109549	8.0	504.0	1.349264507105687	-5.543644116950841	0.05817043989655889
1.349264507105687	-5.543644116950841	8.0	504.0	1.421622019782496	-5.12688886582505	0.05089785587865296
1.421622019782496	-5.12688886582505	8.0	504.0	1.4878660417729632	-4.70624345631072	0.0445228401822585
1.4878660417729632	-4.70624345631072	8.0	504.0	1.5481123788717468	-4.289713467529808	0.038915997262867104
1.5481123788717468	-4.289713467529808	8.0	504.0	1.6025631153041904	-3.884283723736978	0.03397728046555472

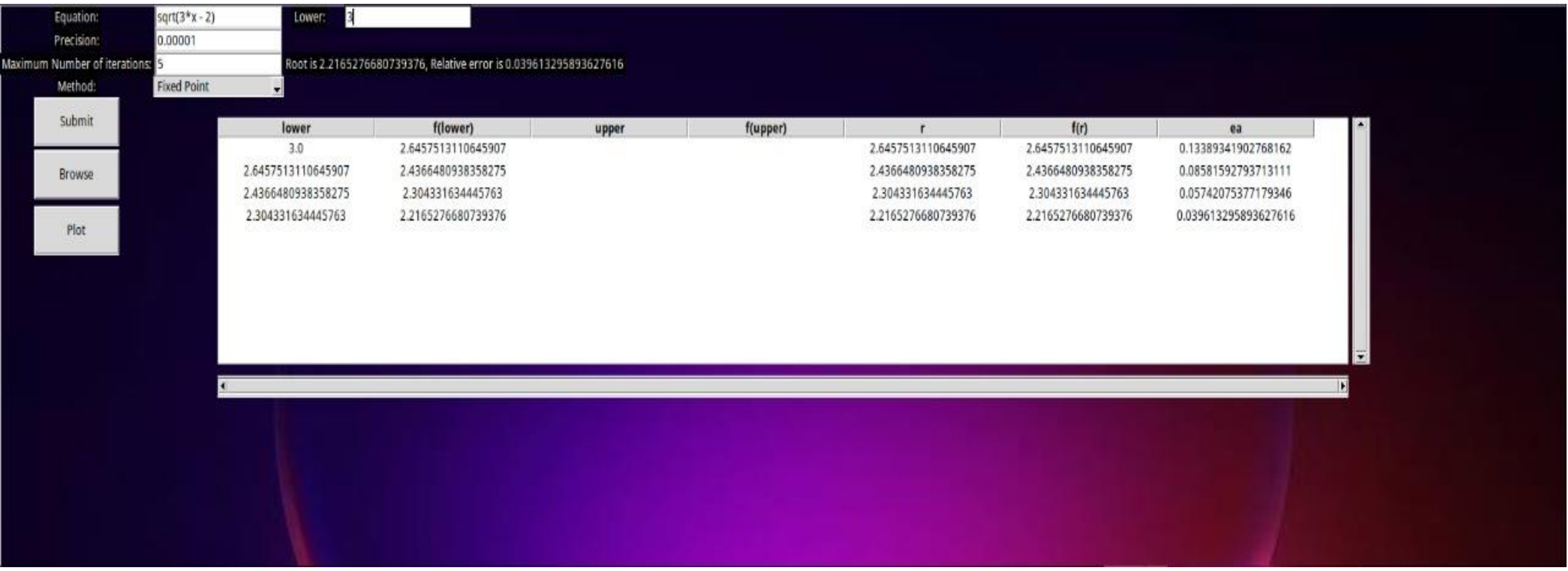
3. Fixed Point Iteration Method:

Steps:

Mapping f(x) = 0 to x = g(x) so that it becomes an iterative method, where both relations will have roots at the same locations.

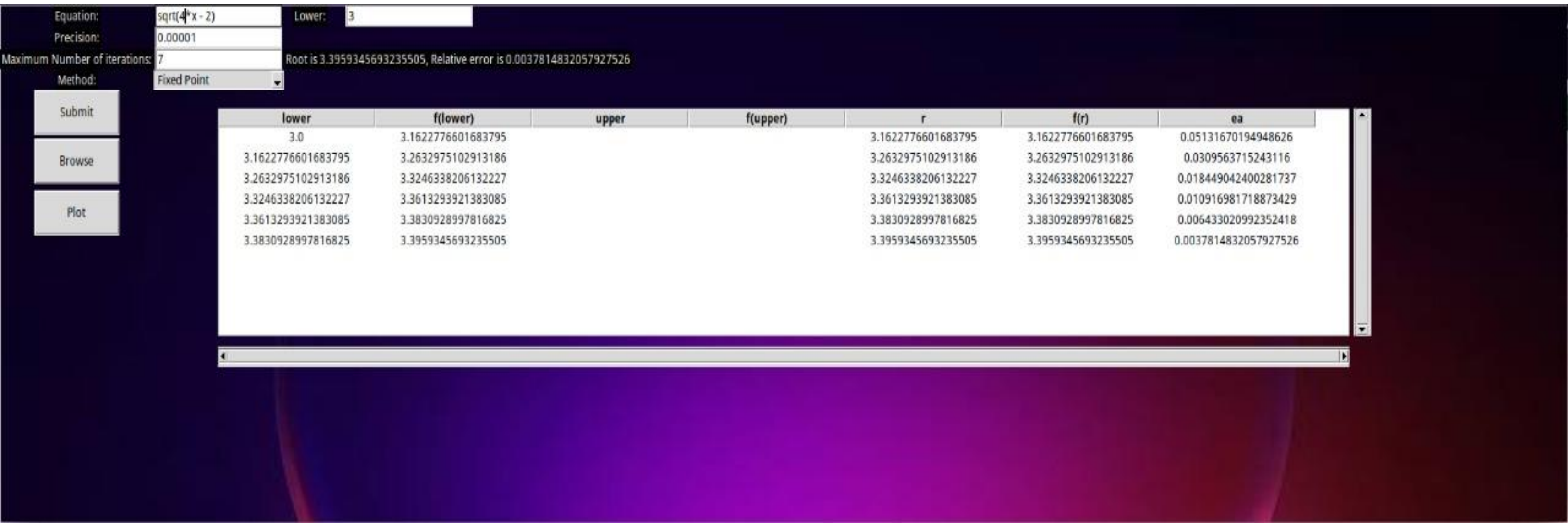
Examples:

#Iterations = 5:

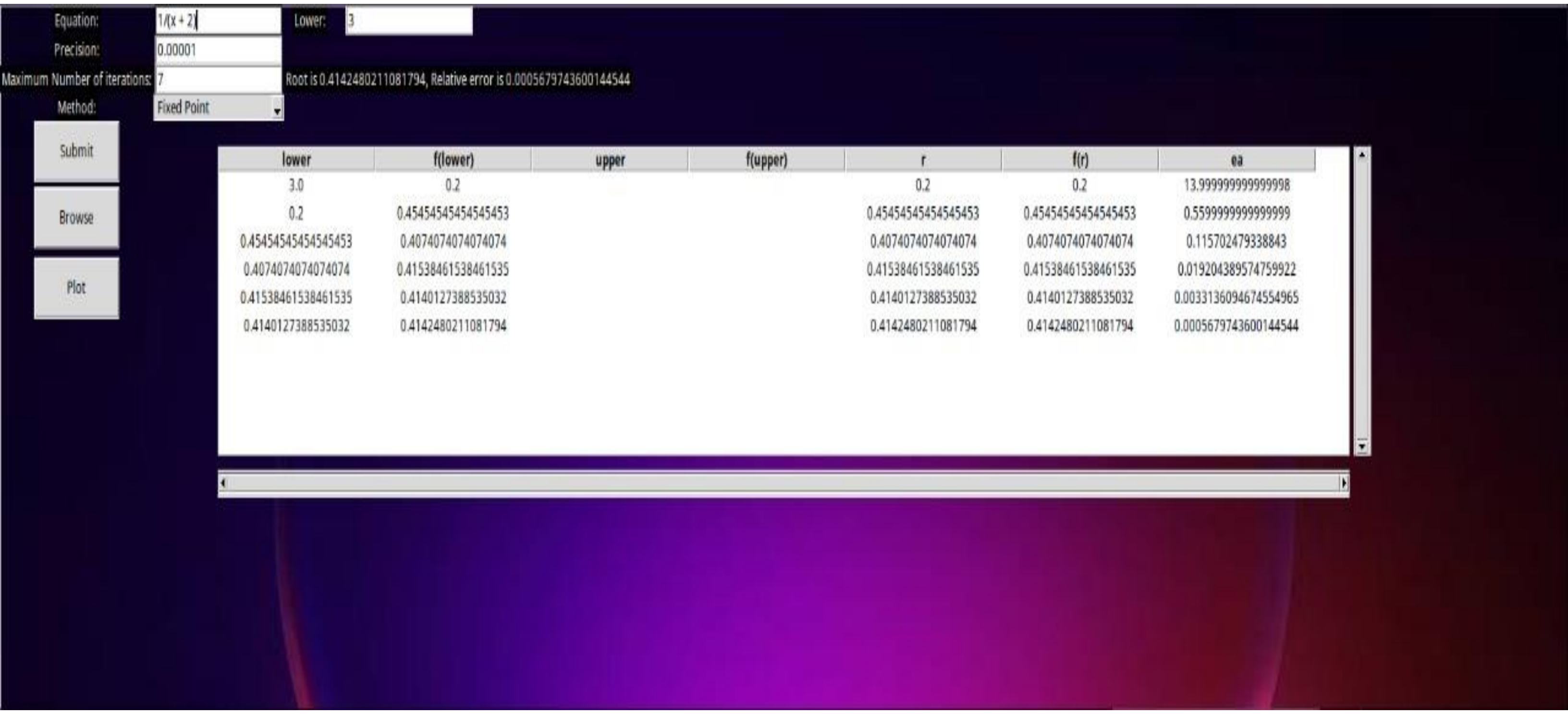


lower	f(lower)	upper	f(upper)	r	f(r)	ea
3.0	2.6457513110645907			2.6457513110645907	2.6457513110645907	0.13389341902768162
2.6457513110645907	2.4366480938358275			2.4366480938358275	2.4366480938358275	0.08581592793713111
2.4366480938358275	2.304331634445763			2.304331634445763	2.304331634445763	0.05742075377179346
2.304331634445763	2.2165276680739376			2.2165276680739376	2.2165276680739376	0.039613295893627616

#Iterations = 7:



lower	f(lower)	upper	f(upper)	r	f(r)	ea
3.0	3.1622776601683795			3.1622776601683795	3.1622776601683795	0.05131670194948626
3.1622776601683795	3.2632975102913186			3.2632975102913186	3.2632975102913186	0.0309563715243116
3.2632975102913186	3.3246338206132227			3.3246338206132227	3.3246338206132227	0.018449042400281737
3.3246338206132227	3.3613293921383085			3.3613293921383085	3.3613293921383085	0.010916981718873429
3.3613293921383085	3.3830928997816825			3.3830928997816825	3.3830928997816825	0.006433020992352418
3.3830928997816825	3.3959345693235505			3.3959345693235505	3.3959345693235505	0.0037814832057927526



lower	f(lower)	upper	f(upper)	r	f(r)	ea
3.0	0.2			0.2	0.2	13.999999999999998
0.2	0.45454545454545453			0.45454545454545453	0.45454545454545453	0.5599999999999999
0.45454545454545453	0.4074074074074074			0.4074074074074074	0.4074074074074074	0.115702479338843
0.4074074074074074	0.41538461538461535			0.41538461538461535	0.41538461538461535	0.019204389574759922
0.41538461538461535	0.4140127388535032			0.4140127388535032	0.4140127388535032	0.0033136094674554965
0.4140127388535032	0.4142480211081794			0.4142480211081794	0.4142480211081794	0.0005679743600144544

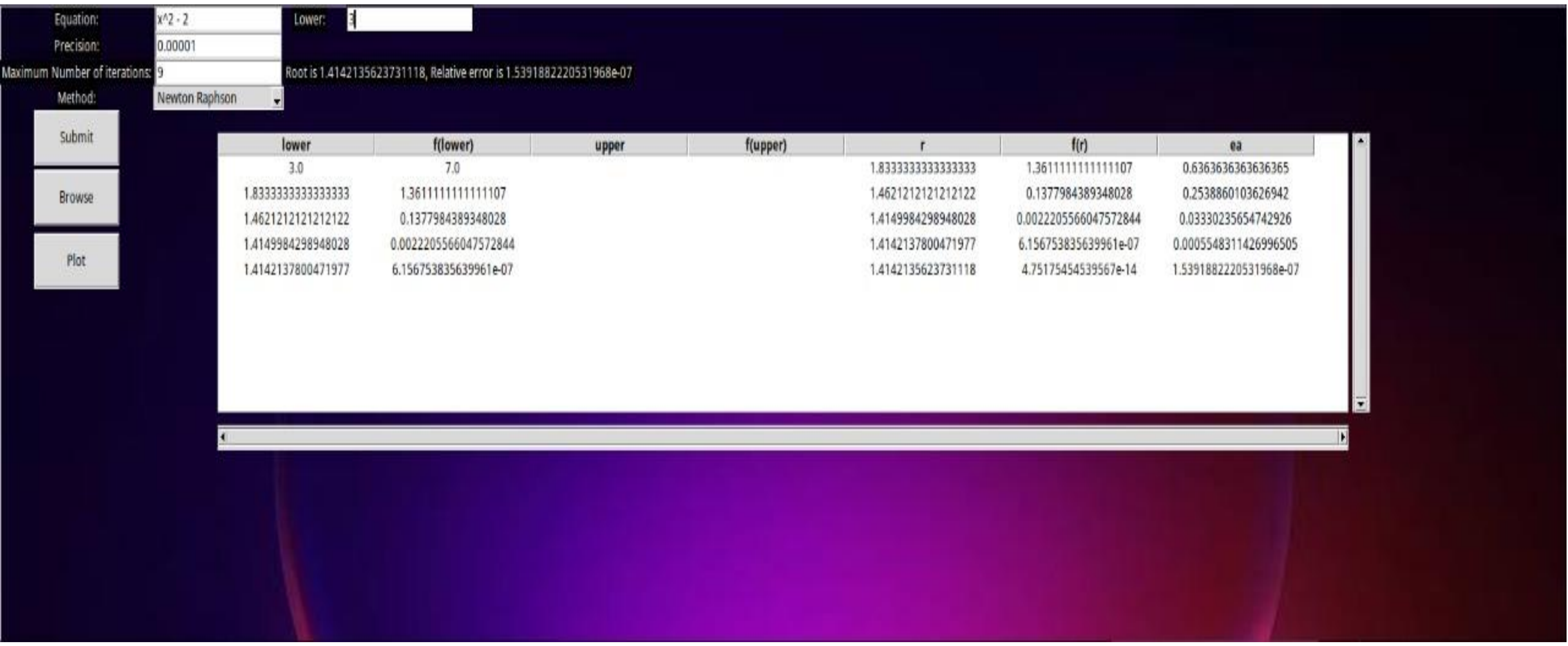
4. Newton Raphson Method:

Explanation:

It uses the idea that a continuous and differentiable function can be approximated by a straight line tangent to it.

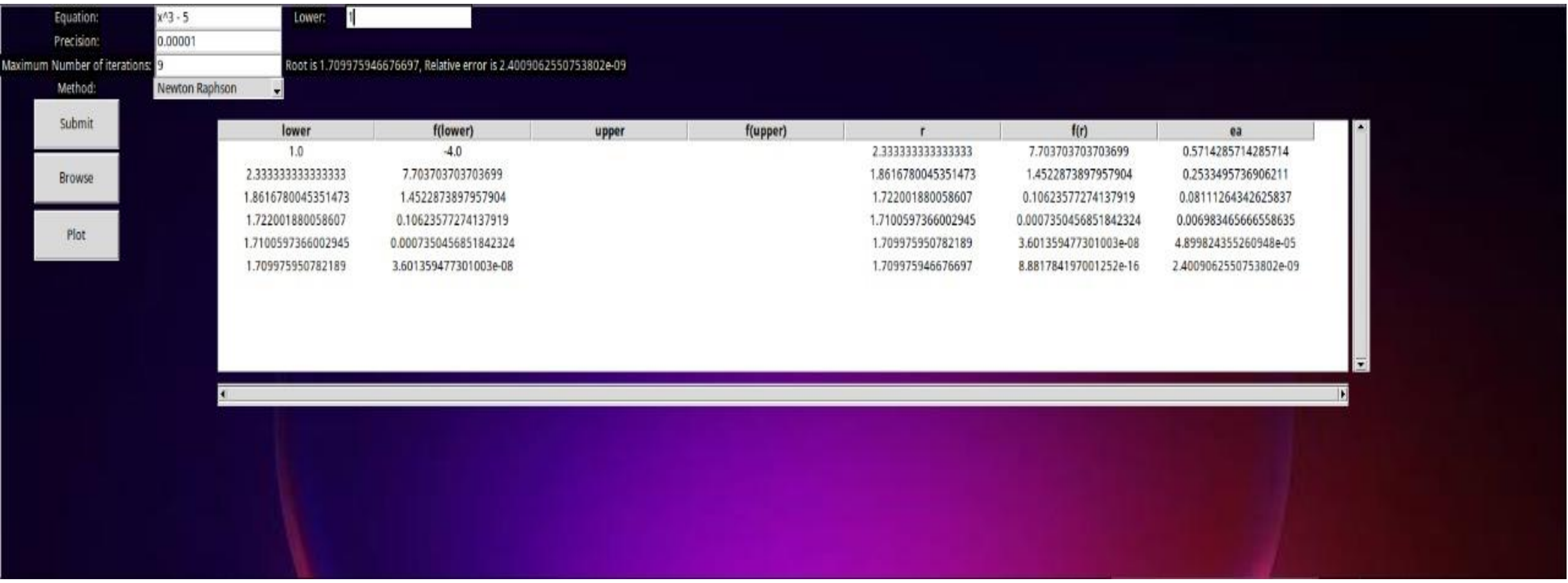
Examples:

#Iterations = 9:

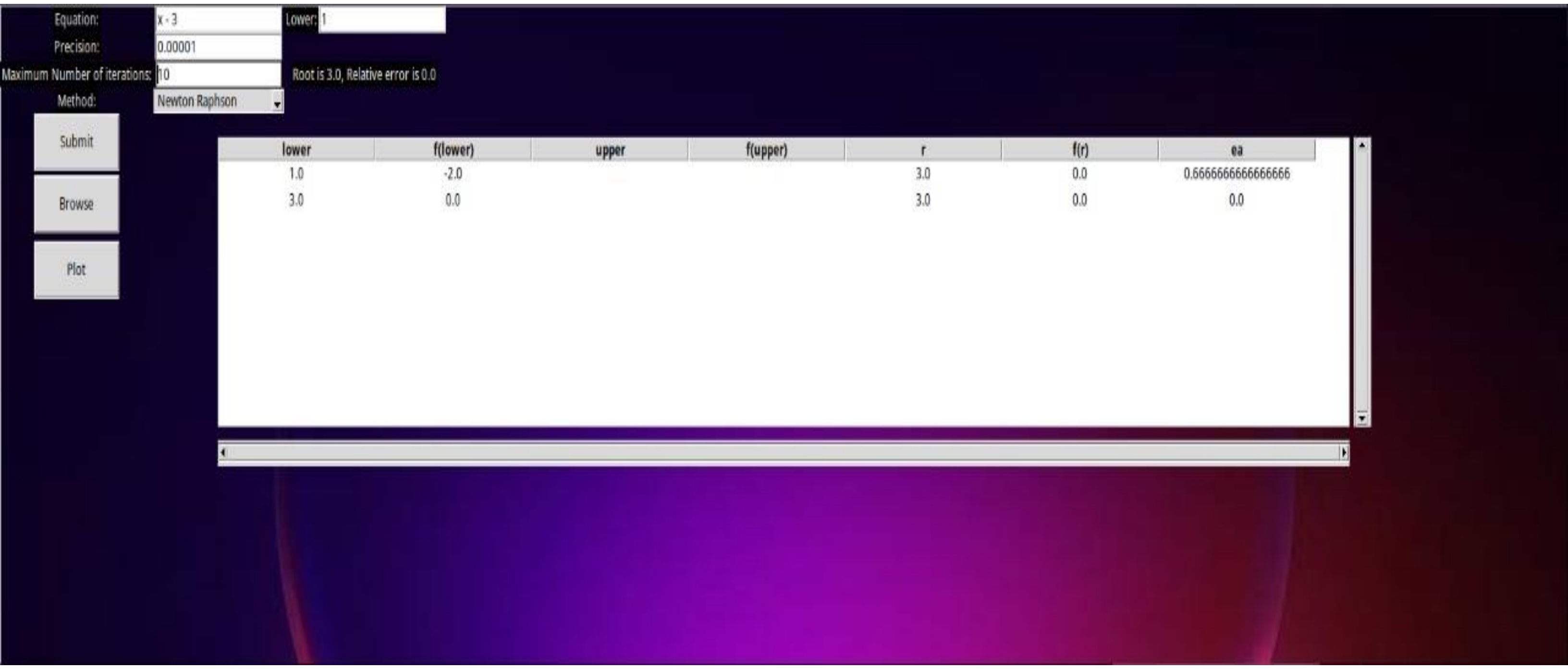


lower	f(lower)	upper	f(upper)	r	f(r)	ea
3.0	7.0			1.8333333333333333	1.3611111111111107	0.6363636363636365
1.8333333333333333	1.3611111111111107			1.4621212121212122	0.1377984389348028	0.2538860103626942
1.4621212121212122	0.1377984389348028			1.4149984298948028	0.0022205566047572844	0.03330235654742926
1.4149984298948028	0.0022205566047572844			1.4142137800471977	6.156753835639961e-07	0.0005548311426996505
1.4142137800471977	6.156753835639961e-07			1.4142135623731118	4.751754545395 67e-14	1.5391882220531968e-07

#Iterations = 9:



#Iterations = 10:



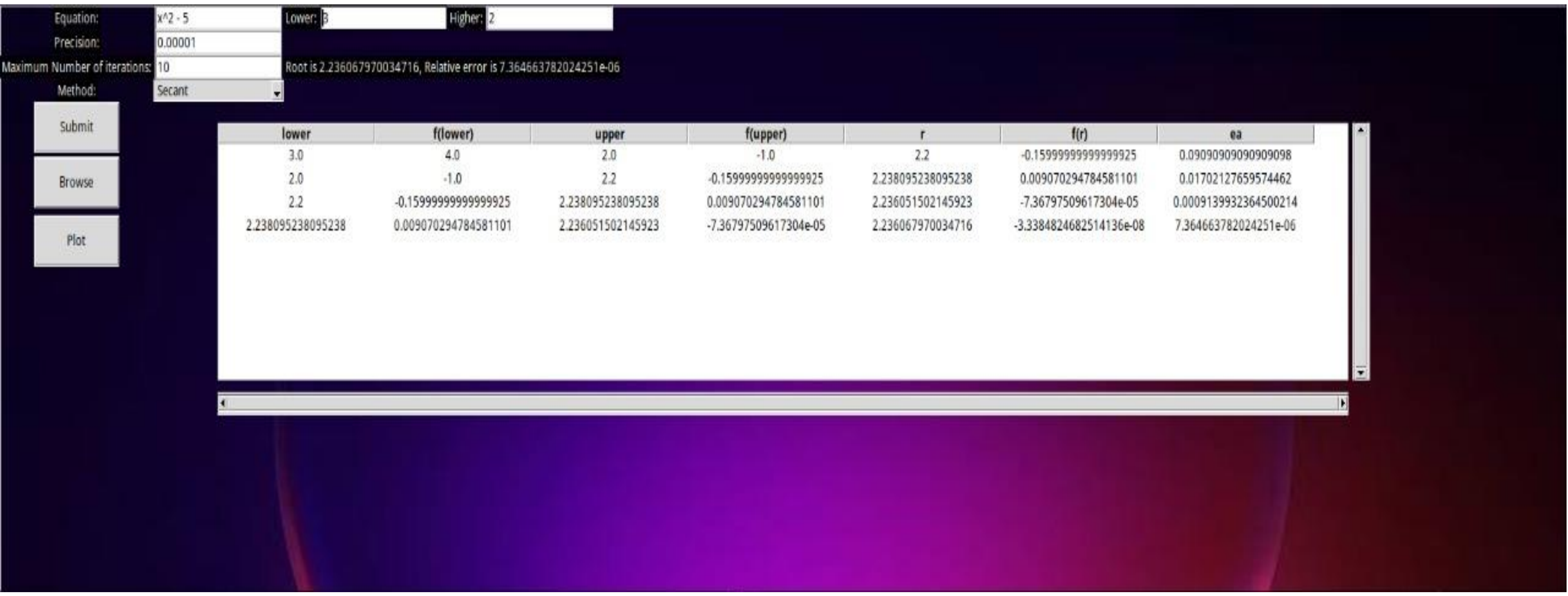
5. Secant Method

Explanation:

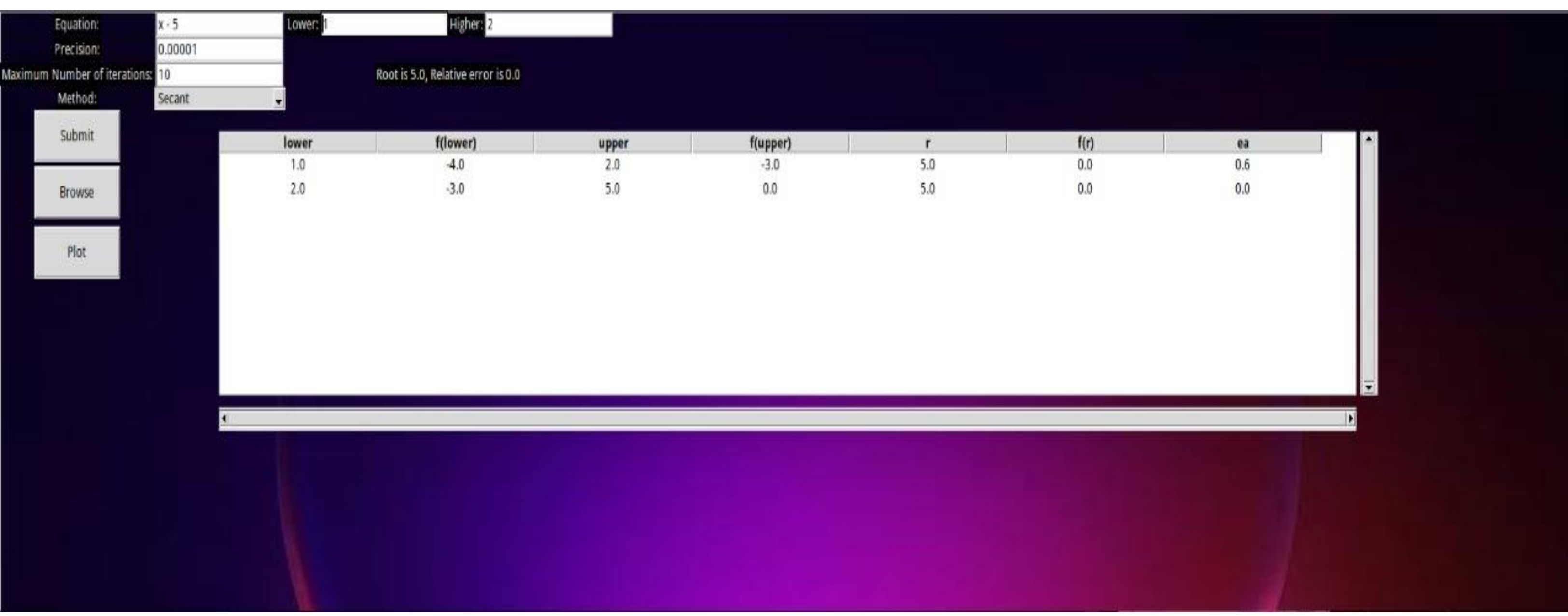
Same as Newton Raphson but it uses approximation of derivative: f'(x) = (f(xi-1) – f(xi)) / (xi-1 - xi)

Examples:

#Iterations = 10:

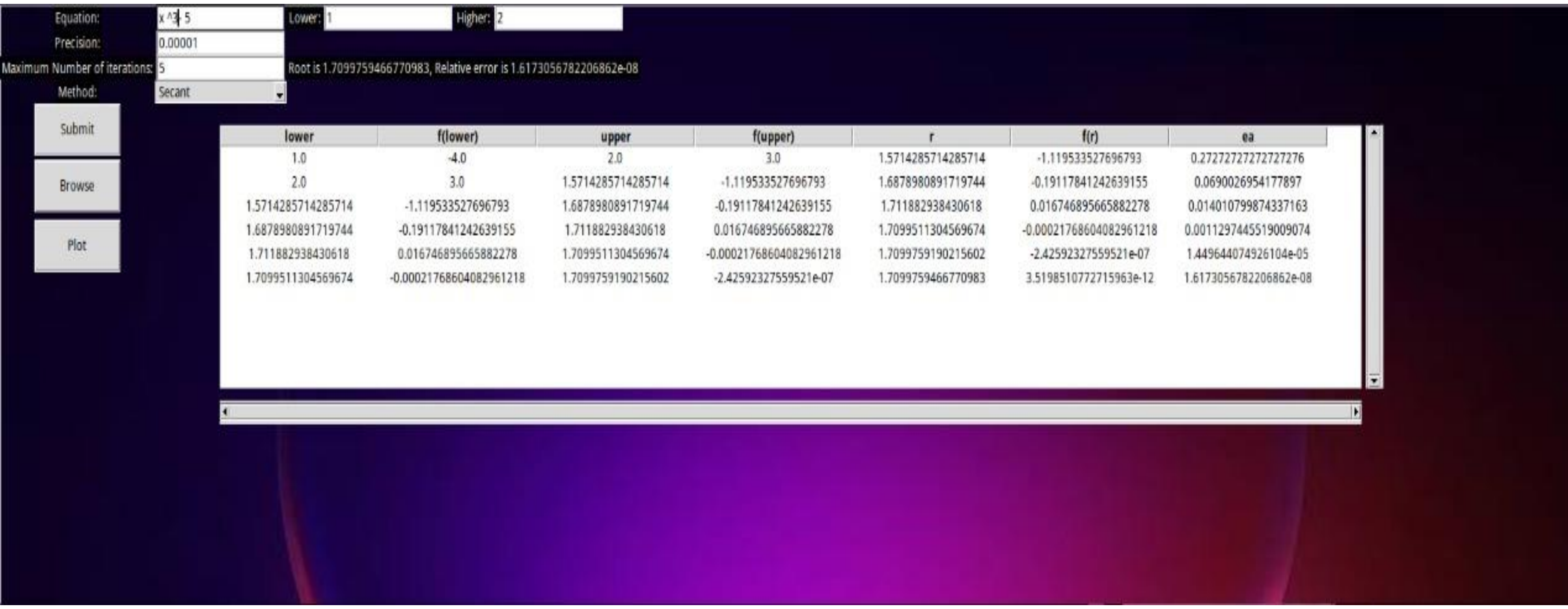


lower	f(lower)	upper	f(upper)	r	f(r)	ea
3.0	4.0	2.0	-1.0	2.2	-0.15999999999999925	0.09090909090909098
2.0	-1.0	2.2	-0.15999999999999925	2.238095238095238	0.009070294784581101	0.01702127659574462
2.2	-0.15999999999999925	2.238095238095238	0.009070294784581101	2.236051502145923	-7.36797509617304e-05	0.0009139932364500214
2.238095238095238	0.009070294784581101	2.236051502145923	-7.36797509617304e-05	2.236067970034716	-3.3384824682514136e-08	7.364663782024251e-06



lower	f(lower)	upper	f(upper)	r	f(r)	ea
1.0	-4.0	2.0	-3.0	5.0	0.0	0.6
2.0	-3.0	5.0	0.0	5.0	0.0	0.0

#Iterations = 5:

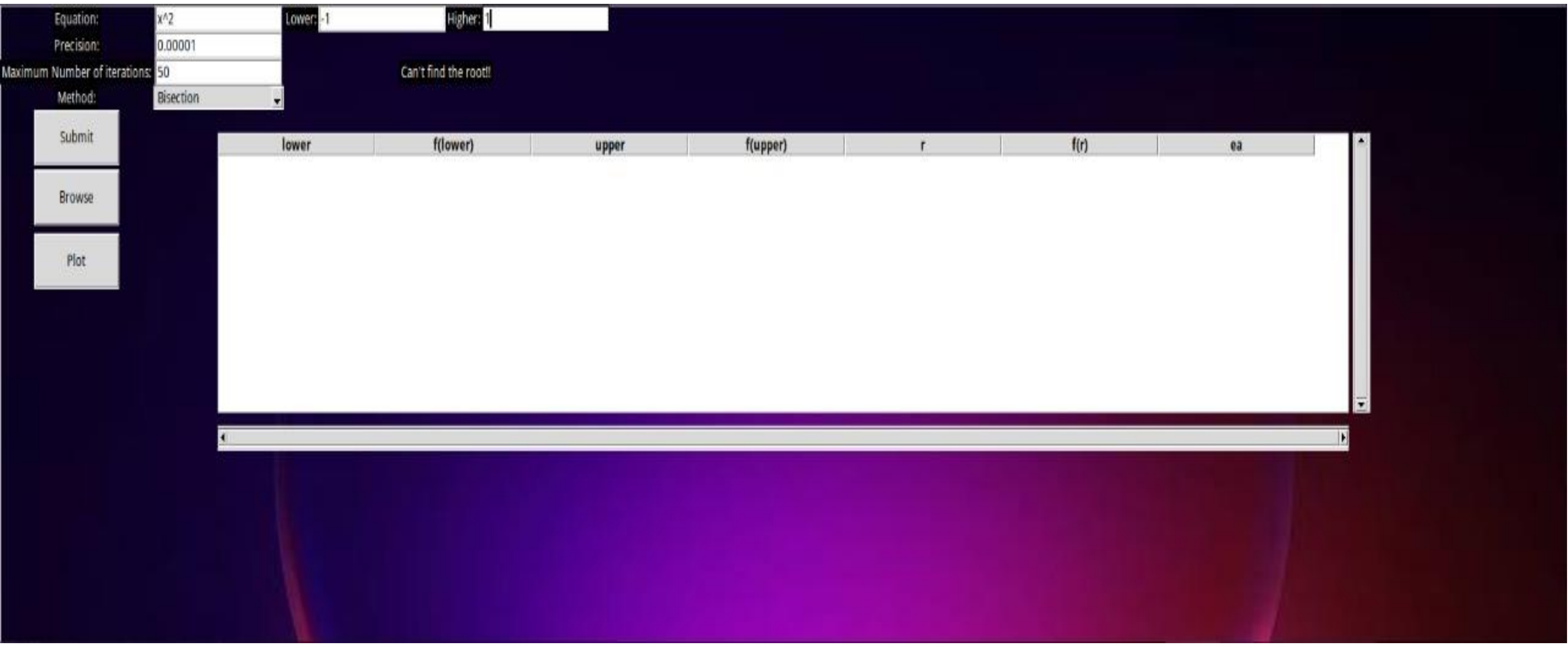


lower	f(lower)	upper	f(upper)	r	f(r)	ea
1.0	-4.0	2.0	3.0	1.5714285714285714	-1.119533527696793	0.27272727272727276
2.0	3.0	1.5714285714285714	-1.119533527696793	1.6878980891719744	-0.19117841242639155	0.0690026954177897
1.5714285714285714	-1.119533527696793	1.6878980891719744	-0.19117841242639155	1.711882938430618	0.016746895665882278	0.014010799874337163
1.6878980891719744	-0.19117841242639155	1.711882938430618	0.016746895665882278	1.7099511304569674	-0.00021768604082961218	0.0011297445519009074
1.711882938430618	0.016746895665882278	1.7099511304569674	-0.00021768604082961218	1.7099759190215602	-2.42592327559521e-07	1.449644074926104e-05
1.7099511304569674	-0.00021768604082961218	1.7099759190215602	-2.42592327559521e-07	1.7099759466770983	3.5198510772715963e-12	1.6173056782206862e-08

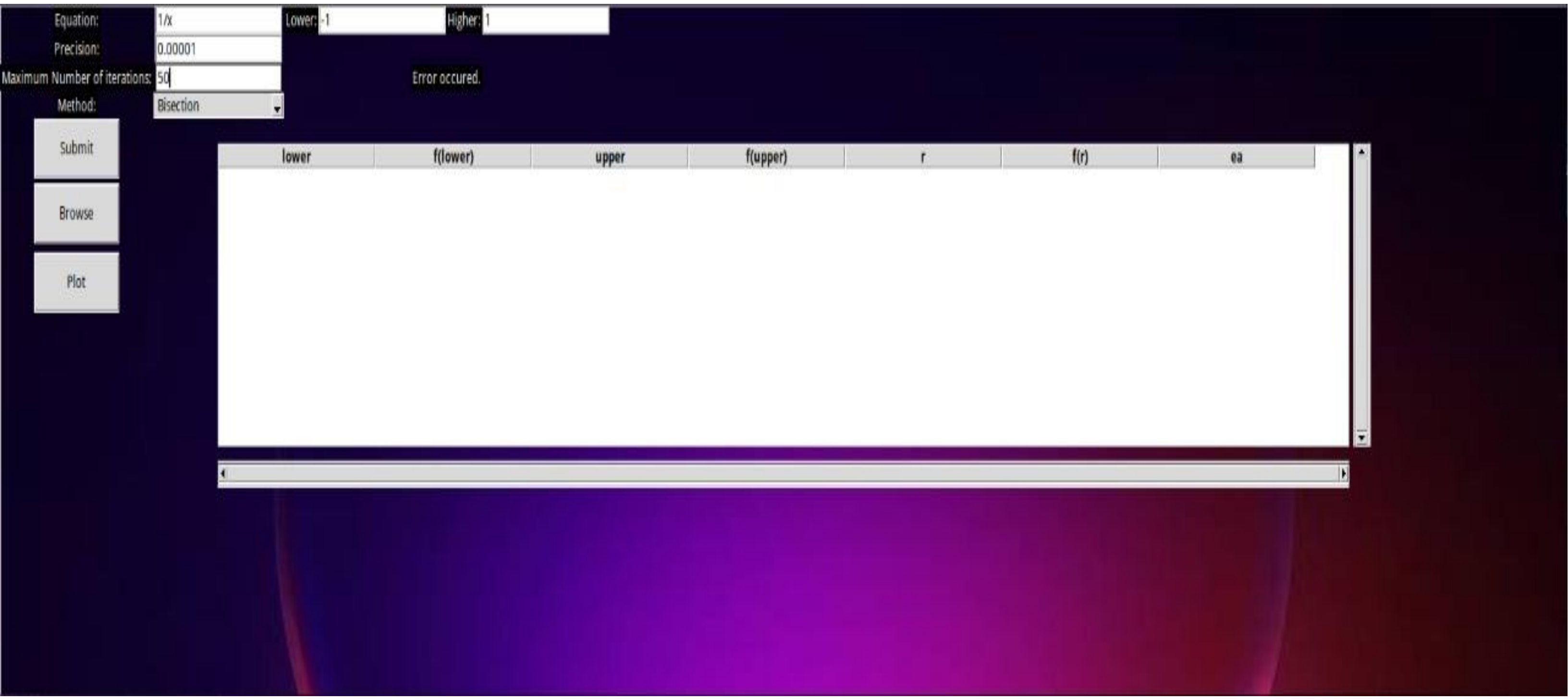
➢ Problematic functions (Pitfalls):

1. Bisection Method:

- Function doesn’t cross X-axis (tangent to X-axis)

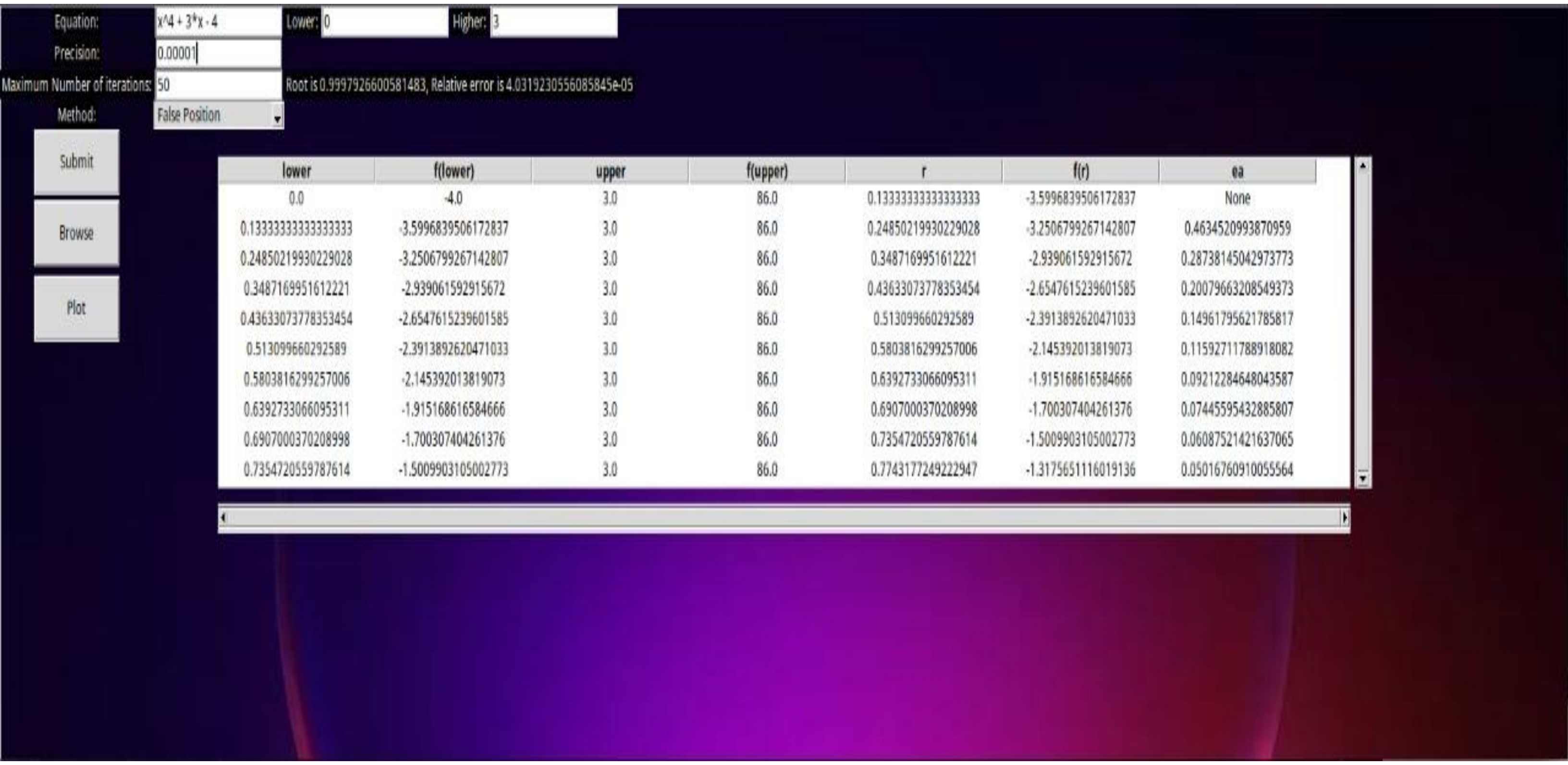


- F(x) = 1/x (discontinuous)

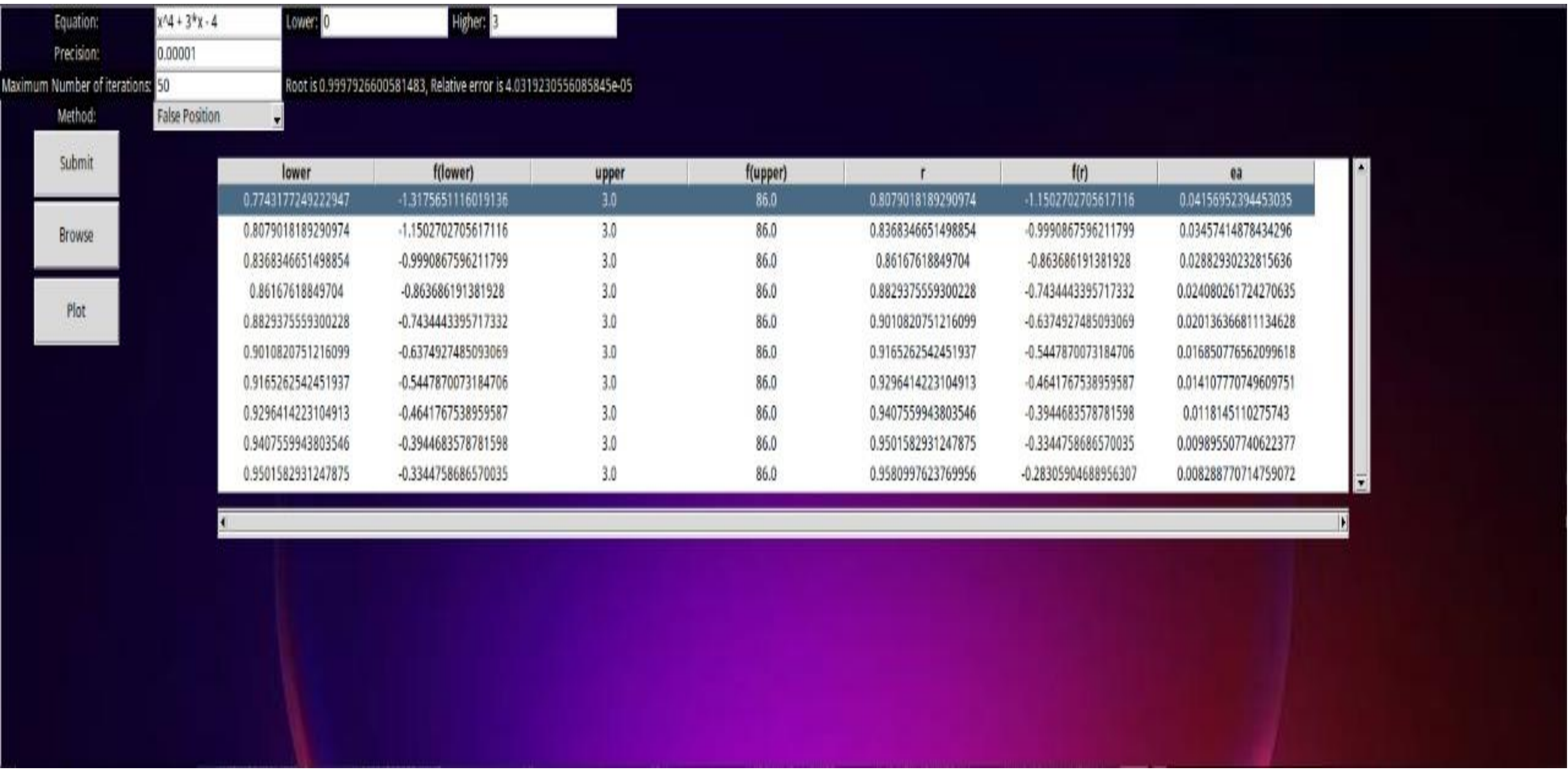


2. False Position Method:

- Steep function



lower	f(lower)	upper	f(upper)	r	f(r)	ea
0.0	-4.0	3.0	86.0	0.1333333333333333	-3.5996839506172837	None
0.1333333333333333	-3.5996839506172837	3.0	86.0	0.24850219930229028	-3.2506799267142807	0.4634520993870959
0.24850219930229028	-3.2506799267142807	3.0	86.0	0.3487169951612221	-2.939061592915672	0.28738145042973773
0.3487169951612221	-2.939061592915672	3.0	86.0	0.43633073778353454	-2.6547615239601585	0.20079663208549373
0.43633073778353454	-2.6547615239601585	3.0	86.0	0.513099660292589	-2.3913892620471033	0.14961795621785817
0.513099660292589	-2.3913892620471033	3.0	86.0	0.5803816299257006	-2.145392013819073	0.11592711788918082
0.5803816299257006	-2.145392013819073	3.0	86.0	0.6392733066095311	-1.915168616584666	0.09212284648043587
0.6392733066095311	-1.915168616584666	3.0	86.0	0.6907000370208998	-1.700307404261376	0.07445595432885807
0.6907000370208998	-1.700307404261376	3.0	86.0	0.7354720559787614	-1.5009903105002773	0.06087521421637065
0.7354720559787614	-1.5009903105002773	3.0	86.0	0.7743177249222947	-1.3175651116019136	0.05016760910055564



lower	f(lower)	upper	f(upper)	r	f(r)	ea
0.7743177249222947	-1.3175651116019136	3.0	86.0	0.8079018189290974	-1.1502702705617116	0.04156952394453035
0.8079018189290974	-1.1502702705617116	3.0	86.0	0.8368346651498854	-0.9990867596211799	0.03457414878434296
0.8368346651498854	-0.9990867596211799	3.0	86.0	0.86167618849704	-0.863686191381928	0.02882930232815636
0.86167618849704	-0.863686191381928	3.0	86.0	0.8829375559300228	-0.7434443395717332	0.024080261724270635
0.8829375559300228	-0.7434443395717332	3.0	86.0	0.9010820751216099	-0.6374927485093069	0.020136366811134628
0.9010820751216099	-0.6374927485093069	3.0	86.0	0.9165262542451937	-0.5447870073184706	0.016850776562099618
0.9165262542451937	-0.5447870073184706	3.0	86.0	0.9296414223104913	-0.4641767538959587	0.014107770749609751
0.9296414223104913	-0.4641767538959587	3.0	86.0	0.9407559943803546	-0.3944683578781598	0.0118145110275743
0.9407559943803546	-0.3944683578781598	3.0	86.0	0.9501582931247875	-0.3344758686570035	0.009895507740622377
0.9501582931247875	-0.3344758686570035	3.0	86.0	0.9580997623769956	-0.28305904688956307	0.008288770714759072

Equation: x^4 + 3*x - 4 Lower: 0 Higher: 3

Precision: 0.00001

Maximum Number of iterations: 50 Root is 0.9997926600581483, Relative error is 4.0319230556085845e-05

Method: False Position

Submit

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Plot

lower	f(lower)	upper	f(upper)	r	f(r)	ea
0.9580997623769956	-0.28305904688956307	3.0	86.0	0.9647983929249812	-0.23914927590849544	0.006943036594077788
0.9647983929249812	-0.23914927590849544	3.0	86.0	0.9704421984906255	-0.20176515980173138	0.005815705020270598
0.9704421984906255	-0.20176515980173138	3.0	86.0	0.9751926122830723	-0.17001996289046462	0.004871256952332065
0.9751926122830723	-0.17001996289046462	3.0	86.0	0.9791877103121575	-0.14312299127228956	0.004080012429702l125
0.9791877103121575	-0.14312299127228956	3.0	86.0	0.9825452006103584	-0.12037675458971053	0.003417135716621795
0.9825452006103584	-0.12037675458971053	3.0	86.0	0.9853651448608812	-0.10117140410521008	0.002861826669260 6847
0.9853651448608812	-0.10117140410521008	3.0	86.0	0.9877323999600841	-0.08497759836785646	0.00239665632037437
0.9877323999600841	-0.08497759836785646	3.0	86.0	0.9897187821686343	-0.0713386400461391	0.0020070167853111776
0.9897187821686343	-0.0713386400461391	3.0	86.0	0.9913849666437027	-0.059862472777012066	0.0016806634467226135
0.9913849666437027	-0.059862472777012066	3.0	86.0	0.9927821413462747	-0.05021392708974792	0.001407332630578347

Equation: x^4 + 3*x - 4 Lower: 0 Higher: 3

Precision: 0.00001

Maximum Number of iterations: 50 Root is 0.9997926600581483, Relative error is 4.0319230556085845e-05

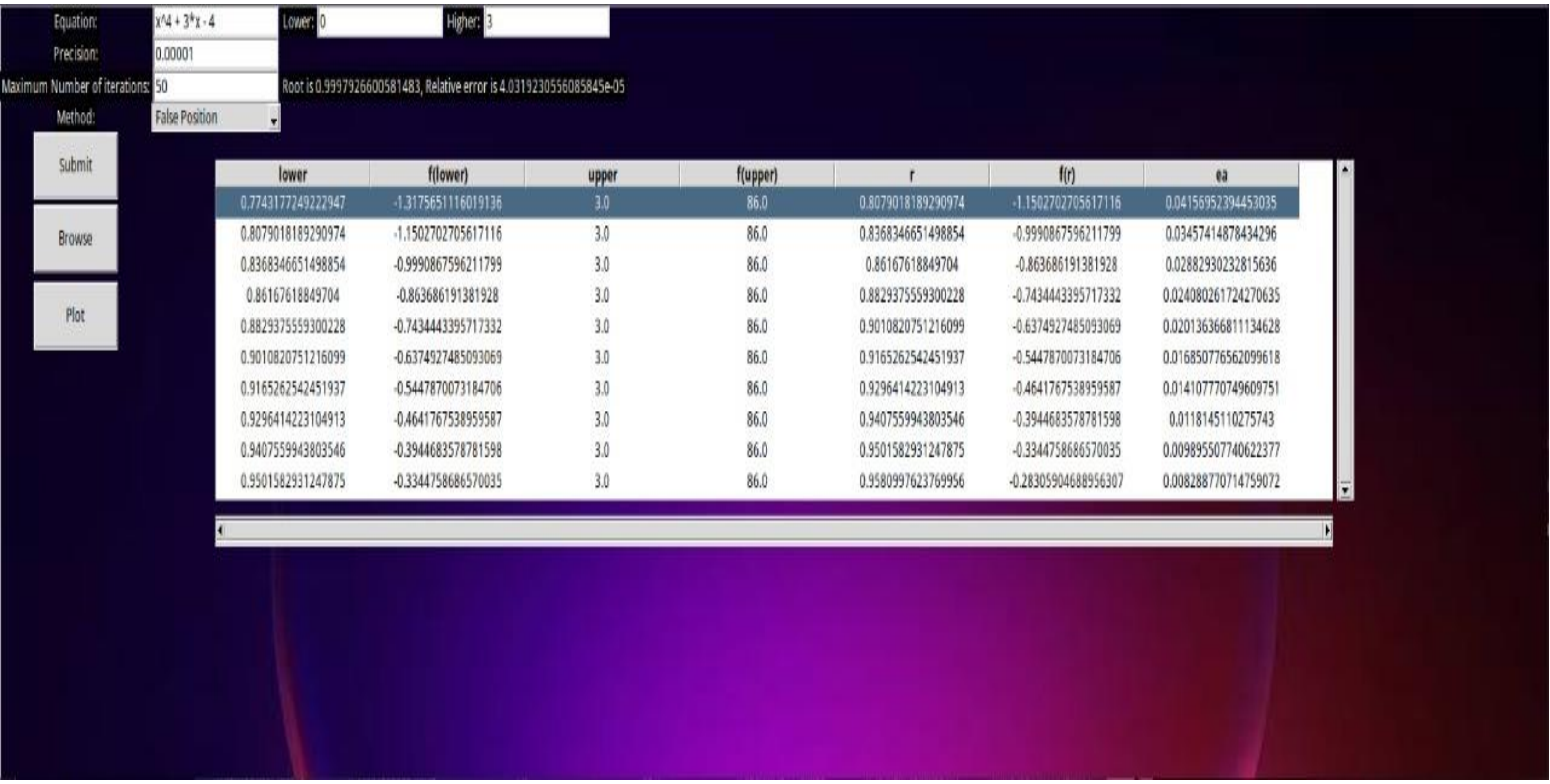
Method: False Position

Submit

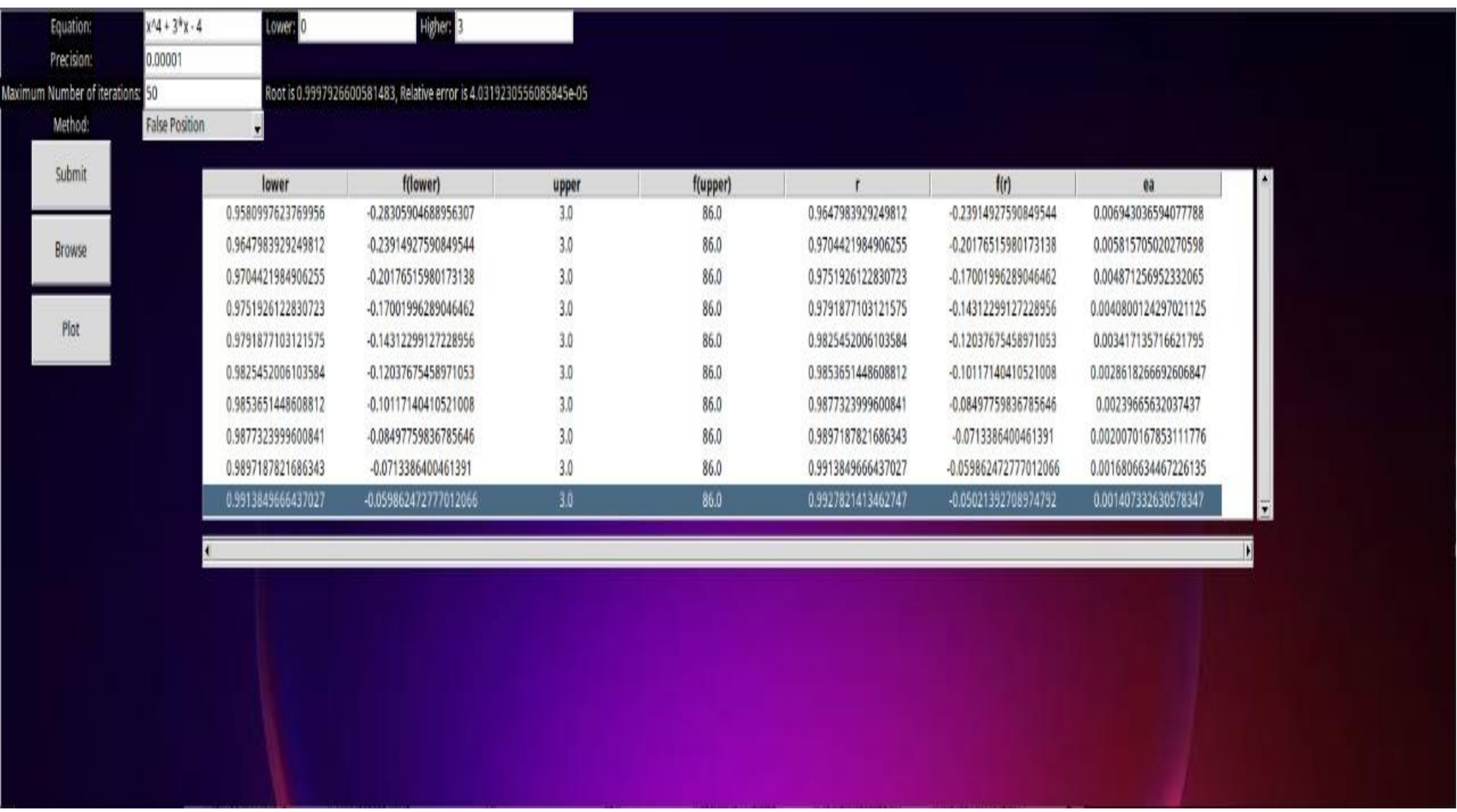
Browse

Plot

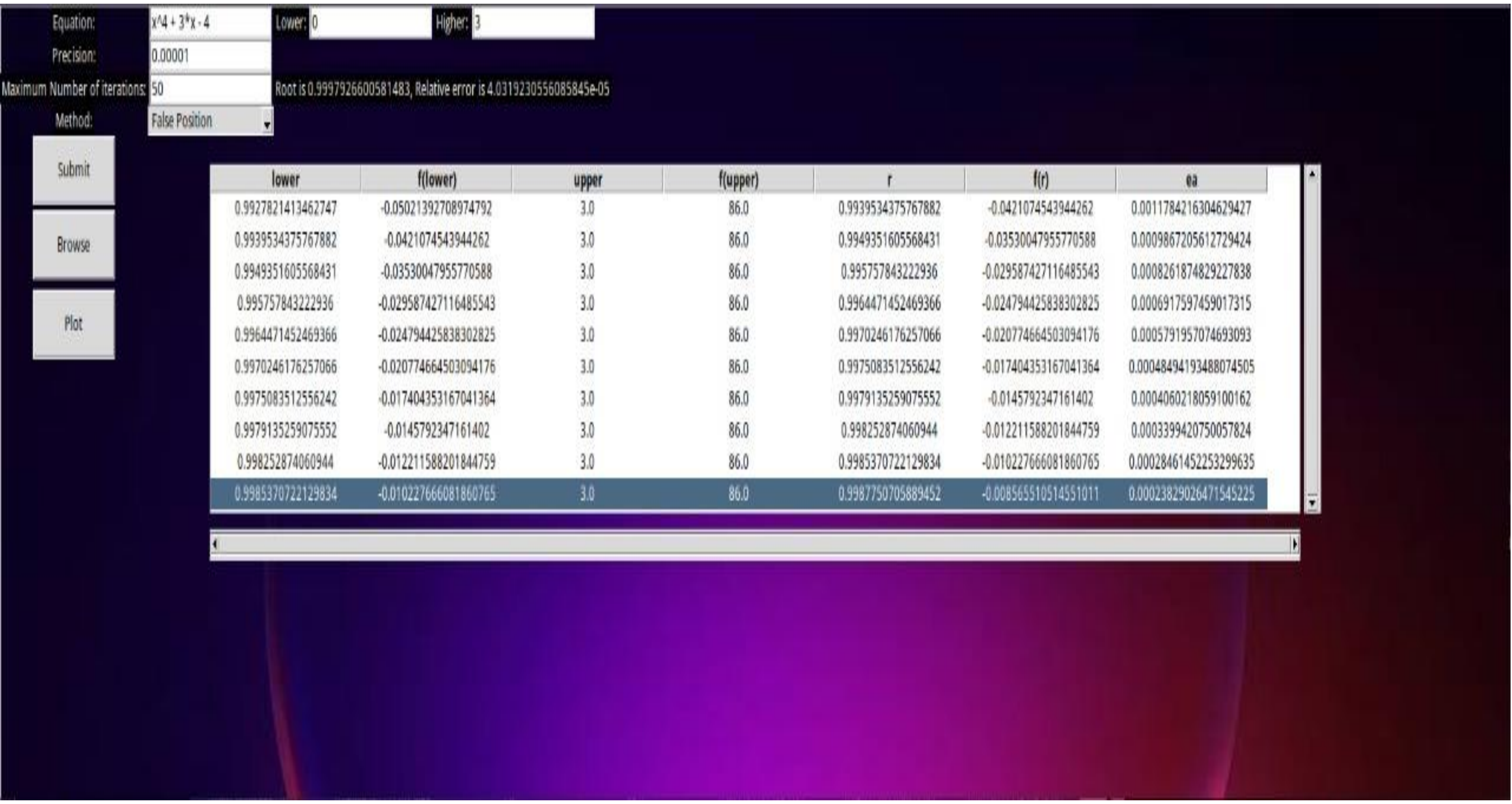
lower	f(lower)	upper	f(upper)	r	f(r)	ea
0.9927821413462747	-0.05021392708974792	3.0	86.0	0.9939534375767882	-0.0421074543944262	0.0011784216304629427
0.9939534375767882	-0.0421074543944262	3.0	86.0	0.9949351605568431	-0.03530047955770588	0.0009867205612729424
0.9949351605568431	-0.03530047955770588	3.0	86.0	0.995757843222936	-0.029587427116485543	0.0008261874829227838
0.995757843222936	-0.029587427116485543	3.0	86.0	0.9964471452469366	-0.024794425838302825	0.0006917597459017315
0.9964471452469366	-0.024794425838302825	3.0	86.0	0.9970246176257066	-0.020774664503094176	0.0005791957074693093
0.9970246176257066	-0.020774664503094176	3.0	86.0	0.9975083512556242	-0.017404353167041364	0.00048494193488074505
0.9975083512556242	-0.017404353167041364	3.0	86.0	0.9979135259075552	-0.0145792347161402	0.0004060218059100162
0.9979135259075552	-0.0145792347161402	3.0	86.0	0.998252874060944	-0.012211588201844759	0.0003399420750057824
0.998252874060944	-0.012211588201844759	3.0	86.0	0.9985370722129834	-0.010227666081860765	0.00028461452253299635
0.9985370722129834	-0.010227666081860765	3.0	86.0	0.9987750705889452	-0.008565510514551011	0.00023829026471545225



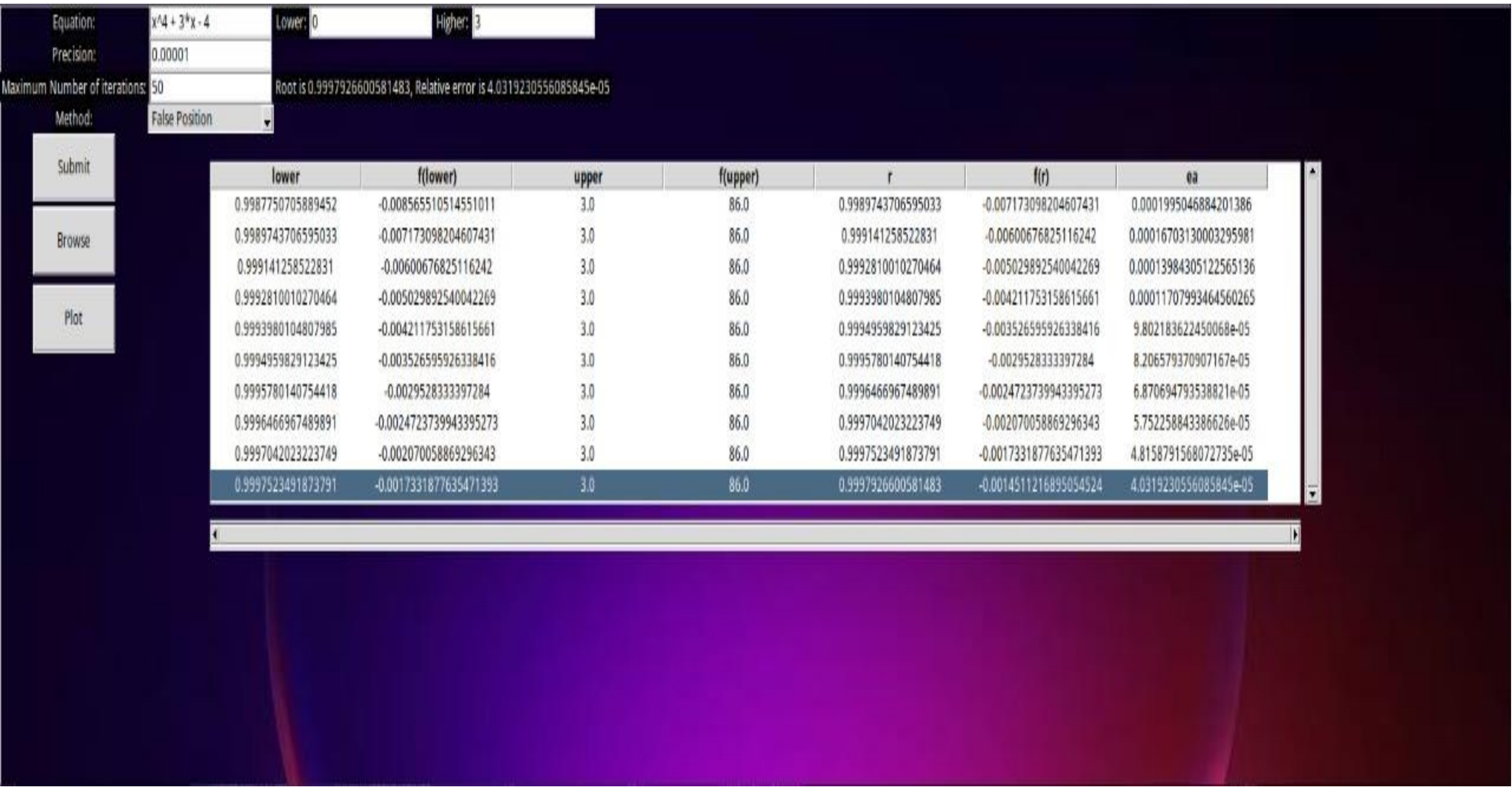
lower	f(lower)	upper	f(upper)	r	f(r)	ea
0.7743177249222947	-1.3175651116019136	3.0	86.0	0.8079018189290974	-1.1502702705617116	0.04156952394453035
0.8079018189290974	-1.1502702705617116	3.0	86.0	0.8368346651498854	-0.9990867596211799	0.03457414878434296
0.8368346651498854	-0.9990867596211799	3.0	86.0	0.86167618849704	-0.863686191381928	0.02882930232815636
0.86167618849704	-0.863686191381928	3.0	86.0	0.8829375559300228	-0.7434443395717332	0.024080261724270635
0.8829375559300228	-0.7434443395717332	3.0	86.0	0.9010820751216099	-0.6374927485093069	0.020136366811134628
0.9010820751216099	-0.6374927485093069	3.0	86.0	0.9165262542451937	-0.5447870073184706	0.016850776562099618
0.9165262542451937	-0.5447870073184706	3.0	86.0	0.9296414223104913	-0.4641767538959587	0.014107770749609751
0.9296414223104913	-0.4641767538959587	3.0	86.0	0.9407559943803546	-0.3944683578781598	0.0118145110275743
0.9407559943803546	-0.3944683578781598	3.0	86.0	0.9501582931247875	-0.3344758686570035	0.009895507740622377
0.9501582931247875	-0.3344758686570035	3.0	86.0	0.9580997623769956	-0.28305904688956307	0.008288770714759072



lower	f(lower)	upper	f(upper)	r	f(r)	ea
0.9580997623769956	-0.28305904688956307	3.0	86.0	0.9647983929249812	-0.23914927590849544	0.006943036594077788
0.9647983929249812	-0.23914927590849544	3.0	86.0	0.9704421984906255	-0.20176515980173138	0.005815705020270598
0.9704421984906255	-0.20176515980173138	3.0	86.0	0.9751926122830723	-0.17001996289046462	0.004871256952332065
0.9751926122830723	-0.17001996289046462	3.0	86.0	0.9791877103121575	-0.14312299127228956	0.0040800124297021125
0.9791877103121575	-0.14312299127228956	3.0	86.0	0.9825452006103584	-0.12037675458971053	0.003417135716621795
0.9825452006103584	-0.12037675458971053	3.0	86.0	0.9853651448608812	-0.10117140410521008	0.0028618266692606847
0.9853651448608812	-0.10117140410521008	3.0	86.0	0.9877323999600841	-0.08497759836785646	0.00239665632037437
0.9877323999600841	-0.08497759836785646	3.0	86.0	0.9897187821686343	-0.0713386400461391	0.0020070167853111776
0.9897187821686343	-0.0713386400461391	3.0	86.0	0.9913849666437027	-0.059862472777012066	0.0016806634467226135
0.9913849666437027	-0.059862472777012066	3.0	86.0	0.9927821413462747	-0.05021392708974792	0.001407332630578347

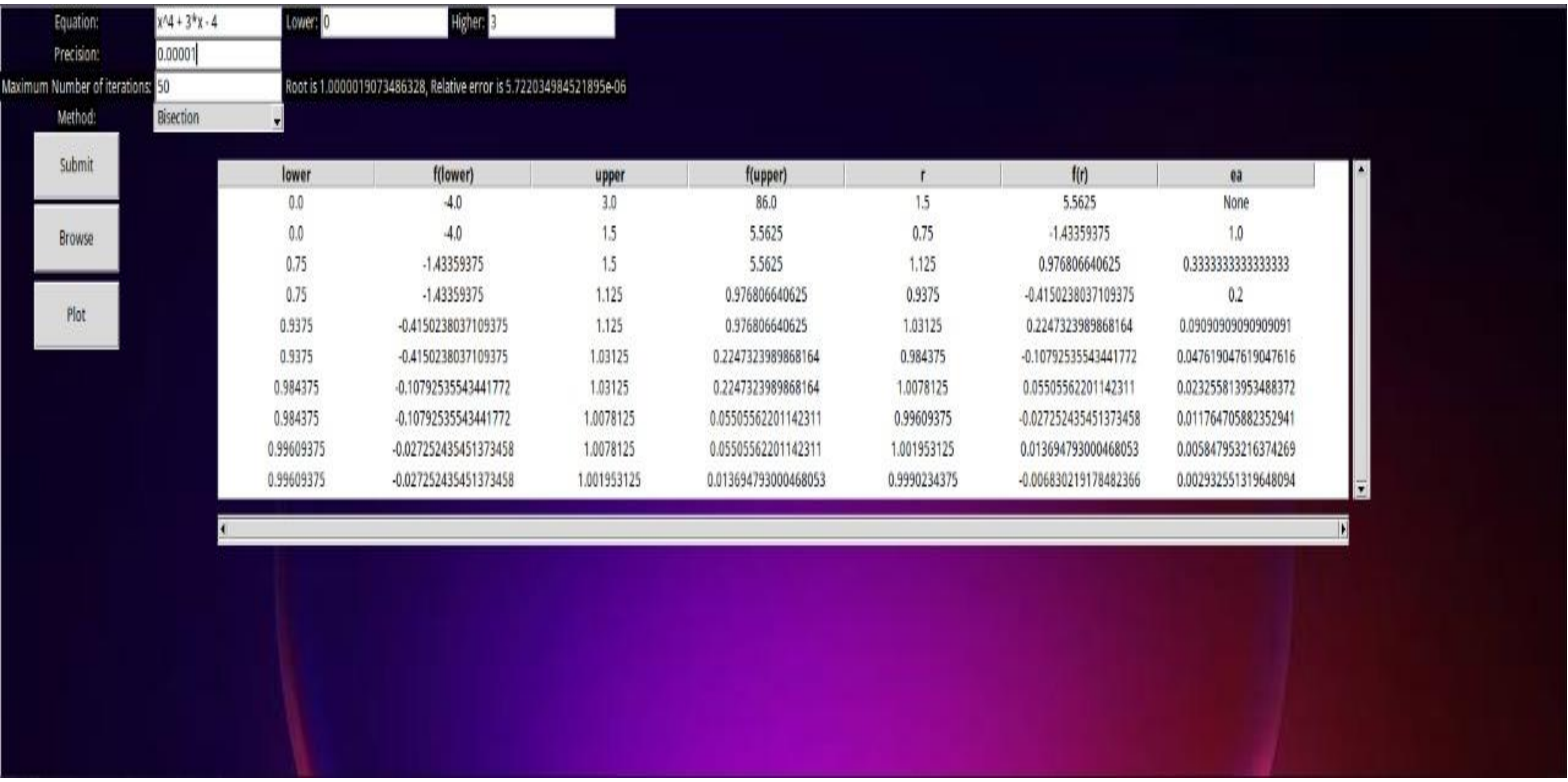


lower	f(lower)	upper	f(upper)	r	f(r)	ea
0.9927821413462747	-0.05021392708974792	3.0	86.0	0.9939534375767882	-0.0421074543944262	0.0011784216304629427
0.9939534375767882	-0.0421074543944262	3.0	86.0	0.9949351605568431	-0.03530047955770588	0.0009867205612729424
0.9949351605568431	-0.03530047955770588	3.0	86.0	0.995757843222936	-0.029587427116485543	0.0008261874829227838
0.995757843222936	-0.029587427116485543	3.0	86.0	0.9964471452469366	-0.024794425838302825	0.0006917597459017315
0.9964471452469366	-0.024794425838302825	3.0	86.0	0.9970246176257066	-0.020774664503094176	0.0005791957074693093
0.9970246176257066	-0.020774664503094176	3.0	86.0	0.9975083512556242	-0.017404353167041364	0.00048494193488074505
0.9975083512556242	-0.017404353167041364	3.0	86.0	0.9979135259075552	-0.0145792347161402	0.0004060218059100162
0.9979135259075552	-0.0145792347161402	3.0	86.0	0.998252874060944	-0.012211588201844759	0.0003399420750057824
0.998252874060944	-0.012211588201844759	3.0	86.0	0.9985370722129834	-0.010227666081860765	0.00028461452253299635
0.9985370722129834	-0.010227666081860765	3.0	86.0	0.9987750705889452	-0.008565510514551011	0.00023829026471545225

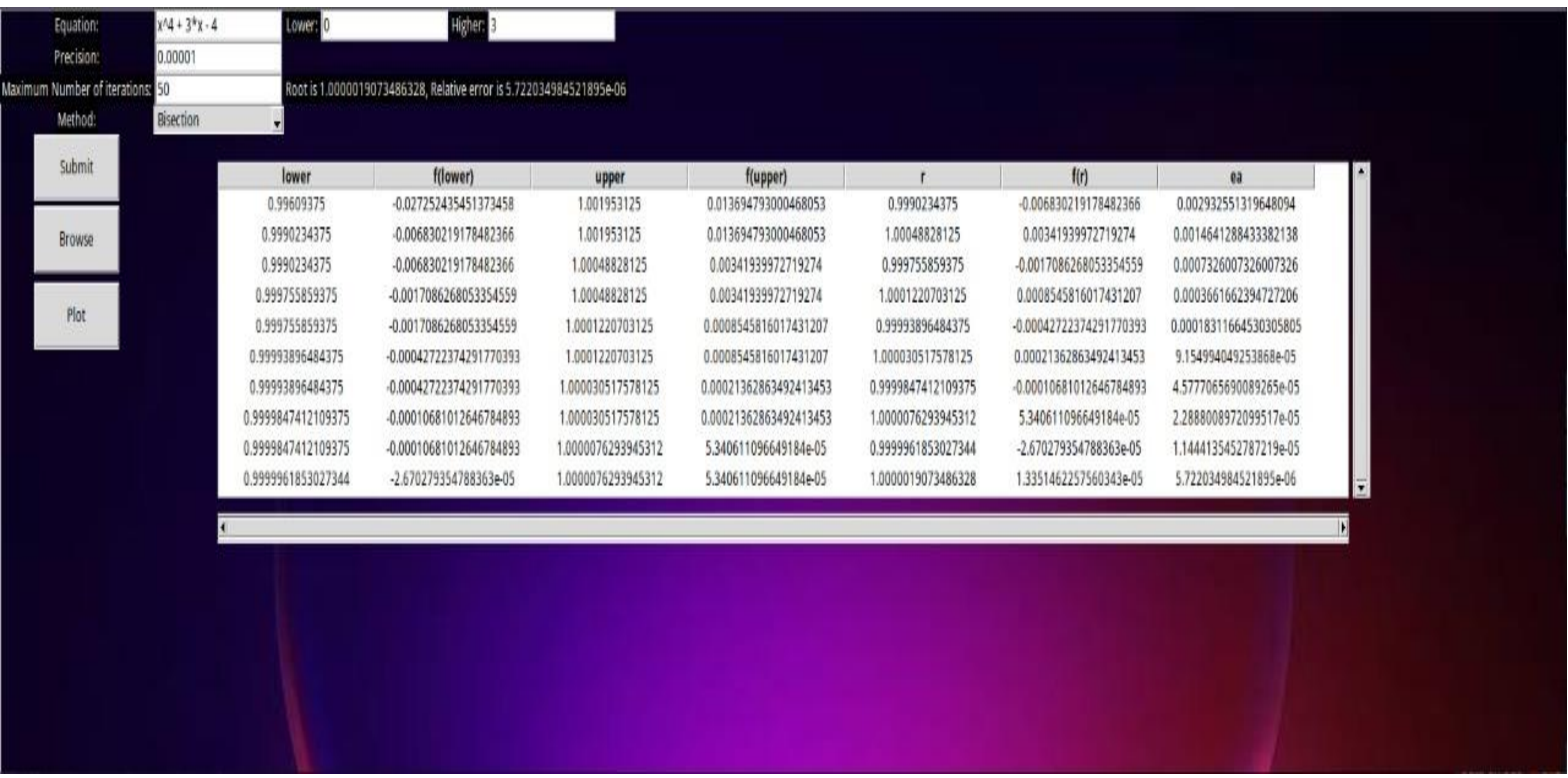


lower	f(lower)	upper	f(upper)	r	f(r)	ea
0.9987750705889452	-0.008565510514551011	3.0	86.0	0.9989743706595033	-0.007173098204607431	0.0001995046884201386
0.9989743706595033	-0.007173098204607431	3.0	86.0	0.999141258522831	-0.00600676825116242	0.00016703130003295981
0.999141258522831	-0.00600676825116242	3.0	86.0	0.9992810010270464	-0.005029892540042269	0.00013984305122565136
0.9992810010270464	-0.005029892540042269	3.0	86.0	0.9993980104807985	-0.004211753158615661	0.00011707993464560265
0.9993980104807985	-0.004211753158615661	3.0	86.0	0.9994959829123425	-0.003526595926338416	9.802183622450068e-05
0.9994959829123425	-0.003526595926338416	3.0	86.0	0.9995780140754418	-0.0029528333397284	8.206579370907167e-05
0.9995780140754418	-0.0029528333397284	3.0	86.0	0.9996466967489891	-0.0024723739943395273	6.870694793538821e-05
0.9996466967489891	-0.0024723739943395273	3.0	86.0	0.9997042023223749	-0.002070058869296343	5.752258843386626e-05
0.9997042023223749	-0.002070058869296343	3.0	86.0	0.9997523491873791	-0.0017331877635471393	4.8158791568072735e-05
0.9997523491873791	-0.0017331877635471393	3.0	86.0	0.9997926600581483	-0.0014511216895054524	4.0319230556085845e-05

- **Note:**
  Bisection is faster than false position in case of steep function



lower	f(lower)	upper	f(upper)	r	f(r)	ea
0.0	-4.0	3.0	86.0	1.5	5.5625	None
0.0	-4.0	1.5	5.5625	0.75	-1.43359375	1.0
0.75	-1.43359375	1.5	5.5625	1.125	0.976806640625	0.3333333333333333
0.75	-1.43359375	1.125	0.976806640625	0.9375	-0.4150238037109375	0.2
0.9375	-0.4150238037109375	1.125	0.976806640625	1.03125	0.2247323989868164	0.09090909090909091
0.9375	-0.4150238037109375	1.03125	0.2247323989868164	0.984375	-0.10792535543441772	0.047619047619047616
0.984375	-0.10792535543441772	1.03125	0.2247323989868164	1.0078125	0.05505562201142311	0.023255813953488372
0.984375	-0.10792535543441772	1.0078125	0.05505562201142311	0.99609375	-0.027252435451373458	0.011764705882352941
0.99609375	-0.027252435451373458	1.0078125	0.05505562201142311	1.001953125	0.013694793000468053	0.005847953216374269
0.99609375	-0.027252435451373458	1.001953125	0.013694793000468053	0.9990234375	-0.006830219178482366	0.002932551319648094



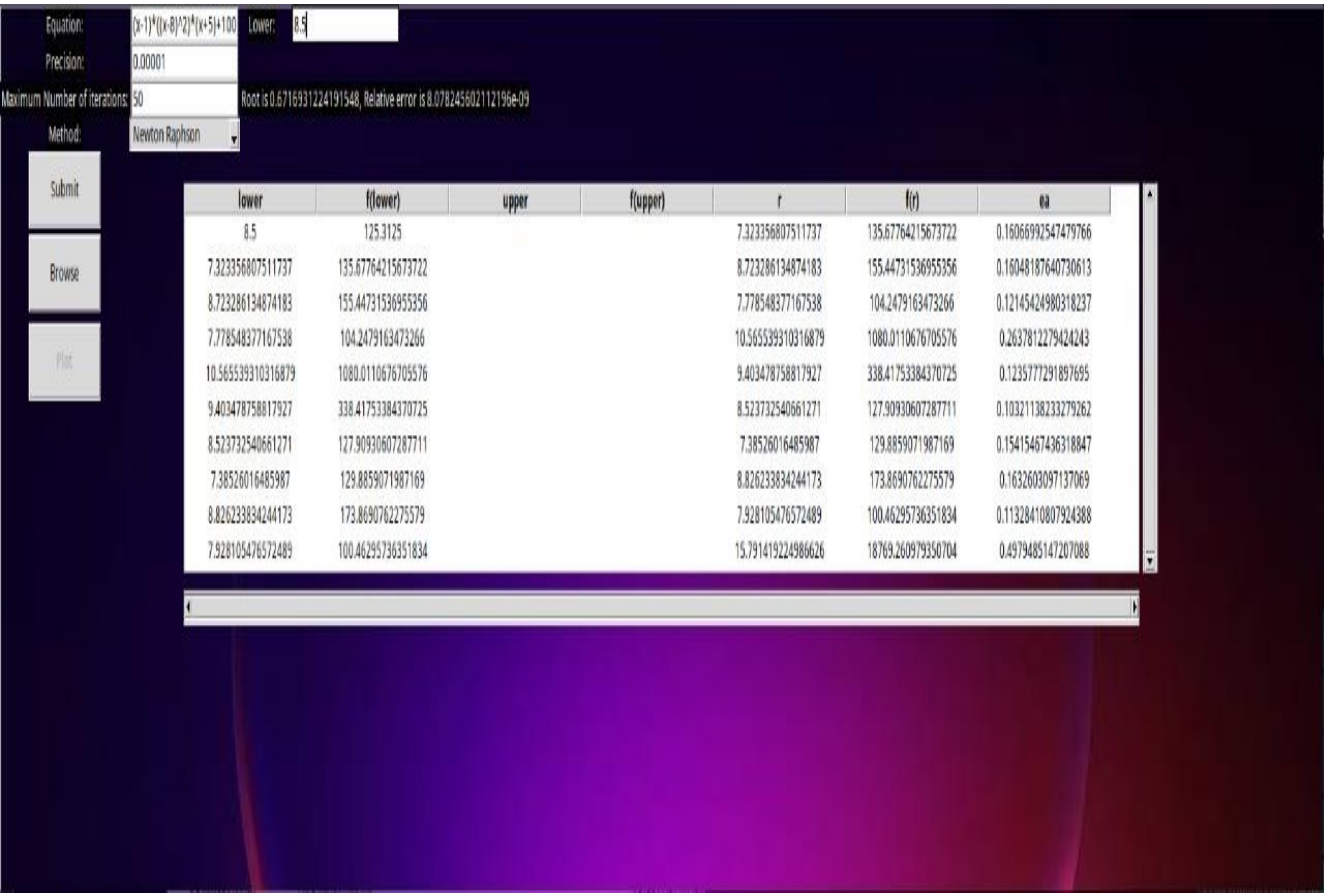
lower	f(lower)	upper	f(upper)	r	f(r)	ea
0.99609375	-0.027252435451373458	1.001953125	0.013694793000468053	0.9990234375	-0.006830219178482366	0.002932551319648094
0.9990234375	-0.006830219178482366	1.001953125	0.013694793000468053	1.00048828125	0.00341939972719274	0.0014641288433382138
0.9990234375	-0.006830219178482366	1.00048828125	0.00341939972719274	0.999755859375	-0.0017086268053354559	0.0007326007326007326
0.999755859375	-0.0017086268053354559	1.00048828125	0.00341939972719274	1.0001220703125	0.0008545816017431207	0.0003661662394727206
0.999755859375	-0.0017086268053354559	1.0001220703125	0.0008545816017431207	0.99993896484375	-0.00042722374291770393	0.00018311664530305805
0.99993896484375	-0.00042722374291770393	1.0001220703125	0.0008545816017431207	1.000030517578125	0.00021362863492413453	9.154994049253868e-05
0.99993896484375	-0.00042722374291770393	1.000030517578125	0.00021362863492413453	0.9999847412109375	-0.00010681012646784893	4.5777065690089265e-05
0.9999847412109375	-0.00010681012646784893	1.000030517578125	0.00021362863492413453	1.0000076293945312	5.340611096649184e-05	2.2888008972099517e-05
0.9999847412109375	-0.00010681012646784893	1.0000076293945312	5.340611096649184e-05	0.9999961853027344	-2.670279354788363e-05	1.1444135452787219e-05
0.9999961853027344	-2.670279354788363e-05	1.0000076293945312	5.340611096649184e-05	1.0000019073486328	1.3351462257560343e-05	5.722034984521895e-06

3. Fixed Point Iteration Method:

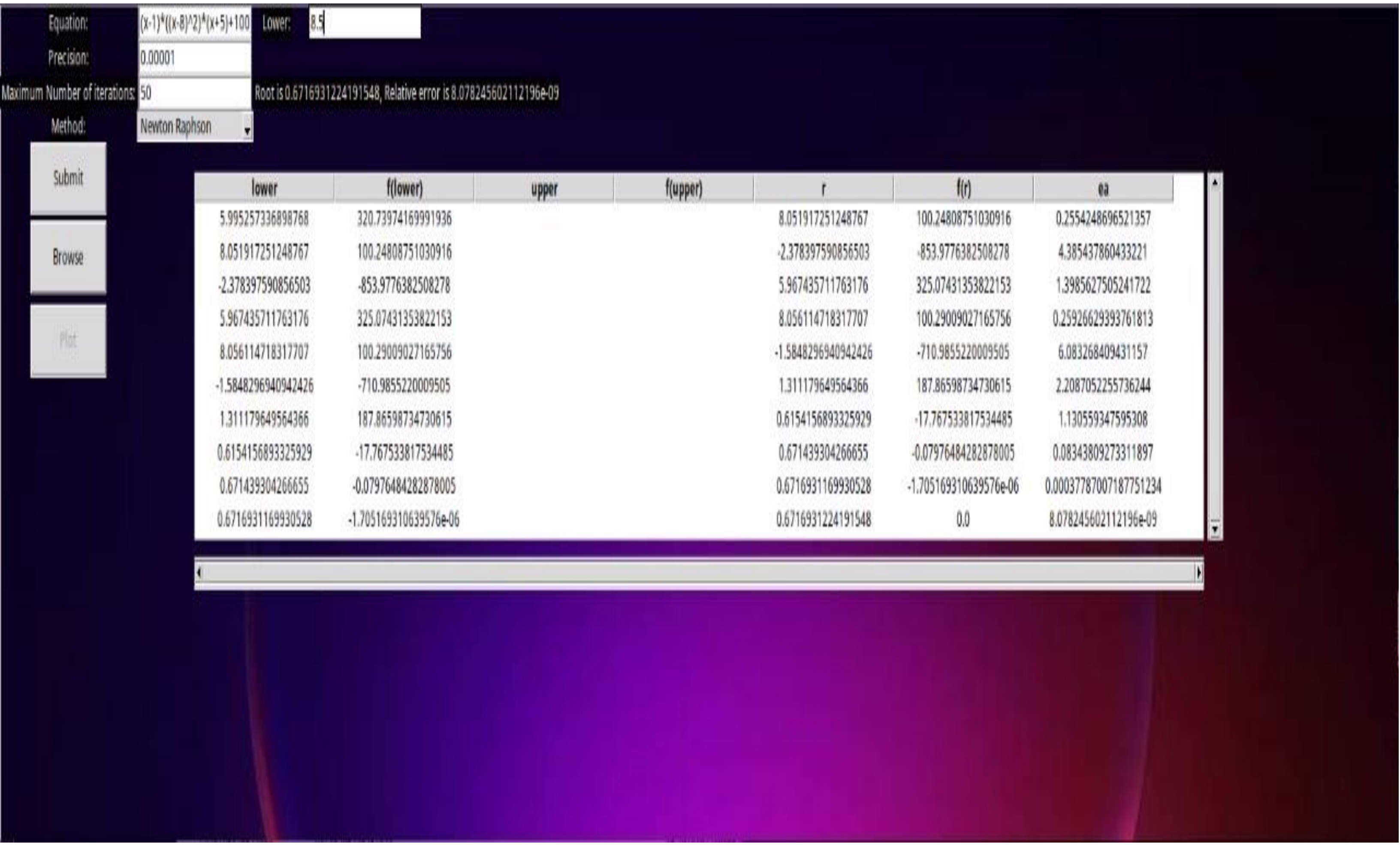
- If g(x) is chosen such that: $|g'(c)| > 1$, this method will diverge.

4. Newton Raphson Method:

- If $f'(x) = 0$, which leads to division by zero.



lower	f(lower)	upper	f(upper)	r	f(r)	ea
8.5	125.3125			7.323356807511737	135.67764215673722	0.16066992547479766
7.323356807511737	135.67764215673722			8.723286134874183	155.44731536955356	0.16048187640730613
8.723286134874183	155.44731536955356			7.778548377167538	104.2479163473266	0.12145424980318237
7.778548377167538	104.2479163473266			10.565539310316879	1080.0110676705576	0.263781227942424 3
10.565539310316879	1080.0110676705576			9.403478758817927	338.41753384370725	0.1235777291897695
9.403478758817927	338.41753384370725			8.523732540661271	127.90930607287711	0.10321138233279262
8.523732540661271	127.90930607287711			7.38526016485987	129.8859071987169	0.15415467436318847
7.38526016485987	129.8859071987169			8.826233834244173	173.8690762275579	0.1632603097137069
8.826233834244173	173.8690762275579			7.928105476572489	100.46295736351834	0.11328410807924388
7.928105476572489	100.46295736351834			15.791419224986626	18769.260979350704	0.4979485147207088



lower	f(lower)	upper	f(upper)	r	f(r)	ea
5.995257336898768	320.73974169991936			8.051917251248767	100.24808751030916	0.2554248696521357
8.051917251248767	100.24808751030916			-2.378397590856503	-853.9776382508278	4.385437860433221
-2.378397590856503	-853.9776382508278			5.967435711763176	325.07431353822153	1.3985627505241722
5.967435711763176	325.07431353822153			8.056114718317707	100.29009027165756	0.25926629393761813
8.056114718317707	100.29009027165756			-1.5848296940942426	-710.9855220009505	6.083268409431157
-1.5848296940942426	-710.9855220009505			1.311179649564366	187.86598734730615	2.2087052255736244
1.311179649564366	187.86598734730615			0.6154156893325929	-17.767533817534485	1.130559347595308
0.6154156893325929	-17.767533817534485			0.671439304266655	-0.07976484282878005	0.08343809273311897
0.671439304266655	-0.07976484282878005			0.6716931169930528	-1.705169310639576e-06	0.00037787007187751234
0.6716931169930528	-1.705169310639576e-06			0.6716931224191548	0.0	8.078245602112196e-09

- **Sample Runs:**
  - ✓ As shown above.