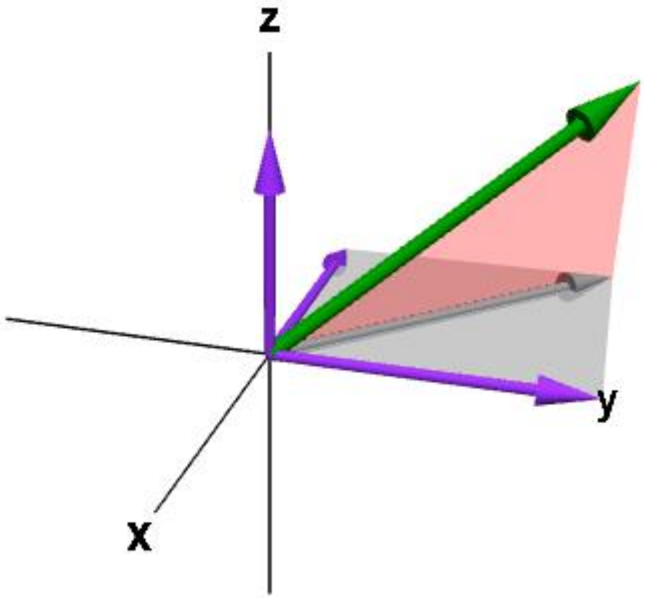
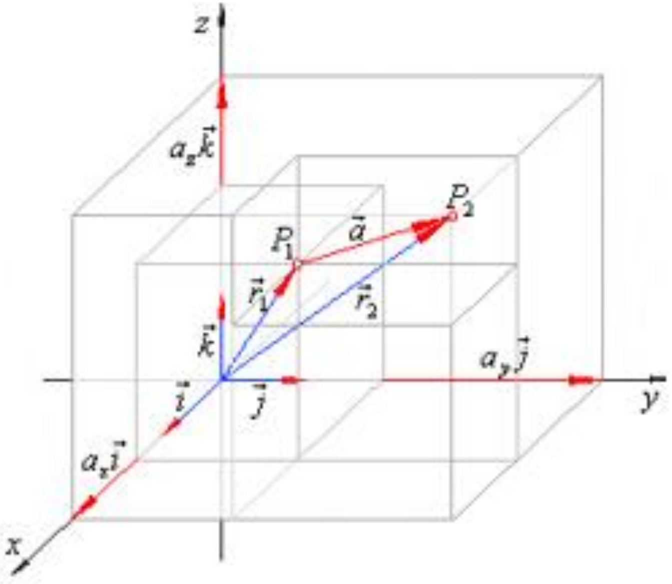
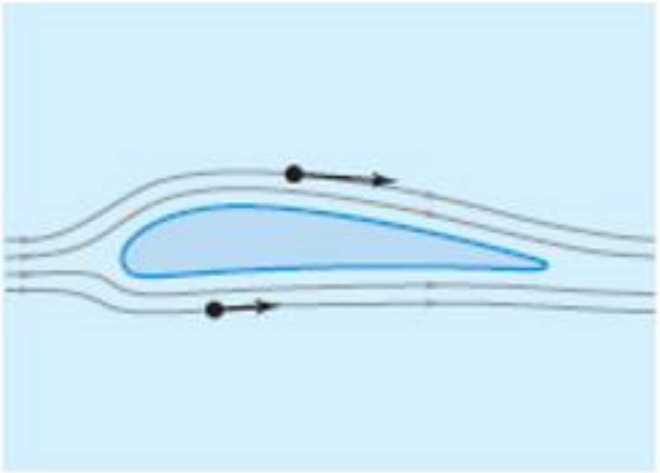
Lecture 6 - Chapter 10 – Sec. 10.2

Vectors







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Learning Objectives

- *Determine the components of a vector and perform vector operations in* $\mathcal{R}^3$.
- *Norm of a vector and distance between the points in* $\mathcal{R}^3$.
- *Dot product and angle between two vectors in* $\mathcal{R}^3$
- *Projection of a vector onto another vector in* $\mathcal{R}^3$
- *Application of dot product in Engineering*

Why do we need Vectors in Engineering Sciences?

- *Vector space has been particularly responsive to engineering science as vectors play a significant role in many important engineering science undertakings.*

- *A few well-known examples are:*
  - *Graph analysis*
  - *Machine learning*
  - *Image processing*
  - *Bioinformatics*
  - *Scientific computing*
  - *Data mining*
  - *Parallel computing*

Use rotational and curvature properties of vector fields to identify critical features of an image. Using vector analysis and differential geometry, we establish the properties needed.

- *Vectors are used in various machine learning algorithms, including regression, classification, clustering, and dimensionality reduction.*

What is a Vector?

- *An Euclidean vector is a directed line segment with direction and magnitude.*
- *A three-dimensional vector is a line segment drawn in a 3-D plane*
- *Three-dimensional vectors can also be r in component form. The notation*

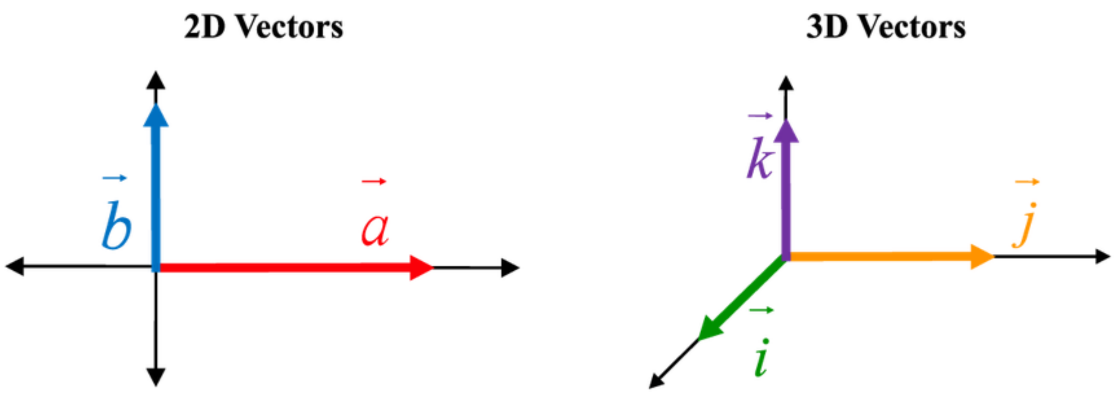
$$\vec{A} = \langle A_x, A_y, A_z \rangle$$

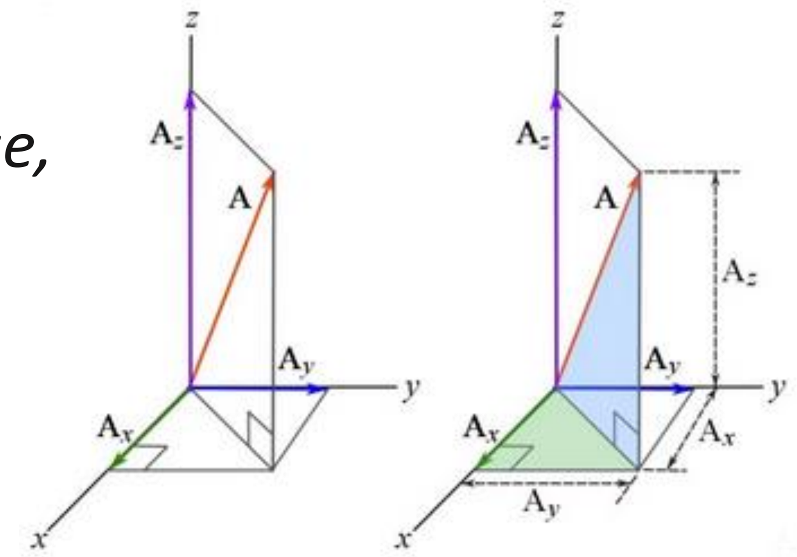
is a natural extension of the two-dimensional case,

$$\vec{A} = \langle A_x, A_y \rangle.$$

The length of the three-dimensional vector $\vec{A}$ is

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$



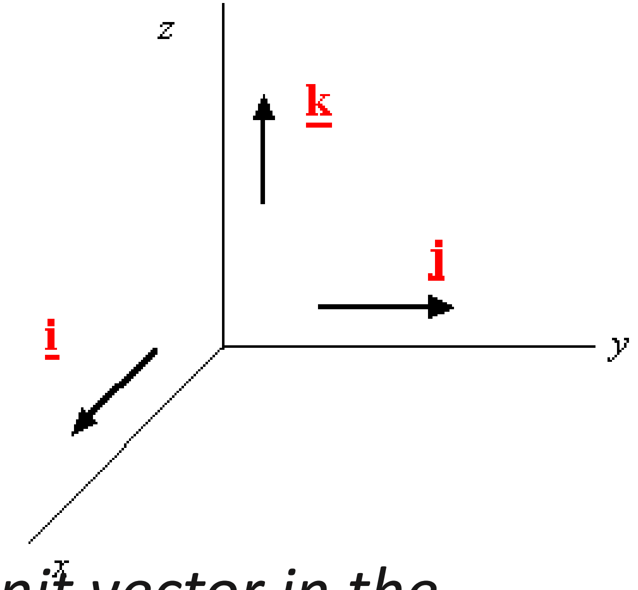


What is a Vector?

Unit Vector

- *A vector of unit length.*
- *Unit vectors are often chosen to form the basis of a vector space, and every vector in the space may be written as a linear combination of unit vectors.*

$$\vec{A} = \hat{\imath} A_x + \hat{\jmath}\, A_y + \hat{k}\, A_z \;=\; \langle A_x, A_y, A_z \rangle$$



- *The **normalized vector** $\hat{A}$ of a non-zero vector $\vec{A}$ is the unit vector in the direction of $\vec{A}$, i.e.,*

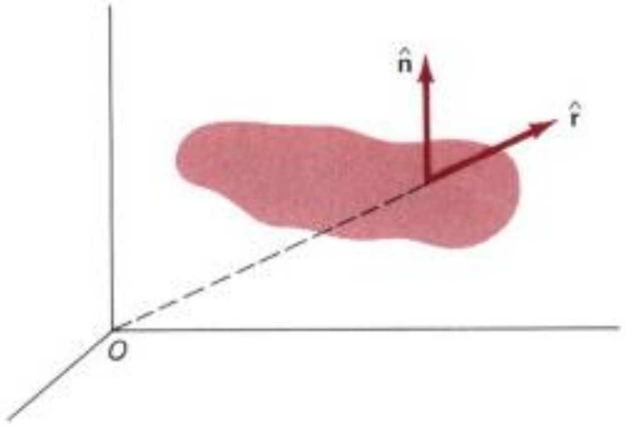
$$\widehat{\boldsymbol{A}} = \frac{\vec{\boldsymbol{A}}}{||\,\vec{\boldsymbol{A}}\,||}$$

Where $||\,\vec{A}\,||$ is the norm (length) of $\vec{A}$.

Radial and Normal Unit Vectors

- *The three Cartesian unit vectors* $\hat{\imath}, \hat{\jmath}$, *and* $\hat{k}$ *do not vary in direction.*
- *In polar or spherical coordinates, the unit vector may vary in direction because the position vector* $\vec{r}$ *varies in direction.*
- *The unit vector* $\hat{r}$ *is formed by taking the position vector* $\vec{r}$ *and dividing it by its magnitude* $|\ \vec{r}\ |$. *This procedure gives* $\hat{r}$ *unit magnitude but preserves the radial direction of* $\vec{r}$**.**
- *In addition to* $\hat{\imath}, \hat{\jmath}$, *and* $\hat{k}$, *two other unit vectors are*

  - *Normal Unit vector* - *Radial Unit Vector*



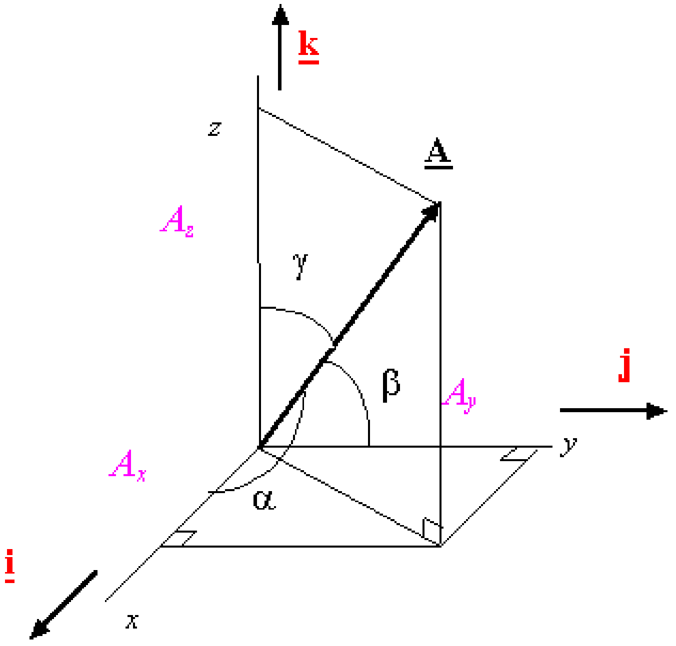
- *The unit vector* $\hat{n}$ *is normal, or perpendicular, to a surface at a given point.*
- *For the special case of a spherical surface (origin at the centre) the normal vector* $\hat{n}$ *is radial and* $\hat{n} = \hat{r}$.

Direction Cosines of a Vector

- *The direction cosines are the cosines of the angle subtended by a vector or line with the x-axis, y-axis, and z-axis respectively.*
- *If the angles subtended by the line with the three axes are α, β, and γ, then the direction cosines are $\cos\alpha$, $\cos\beta$, and, $\cos\gamma$ respectively.*

$$\cos\alpha = A_x/|\vec{A}|, \cos\beta = A_y/\vec{A}|, \cos\gamma = A_z/|\vec{A}|,$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$



- *Taking direction cosines makes it easy to represent the direction of a vector in terms of angles with respect to the reference.*
- *In terms of unit vectors*

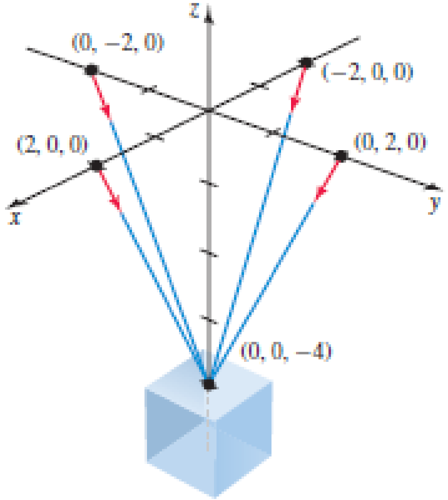
$$\vec{A} = \hat{\imath}\cos\alpha + \hat{\jmath}\cos\beta + \hat{k}\cos\gamma$$

Practice Problem: Find a unit vector with the same direction as $8\hat{\imath} - \hat{\jmath} + 4\hat{k}$..

Example – Ex. 10.2

Four-cable load –

A 500-lb load hangs from four cables of equal length that are anchored at the points $(\pm 2, 0, 0)$ and $(, \pm 2, 0)$. The load is located at $(0, 0, -4)$. Find the vectors describing the forces on the cables due to the load.

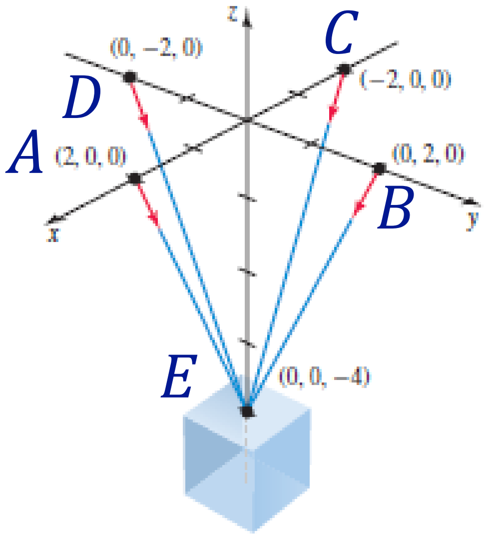


Solution:

Let $A(2,0,0), B(0,2,0), C(-2,0,0), D(0,-2,0)$ *and* $E(0,0,-4)$ *be the given points.*

Express the cables in vector form, we have

$$\overrightarrow{AE} = \langle -2, 0, -4 \rangle, \qquad \overrightarrow{BE} = \langle 0, -2, -4 \rangle$$
$$\overrightarrow{CE} = \langle 2, 0, -4 \rangle, \qquad \overrightarrow{DE} = \langle 0, 2, -4 \rangle$$



Example – Ex. 10.2

Solution:

We are seeking x *so that* $x(\overrightarrow{AE} + \overrightarrow{BE} + \overrightarrow{CE} + \overrightarrow{DE}) = -500\,\hat{k}$

Thus we require $-16x = -500 \Rightarrow x = 125/4$. *Therefore,*

$$x\overrightarrow{AE} = \frac{125}{4}\langle -2, 0, -4\rangle$$
$$x\overrightarrow{BE} = \frac{125}{4}\langle 0, -2, -4\rangle$$
$$x\,\overrightarrow{CE} = \frac{125}{4}\,\langle 2, 0, -4\rangle$$
$$x\overrightarrow{DE} = \frac{125}{4}\langle 0, 2, -4\rangle$$

Homework 1 – Ex. 10.2

Three-cable load - *A 500-lb load hangs from three cables of equal length that are anchored at the points* $(-2,0,0)$, $\left(1, \sqrt{3}, 0\right)$, *and* $(1, -\sqrt{3}, 0)$. *The load is located at* $(0,0,-2\sqrt{3})$. *Find the vectors describing the forces on the cables due to the load.*

Example – Q. 26. Ex. 10.2

The magnitude of a velocity vector is called speed. Suppose that a wind is blowing from the direction N45°W at a speed of 50 km/h. (This means that the direction from which the wind blows is 45° west of the northerly direction.) A pilot is steering a plane in the direction N60°E at an airspeed (speed in still air) of 250 km/h. The true course, or track, of the plane is the direction of the resultant of the velocity vectors of the plane and the wind. The ground speed of the plane is the magnitude of the resultant. Find the true course and the ground speed of the plane.

Solution:

Recognize the principle: There is relative motion between two objects, so the resultant velocity is obtained from relative velocities. Our coordinate axes represent north as positive y-direction and east as positive x-direction.

Example

Execute: The wind is blowing at 50 km/h 45° in the west of the northerly direction, so its velocity vector is 50 km/h east of southerly. This can be written as

$$\vec{v}_{wind} = 50(\hat{\imath}\cos 45 - \hat{\jmath}\sin 45)$$

The velocity vector of the plane, with respect to still air, is 250 km/h $N60^o E$ *or equivalently*

$$\vec{v}_{plane} = 250(\hat{\imath}\cos 30 + \hat{\jmath}\sin 30)$$

The velocity of the plane relative to the ground is $\vec{v} = \vec{v}_{wind} + \vec{v}_{plane}$

$$\vec{v} = 50(\hat{\imath}\cos 45 - \hat{\jmath}\sin 45) + 250(\hat{\imath}\cos 30 + \hat{\jmath}\sin 30) = \hat{\imath}\,251.9 + \hat{\jmath}\,89.6$$

The ground speed is $|\,\vec{v}\,| = 267\ km/h$. *The direction of velocity with the x-axis is* $\theta = \tan^{-1} 89.6251.9 = 20^o$.

Therefore, the true course of plane is $N(90^o - 20^o) = N70^o E$.

Homework 2 – Ex. 10.2

(a) . A boatman wants to cross a canal that is 3 km wide and wants to land at a point 2 km upstream from his starting point. The current in the canal flows at 3.5 km/h and the speed of his boat is 13 km/h.

i. In what direction should he steer?

ii. How long will the trip take?

(b) A small plane is flying horizontally due east in calm air at 250 km/hr when it encounters a horizontal crosswind blowing southwest at 50 km/hr and a 30 km/hr updraft.

Find the resulting speed of the plane, and describe with a sketch the approximate direction of the velocity relative to the ground.

Example – Q. 34. Ex. 10.2

a) Find the unit vectors that are parallel to the tangent line to the curve $y = 2\sin x$ at the point $(\pi/6, 1)$.

(b) Find the unit vectors that are perpendicular to the tangent line.

(c) Sketch the curve $y = 2\sin x$ and the vectors in parts (a) and (b), all starting at $(\pi/6, 1)$.

Solution:

(a) The slope of the tangent line to the graph at the given point is

$$\frac{dy}{dx}\Big|_{x=\pi/6} = 2\cos x = \sqrt{3}$$

Equation of the tangent at $(\pi/6,1)$ *is*

$$y - 1 = \sqrt{3}\,(x - \pi/6)$$

Equation of the line parallel to the tangent and passing through the origin

$$y = \sqrt{3}\,x, \qquad z = 0.$$

Example – Q. 34. Ex. 10.2

Therefore,

$$\frac{x}{1} = \frac{y}{\sqrt{3}}, \qquad z = 0$$

The direction ratios of this are $1, \sqrt{3}, \ 0.$ *Therefore, its direction cosines are*

$$\pm\frac{1}{\sqrt{1^2 + \left(\sqrt{3}\right)^2 + 0^2}}, \qquad \pm\frac{\sqrt{3}}{\sqrt{1^2 + \left(\sqrt{3}\right)^2 + 0^2}}, 0$$

$$\pm\frac{1}{2}, \qquad \pm\frac{\sqrt{3}}{2}, \qquad 0$$

Therefore, unit vectors parallel to the tangent line at $(\pi/6, 1)$ *are*

$$\pm\frac{1}{2}\left(\hat{\imath} + \sqrt{3}\,\hat{\jmath}\right)$$

Note: For any real number m*, the vector* $(1, m)$ *determines a line of slope* m *through the origin:*

Example – Q. 34. Ex. 10.2

(b) Find the unit vectors that are perpendicular to the tangent line.

(c) Sketch the curve $y = 2\sin x$ and the vectors in parts (a) and (b), all starting at $(\pi/6, 1)$.

Solution:

(b) The slope of the tangent line is $\sqrt{3}$, so the slope of a line perpendicular to the tangent line is $-1/\sqrt{3}$.

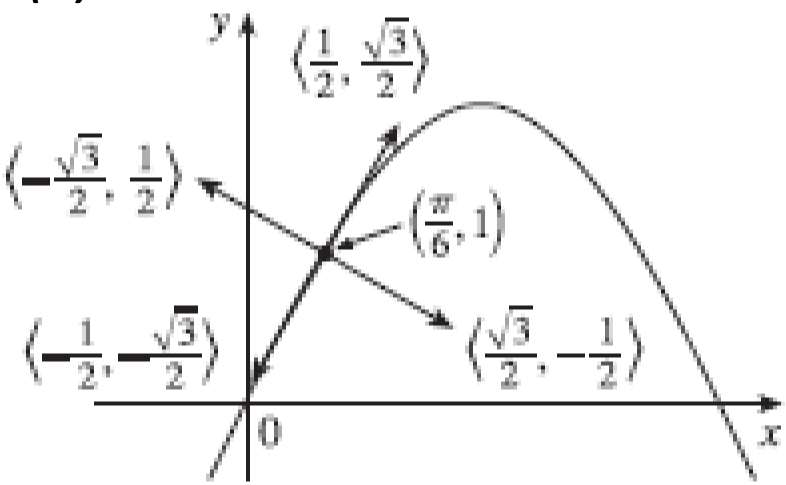
The vector in this direction is

$\sqrt{3}\,\hat{\imath} - \hat{\jmath}$.

Since $|\sqrt{3}\,\hat{\imath} - \hat{\jmath}| = 2$, *the unit vectors Perpendicular to the tangent line are*

$\pm\frac{1}{2}(\sqrt{3}\,\hat{\imath} - \hat{\jmath})$.

(c)



Homework 3– Ex. 10.2

A clothesline is tied between two poles, 8 m apart. The line is quite taut (pulled tight) and has negligible sag (sink). When a wet shirt with a mass of 0.8 kg is hung at the middle of the line, the midpoint is pulled down 8 cm. Find the tension in each half of the clothesline.