Lecture 1 (Chapter 9 – Sec. 9.1)

Parametric Equations & Curves

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Learning Objectives

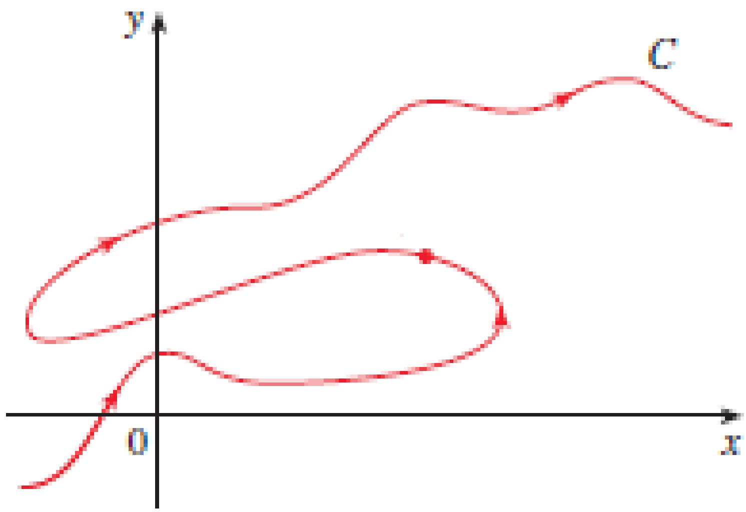
- *Plot a curve described by parametric equations.*
- *Convert the parametric equations of a curve into the form* $y = f(x)$
- *Recognize the parametric equations of basic curves, such as a line and a circle.*
- *Recognize the parametric equations of a cycloid, Witch of Maria Agnesi and Other Parametric Curves*

Intuition For Parametric Equations

- *The plane curves are described by giving y as a function of x $[y = f(x)]$ or x as a function of y $[x = f(y)]$ or by giving a relation between x and y that defines y implicitly as a function of x $[f(x, y) = 0]$.*

- *Such equations assume an "input to output" connection.*

- *Imagine that a particle moves along the curve C. It is impossible to describe C by an equation of the form $y = f(x)$ because C fails the Vertical Line Test*



Intuition For Parametric Equations

- *Now look at the data: For every degree above 40, our convenience store sells S bottles of sunscreen and I pints of ice cream.*

Sunscreen (S)	*Ice Cream (I)*
2	4
5	25
4	16
8	64
3	9

The data obey

$$I = S^2$$

Ice cream = (Sunscreen)^2

And it's correct ….. But misleading

- *The equation implies sunscreen directly changes the demand for ice cream when it is the hidden variable (temperature) that changed them both!*

Intuition For Parametric Equations

It's much better to write two separate equations:

$$Sunscreen = (Temperature\ - 40)$$
$$Ice\ cream = (Temperature - 40)^2$$

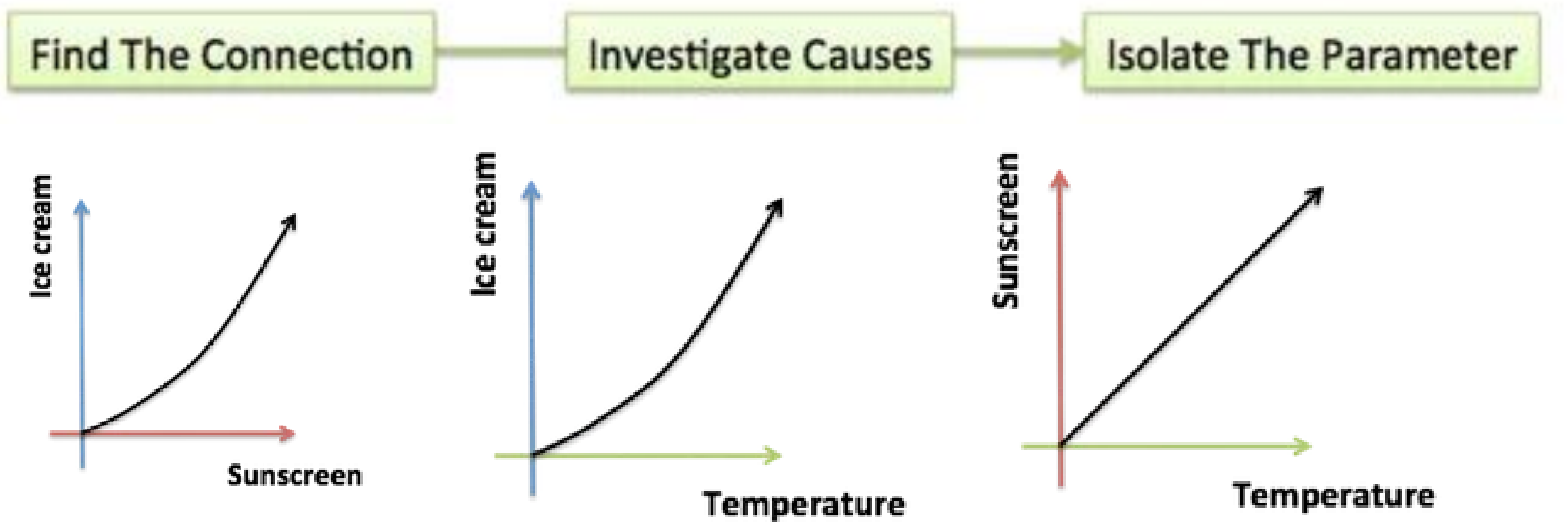
that directly point out the causality.

- *The ideas "temperature impacts ice cream" and "temperature impacts sunscreen" clarify the situation.*

- *we lose information by trying to factor away the common "temperature" portion.*

- *Parametric equations get us closer to the real-world relationship.*

Intuition For Parametric Equations



- *We get so hammered with "parametric equations involve time" that we forget the key insight: parameters point to the cause.*
- *Why did it change? (Maybe it was time, or temperature, or perhaps sunscreen does make you hungry for ice cream.)*

Parametric Equations

Definition

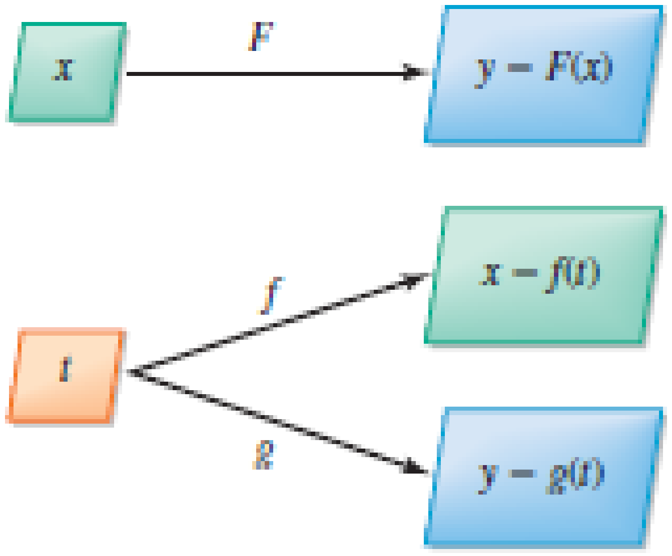
If x and y are continuous functions of t on an interval I, then the equations

$$x = f(t), \qquad y = g(t)$$

are called parametric equations, and t is called the parameter.

- *The set of points $(x, y) = (f(t),\ g(t))$ obtained as t varies over the interval I is called the graph of the parametric equations.*

- *The graph of parametric equations is also called a parametric curve or plane curve, and is denoted by C.*



Graphing a Parametrically Defined Curve

Example - Parametric Parabola

Graph and analyze the parametric equations

$$x = f(t) = 2t, \qquad y = \frac{1}{2}t^2 - 4, \qquad for\ 0 \le t \le 8.$$

Solution

Plotting individual points often helps in visualising a parametric curve. The table shows the values of x and y corresponding to several values of t on the interval $0 \le t \le 8$.

By plotting the (x, y) pairs in the table and connecting them with a smooth curve, we obtain the graph.

Table

t	x	y	(x, y)
0	0	-4	$(0, -4)$
1	2	$-\frac{7}{2}$	$(2, -\frac{7}{2})$
2	4	-2	$(4, -2)$
3	6	$\frac{1}{2}$	$(6, \frac{1}{2})$
4	8	4	$(8, 4)$
5	10	$\frac{17}{2}$	$(10, \frac{17}{2})$
6	12	14	$(12, 14)$
7	14	$\frac{41}{2}$	$(14, \frac{41}{2})$
8	16	28	$(16, 28)$

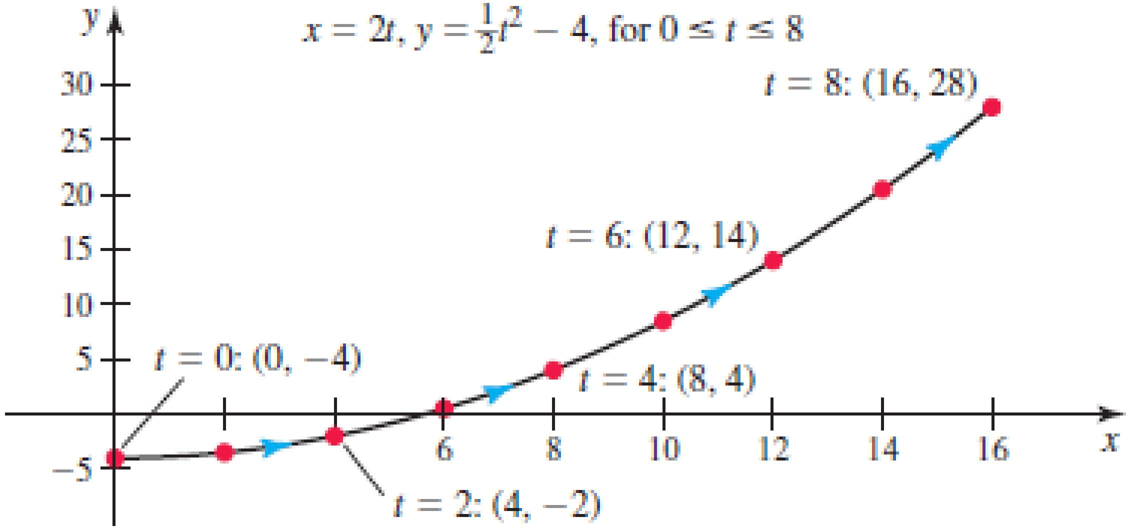
Graphing a Parametrically Defined Curve

Solution

As t increases from its initial value of t = 0 to its final value of t = 8, the curve is generated from the initial point $(0, -4)$ *to the final* $(16,\ 28)$. *Notice that the values of the parameter do not appear in the graph.*

The only signature of the parameter is the direction in which the curve is generated: In this case, it unfolds upward and to the right, as indicated by the arrows on the curve.

The direction in which a parametric curve is generated as the parameter increases is called the ***positive orientation*** *of the curve (and is indicated by arrows on the curve).*



Practice Problem 1 – 9.1

Exercise: 9.1 - Question 4

Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases. Briefly interpret the graph.

$$x = e^{-t} + t, \qquad y = e^{t} - t, \qquad -2 \leq t \leq 2$$

Eliminating the Parameter to Define a Curve

Example – Q. 10. Ex. 9.1

(a) Eliminate the parameter to find a Cartesian equation of the curve.

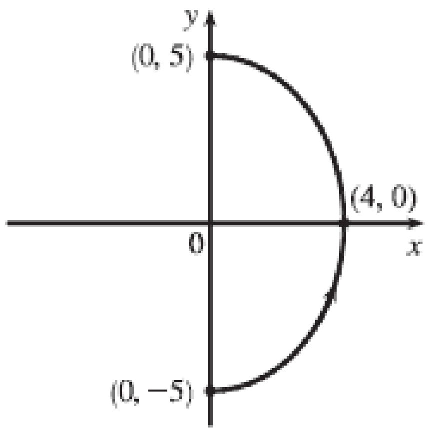
$$x = 4\cos\theta, \qquad y = 5\sin\theta, \qquad -\pi/2 \le \theta \le \pi/2$$

Solution

One can solve either of the equations for the parameter to eliminate the parameter. However, sometimes it is necessary to be a bit creative in eliminating the parameter.

Solving either equation for θ directly is not advisable because sine and cosine are not one-to-one functions. However, squaring and adding two equations gives us

$$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{5}\right)^2 = \cos^2\theta + \sin^2\theta = 1$$



Which is an ellipse with x-intercept $(\pm 4, 0)$ *and y-intercept* $(0, \pm 5)$. *We obtain the portion of ellipse with* $x \ge 0$ *since* $4\cos\theta \ge 0$ *for* $-\pi/2 \le \theta \le \pi/2$.

Homework 1 – 9.1

Question 11; Exercise 9.1

(a) Eliminate the parameter to find a Cartesian equation of the curve.

(b) Sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases.

$$x = \sin t, \qquad y = \csc t, \qquad 0 < t < \pi/2$$

(c) Describe the similarities between the graphs of the parametric equations

$$x = \sin^2 t, \qquad y = \sin t, \qquad 0 \le t \le \pi/2 \quad \textit{and}$$

$$x = \sin^2 t, \qquad y = \sin t, \qquad \pi/2 \le t \le \pi$$

Describing the Motion

Example - Parametric circle

(a) Confirm that the parametric equations

$$x = 4\cos 2\pi t, \qquad y = 4\sin 2\pi t, \qquad 0 \le t \le 1$$

describe a circle of radius 4 centred at the origin.

(b) Suppose a turtle walks with constant speed in the counterclockwise direction on the circular path from part (a). Starting from the point $(4,0)$, the turtle completes one lap in 30 minutes. Find a parametric description of the path of the turtle at any time $t \ge 0$, where t is measured in minutes.

Describing the Motion

Solution

(a) For each value of t, the corresponding ordered pairs (x, y) *are recorded. Plotting these points as t increases from t = 0 to t = 1 results in a graph that appears to be a circle of radius 4; it is generated with positive orientation in the counterclockwise direction, beginning and ending at* $(4,0)$.

Letting t increase beyond t = 1 would simply retrace the same curve.

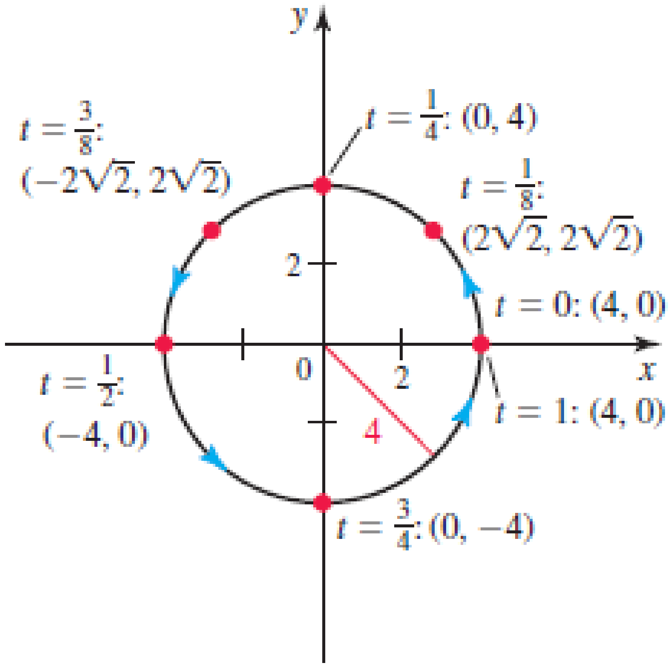
Table

t	(x, y)
0	$(4, 0)$
$\frac{1}{8}$	$(2\sqrt{2}, 2\sqrt{2})$
$\frac{1}{4}$	$(0, 4)$
$\frac{3}{8}$	$(-2\sqrt{2}, 2\sqrt{2})$
$\frac{1}{2}$	$(-4, 0)$
$\frac{3}{4}$	$(0, -4)$
1	$(4, 0)$

To identify the curve conclusively, the parameter t is eliminated by observing that

$$x^2 + y^2 = (4\cos 2\pi t)^2 + (4\sin 2\pi t)^2$$
$$x^2 + y^2 = 16$$

We have confirmed that the graph of the parametric equations is the circle of radius 4.



Describing the Motion

(b) Duplicating the calculations at the end of part (a) with any nonzero real number b, it can be shown that the parametric equations

$$x = 4\cos bt, \qquad y = 4\sin bt$$

also describe a circle of radius 4. When $b > 0$, the graph is generated in the counterclockwise direction.

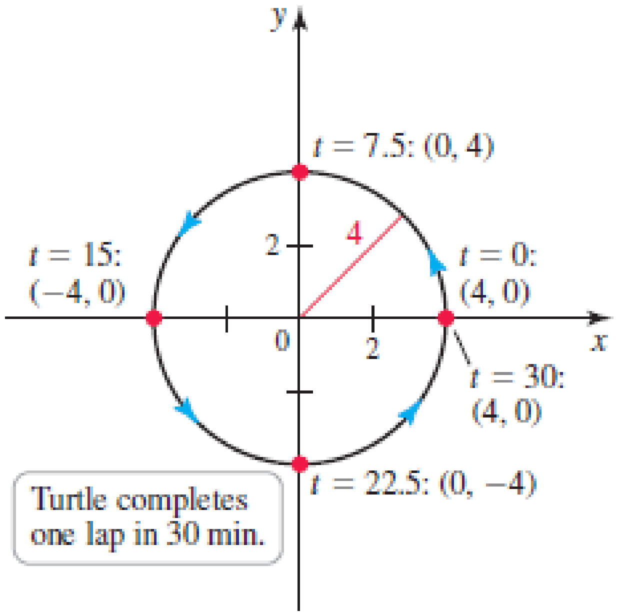
The angular frequency b must be chosen so that as t varies from 0 to 30, the product bt varies from 0 to 2π. Specificall

when t = 30, we must have

$30b = 2\pi \Rightarrow b = \pi/15\ rad/min$ *(since $b = \omega = 2\pi/T$)*

Therefore, the parametric equations for the turtle's motion

$$x = 4\cos\frac{\pi t}{15}, \qquad y = 4\sin\frac{\pi t}{15}, \qquad 0 \le t \le 30.$$



Describing the Motion

Example – Q. 18. Ex. 9.1

Describe the motion of a particle with position (x, y) as t varies in the given interval.

$$x = \sin t\,, \qquad y = \cos^2 t \qquad -2\pi \le t \le 2\pi$$

Solution

$$y = cos^2 t = 1 - \sin^2 t = 1 - x^2$$

The motion of the particle takes place on the parabola, $y = 1 - x^2$.

As t goes from -2π to $-\pi$, the particle starts at the point (0, 1), moves to (1, 0), and goes back to (0, 1).
As t goes from $-\pi$ to 0, the particle moves to (-1, 0) and goes back to (0, 1).
The particle repeats this motion as t goes from 0 to 2π.

Practice Problem 2 – 9.1

Q. 15: Ex. 9.1

Describe the motion of a particle with position (x, y) as t varies in the given interval.

$$x = 3 + 2\cos t, \qquad y = 1 + 2\sin t \qquad \pi/2 \le t \le 3\pi/2$$

Describing the Motion

Example - Find parametric equations that describe the circular path of the following objects. Assume (x, y) denotes the position of the object relative to the origin at the centre of the circle.

A bicyclist rides counterclockwise with constant speed around a circular velodrome track with a radius of 50 m, completing one lap in 24 seconds.

Solution: *Let t be time in seconds, so*

$$0 \leq t \leq 24.$$

Assuming the position (x,y) of the object is defined relative to the origin at the centre of the circle, we let,

$$x = r\cos\theta = rcos\ \omega t = rcos\ (2\pi/T)t = 50\cos(\pi/12)t$$

$$y = 50\sin(\pi/12)t$$

Then because

$$x^2 + y^2 = 50^2,$$

The path is a circle of radius 50. Note that the values of x and y at the same at t=0 and t=24, and that the circle is traversed counterclockwise.

Homework 2 – 9.1

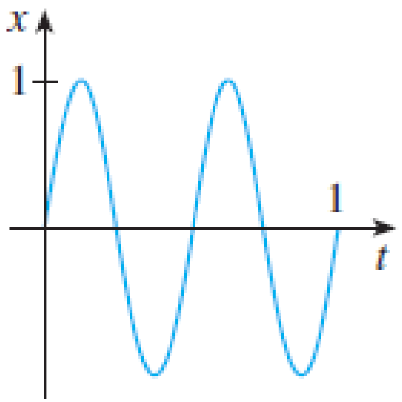
Find parametric equations that describe the circular path of the following objects. Assume (x, y) denotes the position of the object relative to the origin at the centre of the circle.

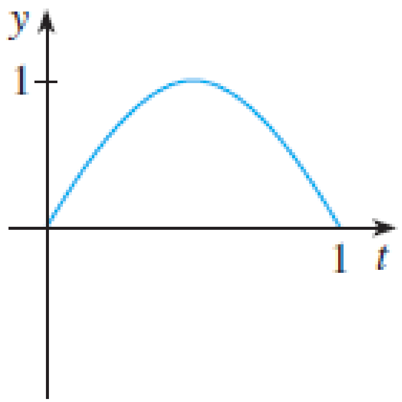
A Ferris wheel has a radius of 20 m and completes a revolution in the clockwise direction at constant speed in 3 min. Assume x and y measure the horizontal and vertical positions of a seat on the Ferris wheel relative to a coordinate system whose origin is at the low point of the wheel. Assume the seat begins moving at the origin.

Example

Q. 21: Ex. 9.1

Use the graphs of $x=f(t)$ and $y=g(t)$ to sketch the parametric curve $x=f(t)$, $y=g(t)$, indicate with arrows the direction in which the curve is traced as t increases.





Solution

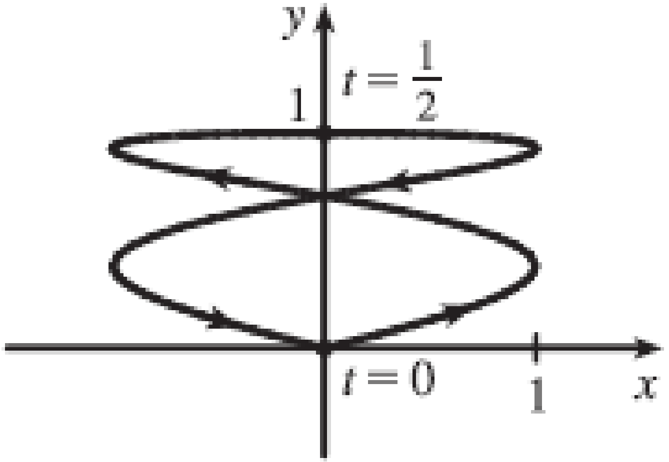
When $t = 0$ we see that $x = 0$ and $y= 0$, so the curve starts at the origin. As t increases from 0 to 1/2, the graphs show that y increases from 0 to 1 while x increases from 0 to 1, decreases to 0 and to -1, then increases back to 0, so we arrive at the point (0, 1).

Example

Solution (Contd.)

Similarly, as t increases from ½ to 1, y decreases from 1 to 0 while x repeats its pattern, and we arrive back at the origin.

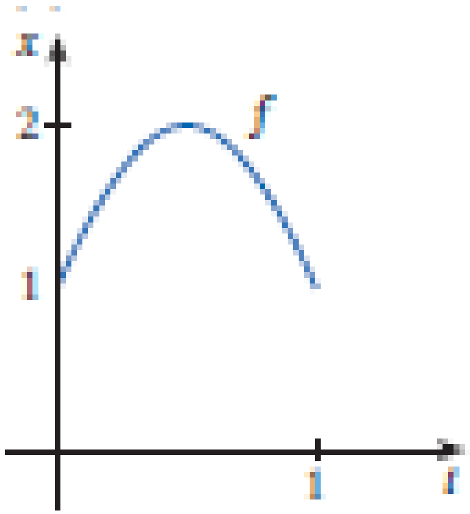
We could achieve greater accuracy by estimating x- and y-values for selected values of t from the given graphs and plotting the corresponding points.



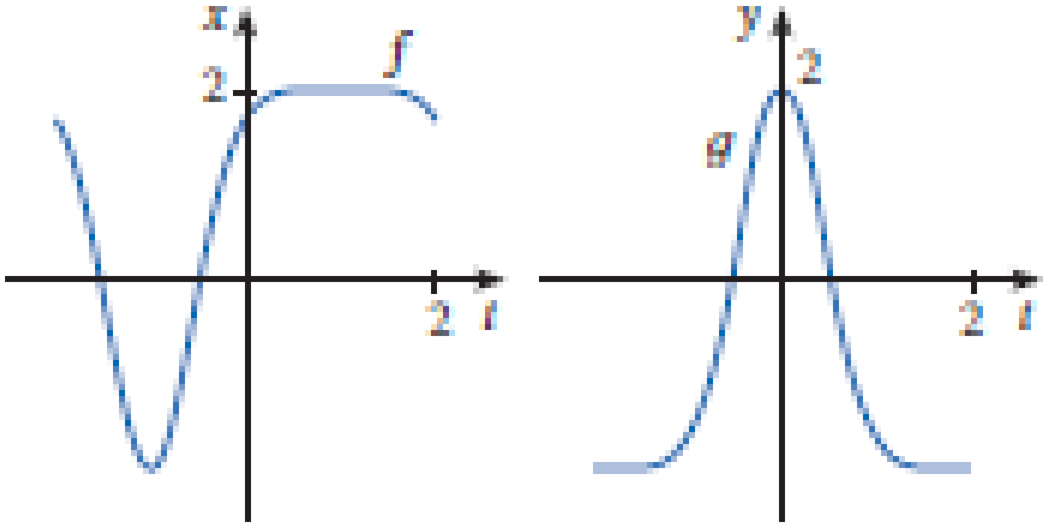
Homework 3 – 9.1

Use the graphs of x=f(t) and y=g(t) to sketch the parametric curve x=f(t), y=g(t), indicate with arrows the direction in which the curve is traced as t increases.

(a)



(b)



Parameterizing a Curve

Find a parametric representation of the segment of the parabola

$$y = 9 - x^2, \qquad -1 \leq x \leq 3$$

Solution

The simplest way to represent a curve parametrically is to let $x = t$ *and* $y = f(t)$, *where* t *is the parameter.*

We must then find the appropriate interval for the parameter. Using this approach, the curve has the parametric representation

$$x(t) = t, \qquad y(t) = 9 - t^2, \qquad -1 \leq t \leq 3$$

Parameterizing a Curve

- *What if we would like to start with the equation of a curve and determine a pair of parametric equations for that curve?*
- *This is certainly possible, and in fact, it is possible to do so in many different ways for a given curve. The process is known as* ***parameterization of a curve.***

Example

Find two different pairs of parametric equations to represent the graph

$$y = 2x^2 - 3$$

Parameterizing a Curve

Solution

First, it is always possible to parameterize a curve by defining $x(t) = t$, *then replacing* x *by* t *in the equation for* $y(t)$.

$$x(t) = t, \qquad y(t) = 2t^2 - 3$$

- *Since there is no restriction on the domain in the original graph, there is no restriction on the values of* t. *We have complete freedom in the choice for the second parameterization.* For example, we can choose

$$x(t) = 3t - 2$$

The only thing we need to check is that there are no restrictions imposed on x *that is, the range of* $x(t)$ *is all real numbers*. *This is the case for* $x(t) = 3t - 2$.

Therefore, the second parameterization is

$$x(t) = 3t - 2, \qquad y(t) = 18t^2 - 24t + 6$$

Graphing a Parametrically Defined Curve

Parametric Equations of a Line

Among the most important of all parametric equations are

$$x = x_0 + at, \qquad y = y_0 + bt, \qquad -\infty < t < \infty$$

where $x_0, y_0, a,$ *and* b *are constants with* $a \neq 0$.

The curve described by these equations is found by eliminating the parameter.

$$y = y_0 + bt = y_0 + \frac{b}{a}(x - x_0), \qquad -\infty < t < \infty$$

This equation describes a line with slope b/a through (x_0, y_0).

If $a = 0$ *and* $b \neq 0$*, the line is vertical.*

Graphing a Parametrically Defined Curve

Example - Find parametric equations for the piecewise linear path connecting P(-2, 0) to Q(0, 3) to R(4, 0) (in that order), where the parameter varies over the interval $0 \leq t \leq 2$.

Solution:

The path consists of two line segments that can be parameterized separately in the form

$$x = x_0 + at, \qquad \textit{and} \qquad y = y_0 + bt$$

The line segment PQ originates at P(-2, 0) and unfolds in the positive x-direction with slope 3/2 ($\Rightarrow b = 3, a = 2$*). It can be represented as*

$$x = -2 + 2t, \qquad y = 3t, \qquad 0 \leq t \leq 1 \quad (since\ x_0 = -2,\ \ y_0 = 0)$$

Finding the parametric equations for the line segment QR requires some ingenuity. We want the line segment to originate at Q(0, 3) when t = 1 and end at R(4, 0) when t = 2.

Graphing a Parametrically Defined Curve

Observe that when t = 1, x = 0 and when t = 2, x = 4. Substituting these pairs of values into the general x-equation $x = x_0 + at$, we obtain the equations

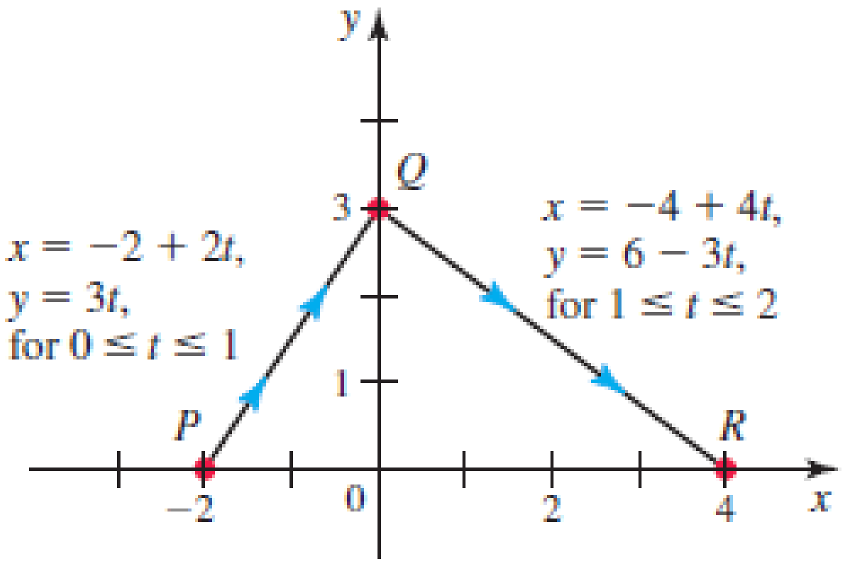
$$x_0 + a = 0, \qquad x = 0 \quad when \quad t = 1$$
$$x_0 + a = 0, \qquad x = 4 \quad when \quad t = 2$$
$$\Rightarrow \quad x_0 = -4 \quad and\ a = 4$$

Similarly,

$$y_0 + b = 3, \qquad y = 3 \quad when \quad t = 1$$
$$y_0 + 2b = 0, \qquad x = 0 \quad when \quad t = 2$$
$$\Rightarrow \quad y_0 = 6 \quad and \quad b = -3$$

Therefore, the equations for the line segment QR are

$$x = -4 + 4t, \qquad y = 6 - 3t, \qquad 1 \le t \le 2$$

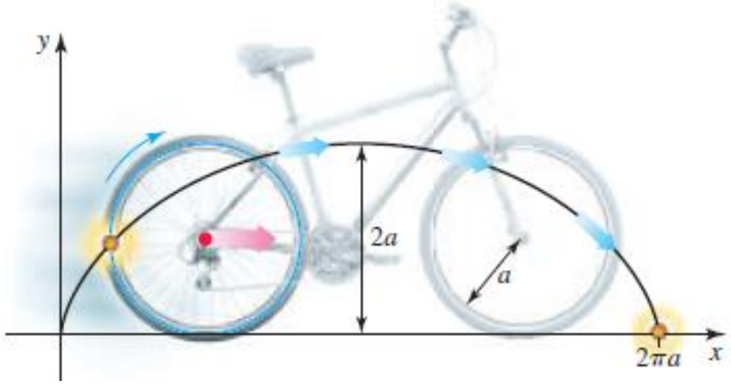


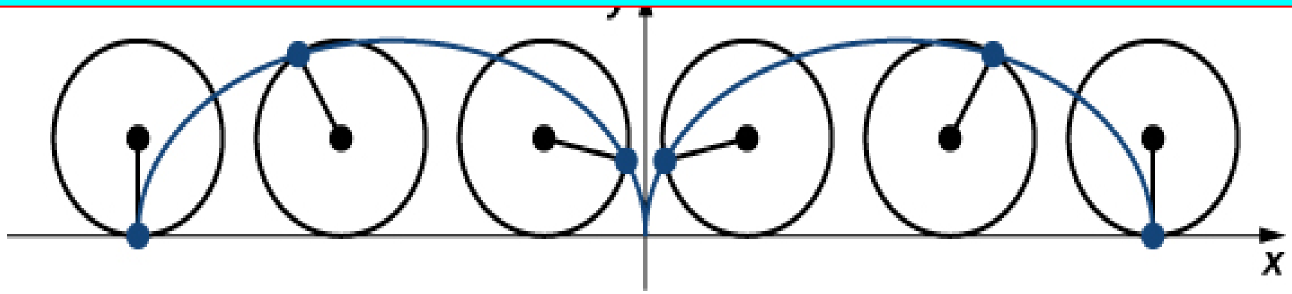
Cycloids and Other Parametric Curves

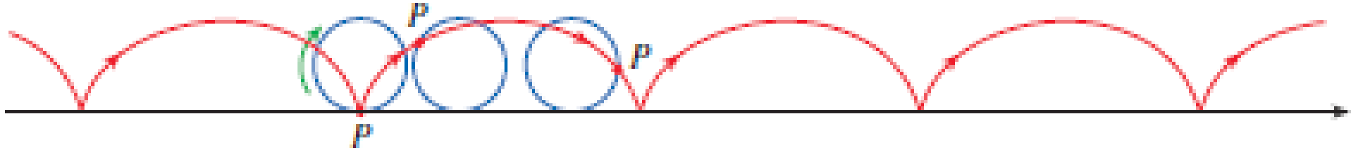
Parametric Equations of Cycloid

- *Imagine going on a bicycle ride through the country. The tires stay in contact with the road and rotate in a predictable pattern.*
- *Now suppose a very determined ant is tired after a long day and wants to get home. So he hangs onto the side of the tire and gets a free ride. The path that this ant travels down a straight road is called cycloid.*

*The curve traced out by a point P on the circumference of a circle as the circle rolls along a straight line is called a **cycloid**.*







Parametric Equations of Cycloid

Parametric Equations of Cycloid

Consider the path that the centre of the wheel takes. The centre moves along the x-axis at a constant height equal to the radius of the wheel.

If the radius is r, then the coordinates of the centre can be given by the equations:

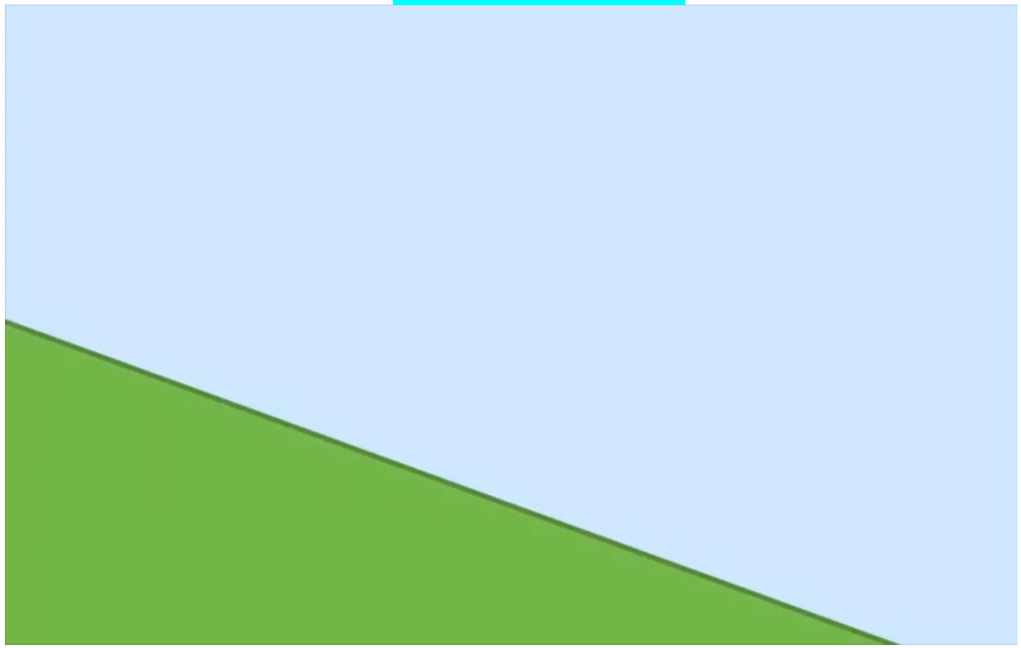
$$(r\theta, r)$$

for any time t.

Play Me

The parametric equations of a cycloid are given by

$$\boxed{\begin{aligned} x &= r(\theta - \sin\theta) \\ y &= r(1 - \cos\theta) \end{aligned}}$$



Practice Problem 3– 9.1

Example

- *When the x-coordinate of P is $2.5r$, what is its y-coordinate? initially the lowest point on the circle, P, coincides with the origin of coordinates O.*

Ans: $0.9316\, r$

Hint: Use Newton-Raphson iteration. For $n > 0$,

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$$

x_0 *is the estimate root of* $f(x) = 0$.

Caution: initially the lowest point on the circle, P, coincides with the origin of coordinates O. Think Carefully about the angle here .....

Example – Cycloid

A circle of diameter 40mm rolls along a straight line without slipping. Draw the curve traced by a point on the circumference for one complete revolution.

Solution

Circumference of the circle is $2\pi r = 125.66\, mm$

A point on this circle traces a cycloid with radius 20 mm and arch-length 125.664mm.

Equation of the cycloid is given by

$$x = r\cos^{-1}\left(\frac{r-y}{r}\right) - \sqrt{(2ry - y^2)}$$

$$x = 20\cos^{-1}(1 - y/20) - \sqrt{(40y - y^2)}$$

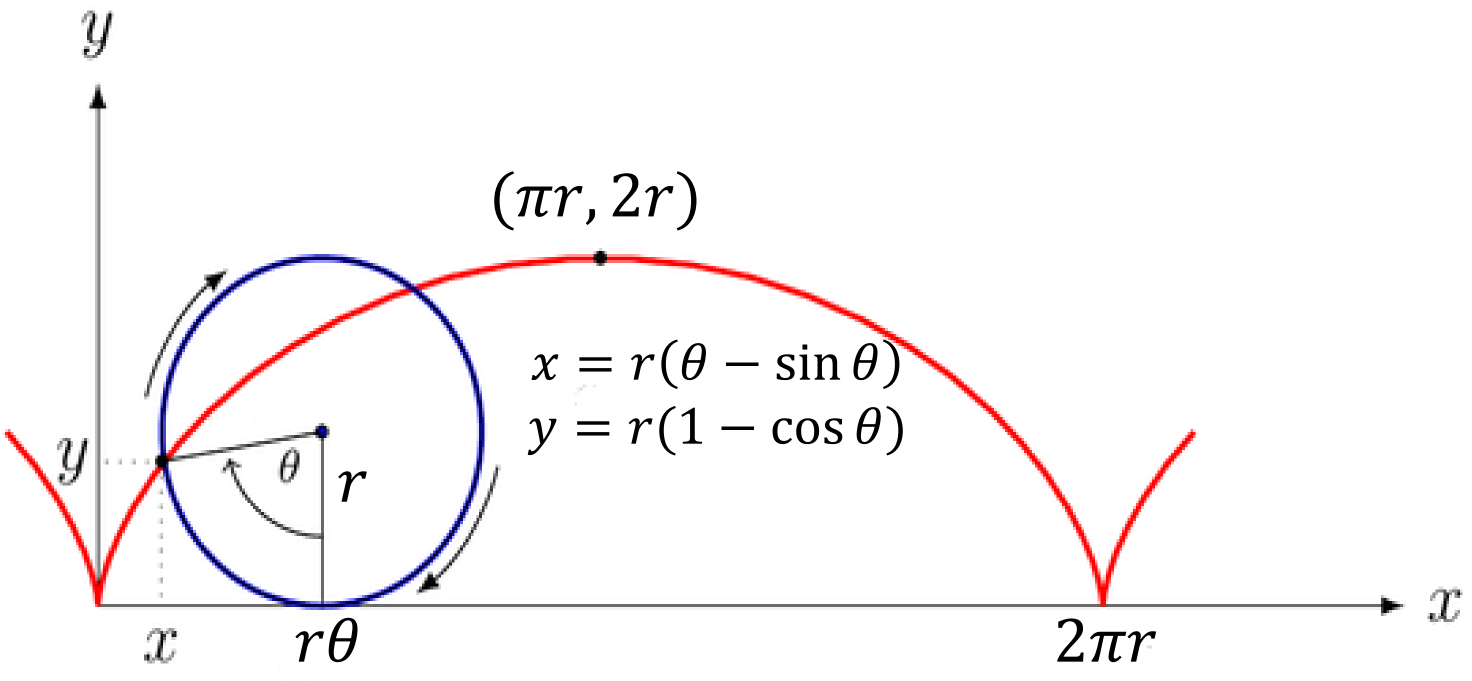
The parametric equations are

$$x = 20(\theta - \sin\theta), \qquad y = 20(1 - \cos\theta)$$

Example – Cycloid

Solution (Contd.)

The arch of the cycloid with r=20mm is shown approximately in the figure below



Example – Half Horizontal & Vertical Revolutions

A circle of 50 mm diameter rolls on a horizontal line. Draw the curve traced out by a point P on the circumference for one-half revolution of the circle. For the remaining half revolution, the circle rolls on a vertical line. The point P is at the bottom of the circle.

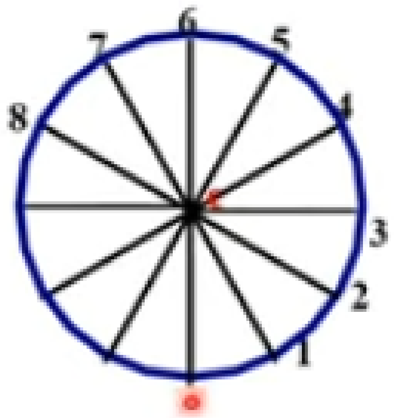
Solution

Diameter of the rolling circle = 50 mm

Length of the horizontal directing line for half revolution =length of the vertical directing line for half revolution = $\pi D/2 = 157/2 = 78.5mm$

Use this information to construct the curve.

- *With C as centre and rolling radius (25 mm) draw a circle and divide the circle into 12 equal parts*
- *Locate a point P on the circumference of the circle*



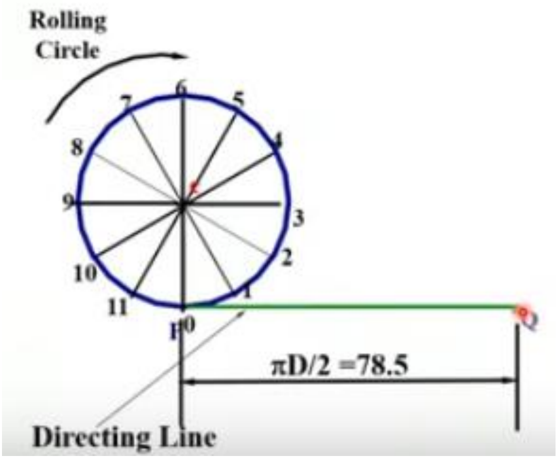
Example – Half Horizontal & Vertical Revolutions

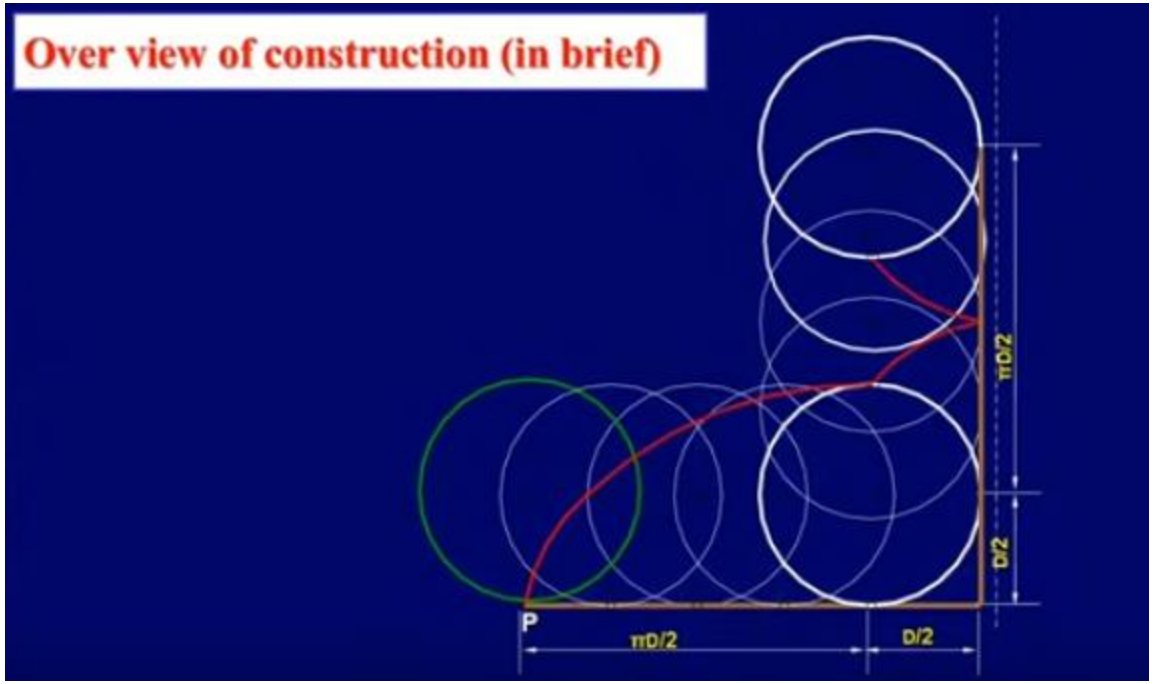
Solution

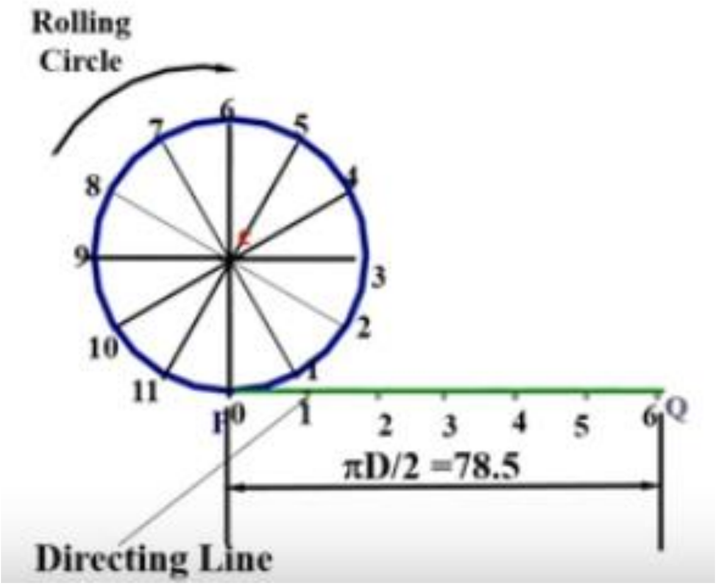
- *Draw a tangent to the circle in the direction of directing line PQ*

(Circle rolls horizontally on directed line for half a revolution, so the length of the directed line is $\pi D/2 = 78.5\ mm$*)*

- *Divide the directed line in PQ into six equal parts*



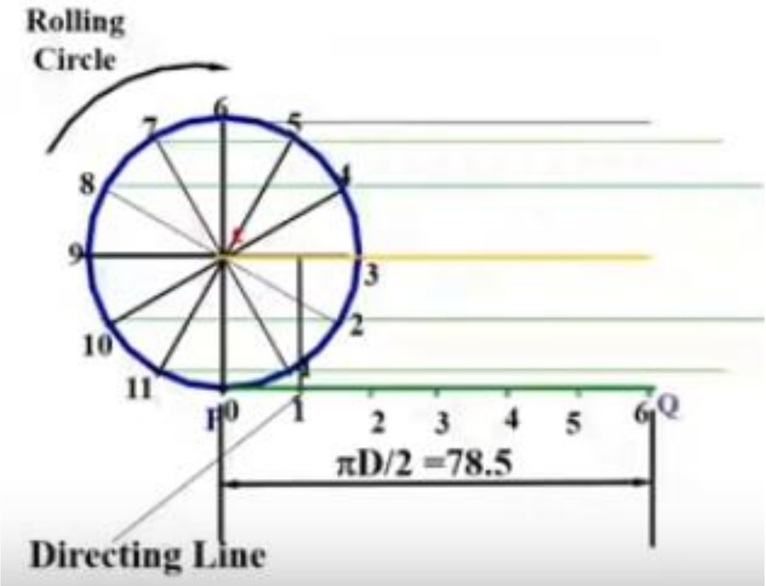




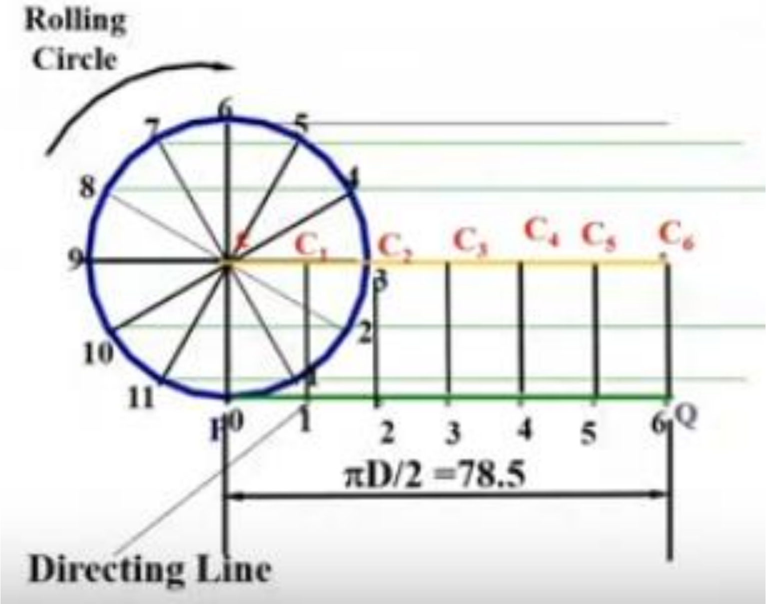
Example – Half Horizontal & Vertical Revolutions

Solution

- *Draw lines parallel to the directing line passing through the division points on the circle*
- *Draw locus of the centre of the generating circle passing through the centre of the rolling circle and parallel to the directing line*



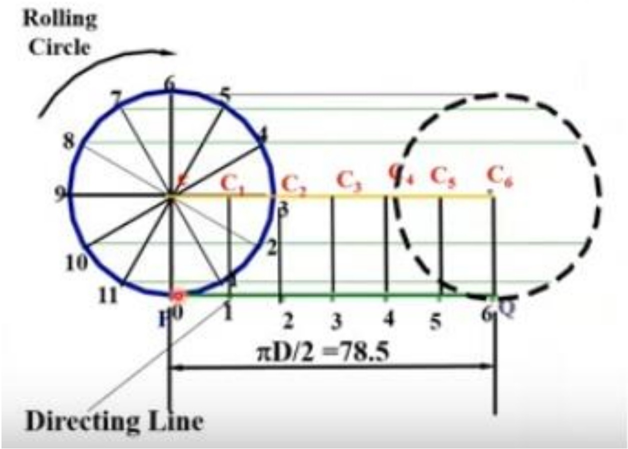
- *Divide the locus of path line into six equal parts with reference to directing line and name the parts ($C_1 \ldots \ldots C_6$).*

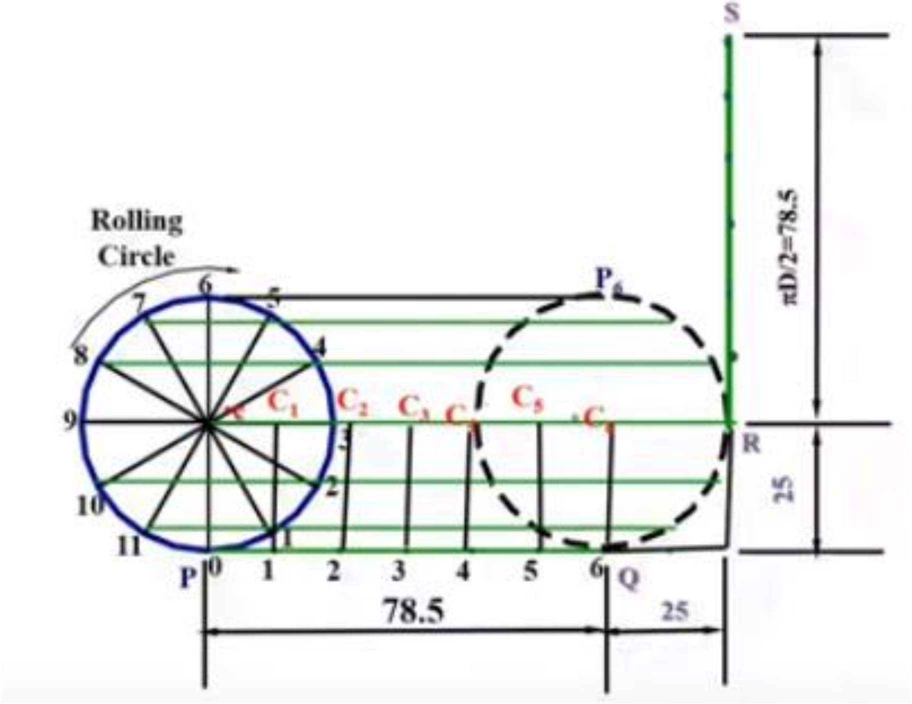


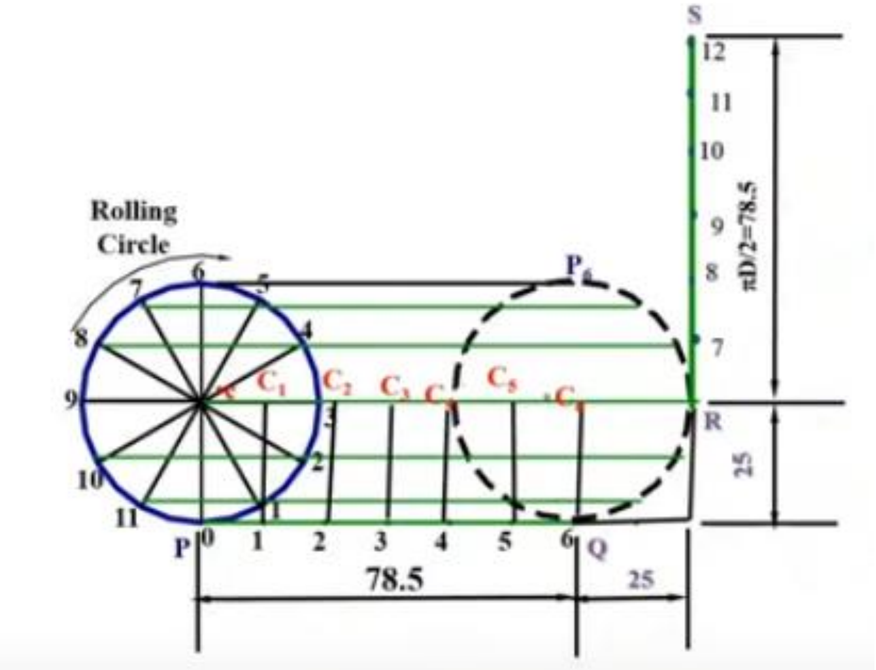
Example – Half Horizontal & Vertical Revolutions

Solution

- *Draw a vertical directing line RS to the rolling circle after the half revolution*
- *Length of the directing line is* $\pi D/2 = 78.5mm$
- *Divide the directing line RS into 6 equal divisions (7-12)*



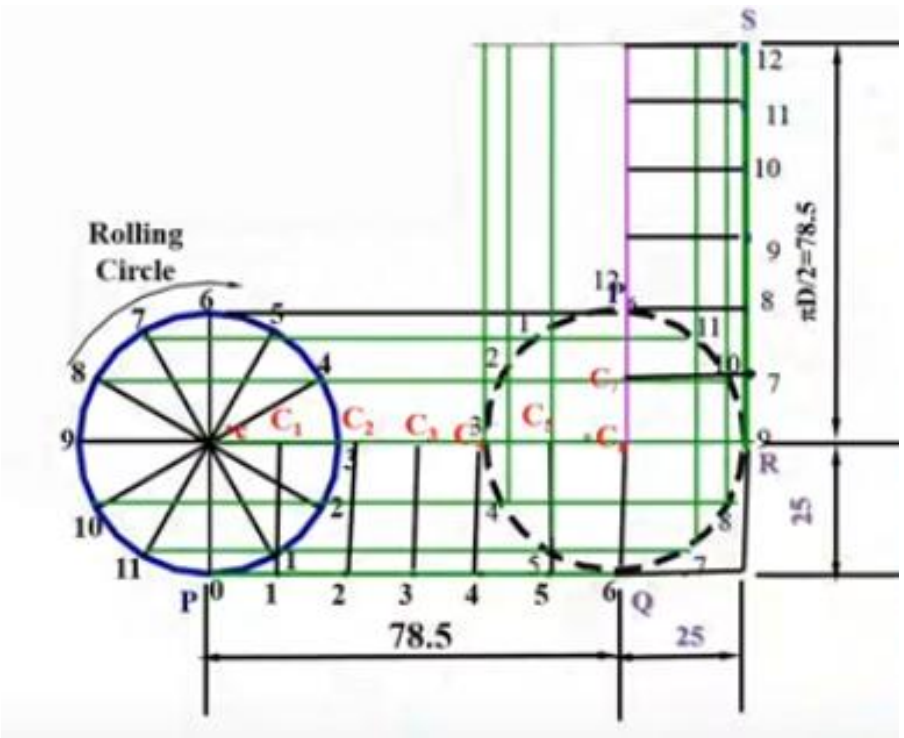


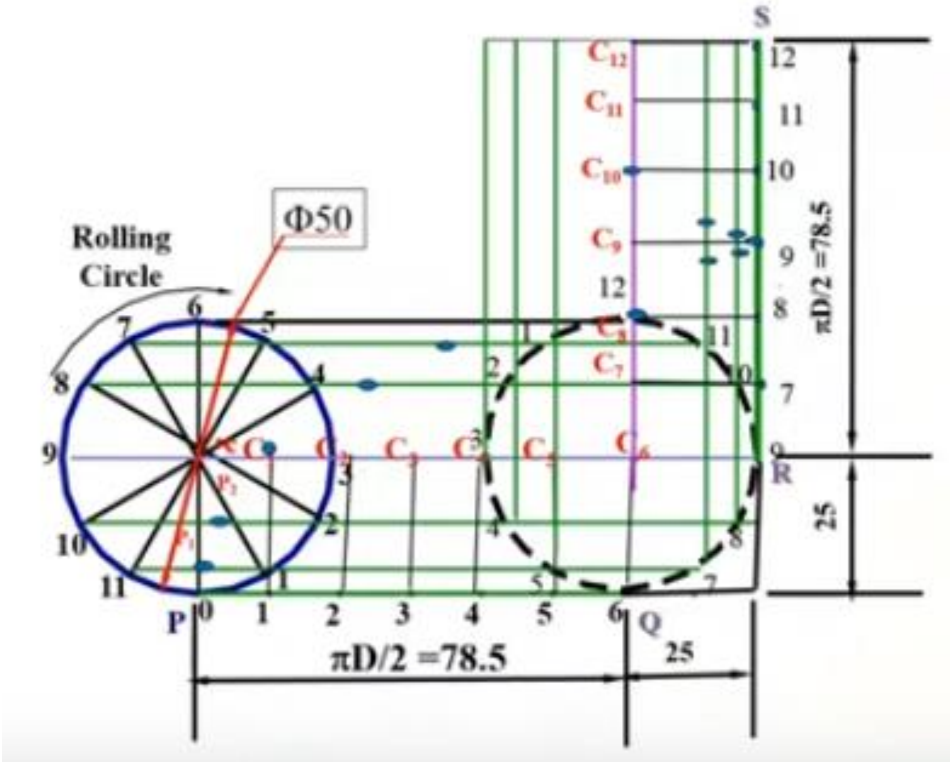


Example – Half Horizontal & Vertical Revolutions

Solution

- *Draw lines parallel to the vertical directing line passing through the division points on the rolling circle*
- *Divide the locus of path line into six equal divisions with reference to directing line. Name the points as C_7 …. C_{12}.*



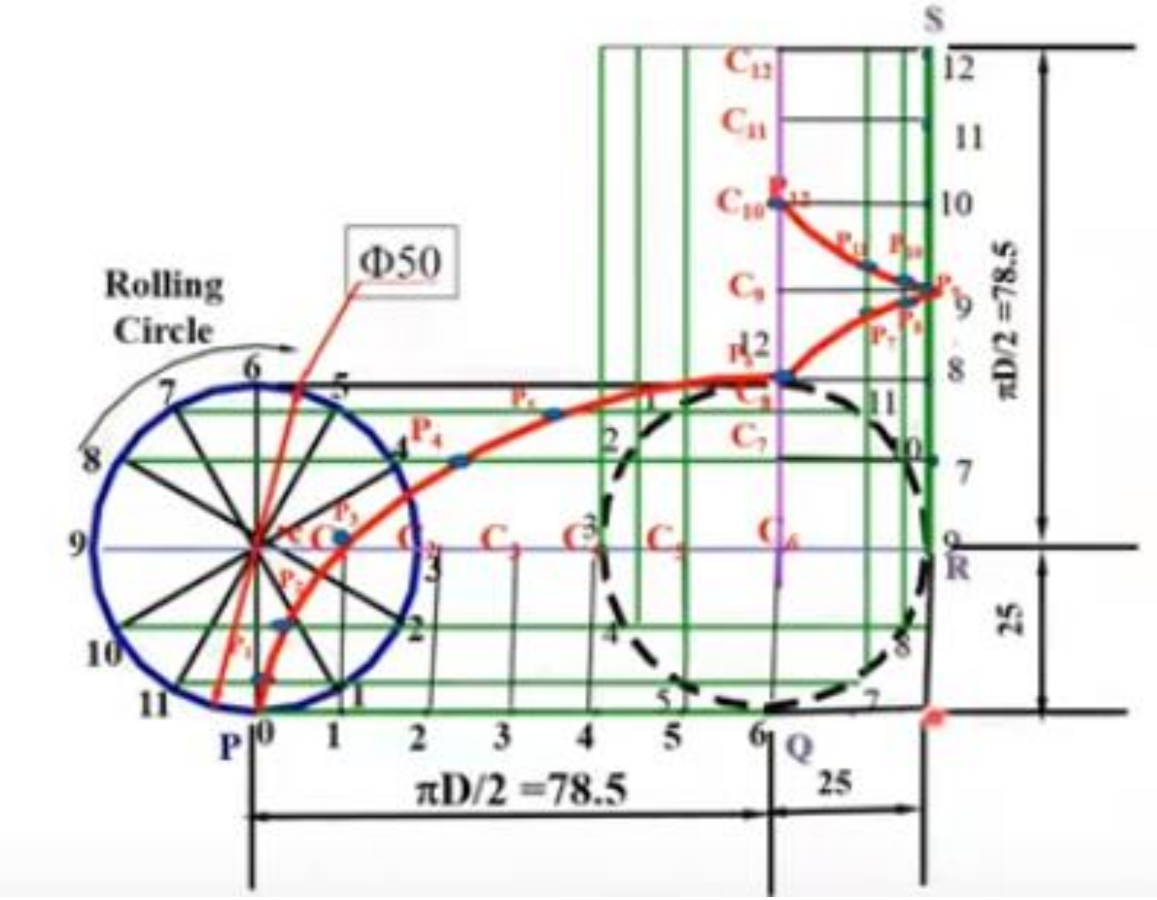


Example – Half Horizontal & Vertical Revolutions

Solution (Contd.)

- *With C_1, C_2, C_3, etc. as the centres and radius = 25 mm, cut an arc on the lines through 1, 2, 3 etc. to locate P_1, P_2, P_3 etc.*

- *Join $P, P_1, P_2, \ldots\ldots P_{12}$ by a smooth curve.*



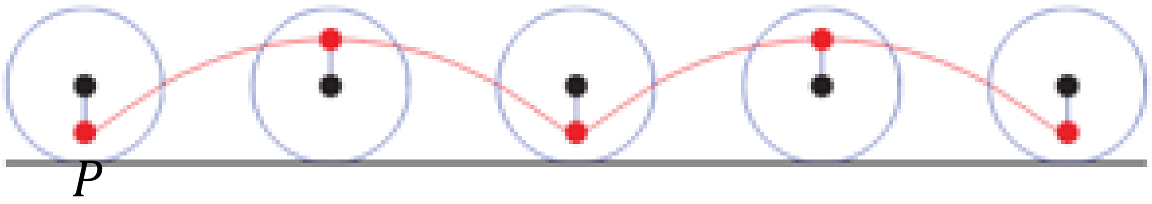
Homework 4 – 9.1

A circle of 50 mm diameter rolls on a horizontal line. Draw the curve traced out by a point P on the circumference for one half revolution of the circle. For the remaining half revolution, the circle rolls on a vertical line. The point P is vertically above the centre of the circle in the starting position.

Trochoid

A trochoid is the locus of a point at a distance d from the centre of a circle of radius r rolling on a fixed line.

The curve traced out by a point P as the circle rolls along a straight line is called trochoid.



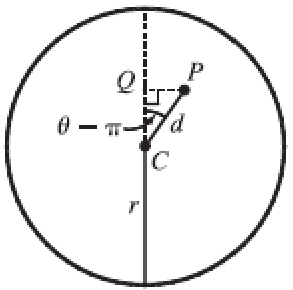
Simulation

The curve traced out by a point P as the circle rolls along a straight line is called trochoid.

Assuming that the line is the x-axis and $\theta = 0$ when P is at one of its lowest points.

Trochoid

The coordinates of centre C of the circle are $(r\theta, r)$.



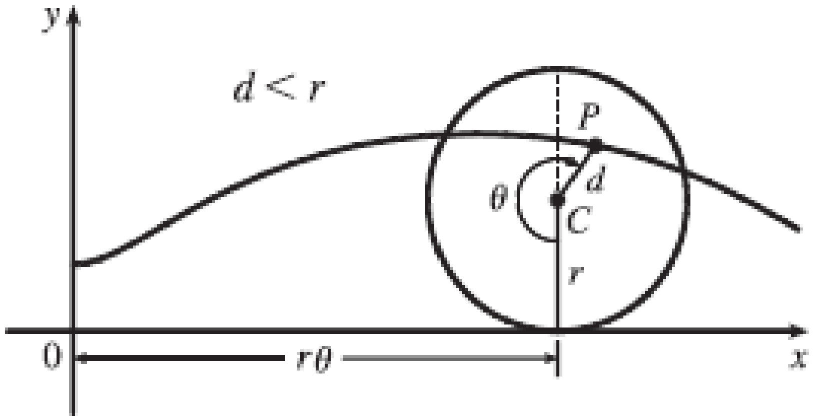
(i) $d < r$.

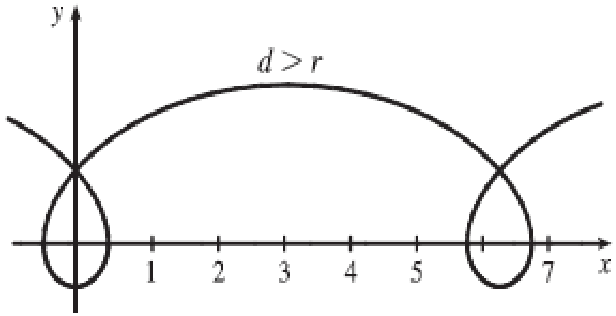
Point Q has coordinates

$$(r\theta, r + d\cos(\theta - \pi) = (r\theta, r - d\sin\theta)$$

So a typical point P of the trochoid has coordinates

$$(r\theta + d sin(\theta - \pi), \qquad r - d cos\theta)$$
$$x = r\theta - d sin\theta, \qquad y = r - d cos\theta$$



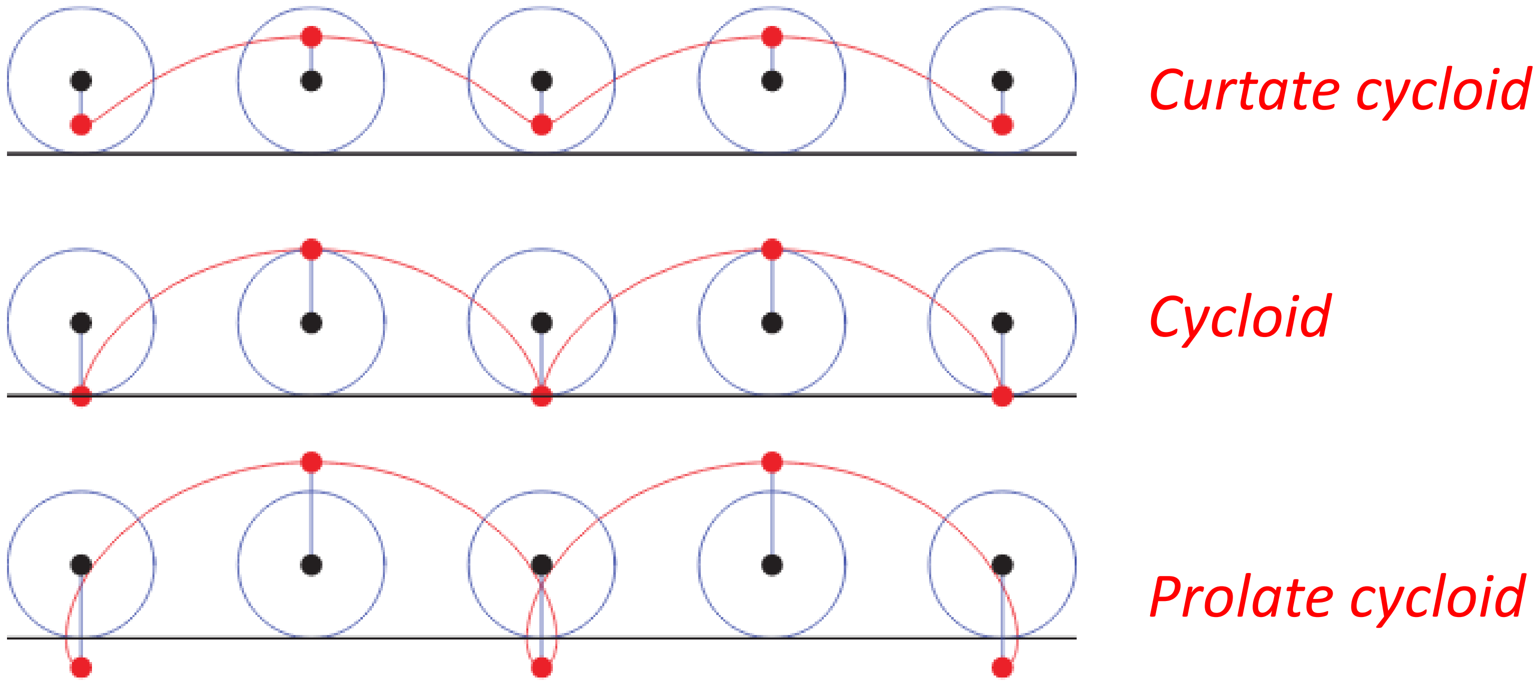


Trochoid

(i) *If d< r, the trochoid is known as a curtate cycloid*

(ii) *If d=r, it is a cycloid*

(iii) *If d> r, the curve is a prolate cycloid.*

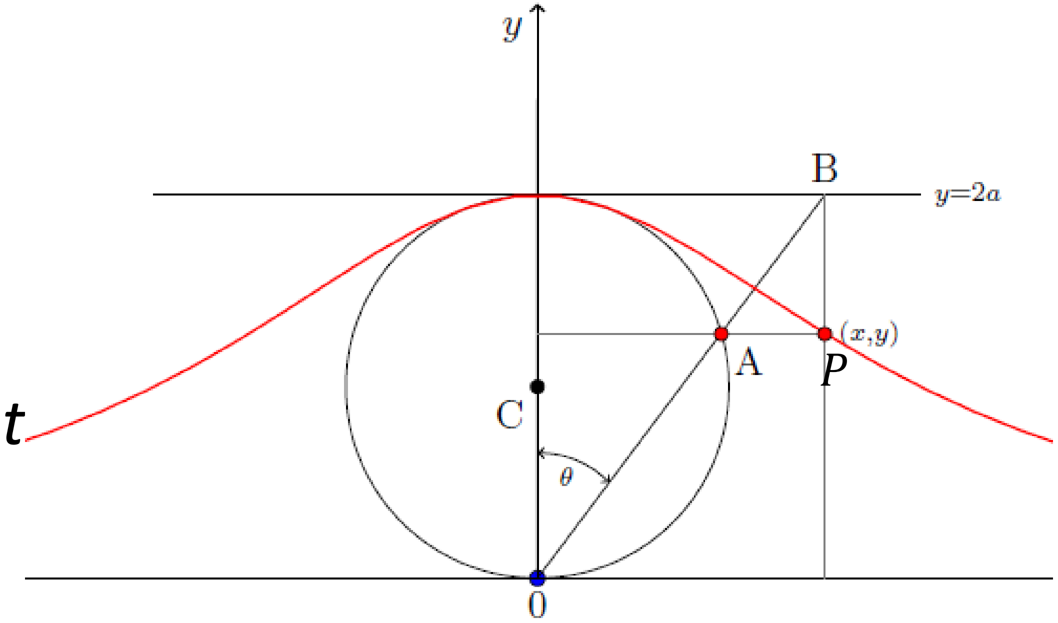


[*Simulation*]

Witch of Maria Agnesi

The figure below shows a circle of radius a centred at the point $C(0, a)$. The points (x, y) are obtained as follows:

- *Draw a line from the point $O(0,0)$ to any point (B) on the line $y = 2a$. The line drawn intersects the circle at the point A.*
- *Draw a horizontal line through A and a vertical line through B.*
- *The point where these two lines intersect is the point (x; y).*
- *As the point A varies, the path that the point P travels is the witch of Agnesi curve for the given circle.*



Witch of Agnesi curves have applications in physics, including modelling water waves and distributions of spectral lines. In probability theory, the curve describes the probability density function of the Cauchy distribution.

Parametric Equation for Witch of Maria Agnesi

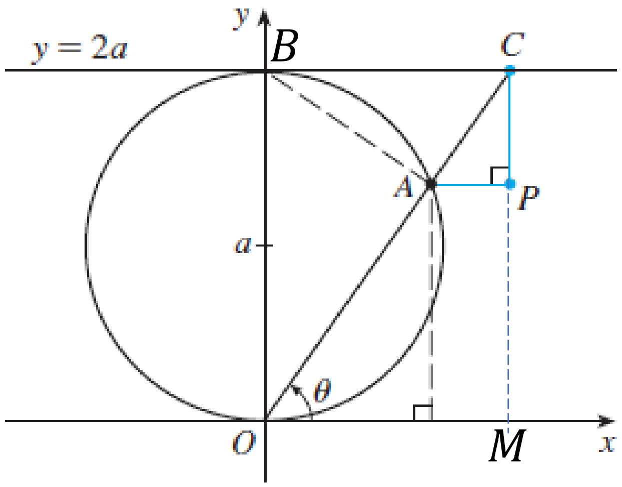
Q. 36 – Ex. 9.1

The radius of the circle is a. The construction leads us to the equation

$$\cot\theta = 0M/CM \Rightarrow 0M = 2a\cot\theta$$

Thus, the coordinates of C are $(2a\cot\theta, 2a)$. So, the x −coordinate of P is $2a\cot\theta$.

To find the y-coordinate of P, consider the triangle OAB.



B is (0,2a), then $\angle OAB$ is a right angle and $\angle OBA = \theta$, so

$$|OA| = 2a\sin\theta$$

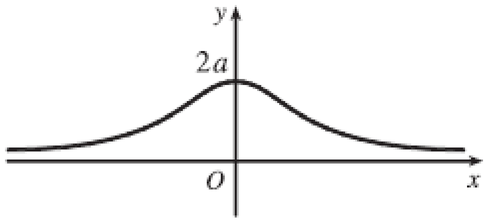
and

$$A = ((2a\sin\theta)cos\theta, (2a\sin\theta)sin\theta)$$
$$= (a\sin 2\theta, \qquad 2a\sin^2\theta)$$

Thus y-coordinate of P is $2a\sin^2\theta$.

Therefore, the parametric equations are

$$x = 2a\cot\theta, \qquad y = 2a\sin^2\theta$$



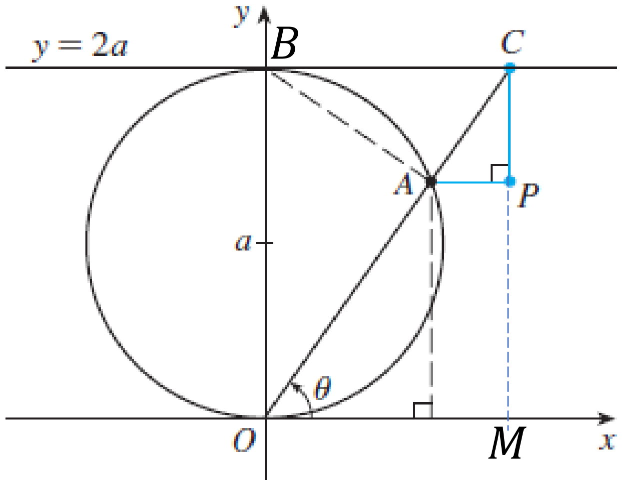
Parametric Equation for Witch of Maria Agnesi

Q. 36 – Ex. 9.1

A curve, called a Witch of Maria Agnesi, conisists of all possible positions of the point P in the figure. Show that parametric equations for the curve can be written as

$$x = 2a\cot\theta, \qquad y = 2a\sin^2\theta$$

Sketch the curve.



Parametric Equation for Witch of Maria Agnesi

Solution: *The radius of the circle is a. The construction leads us to the equation*

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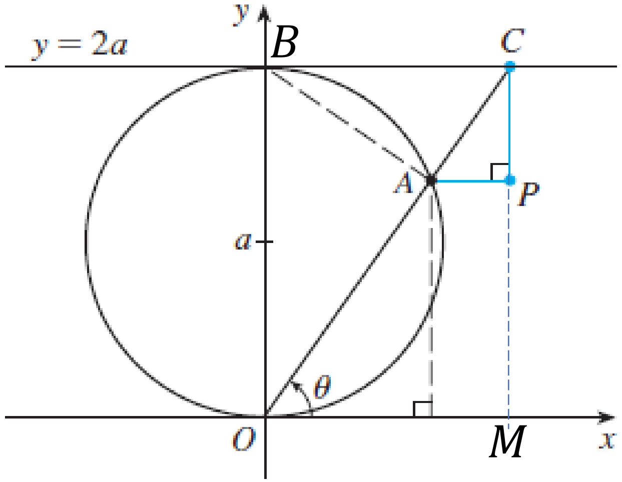

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Therefore, the parametric equations are

$$x = 2a\cot\theta, \qquad y = 2a\sin^2\theta$$