# Numerical Analysis

# Project 2: Numerical Methods

## ➢ Pseudocode:

## 1. Gauss Elimination Method:

```
def gaussElimination(a, x, n):
    # Forward Elimination
    for i in range(n):
        if a[i][i] == 0.0:
            root_label.config(text="Division by zero")
            root_label.grid(row=2, column=5)
            return None

        for j in range(i + 1, n):
            ratio = a[j][i] / a[i][i]

            for k in range(n + 1):
                a[j][k] = a[j][k] - ratio * a[i][k]

    # Back Substitution
    x[n - 1] = a[n - 1][n] / a[n - 1][n - 1]

    for i in range(n - 2, -1, -1):
        x[i] = a[i][n]

        for j in range(i + 1, n):
            x[i] = x[i] - a[i][j] * x[j]

        x[i] = x[i] / a[i][i]
    return x
```

## 2. **LU Decomposition Method:**

```
def LUdecomposition(a, x, n):
    L = np.zeros((n, n))
    U = np.zeros((n, n))
    for i in range(n):
        for j in range(n):
            U[i][j] = a[i][j]
            if i == j:
                L[i][j] = 1
            else:
                L[i][j] = 0

    # Forward Elimination
    for i in range(n):
        if U[i][i] == 0.0:
            root_label.config(text="Division by zero")
            root_label.grid(row=2, column=5)
            return None

        for j in range(i + 1, n):
            ratio = U[j][i] / U[i][i]
            L[j][i] = ratio

            for k in range(n):
                U[j][k] = U[j][k] - ratio * U[i][k]

    # Forward Substitution
    y = np.zeros(n)
    y[0] = a[0][n]

    for i in range(1, n):
        y[i] = a[i][n]

        for j in range(i):
            y[i] = y[i] - L[i][j] * y[j]

    # Back Substitution
    x[n - 1] = y[n - 1] / U[n - 1][n - 1]

    for i in range(n - 2, -1, -1):
        x[i] = y[i]

        for j in range(i + 1, n):
            x[i] = x[i] - U[i][j] * x[j]

        x[i] = x[i] / U[i][i]
    return x
```

## 3. **Gauss Jordan Method:**

```
def gaussJordan(a, x, n):
    for i in range(n):
        if a[i][i] == 0.0:
            root_label.config(text="Division by zero")
            root_label.grid(row=2, column=5)
            return None

        temp = a[i][i]
        for norm in range(n + 1):
            a[i][norm] = a[i][norm] / temp

        for j in range(n):
            if i == j:
                continue
            else:
                ratio = a[j][i]  # a[i][i] = 1 (so, no need to divide)

                for k in range(n + 1):
                    a[j][k] = a[j][k] - ratio * a[i][k]

    for i in range(n):
        x[i] = a[i][n]

    return x
```

## 4. **Gauss Seidel Method:**

```
def gaussSiedel(a, x, n, prec, iter, ea, old_x, iter_num=1):
    global iterVar
    for i in range(n):
        old_x[i] = x[i]
        eachIter[i].append(x[i])
        iterVar = iterVar + 1

    for j in range(n):
        # temp variable b to store b[j]
        b = a[j][n]

        for i in range(n):
            if (j != i):
                b -= a[j][i] * x[i]

        x[j] = b / a[j][j]
    for i in range(n):
        ea[i] = abs((x[i] - old_x[i]) / x[i])
        eachEa[i].append(ea[i])
    if max(ea) <= prec or iter == 1:

        return x, ea, iter_num
    else:
        return gaussSiedel(a, x, n, prec, iter - 1, ea, old_x, iter_num + 1)
```

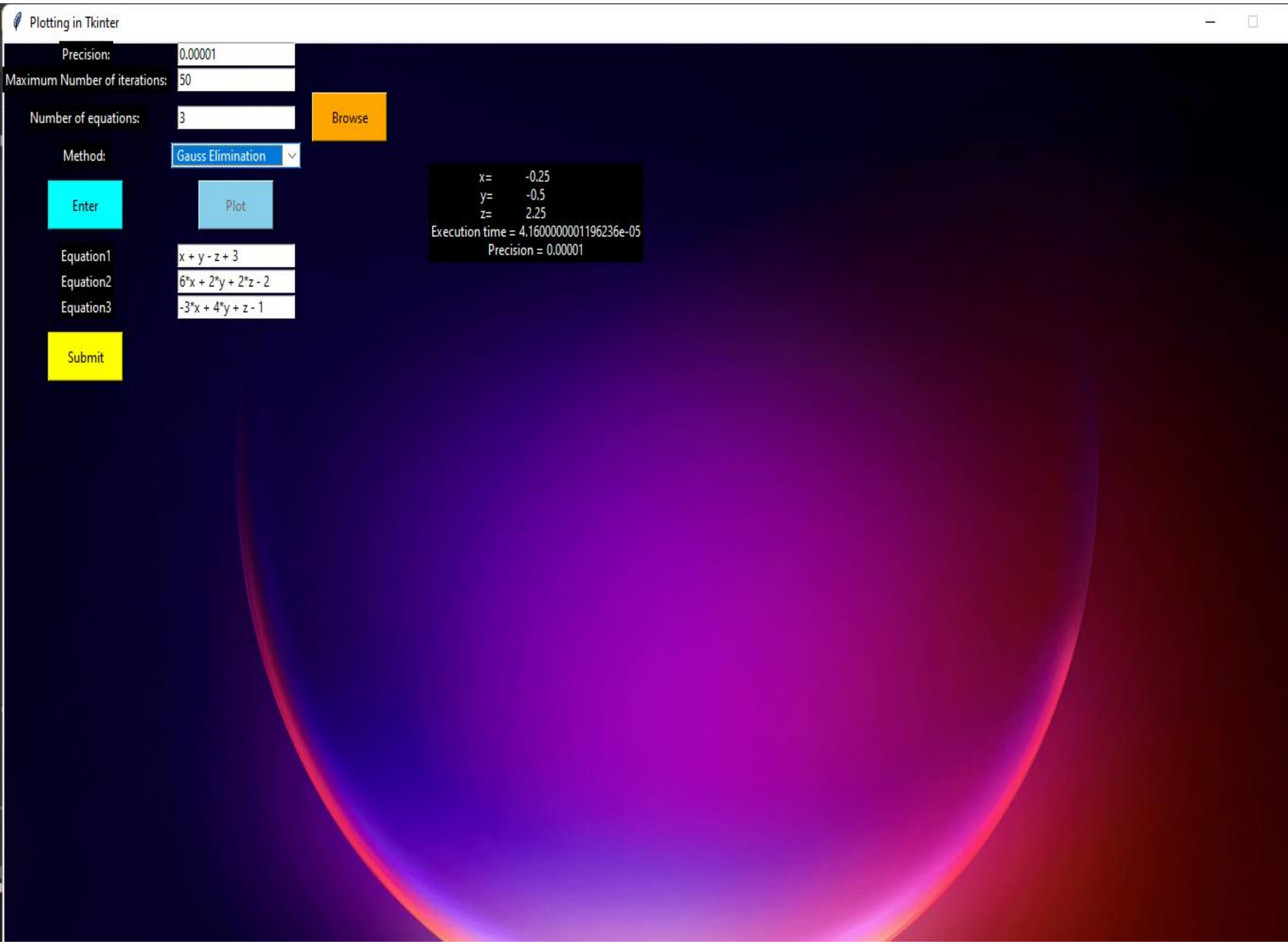
## ➢ Analysis & Screenshots:

**Sample Run #1**:

Equations:

x + y - z + 3
6*x + 2*y + 2*z - 2
-3*x + 4*y + z – 1

- **Gauss Elimination:**

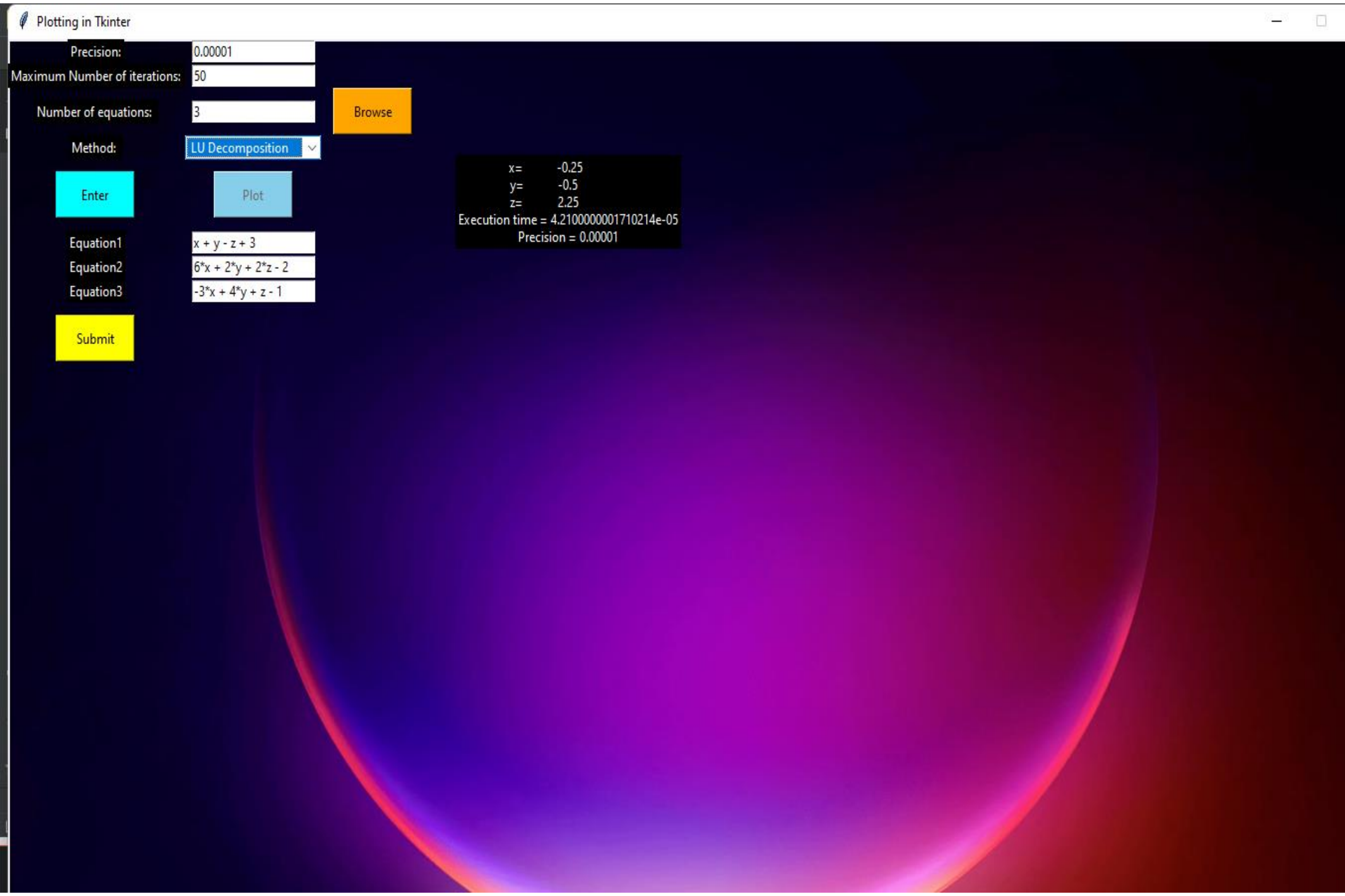


Precision: 0.00001
Maximum Number of iterations: 50
Number of equations: 3
Method: Gauss Elimination

Equation1 x + y - z + 3
Equation2 6*x + 2*y + 2*z - 2
Equation3 -3*x + 4*y + z - 1

```
x=      -0.25
y=      -0.5
z=      2.25
Execution time = 4.1600000001196236e-05
Precision = 0.00001
```

- **LU Decomposition:**



Precision: 0.00001

Maximum Number of iterations: 50

Number of equations: 3

Method: LU Decomposition

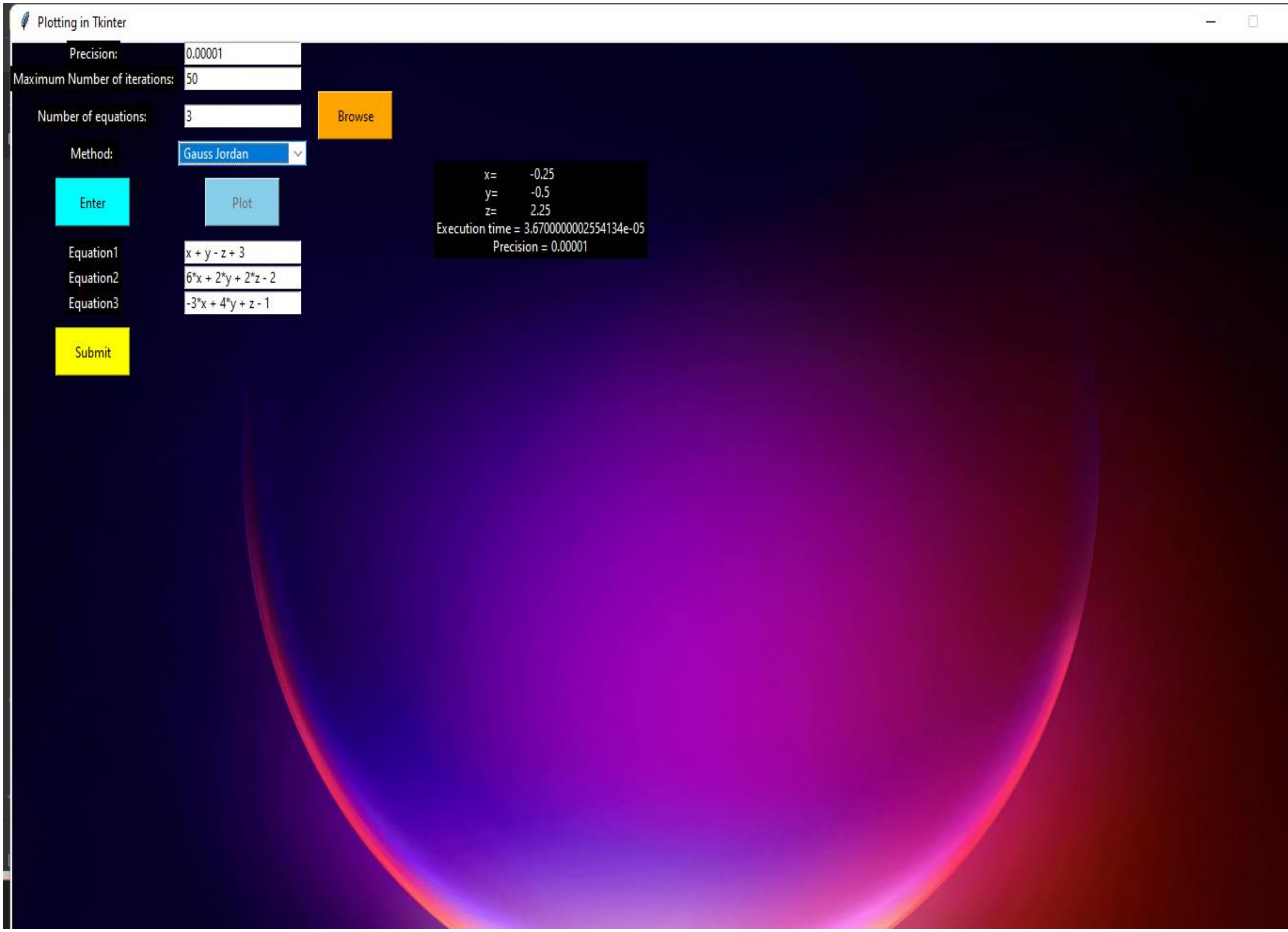
Equation1 x + y - z + 3

Equation2 6*x + 2*y + 2*z - 2

Equation3 -3*x + 4*y + z - 1

```
x=          -0.25
y=          -0.5
z=          2.25
Execution time = 4.2100000001710214e-05
Precision = 0.00001
```

- **Gauss Jordan:**



Precision: 0.00001

Maximum Number of iterations: 50

Number of equations: 3

Method: Gauss Jordan

Equation1 x + y - z + 3

Equation2 6*x + 2*y + 2*z - 2

Equation3 -3*x + 4*y + z - 1

```
x=        -0.25
y=        -0.5
z=        2.25
Execution time = 3.6700000002554134e-05
Precision = 0.00001
```

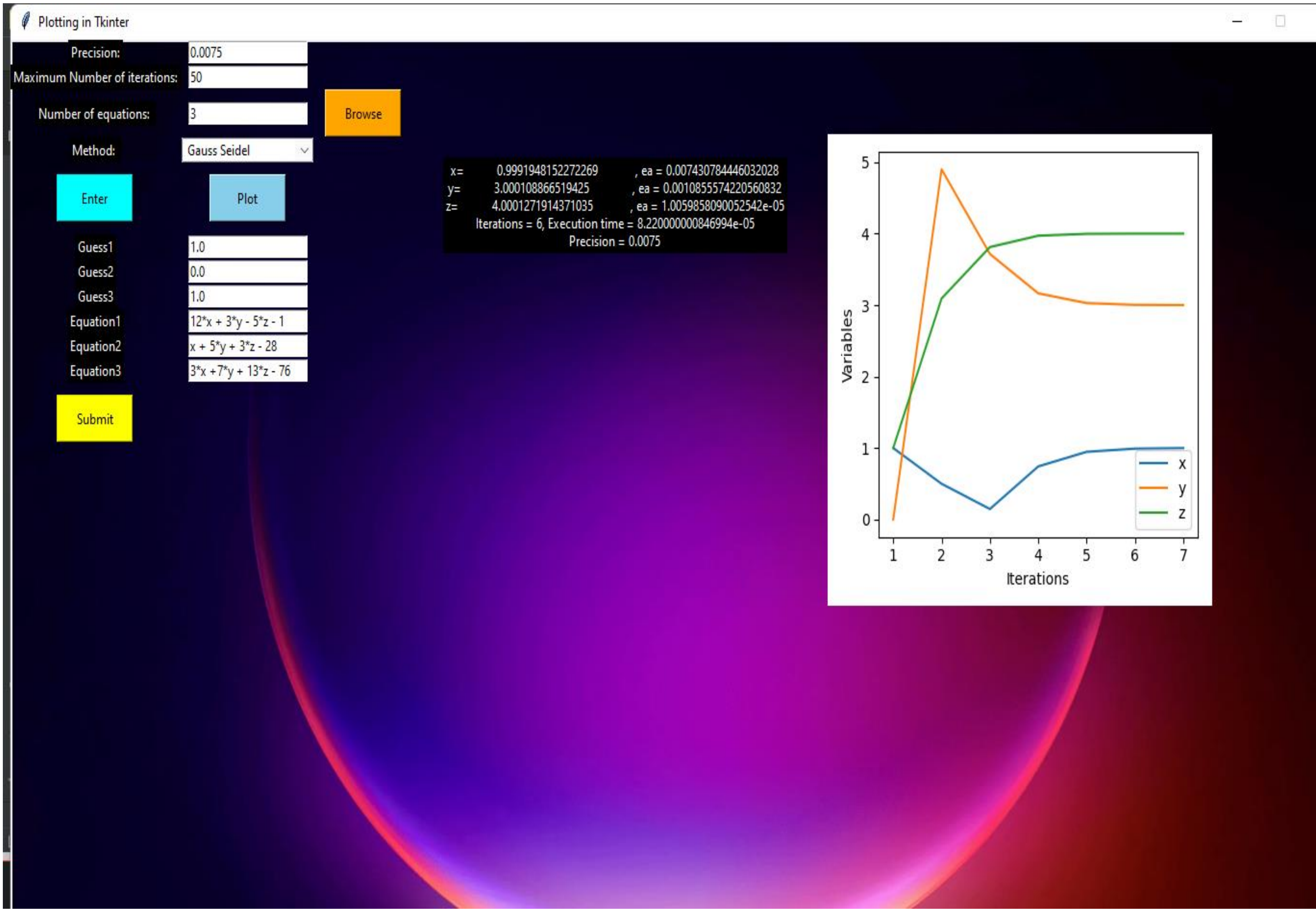
**<u>Sample Run #2:</u>**

Equations:

12*x + 3*y - 5*z - 1
x + 5*y + 3*z - 28
3*x +7*y + 13*z - 76

Initial Guesses:
x = 1
y = 0
z = 1

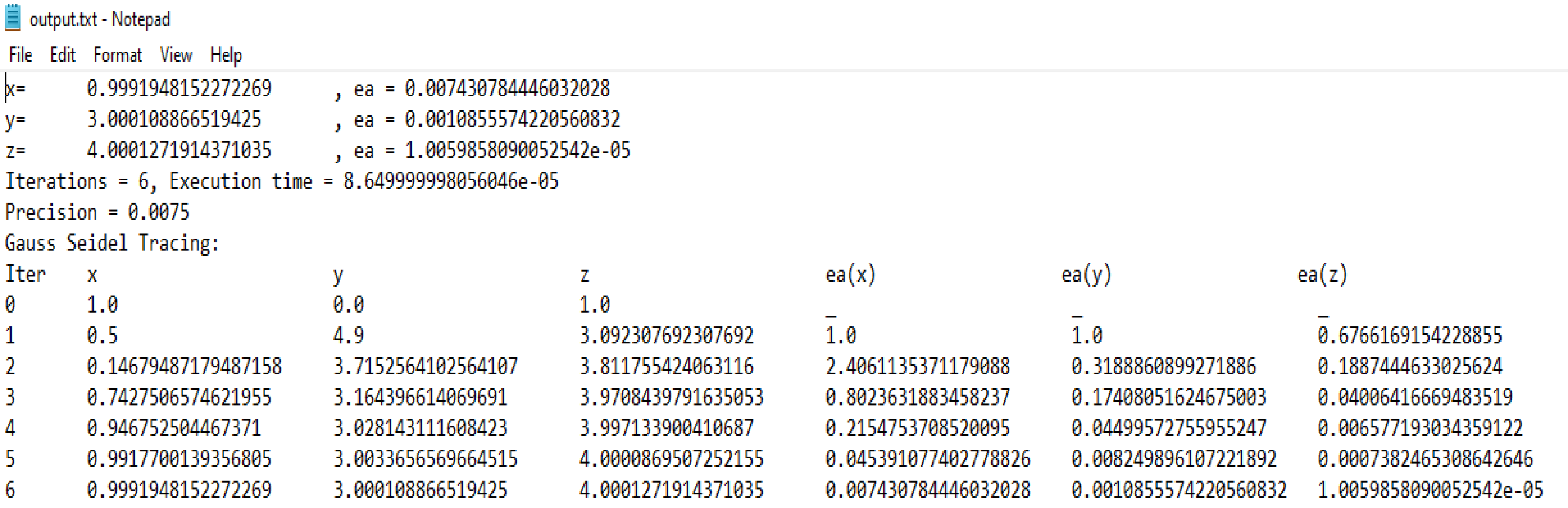
- **Gauss Seidel:**



Precision: 0.0075
Maximum Number of iterations: 50
Number of equations: 3
Method: Gauss Seidel
Guess1 1.0
Guess2 0.0
Guess3 1.0
Equation1 12*x + 3*y - 5*z - 1
Equation2 x + 5*y + 3*z - 28
Equation3 3*x +7*y + 13*z - 76

```
x=        0.9991948152272269          , ea = 0.007430784446032028
y=        3.000108866519425           , ea = 0.0010855574220560832
z=        4.0001271914371035          , ea = 1.0059858090052542e-05
     Iterations = 6, Execution time = 8.220000000846994e-05
                       Precision = 0.0075
```

**Output File:**



```
x=      0.9991948152272269      , ea = 0.007430784446032028
y=      3.000108866519425       , ea = 0.0010855574220560832
z=      4.0001271914371035      , ea = 1.0059858090052542e-05
Iterations = 6, Execution time = 8.649999998056046e-05
Precision = 0.0075
Gauss Seidel Tracing:
Iter    x                       y                       z                       ea(x)                  ea(y)                  ea(z)
0       1.0                     0.0                     1.0                     _                       _                       _
1       0.5                     4.9                     3.092307692307692       1.0                     1.0                     0.676616915422885
2       0.14679487179487158     3.7152564102564107      3.811755424063116       2.4061135371179088      0.3188860899271886      0.1887444633025624
3       0.7427506574621955      3.164396614069691       3.9708439791635053      0.8023631883458237      0.17408051624675003     0.040064166694835l9
4       0.946752504467371       3.028143111608423       3.997133900410687       0.2154753708520095      0.04499572755955247     0.006577193034359122
5       0.9917700139356805      3.0033656569664515      4.0000869507252155      0.045391077402778826    0.008249896107221892    0.0007382465308642646
6       0.9991948152272269      3.000108866519425       4.0001271914371035      0.007430784446032028    0.0010855574220560832   1.0059858090052542e-05
```

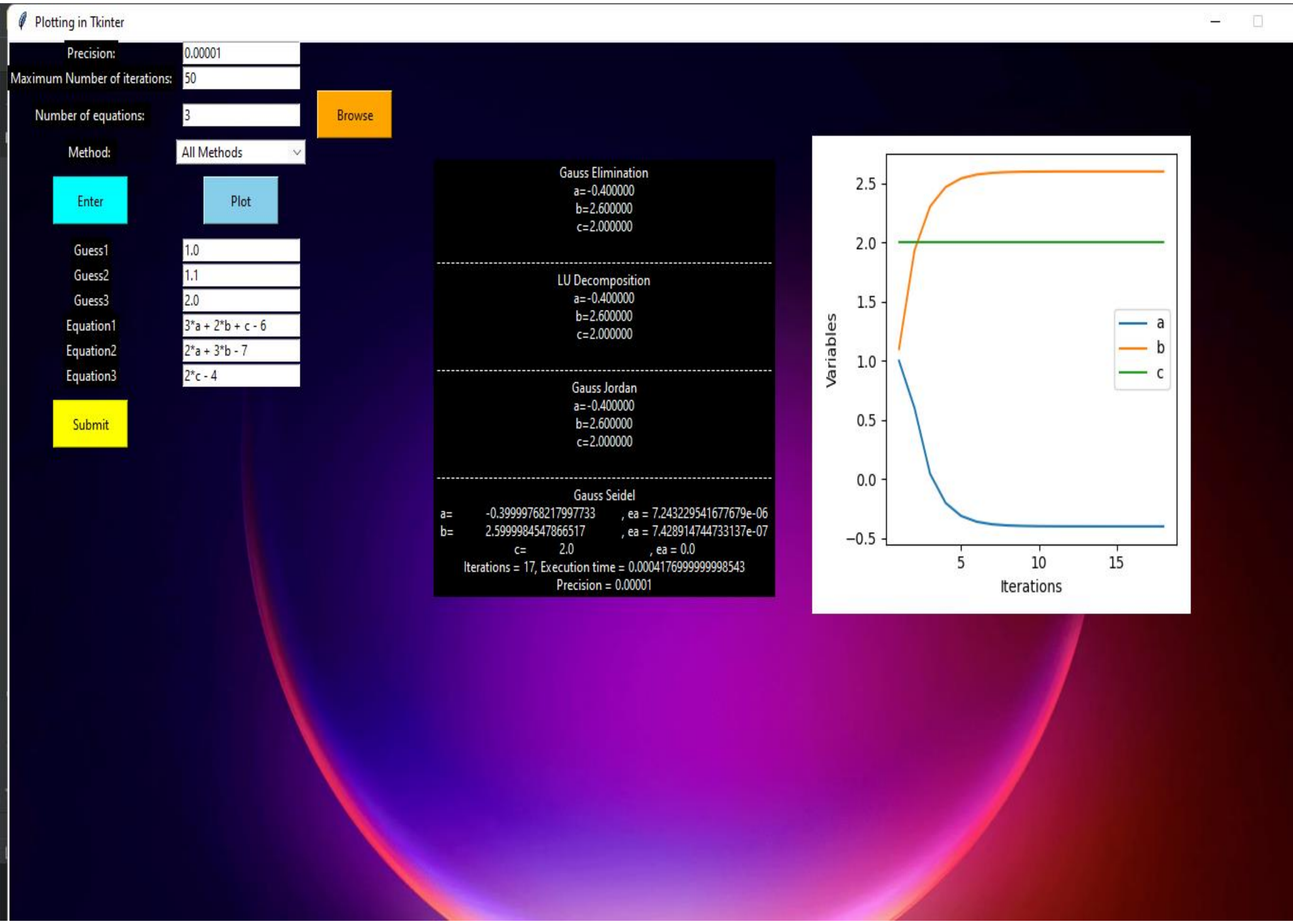
**<u>Sample Run #3: (Choosing All methods)</u>**

Equations:

3*a + 2*b + c - 6
2*a + 3*b - 7
2*c - 4

Initial Guesses:
x = 1
y = 1.1
z = 2

**Output:**



Precision: 0.00001
Maximum Number of iterations: 50
Number of equations: 3
Method: All Methods
Guess1 1.0
Guess2 1.1
Guess3 2.0
Equation1 3*a + 2*b + c - 6
Equation2 2*a + 3*b - 7
Equation3 2*c - 4

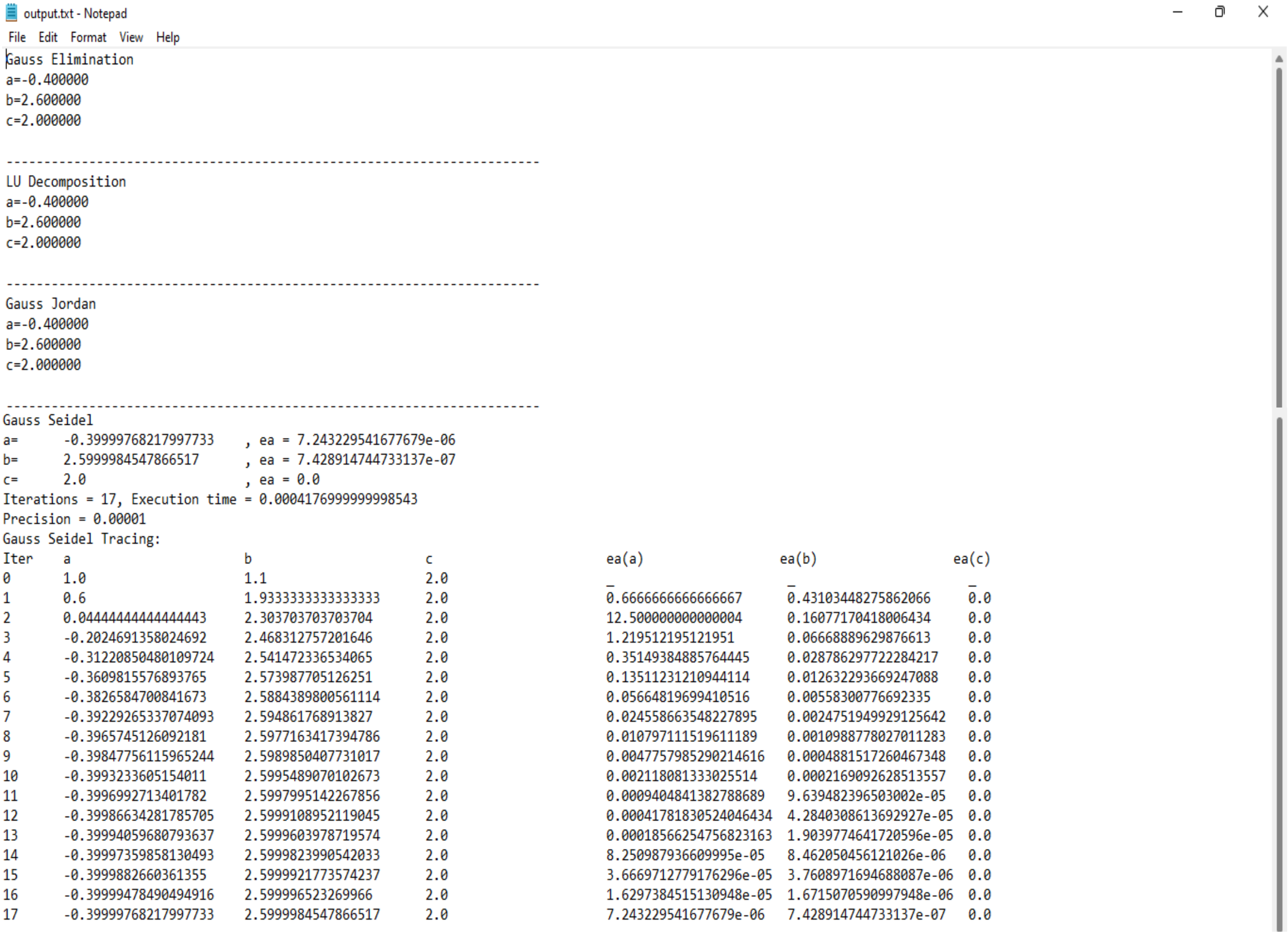
```
Gauss Elimination
a=-0.400000
b=2.600000
c=2.000000

--------------------------------------------------------------------------
LU Decomposition
a=-0.400000
b=2.600000
c=2.000000

--------------------------------------------------------------------------
Gauss Jordan
a=-0.400000
b=2.600000
c=2.000000

--------------------------------------------------------------------------
Gauss Seidel
a=      -0.39999768217997733      , ea = 7.243229541677679e-06
b=      2.5999984547866517        , ea = 7.428914744733137e-07
          c=      2.0                   , ea = 0.0
Iterations = 17, Execution time = 0.0004176999999998543
Precision = 0.00001
```

## Output file:



```
Gauss Elimination
a=-0.400000
b=2.600000
c=2.000000

----------------------------------------------------------------------
LU Decomposition
a=-0.400000
b=2.600000
c=2.000000

----------------------------------------------------------------------
Gauss Jordan
a=-0.400000
b=2.600000
c=2.000000

----------------------------------------------------------------------
Gauss Seidel
a=      -0.39999768217997733    , ea = 7.243229541677679e-06
b=      2.5999984547866517      , ea = 7.428914744733137e-07
c=      2.0                     , ea = 0.0
Iterations = 17, Execution time = 0.0004176999999998543
Precision = 0.00001
Gauss Seidel Tracing:
Iter    a                       b                       c                       ea(a)                   ea(b)                   ea(c)
0       1.0                     1.1                     2.0                     _                       _                       _
1       0.6                     1.9333333333333333      2.0                     0.6666666666666667      0.43103448275862066     0.0
2       0.04444444444444443     2.303703703703704       2.0                     12.500000000000004      0.16077170418006434     0.0
3       -0.2024691358024692     2.468312757201646       2.0                     1.219512195121951       0.06668889629876613     0.0
4       -0.31220850480109724    2.541472336534065       2.0                     0.35149384885764445     0.028786297722284217    0.0
5       -0.3609815576893765     2.573987705126251       2.0                     0.13511231210944114     0.012632293669247088    0.0
6       -0.3826584700841673     2.5884389800561114      2.0                     0.05664819699410516     0.00558300776692335     0.0
7       -0.39229265337074093    2.594861768913827       2.0                     0.024558663548227895    0.0024751949929125642   0.0
8       -0.3965745126092181     2.5977163417394786      2.0                     0.010797111519611189    0.0010988778027011283   0.0
9       -0.39847756115965244    2.5989850407731017      2.0                     0.0047757985290214616   0.0004881517260467348   0.0
10      -0.3993233605154011     2.5995489070102673      2.0                     0.002118081333025514    0.0002169092628513557   0.0
11      -0.3996992713401782     2.5997995142267856      2.0                     0.0009404841382788689   9.639482396503002e-05   0.0
12      -0.39986634281785705    2.5999108952119045      2.0                     0.00041781830524046434  4.2840308613692927e-05  0.0
13      -0.39994059680793637    2.5999603978719574      2.0                     0.00018566254756823163  1.9039774641720596e-05  0.0
14      -0.39997359858130493    2.5999823990542033      2.0                     8.250987936609995e-05   8.462050456121026e-06   0.0
15      -0.3999882660361355     2.5999921773574237      2.0                     3.6669712779176296e-05  3.7608971694688087e-06  0.0
16      -0.39999478490494916    2.599996523269966       2.0                     1.6297384515130948e-05  1.6715070590997948e-06  0.0
17      -0.39999768217997733    2.5999984547866517      2.0                     7.243229541677679e-06   7.428914744733137e-07   0.0
```