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Receptivity Analysis of Transonic Multi-Fluid Flow Past an Aircraft Wing

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1. Motivation

The transition of airflow from laminar to turbulent is imperative to aviation industry. In particular, the stability analysis of airflow in rainy condition is crucial for flow control purposes.

The stability analysis is the primary step to study turbulent behaviour of airflow and the following mathematical model is applied to commercial aircrafts which travel with transonic speed.

2. Method

We describe the behaviour of fluids with the Navier-Stokes equations. By means of asymptotic methods we reduce the Navier-Stokes equations to simpler partial differential equations. Assuming that the wing vibrates and produces disturbance waves we conduct receptivity analysis. With key assumptions:

$$Re = \frac{\rho_\infty V_\infty L}{\mu_{air}} \longrightarrow \infty \qquad M_\infty = \frac{V_\infty}{\sqrt{\Upsilon p_\infty / \rho_\infty}} \approx 1$$

$$\sigma_\mu = \frac{\mu_{air}}{\mu_{water}} \ll 1 \qquad \delta = Re^{-11/18}\, \sigma_\mu^{-1/9}$$

By linearising the reduced Navier-Stokes equations and assuming periodic solutions in time, we are able to apply Fourier transformation. Consequently, a system of ordinary differential equations is derived which their solutions lead to the receptivity coefficient.

Region 2 has no displacement effect on the streamlines hence it is disregarded in the mathematical model.

3. Mathematical Model

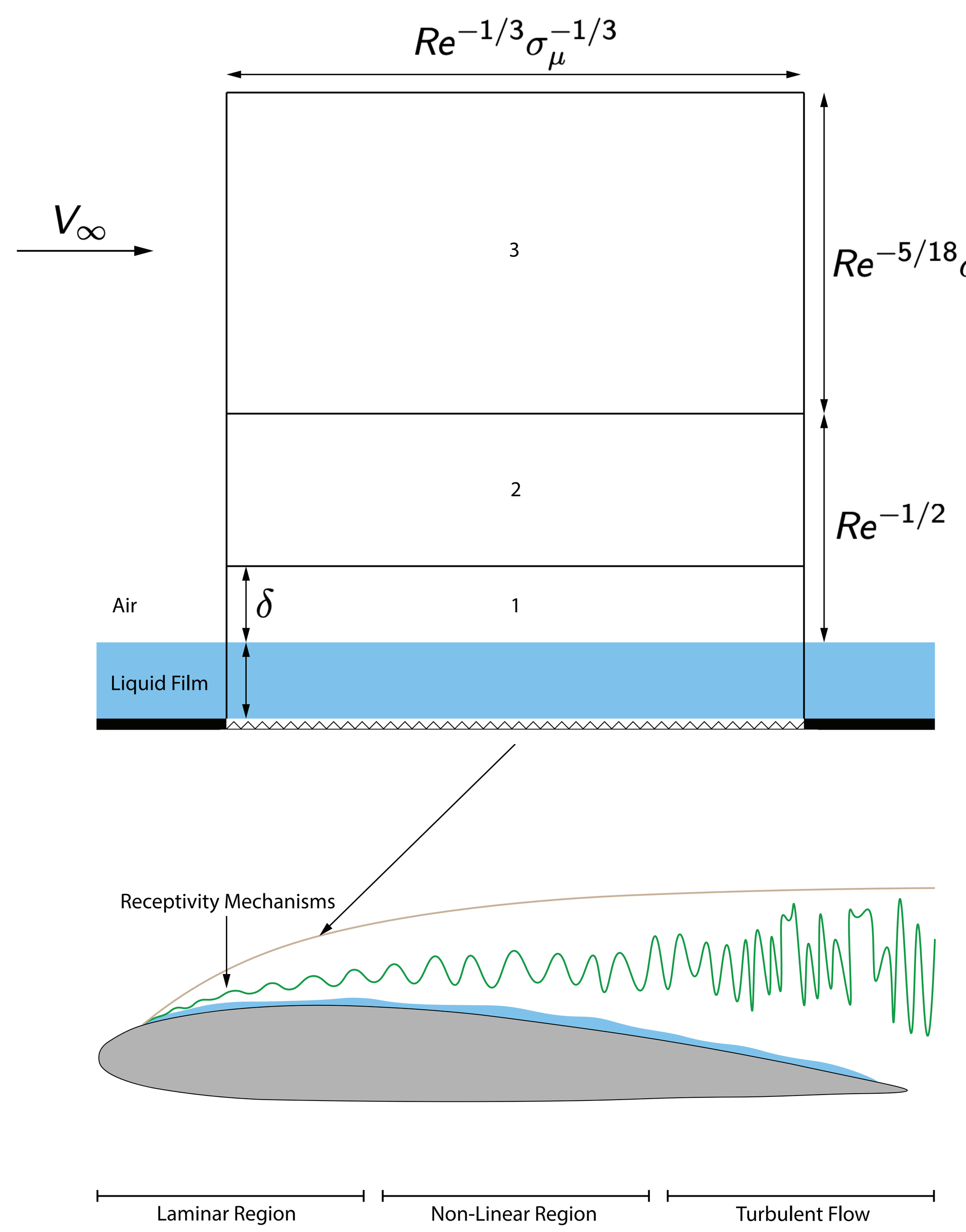
The governing equations are:

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{dP}{dX} + \frac{\partial^2 U}{\partial Y^2}, \quad \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \qquad \text{Region 1}$$

$$\frac{\partial^2 P}{\partial X \partial T} + M^*_\infty \frac{\partial^2 P}{\partial X^2} - \frac{\partial^2 P}{\partial Y^2} = 0 \qquad \text{Region 3}$$

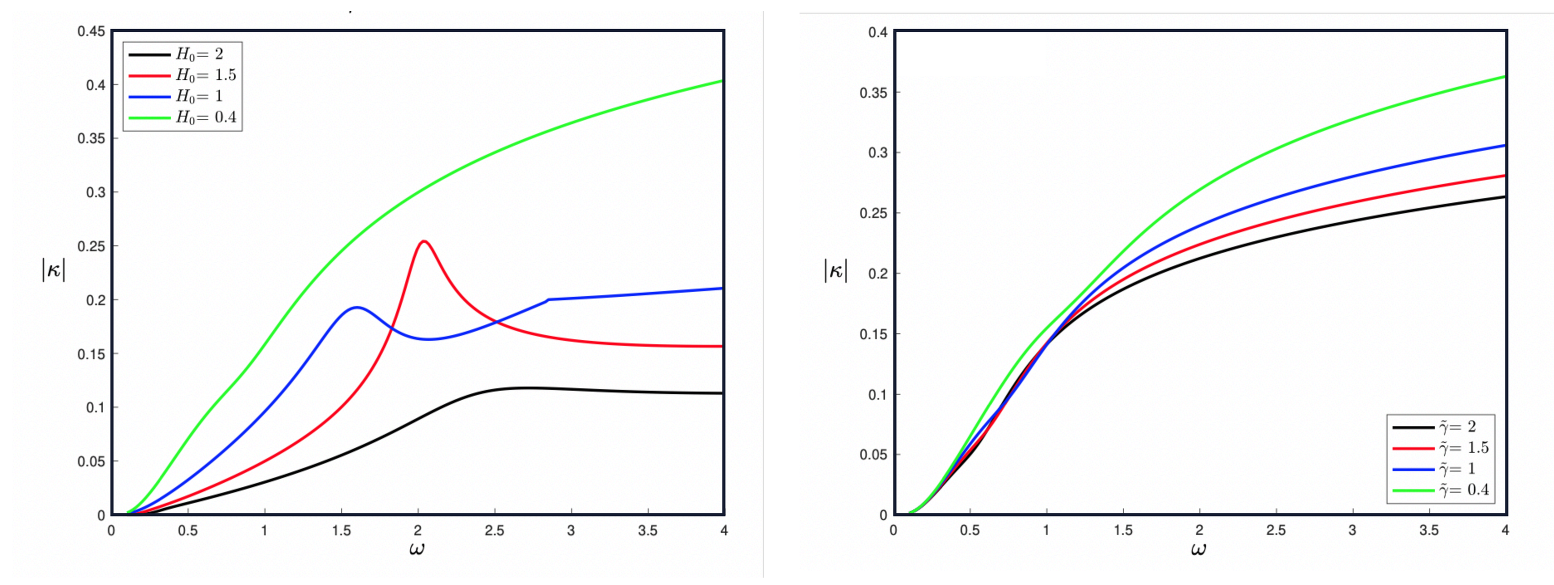
$$\frac{\partial^2 U'}{\partial Y^2} = \frac{dP'}{dX}, \quad \frac{\partial U'}{\partial X} = -\frac{\partial V'}{\partial Y} \qquad \text{Liquid Film}$$

The visual representation of the governing equations is shown below.



4. Results

The figures below display the modulus of receptivity coefficient against frequency. In the first figure, the surface tension is fixed and the initial film thickness is varied. And in the second figure, the initial film thickness is fixed and the surface tension is varied.



5. Conclusion

- The aim of these analyses is to find the initial amplitude of the disturbance waves that penetrate the boundary layer.
- The initial amplitude is represented by the modulus of receptivity coefficient $\kappa(\omega)$.
- Receptivity coefficient is dependent of surface tension $\tilde{\gamma}$ and frequency ω .
- Surface tension reduces the value of receptivity coefficient modulus.

References

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