Sec 10.5

Homework 1 - 10.5

L_1: $x_1 = 2+t_1$ $\quad$ L_2: $x_2 = 3+t_2$

$y_1 = 3-2t_1$ $\quad$ $y_2 = -4+3t_2$

$z_1 = 1-3t_1$ $\quad$ $z_2 = 2-7t_2$

the direction vector of these two lines:

$\vec{v_1} = \langle 2,3,1\rangle$. $\vec{v_2} = \langle 3,-4,2\rangle$

$\vec{v_1} \neq k\vec{v_2}$, where $k\in R$. so they're not parallel.

$$\begin{cases} 2+t_1 = 3+t_2 \\ 3-2t_1 = -4+3t_2 \end{cases} \Rightarrow \begin{cases} t_1 = 2 \\ t_2 = 1 \end{cases}$$

then. $z_1 = 1-3\times 2 = z_2 = 2-7\times 1 = -5$

So, they intersect.

and the intersect point is:

$p = (4,-1,-5)$.

Homework 2 - 10.5

the normal vector of the desired plane is also the normal vector of

$2x+4y+8z=17$

$\Rightarrow \vec{n} = \langle 2,4,8\rangle = \langle 1,2,4\rangle$

$$\begin{cases} x = 3+2t \\ y = t \\ z = 8-t \end{cases}$$ suppose $t=0$ $\Rightarrow$ the plane passes the point $(3,0,8)$.

So, the desired plane

$1\cdot(x-3) + 2(y-0) + 4(z-8) = 0$.

$\Rightarrow x+2y+4z-35 = 0$.

Homework 3 - 10.5

(a). $$\begin{cases} x-4y+2z=0 \\ 2x-8y+4z=-1 \end{cases}$$

the normal vectors of the two planes:

$\vec{n_1} = \langle 1,-4,2\rangle$ $\quad$ $\vec{n_2} = \langle 2,-8,4\rangle = \langle 1,-4,2\rangle$

$\Rightarrow \vec{n_1} = \vec{n_2}$, which means they're parallel.

(b). that means the plane passes through $A\cdot(a,0,0)$. $B(0,b,0)$. $C(0,0,c)$.

first we find a normal vector to the plane.

$\vec{AB} = \langle -a,b,0\rangle$. $\quad$ $\vec{BC} = \langle 0,-b,c\rangle$

suppose the normal vector is $\vec{n}$ $= \langle n_1, n_2, n_3\rangle$

such that $\vec{AB}\cdot\vec{n} = 0$, $\vec{BC}\cdot\vec{n} = 0$.

$$\begin{cases} -an_1 + bn_2 = 0 \\ -bn_2 + cn_3 = 0 \end{cases}$$ we let $n_2 = 1$.

$$\Rightarrow \begin{cases} n_1 = \frac{b}{a} \\ n_3 = \frac{b}{c} \end{cases} \Rightarrow \vec{n} = \langle \frac{b}{a}, 1, \frac{b}{c}\rangle.$$

we choose point A, so the equation of the plane can be written as:

$\frac{b}{a}(x-a) + y + \frac{b}{c}z = 0$

$\Rightarrow \frac{b}{a}x + y + \frac{b}{c}z = b$

Homework 4 - 10.5

(a). $D = \frac{|3\times 1 - 2\times 2 + 6\times 4 - 5|}{\sqrt{3^2+2^2+6^2}} = \frac{18}{7}$

(b). we let $x=y=0$ for the first plane. then we got the point $(0,0,0)$.

$D = \frac{1}{\sqrt{3^2+6^2+9^2}} = \frac{\sqrt{14}}{42}$

Homework 5 - 10.5

L_1: $\vec{v_1} = \langle 2-0, 0-0, -1-0\rangle = \langle 2,0,-1\rangle$

parametric equation

$x_1 = 2t_1$, $y_1 = 0$, $z_1 = -t_1$

L_2: $\vec{v_2} = \langle 4-1, 1+1, 3-1\rangle = \langle 3,2,2\rangle$

parametric equation

$x_2 = 1+3t_2$ $y_2 = -1+2t_2$ $z_2 = 1+2t_2$

$\vec{v_1} \neq k\vec{v_2}$, where k is real number

$\Rightarrow L_1$ is not parallel to L_2

$$\begin{cases} 2t_1 = 1+3t_2 \\ 0 = -1+2t_2 \end{cases} \Rightarrow \begin{cases} t_1 = \frac{5}{4} \\ t_2 = \frac{1}{2} \end{cases}$$

$z_1 = -\frac{5}{4} \neq z_2 = 1+2\times\frac{1}{2} = 2$

So they don't intersect

$\Rightarrow$ they are skew.

we find $\vec{n} = \vec{v_1}\times\vec{v_2} = \langle 2,-7,4\rangle$

pick any point on the two lines:

$P_1: (0,0,0)$. $\quad$ $P_2(1,-1,1)$

$\Rightarrow \vec{P_1P_2} = \langle 1,-1,1\rangle$

distance is projection of $\vec{P_1P_2}$ along $\vec{n}$:

$D = \frac{|\vec{n}\cdot\vec{b}|}{|\vec{n}|} = \frac{|2+7+4|}{\sqrt{4+49+16}} = \frac{13}{\sqrt{69}}$

$= \frac{13\sqrt{69}}{69}$

So, the distance between L_1 and L_2 is $\frac{13\sqrt{69}}{69}$

Sec 10.6.

Homework 1-10.6

first we can divide all these functions into 2 groups:
whether they can passes through the origin.

$\begin{cases} x^2+4y^2+9z^2=1 \\ 9x^2+4y^2+z^2=1. \end{cases}$ ✓

$\begin{cases} x^2-y^2+z^2=1 \\ -x^2+y^2-z^2=1. \end{cases}$ ✓

$x^2+2z^2=1$ ✓

↓

$y=2x^2+z^2$ ✓

$y^2=x^2+2z^2$

$y=x^2-z^2$ ✓

↓

① we can know that in $y=2x^2+\cancel{z}z^2$, $y\geq 0$,
So we match it with VI.

② Then we take a look at $x^2+2z^2=1$,
no matter what value y is, the trace is always a ~~ci ellipitcal~~ elliptical:

$$x^2+2z^2=1$$

and strech in y-axis, so it's VIII.

③. all traces of $y=x^2-z^2$ are parabolas.
that matches graph V.

④. for $\begin{cases} x^2+4y^2+9z^2=1 \\ 9x^2+4y^2+z^2=1 \end{cases}$

the horizontal traces are ellipses

$x^2+4y^2=1-9z^2 \rightarrow -\frac{1}{3}\leq z\leq \frac{1}{3}$

$9x^2+4y^2=\underbrace{1-z^2}$

↓

$-1\leq z\leq 1$

that's a ellipsoid, from the domain of z we can determine that:

$x^2+4y^2+9z^2=1$ matches VII

$9x^2+4y^2+z^2=1$ matches IV.

⑤. for $\begin{cases} x^2-y^2+z^2=1 \\ -x^2+y^2-z^2=1 \end{cases}$

x-z traces: $\begin{cases} \underline{x^2+z^2}=\underline{1+y^2} \rightarrow \text{a circle.} \\ x^2+z^2=y^2-1 \end{cases}$

↓

$y\geq 1$ or $y\leq -1$

$\Rightarrow$ matches III

x-y traces:
$x^2-y^2=1-z^2$

y-z traces:
$z^2-y^2=1-x^2$

} they are hyperbola

So, it matches graph II.

⑥. for $y^2=x^2+2z^2$

x-z trace is a ellipse.

x-y trace is hyperbola

y-z trace is also hyperbola

$y^2-2z^2=x^2\ \underline{\geq 0}.$

and they can passes throug the origin.

$\Rightarrow$ that matches graph I.

In all: the situations are:

$x^2+4y^2+9z^2=1$	VII
$x^2-y^2+z^2=1$	II
$9x^2+4y^2+z^2=1$	IV
$-x^2+y^2-z^2=1$	III
$y=2x^2+z^2$	VI
$y^2=x^2+2z^2$	I
$x^2+2z^2=1.$	VIII
$y=x^2-z^2$	V

Homework 2-10.6

Let $P(x,y,z)$ be an arbitrary points.

the distance from P to the x-axis is ~~$\sqrt{y^2+z^2}$ x~~ $\sqrt{y^2+z^2}$

the distance from P to the y-z plane is $|x|$

from the question, we obtain:

$$\sqrt{y^2+z^2}=2|x|$$

$\Rightarrow y^2+z^2=4x^2 \Rightarrow y^2+z^2-4x^2=0.$

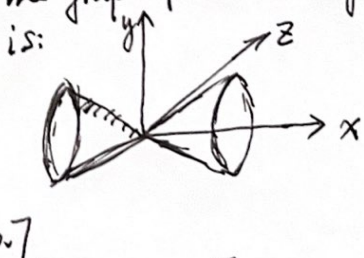
x-y traces are hyperbolas

x-z traces are hyperbolas

y-z traces are circles

and the graph passes through $(0,0,0)$.

that is:



Sec 10.7

Homework 1- Ex 10.7.

$r_1(t)=\langle t^2, 7t-12, t^2\rangle$

$r_2(t)=\langle 4t-3, t^2, 5t-6\rangle$

we let $t^2=4t-3$

$\Rightarrow t=1$ or $t=3$

when $t=1$, $r_1(1)=\langle 1,-5,1\rangle$
$r_2(1)=\langle 1,1,-1\rangle$ they won't intersect

when $t=3$, $r_1(3)=\langle 9,9,9\rangle=r_2(3)$
they will intersect.

when $t=3$, this two particles will intersect at $\langle 9,9,9\rangle$. they will collide.

Homework 2 - Ex 10.7

$$\int (i t e^{2t} + j\cdot\frac{t}{1-t} + k\cdot\frac{1}{\sqrt{1-t^2}})dt$$
$$= \int i t e^{2t}dt + \int j\frac{t}{1-t}dt + \int k\frac{1}{\sqrt{1-t^2}}dt$$
$$= \frac{te^{2t}}{2} - \frac{e^{2t}}{4} + 1-t-\ln|1-t| + \arcsin t + C$$
$$\Rightarrow r(t) = (\frac{t\cdot e^{2t}}{2} - \frac{e^{2t}}{4})i + (1-t-\ln|1-t|)j + (\arcsin t)k + C$$

Homework 3 - Ex 10.7

$r(t) = u(t)\times v(t)$
$\Rightarrow r'(t) = u'(t)\times v(t) + u(t)\times v'(t)$
So, $r'(2) = u'(2)\times v(2) + u(2)\times v'(2)$
$= \langle 3,0,4\rangle \times v(2) + \langle 1,2,-1\rangle\times v'(2)$
$v'(t) = \langle 1, 2t, 3t^2\rangle$
So, $v(2) = \langle 2,4,8\rangle$, $v'(2) = \langle 1,4,12\rangle$
$\Rightarrow r'(2) = \langle -16,-16,12\rangle + \langle 28,-13,2\rangle$
$= \langle 12,-29,14\rangle$

Homework 4 - Ex 10.7

for a sphere with centre the origin, equation $x^2+y^2+z^2 = c^2$.
as $r(t)$ is always perpendicular to $r'(t) \Rightarrow r(t)\cdot r'(t) = 0$ for $\forall t\in R$.
that means $\frac{d}{dt}(r^2(t)) = 0$
$\Leftrightarrow r'(t)\cdot 2r(t) = 0$.
$\Leftrightarrow 2\cdot r(t)\cdot r'(t) = 0$.
that's what we already know
So, $\int \frac{d}{dt} r^2(t) = \int 0 \Rightarrow r^2(t) = C$
$r^2(t) = (\sqrt{x^2+y^2+z^2})^2 = x^2+y^2+z^2 = C$
$\Rightarrow x^2+y^2+z^2 = C$, which means the points lie on a sphere with centre the origin.

Homework 5 - Ex 10.7

$u(t) = r(t)\cdot[r'(t)\times r''(t)]$
then, $u'(t) = r'(t)\cdot[r'(t)\times r''(t)] + r(t)\cdot[r'(t)\times r''(t)]' = r'(t)\cdot[r'(t)\times r''(t)] + r(t)\cdot[r''(t)\times r''(t) + r'(t)\times r'''(t)]$
$= r'(t)\cdot[r'(t)\times r''(t)] + r(t)\cdot[r''(t)\times r''(t)] + r(t)\cdot[r'(t)\times r'''(t)]$
$\Rightarrow$ as $r''(t)\times r''(t) = \vec{0}$
So, $= r'(t)\cdot[r'(t)\times r''(t)] + r(t)\cdot[r'(t)\times r'''(t)]$
then, as $r'(t)\perp(r'(t)\times r''(t))$,
$r'(t)\cdot[r'(t)\times r''(t)] = 0$ again
$\Rightarrow = r(t)\cdot[r'(t)\times r'''(t)]$
So, in the end, $u'(t) = r(t)\cdot[r'(t)\times r'''(t)]$

Sec 10.8

Homework 1 - 10.8

$r(t) = i2t + j(1-3t) + k(5+4t)$
$\Rightarrow r'(t) = 2i - 3j + 4k$
So, $|r'(t)| = \sqrt{2^2+3^2+4^2} = \sqrt{29}$
$s = s(t) = \int_0^t |r'(t)|dt = \int_0^t \sqrt{29}\,dt$
$= \sqrt{29}t\Big|_0^t$
$= \sqrt{29}t$
$\Rightarrow t = \frac{s}{\sqrt{29}}$
substituting,
$r(t(s)) = \frac{2s}{\sqrt{29}}i + (1-\frac{3s}{\sqrt{29}})j + (5+\frac{4s}{\sqrt{29}})k$

Homework 2 - 10.8

the curvature is given by:
$$k = \frac{|r'(t)\times r''(t)|}{|r'(t)|^3}$$
$r(t) = \langle e^t\cos t, e^t\sin t, t\rangle$ at $(1,0,0)$
which correspond to $t=0$
$\Rightarrow r'(t) = \langle e^t(\cos t - \sin t), e^t(\cos t+\sin t), 1\rangle$
$r''(t) = \langle e^t(-2\sin t), e^t(2\cos t), 0\rangle$
So, $r'(0) = \langle 1,1,1\rangle$
$r''(0) = \langle 0,2,0\rangle$
$\Rightarrow |r'(0)\times r''(0)| = |\langle -2,0,2\rangle| = \sqrt{4+4} = 2\sqrt{2}$
$|r'(t)|^2 = (\sqrt{3})^2 = 3$
$\Rightarrow k = \frac{2\sqrt{2}}{3} \Rightarrow$ the curvature of the curve at this point is $\frac{2\sqrt{2}}{3}$

Homework 3 - 10.8

$k(x) = \frac{|f''(x)|}{[1+(f'(x))^2]^{\frac{3}{2}}}$, $f(x) = e^x$
$\Rightarrow f'(x) = f''(x) = e^x$
$\Rightarrow k(x) = \frac{e^x}{(1+e^{2x})^{\frac{3}{2}}} \Rightarrow$ find $k'(x)$
$k'(x) = \frac{(1+e^{2x})\cdot e^x - 1.5e^x\cdot 2e^{2x}}{(1+e^{2x})^{2.5}} = 0$

$$\Rightarrow \frac{e^x - 2e^{3x}}{(1+e^{2x})^2 \times \sqrt{1+e^{2x}}} = 0$$

$$\Rightarrow e^x = 2e^{3x}$$

$$x = -\frac{1}{2}\ln 2$$

So, when $x = -\frac{1}{2}\ln 2$, $k(x)$ has a maximum.

$x = -\frac{1}{2}\ln 2$, $y = e^{-\frac{1}{2}\ln 2} = \frac{\sqrt{2}}{2}$.

So, at $(-\frac{1}{2}\ln 2, \frac{\sqrt{2}}{2})$, the curve has the maximum curvature.

$$k(-\frac{1}{2}\ln 2) = \frac{2\sqrt{3}}{9}.$$

$$\lim_{x\to\infty} k(x) = \lim_{x\to\infty} \frac{e^x}{(1+e^{2x})^{\frac{3}{2}}}$$

$$= \lim_{x\to\infty} \frac{e^x}{3\sqrt{1+e^{2x}} \cdot e^{2x}} = \lim_{x\to\infty} \frac{1}{3\sqrt{1+e^{2x}} \cdot e^x}$$

$$\lim_{x\to\infty} 3\sqrt{1+e^{2x}} \cdot e^{2x} = +\infty$$

$$= \frac{1}{\infty}$$

by L'Hopital's rule,

$$\lim_{x\to\infty} k(x) = 0.$$

So, when $x \to \infty$, $k(x) = 0$.

Homework 4 - Ex 10.8

$x = t$, $y = t^2$, $z = t^3$ at $(1,1,1)$.

for this, $t = 1$

$r(t) = \langle t, t^2, t^3 \rangle$

$\Rightarrow r'(t) = \langle 1, 2t, 3t^2 \rangle$.

the normal plane is determined by vectors $\vec{B}$ and $\vec{N}$.

$T(t) = \frac{r'(t)}{|r'(t)|}$, $r'(1) = \langle 1, 2, 3 \rangle$

So, $T(t) = \frac{1}{\sqrt{14}} \langle 1, 2, 3 \rangle$

the equation of the normal plane:

$$x - 1 + 2(y-1) + 3(z-1) = 0$$

$$\Rightarrow x + 2y + 3z = 6$$

the osculating plane is determined by $\vec{N}$ and $\vec{T}$

the normal vector of the plane is

$$\vec{B} = \vec{N} \times \vec{T}$$

$$T(t) = \frac{1}{\sqrt{1+4t^2+9t^4}} \langle 1, 2t, 3t^2 \rangle$$

$$\Rightarrow T'(t) \Rightarrow T'(1) = \frac{1}{7\sqrt{14}} \langle 11, 8, -9 \rangle$$

$$N(1) = \frac{T'(1)}{|T'(1)|} = \frac{\frac{1}{7\sqrt{14}} \langle 11, 8, -9 \rangle}{\sqrt{11^2+8^2+9^2}}$$

$$= \frac{\langle 11, 8, -9 \rangle}{\sqrt{266}}$$

the normal vector $\vec{n} = \langle 1, 2, 3 \rangle \times \langle 11, 8, -9 \rangle$

$$= \langle -42, 42, -14 \rangle = \langle 3, -3, 1 \rangle$$

So: the equation of the osculating plane can be written as:

$$3(x-1) - 3(y-1) + (z-1) = 0$$

$$\Rightarrow 3x - 3y + z = 1$$

Homework 5 - Ex 10.8

(a). $\frac{dB}{ds} = \frac{d\,T\times N}{ds}$ as $\vec{B} = \vec{T} \times \vec{N}$

as $\vec{B} \perp \vec{T} \times \vec{N}$ and the ~~de~~ differentiation process will not change the orientation of the vector,

that implies: $\frac{dB}{ds} \perp \vec{B}$

(b). $\frac{dB}{ds} = \frac{d\,T\times N}{ds} = \frac{d}{ds}\left[\frac{r'(t)}{|r'(t)|} \times \frac{r'(t)}{|r'(t)|}\right]$

as $T = \frac{r'(t)}{|r'(t)|}$, $\vec{T}$ is perpendicular to $T \times B$ for sure.

also differentiation process will not change the direction of the vector

$\Rightarrow$ implies $T \perp \frac{dB}{ds}$

(c). $\frac{dB}{ds} = \frac{d}{ds} \cdot T \times N$. ~~$\Rightarrow B \perp T\times N$.~~

~~$N \perp T\times N$ for sure.~~

also differentiation don't change the direction of vectors

~~$\Rightarrow N$ is also perpendicular.~~

as $\frac{dB}{ds} \perp \vec{B}$, $\frac{dB}{ds} \perp T$ and $\vec{B}, \vec{T}, \vec{N}$ are perpendicular to each other,

so, $\vec{N}$ must be parallel to $\frac{dB}{ds}$

Homework 6 - Ex 10.8

$r(t) = \langle \sin t, 3t, \cos t \rangle$, $t = \frac{\pi}{2}$

$r'(t) = \langle \cos t, 3, -\sin t \rangle$

$|r'(t)| = \sqrt{9 + \cos^2 t + \sin^2 t} = \sqrt{10}$

So, $T(t) = \frac{r'(t)}{|r'(t)|} = \frac{1}{\sqrt{10}} \cdot \langle \cos t, 3, -\sin t \rangle$

$T'(t) = \frac{1}{\sqrt{10}} \cdot \langle -\sin t, 0, -\cos t \rangle$

$|T'(t)| = \frac{1}{\sqrt{10}} \cdot \sqrt{\sin^2 t + \cos^2 t} = \frac{1}{\sqrt{10}}$

$\Rightarrow N(t) = \frac{T'(t)}{|T'(t)|} = \frac{1}{\sqrt{10}} \cdot \frac{1}{\sqrt{10}} \cdot \langle -\sin t, 0, -\cos t \rangle$

$B(t) = N(t) \times T(t)$

when $t = \frac{\pi}{2}$, $N(\frac{\pi}{2}) = \langle -1, 0, 0 \rangle$

$T(\frac{\pi}{2}) = \frac{1}{\sqrt{10}} \cdot \langle 0, 3, -1 \rangle$

$$= \frac{1}{\sqrt{10}} \cdot \langle \cos t, 3, -\sin t \rangle \times \langle -\sin t, 0, -\cos t \rangle$$

$$= \frac{1}{\sqrt{10}} \begin{vmatrix} i & j & k \\ \cos t & 3 & -\sin t \\ -\sin t & 0 & -\cos t \end{vmatrix} = \frac{1}{\sqrt{10}} \langle -3\cos t, 1, 3\sin t \rangle$$

$$\Rightarrow B'(t) = \frac{1}{\sqrt{10}} \langle 3\sin t, 0, 3\cos t \rangle$$

So, $B'(\frac{\pi}{2}) = \frac{1}{\sqrt{10}} \langle 3, 0, 0 \rangle$.

By the given formula, we obtain:

$$\tau(t) = \frac{-B(t)\cdot N(t)}{|r'(t)|} \Rightarrow \tau(\tfrac{\pi}{2}) =$$

$$\frac{-\frac{1}{\sqrt{10}}\langle 3,0,0\rangle \cdot \langle -1,0,0\rangle}{|\langle 0,3,-1\rangle|} = \frac{3}{10}$$

So, the torsion at the given value of the curve is $\frac{3}{10}$

Sec 10.9

Homework 1 - 10.9

$r(t) = i(3t - t^3) + j\,3t^2$

first we find the velocity vector

$r'(t) = v(t) = (-3t^2+3)i + 6tj$

$v_s = |r'(t)| = \sqrt{9t^4 - 18t^2 + 9 + 36t^2}$

$= \sqrt{(3t^2+3)^2} = 3t+3$

the acceleration:

$a(t) = r''(t) = (-6t)i + 6j$

$r'(t)\times r''(t) = \langle -3t^2+3, 6t, 0\rangle \times \langle -6t, 6, 0\rangle$

$$= \begin{vmatrix} i & j & k \\ -3t^2+3 & 6t & 0 \\ -6t & 6 & 0 \end{vmatrix} = \langle 0, 0, 6(-3t^2+3)+36t^2\rangle$$

$= \langle 0, 0, 18t^2+18\rangle = \langle 0, 0, t^2+1\rangle$

Then, $a_T = \frac{r'(t)\cdot r''(t)}{|r'(t)|} = \frac{18t^3 - 18t + 36t}{3t+3}$

$= \frac{6(t^3+t)}{t+1}$

$a_N = \frac{|r'(t)\times r''(t)|}{|r'(t)|} = \frac{\sqrt{(t^2+1)^2+0+0}}{3t+3}$

$= \frac{t^2+1}{3t+3}$

So, $a_T = \frac{6t^3+6t^2}{t+1} = 6t$, $a_N = \frac{t+1}{3t+3}$ 6

Homework 2 - 10.9

$v_0 = 80$ m/s, suppose the angle of setting the catapult is α.

$r(t) = r_0 + v_0 t - \frac{1}{2}gt^2$

$v_0 = i\,v_0\cos\alpha + j\sin\alpha\cdot v_0$

Suppose that $r(0) = \cancel{100i}$ 0.

$\Rightarrow r(t) = i(v_0\cos\alpha\,\cancel{t+100}) + j\cdot(v_0\sin\alpha - \frac{1}{2}gt^2)$

So, the parametric equation should be:

$x = (v_0\cos\alpha)\cancel{t+100}\,t$ $\quad y = v_0\sin\alpha t - \frac{1}{2}gt^2$

the catapult reach the wall when $x=100$

$\Rightarrow \begin{cases} 100 = v_0\cos\alpha\cdot t \\ y = 15 = v_0\sin\alpha t - \frac{1}{2}gt^2 \end{cases} \Rightarrow$ when $t = 1.2828$, $\alpha \approx 0.2267$ rad.

or when $t = 16.0869$, $\alpha \approx 1.493$ rad

when $\alpha = 0.2267$ rad, the angle is about $12.988° \approx 13°$.

when $\alpha = 1.493$ rad, the angle is about $85.5425° \approx 85.5°$ ~~(not adapt)~~

So, I should tell them set the catapult between ~~0 and 13°~~. $13° \sim 85.5°$ or $0.2267 \sim 1.493$ rad

Homework 3 - Ex 10.8

(a). $m\frac{dv}{dt} = \frac{dm}{dt}v_e$

$\Rightarrow \frac{dv}{dt} = \frac{1}{m}\frac{dm}{dt}\cdot \vec{v_e}$

$\int_0^t \frac{dv}{dt} = \int_0^t \frac{1}{m}\frac{dm}{dt}\cdot v_e \Rightarrow v(t) - v(0) = \int \frac{dm}{m}$

$\Rightarrow v(t) - v(0) = v_e\cdot \ln m\Big|_{m(0)}^{m(t)}$

$\Rightarrow v(t) - v(0) = v_e[\ln m(t) - \ln m(0)]$

$\Rightarrow v(t) = v(0) + v_e\cdot \ln\frac{m(t)}{m(0)}$

and that's ~~was~~ what we want.

(b). from (a), we obtained that:

$v(t) = v(0) - \ln\frac{m(0)}{m(t)}\cdot\vec{v_e}$

as $v(0) = 0$, $v(t) = -\vec{v_e}\cdot\ln\frac{m(0)}{m(t)}$

speed of the rocket is twice the speed of gases, we have:

$|\vec{v(t)}| = 2|\vec{v_e}|$

$\Rightarrow |\vec{v_e}|\cdot\ln\frac{m(0)}{m(t)} = 2|\vec{v_e}|$

$\Rightarrow \ln\frac{m(0)}{m(t)} = 2$

$\frac{m(0)}{m(t)} = e^2 \Rightarrow m(t) = \frac{m(0)}{e^2}$

So, mass of the fuel burnt is

$m(0) - m(t) = m(0) - \frac{m(0)}{e^2}$

$= m(0)\cdot(1 - \frac{1}{e^2})$

therefore, the fraction of initial mass which the rocket would burn as fuel is:

$$\frac{m(0)-m(t)}{m(0)} = \frac{m(0)\cdot(1-\frac{1}{e^2})}{m(0)}$$

$$= 1 - \frac{1}{e^2}$$

Hence the fraction is $1 - \frac{1}{e^2}$

对于10.9求解三角函数不等式，我们可以用Matlab绘制函数图像并根据根来求解区间，例如：