Calculus . 11.1 ~ 11.4

11.1

homework 1.

(a) $f(x,y) = \cancel{2x+3y-12}\ \sqrt{(y-x)}\ \ln(y+x)$

$y-x \geq 0$ and $y+x > 0$

$\therefore y \geq x$ and $y > -x$

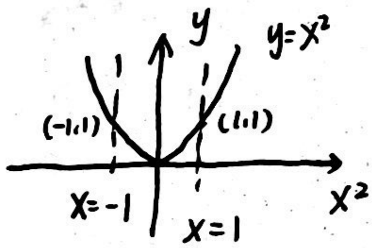
$\therefore$ domain: $D = \{(x,y) \mid y \geq x,\ y > -x\}$.

Range $= \{z ;\ z \in R\}$.

also, $z = f(x,y) = \sqrt{y-x}\ \ln(y+x)$

$\therefore z$ is R

(b) $g(x,y) = \dfrac{\sqrt{y-x^2}}{1-x^2}$



$\therefore 1-x^2 \neq 0,\quad y-x^2 \geq 0 \quad \therefore x \neq \pm 1,\ y \geq x^2$

$\therefore$ domain $= \{(x,y) \mid y \geq x^2,\ x \neq \pm 1\}$.

also, $z = g(x,y) = \dfrac{\sqrt{y-x^2}}{1-x^2}$ $\quad \because \sqrt{y-x^2} \geq 0,\quad 1-x^2 < 1 \quad (x \neq \pm 1)$

$\therefore z \in R$.

Range $= \{z ;\ z \in R\}$.

(c) $f(x,y) = x^2 e^{3xy}$.

domain $= \{(x,y) \mid (x,y) \in R^2\}$ suppose $z = x^2 e^{3xy}$, $\because x^2 \geq 0,\ e^{3xy} > 0 \quad \therefore z \geq 0$

$\therefore$ range $= \{z ;\ z \in \cancel{R\}}\ [0,+\infty)\}$

(d). $h(x,y) = \cos^{-1}(x^2+4y^2-4)$

$\because \quad x^2+4y^2-4 \in [-1,1]$

$\therefore \quad 3 \le x^2+4y^2 \le 5$

$\therefore$ domain $= \{(x,y) \mid 3 \le x^2+4y^2 \le 5\}$.

$\therefore$ range $= \{z \ ; \ \cancel{z \in R\}}. \ 0 \le z \le \cancel{R}\ \pi \}$

suppose $z = \cos^{-1}(x^2+4y^2-4)$

$\therefore \cos z = x^2+4y^2-4$

$\therefore z \in R\ [0,\pi]$

2.

(1). $f(x,y) = 2+\sin(x+y)$

suppose $2+\sin(x+y) = k$

$k=1 \quad \therefore \sin(x+y) = -1$

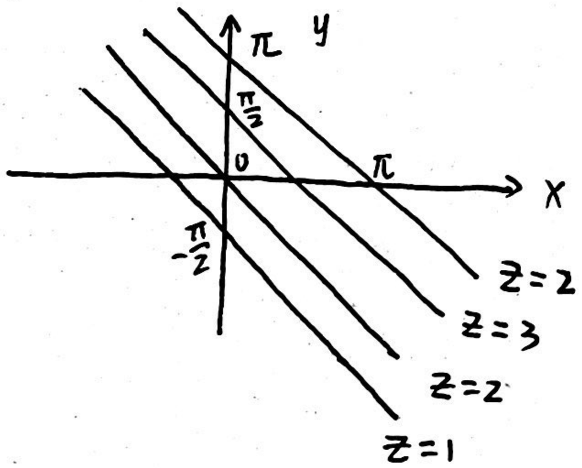
$\therefore x+y = -\frac{\pi}{2}+2k\pi.$

$k=2, \quad \therefore \sin(x+y) = 0.$

$x+y = k\pi$

$k=3. \quad \therefore \sin(x+y) = 1$

$x+y = \frac{\pi}{2}+2k\pi.$



(2) $f(x,y) = e^{x^2+y^2}$

suppose $e^{x^2+y^2} = k$

$k=1, \quad e^{x^2+y^2} = 1$

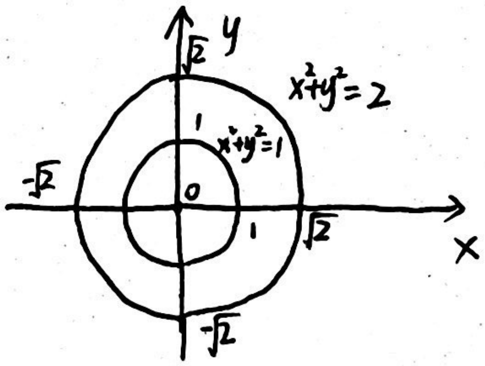
$x^2+y^2 = 0$

$k=e \quad e^{x^2+y^2} = e$

$x^2+y^2 = 1$

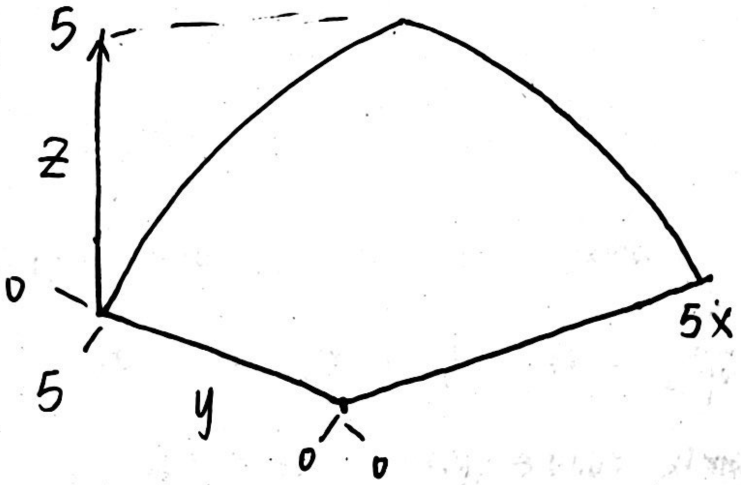
$k=e^2 \quad e^{x^2+y^2} = e^2$

$x^2+y^2 = 2$



3.

(a) Graph



(b) $f(x,y) = \dfrac{1}{\frac{1}{x}+\frac{1}{y}} = \dfrac{xy}{x+y}$ $(0 < x \leq 2,\ 0 < y \leq 2)$

$\therefore f(x,y)_{max} = \dfrac{x}{2}$

when $x = 2$. $f(x,y)_{max} = 1$

(c) $f(a,b) = \dfrac{ab}{a+b}$ $f(b,a) = \dfrac{ba}{b+a}$

$f(a,b) = f(b,a)$ $\therefore f(x,y)$ is symmetric in x and y.

1.2.

homework 1.

$$\lim_{(x,y)\to(0,0)} \frac{y^4}{x^4+3y^4} \quad \text{does not exist.}$$

① $(x,y)\to(0,0)$. along the parobola $x=y^2$.

we have $f(x,y) = f(y^2,y) = \frac{y^4}{4y^4} = \frac{1}{4}$

so $f(x,y) \to \frac{1}{4}$, as $(x,y)\to(0,0)$. along $y^2=x$.

② $(x,y)\to(0,0)$ along the parobola $x=0$.

we have $f(x,y) = f(0,y) = \frac{y^4}{3y^4} = \frac{1}{3}$

so $f(x,y) \to \frac{1}{3}$ as $(x,y)\to(0,0)$ along $x=0$.

Since $f(x,y)$ has 2 different limits along 2 different lines. ∴ does not exist.

2. $$\lim_{(x,y)\to(0,0)} \frac{x^2+\sin^2 y}{2x^2+y^2} \quad \text{does not exist.}$$

① $(x,y)\to(0,0)$. along $y=0$.

we have $f(x,y) = f(x,0) = \frac{x^2}{2x^2} = \frac{1}{2}$

so $f(x,y) \to \frac{1}{2}$ as $(x,y)\to(0,0)$. along $y=0$.

② $(x,y)\to(0,0)$ along $x=0$.

we have $f(x,y) = f(0,y) = \frac{\sin^2 y}{y^2} = 1$

so $f(x,y) \to 1$ as $(x,y)\to(0,0)$ along $x=0$

Since $f(x,y)$ has 2 different limits along 2 different lines.

∴ the limit does not exist.

$\lim\limits_{(x,y)\to(0,0)} \frac{xy\cos y}{3x^2+y^2}$ does not exist.

① $(x,y)\to(0,0)$ along $y=0$

we have $f(x,y) = f(x,y) = 0$

so $f(x,y)\to 0$, as $(x,y)\to(0,0)$ along $y=0$

② $(x,y)\to(0,0)$ along $x=y$

we have $f(x,y) = f(y,y) = \frac{y^2\cos y}{4y^2} = \frac{\cos y}{4} = \frac{1}{4}$.

so $f(x,y)\to\frac{1}{4}$, as $(x,y)\to(0,0)$ along $x=y$.

Since $f(x,y)$ has 2 different limits along 2 different lines.

$\therefore$ the limit does not exist.

4. $\lim\limits_{(x,y)\to(0,0)} \frac{6x^3y}{2x^4+y^4}$ does not exist.

① $(x,y)\to(0,0)$ along $y=0$.

$f(x,y) = f(x,0) = 0$

So $f(x,y)\to 0$ as $(x,y)\to(0,0)$ along $y=0$.

② $(x,y)\to(0,0)$ along $x=y$.

$f(x,y) = f(x,x) = \frac{6x^4}{3x^4} = 2$

so $f(x,y)\to 2$ as $(x,y)\to(0,0)$ along $x=y$.

Since $f(x,y)$ has 2 different limits along 2 different lines

$\therefore$ the limit does not exist.

5. $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy}{\sqrt{x^2+y^2}}$ exist.

let $\varepsilon>0$ we want to find $\delta>0$ such that.

if $0<\sqrt{x^2+y^2}<\delta$ then $\left|\dfrac{xy}{\sqrt{x^2+y^2}}-0\right|<\varepsilon$.

that is if $0<\sqrt{x^2+y^2}<\delta$ then $\dfrac{|xy|}{\sqrt{x^2+y^2}}<\varepsilon$.

$$\frac{|xy|}{\sqrt{x^2+y^2}} \leqslant \frac{x^2y^2}{\sqrt{x^2+y^2}} \leq \frac{(x^2+y^2)^2}{\sqrt{x^2+y^2}} = (x^2+y^2)^{\frac{3}{2}}$$

let $\cancel{\varepsilon=\delta^3}$ $\delta=\varepsilon^{\frac{1}{3}}$

$$\therefore \left|\frac{xy}{\sqrt{x^2+y^2}}-0\right| \leq (x^2+y^2)^{\frac{3}{2}} = \delta^3 = (\varepsilon^{\frac{1}{3}})^3 = \varepsilon.$$

Hence. by Definition 1.

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = 0.$$

6. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4-y^4}{x^2-y^2}$ exist.

~~① let $(x,y)\to(0,0)$ along $y=0$.~~

let $\varepsilon>0$ we want to find $\delta>0$ such that.

if $0<\sqrt{x^2+y^2}<\delta$ then $\left|\dfrac{x^4-y^4}{x^2+y^2}-0\right|<\varepsilon$.

that is $0<\sqrt{x^2+y^2}<\delta$ then $\dfrac{|x^4-y^4|}{x^2+y^2}<\varepsilon$.

$$\frac{|(x^2+y^2)(x^2-y^2)|}{x^2+y^2} = |x^2-y^2| \leq x^2+y^2$$

let $\delta=\varepsilon^{\frac{1}{2}}$ $\left|\dfrac{x^4-y^4}{x^2+y^2}-0\right| \leq x^2+y^2=\delta^2=(\varepsilon^{\frac{1}{2}})^2=\varepsilon$

Hence, $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4-y^4}{x^2-y^2}=0$

7. $\lim_{(x,y)\to(0,0)} \frac{x^2ye^y}{x^4+4y^2}$ does not exist

① $(x,y)\to(0,0)$ along $y=0$.

we have $f(x,y)=f(x,0)=\frac{0}{x^4}=0$.

so $f(x,y)\to 0$ as $(x,y)\to(0,0)$. along $y=0$

② $(x,y)\to(0,0)$. along $y=x^2$

we have $f(x,y)=f(x,x^2)=\frac{x^4e^{x^2}}{5x^4}=\frac{e^{x^2}}{5}=\frac{1}{5}$

so $f(x,y)\to \frac{1}{5}$ as $(x,y)\to(0,0)$ along $y=x^2$

Since $f(x,y)$ has 2 different limits along 2 different lines,

$\therefore$ the limit does not exist.

8. $\lim_{(x,y)\to(0,0)} \frac{x^2\sin^2 y}{x^2+2y^2}$ exist.

Let $\varepsilon>0$ we want to find $\delta>0$ such that

if $0<\sqrt{x^2+y^2}<\delta$ then $\left|\frac{x^2\sin^2 y}{x^2+2y^2}-0\right|<\varepsilon$

also $\left|\frac{x^2\sin^2 y}{x^2+2y^2}\right|<\varepsilon$

$\because \frac{|x^2\sin^2 y|}{x^2+2y^2}\le\frac{x^2}{x^2+2y^2}\le\frac{x^2}{x^2+y^2}\le\frac{x^2+y^2}{\sqrt{x^2+y^2}}=\sqrt{x^2+y^2}$

let $\delta=\varepsilon$.

$\left|\frac{x^2\sin^2 y}{x^2+2y^2}-0\right|\le\sqrt{x^2+y^2}=\delta=\varepsilon$

Hence, $\lim_{(x,y)\to(0,0)} \frac{x^2\sin^2 y}{x^2+2y^2}=0$

9. $\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}$ exist

$\because \lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1} = \lim_{(x,y)\to(0,0)} \frac{(x^2+y^2)(\sqrt{x^2+y^2+1}+1)}{(\sqrt{x^2+y^2+1})^2-1} = \lim_{(x,y)\to(0,0)} \sqrt{x^2+y^2+1}+1 = 2.$

let $\varepsilon>0$ we want to find $\delta>0$ such that

if $0<\sqrt{x^2+y^2}<\delta$ the $\left|\frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}-2\right|<\varepsilon$. $\therefore \left|\frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}\right|<\varepsilon+2$

$\therefore \left|\frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}\right| = \left|\sqrt{x^2+y^2+1}+1\right| \leq x^2+y^2+2$ $\cancel{\therefore x^2+y^2}$

let $\delta=\varepsilon^{\frac{1}{2}}$

$\left|\frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}-2\right| \leq x^2+y^2 = \delta^2 = (\varepsilon^{\frac{1}{2}})^2 = \varepsilon$

Hence, $\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1} = 2$

10. $\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^2+y^8}$ does not exist.

① $(x,y)\to(0,0)$ along $x=y$.

$f(x,y)=f(x,x)=\frac{x^5}{x^2+x^8}=\frac{x^3}{1+x^6}=0$

so $f(x,y)\to 0$ as $(x,y)\to(0,0)$ along $x=y$.

② $(x,y)\to(0,0)$. along $x=y^4$

$f(x,y)=f(y^4,y)=\frac{y^8}{2y^8}=\frac{1}{2}$

so $f(x,y)\to\frac{1}{2}$ as $(x,y)\to(0,0)$ along $x=y^4$

Since $f(x,y)$ has 2 different limits. along 2 different lines.

$\therefore$ the limit does not exist.

homework 2.

$$\lim_{(x,y)\to(1,2)} \frac{\sqrt{y}-\sqrt{x+1}}{y-x-1}$$

$$= \lim_{(x,y)\to(1,2)} \frac{\sqrt{y}-\sqrt{x+1}}{(\sqrt{y}-\sqrt{x+1})(\sqrt{y}+\sqrt{x+1})}$$

$$= \lim_{(x,y)\to(1,2)} \frac{1}{\sqrt{y}+\sqrt{x+1}}$$

$$= \frac{1}{\sqrt{2}+\sqrt{1+1}} = \frac{\sqrt{2}}{4}$$

$$\lim_{(x,y)\to(0,0)} \frac{y}{\sqrt{x^2+y^2}}$$

① $(x,y)\to(0,0)$ along $x=y$

$f(x,y) = f(y,y) = \frac{y}{\sqrt{2}\,y} = \frac{\sqrt{2}}{2}$

$\therefore f(x,y) \to \frac{\sqrt{2}}{2}$ along $x=y$

② $(x,y)\to(0,0)$ along $x=0$.

$f(x,y) = f(0,y) = \frac{y}{\sqrt{y^2}} = 1$ or -1

$\therefore f(x,y) \to \pm 1$ as $(x,y)\to(0,0)$ along $x=0$.

Since $f(x,y)$ has 2 different limits along 2 lines.

$\therefore$ the limit does not exist.

$$f(x,y) = \frac{x^2 y}{x^2+y^2}$$

(a) the x-axis

$(x,y)\to(0,0)$. along $y=0$. $f(x,y) = f(x,0) = \frac{0}{x^2} = 0$

so $f(x,y)\to 0$ as $(x,y)\to(0,0)$ along $y=0$.

(b) the y-axis.

$(x,y)\to(0,0)$ along $x=0$. $f(0,y) = \frac{0}{y^2} = 0$

so $f(x,y)\to 0$ as $(x,y)\to(0,0)$ along $x=0$.

(c) the line $y=x$.

$(x,y)\to(0,0)$ along $x=y$, $f(x,y) = f(x,x) = \frac{x^3}{2x^2} = \frac{x}{2} = 0$

so $f(x,y)\to 0$ as $(x,y)\to(0,0)$ along $x=y$.

(d). the line $y=-x$.

$(x,y)\to(0,0)$ along $y=-x$, $f(x,y) = f(x,-x) = \frac{-x^3}{2x^2} = -\frac{x}{2} = 0$

so $f(x,y)\to 0$ as $(x,y)\to(0,0)$ along $y=-x$.

(e) the parabola $y=x^2$.

$(x,y)\to(0,0)$ along $y=x^2$, $f(x,y)=f(x,x^2)=\frac{x^4}{x^2+x^4}=\frac{x^2}{1+x^2}=0$

so $f(x,y)\to 0$ as $(x,y)\to(0,0)$ alone $y=x^2$

homework 3.

(a) $f(x,y)=e^{x^2-y^2}$

let $t=x^2-y^2$ $\therefore g(t)=e^t$ $g(t)$ is continuous in R.

$t\in R$, $t=x^2-y^2$ is polynomial functions, $\therefore$ it will be continuous everywhere.

$\therefore f(x,y)$ is continuous in R^2 $\therefore \{(x,y)\in R^2\}$.

(b) $g(x,y)=\cos(x^2-y)$

let $t=x^2-y$. $\cos t$ is continuous everywhere

x^2-y is polynomial functions $\therefore$ it is continuous everywhere

$\therefore g(x,y)$ is ~~cou~~ continuous in R^2, $\{(x,y)\in R^2\}$.

(c) $h(x,y)=xy\sin\left(\frac{y}{x}\right)$

xy is polynomial functions. $\therefore$ it continuous everywhere.

$\sin t$ is continuous everywhere.

$\frac{y}{x}$ is the quotient of x and y. $\therefore x\neq 0$.

$\therefore h(x,y)$ is continuous except where $x=0$.

(d) $f(x,y) = \begin{cases} \dfrac{1-\cos(x^2+y^2)}{x^2+y^2}, & \text{if } (x,y) \neq 0 \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$

$\because \cos t$ is continuous everywhere.

x^2+y^2 is polynomial functions. $\therefore$ continuous everywhere.

$\therefore 1-\cos(x^2+y^2)$ continuous everywhere.

$\dfrac{1-\cos(x^2+y^2)}{x^2+y^2}$ is the quotient of 2 continuous function. $\therefore$ it continuous as long as $x^2+y^2 \neq 0$.

$\because (x,y) \neq 0$. $\therefore$ it continuous everywhere.

$$\lim_{(x,y)\to(0,0)} \frac{1-\cos(x^2+y^2)}{x^2+y^2} = 0$$

let $0 < \sqrt{x^2+y^2} < \delta$ then $\left|\dfrac{1-\cos(x^2+y^2)}{x^2+y^2} - 0\right| < \varepsilon$.

$\therefore \dfrac{|1-\cos(x^2+y^2)|}{x^2+y^2} < \varepsilon$ $\quad \therefore \dfrac{|1-\cos(x^2+y^2)|}{x^2+y^2} \leq \dfrac{2}{x^2+y^2}$

$\because \delta = \sqrt{\dfrac{2}{\varepsilon}}$

$\therefore \left|\dfrac{1-\cos(x^2+y^2)}{x^2+y^2} - 0\right| \leq \dfrac{2}{x^2+y^2} = \dfrac{2}{\delta^2} = \dfrac{2}{\left(\frac{2}{\varepsilon}\right)} = \varepsilon$

$\therefore \displaystyle\lim_{(x,y)\to(0,0)} \frac{1-\cos(x^2+y^2)}{x^2+y^2}$ exist and equal to 0

$\therefore f(x,y)$ is continuous in R^2. $\{(x,y) \in R^2\}$.

11.3

homework 1. first partial derivatives.

(a) $f(r,s) = (r+s)\ln(r^2+s^2)$ at $(2,1)$

$$f_r(r,s) = \ln(r^2+s^2) + (r+s)\frac{2r}{r^2+s^2}$$

$$\therefore f_r(2,1) = \ln(4+1) + (2+1)\times\frac{4}{4+1} = \ln 5 + \frac{12}{5}$$

$$f_s(r,s) = \ln(r^2+s^2) + (r+s)\frac{2s}{r^2+s^2}$$

$$\therefore f_s(2,1) = \ln(4+1) + (2+1)\times\frac{2}{4+1} = \ln 5 + \frac{6}{5}$$

(b) $f(u,v) = \frac{(u-v)}{u^2+v^2}$ at $(1,0)$.

$$f_u(u,v) = \frac{(u^2+v^2)-(u-v)(2u)}{(u^2+v^2)^2}$$

$$\therefore f_u(1,0) = \frac{1-(1-0)\times 2}{1} = -1$$

$$f_v(u,v) = \frac{-(u^2+v^2)-(u-v)2v}{(u^2+v^2)^2}$$

$$\therefore f_v(u,v) = \frac{-1-(1-0)\times 0}{1} = -1$$

(c). $g(\alpha,\beta) = \sin\alpha\cos\beta\tan\theta$ at $(0,\pi)$.

$$g_\alpha(\alpha,\beta) = \cos\alpha\cos\beta\tan\theta$$

$$\therefore g_\alpha(0,\pi) = 1\times(-1)\tan\theta = -\tan\theta$$

$$g_\beta(\alpha,\beta) = -\sin\beta\sin\alpha\tan\theta$$

$$\therefore g_\beta(0,\pi) = -0\times 0\times\tan\theta = 0.$$

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(d) $f(x,y) = y^x$ at $(1,1)$

$f_x(x,y) = y^x \ln y$

$\therefore f_x(1,1) = 1^1 \ln 1 = 0$

$f_y(x,y) = x y^{x-1}$

$\therefore f_y(1,1) = 1 \times 1^{1-1} = 1$

(e). $u = \sin(x_1 + 2x_2 + \cdots + nx_n)$

$$\frac{\partial u}{\partial x_i} = i \cos(x_1 + 2x_2 + \cdots + nx_n)$$

(f) $f(x,y) = \int_y^x \cos(t^2)dt$

$$\therefore f_x(x,y) = \frac{\partial}{\partial x}\int_y^x \cos(t^2)dt = \cos(x^2)$$

$$f_y(x,y) = \frac{\partial}{\partial y}\int_y^x \cos(t^2)dt = -\cos(y^2)$$

homework 2.

(a). $f(x,y) = \sin^2(mx+ny)$

$f_x(x,y) = 2\sin(mx+ny) \cdot \cos(mx+ny) \cdot (m)$

$\therefore f_{xx}(x,y) = 2\cos^2(mx+ny)m^2 - 2\sin^2(mx+ny)m^2$

$f_y(x,y) = 2\sin(mx+ny) \cdot \cos(mx+ny) \cdot n$

$\therefore f_{yy}(x,y) = 2\cos^2(mx+ny)n^2 - 2\sin^2(mx+ny)n^2$

also, Clairaut's theorem.

$\because f_{xy}(x,y) = 2\cos^2(mx+ny)mn - 2\sin^2(mx+ny)mn.$

$$f_{yx}(x,y) = 2\cos^2(mx+ny)\,mn \quad - 2\sin^2(mx+ny)\,nm$$

$$\therefore f_{xy}(x,y) = f_{yx}(x,y).$$

(b). $z = g(x,y) = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$$g_x(x,y) = \frac{1}{1+\left(\frac{x+y}{1-xy}\right)^2} \cdot \frac{(1-xy)+y(x+y)}{(1-xy)^2}$$

$$= \frac{1+y^2}{1+x^2y^2+x^2+y^2}$$

$$\therefore g_{xx}(x,y) = \frac{-(2x+2y^2x)(1+y^2)}{(1+x^2y^2+x^2+y^2)^2} = \frac{-2x(1+y^2)^2}{(1+x^2y^2+x^2+y^2)^2}$$

$$g_{xy}(x,y) = \frac{2yx^2-2x^2y}{(1+x^2y^2+x^2+y^2)^2} = 0$$

$$g_y(x,y) = \frac{1}{1+\left(\frac{x+y}{1-xy}\right)^2} \cdot \frac{(1-xy)+x(x+y)}{(1-xy)^2} = \frac{1+x^2}{1+x^2y^2+x^2+y^2}$$

$$\therefore g_{yy}(x,y) = \frac{-(x^2+1)}{(1+x^2y^2+x^2+y^2)^2}\,2y\,(1+x^2)$$

$$g_{yx}(x,y) = \frac{2x(1+x^2y^2+x^2+y^2)-(2y^2x+2x)(1+x^2)}{(1+x^2y^2+x^2+y^2)^2} = 0$$

$$\therefore g_{xy}(x,y) = g_{yx}(x,y)$$

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(c)

$V(x,y) = e^{xe^y}$

$V_x(x,y) = e^y e^{xe^y}$

$V_y(x,y) = e^{xe^y} \cdot xe^y$

$\therefore V_{xx}(x,y) = e^{2y} e^{xe^y}$ $\quad V_{xy}(x,y) = e^{xe^y}(e^y + xe^{2y})$

$V_{yy}(x,y) = 2e^{xe^y} \cdot (xe^y)^2$ $\quad V_{yx}(x,y) = e^{xe^y}(e^y + xe^{2y})$

$\therefore V_{xy}(x,y) = V_{yx}(x,y)$.

(d)

$h(x,y) = \ln(\sqrt{x^2+y^2})$

$h_x(x,y) = \dfrac{\frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}}{\sqrt{x^2+y^2}} \cdot 2x = \dfrac{x}{x^2+y^2}$

$h_{xx}(x,y) = \dfrac{y^2-x^2}{(x^2+y^2)^2}$

$h_{xy}(x,y) = \dfrac{-2yx}{x^2+y^2}$

$h_y(x,y) = \dfrac{y}{x^2+y^2}$ $\quad h_{yy}(x,y) = \dfrac{x^2-y^2}{(x^2+y^2)^2}$

$h_{yx}(x,y) = \dfrac{-2xy}{x^2+y^2}$

$\therefore h_{xy}(x,y) = h_{yx}(x,y)$.

homework 3.

$$u(x,y) = \tan^{-1}\left(\frac{y}{x-1}\right) - \tan^{-1}\left(\frac{y}{x+1}\right)$$

$$u_x(x,y) = \frac{1}{1+\left(\frac{y}{x-1}\right)^2} \cdot \frac{-y}{(x-1)^2} - \frac{1}{1+\left(\frac{y}{x+1}\right)^2} \cdot \frac{-y}{(x+1)^2}$$

$$= \frac{-y}{(x-1)^2+y^2} + \frac{y}{(x+1)^2+y^2}$$

$$u_{xx}(x,y) = \frac{2(x-1)y}{[(x-1)^2+y^2]^2} + \frac{-2(x+1)y}{[(x+1)^2+y^2]^2}$$

$$u_y(x,y) = \frac{x-1}{(x-1)^2+y^2} - \frac{x+1}{(x+1)^2+y^2}$$

$$u_{yy}(x,y) = \frac{-2y(x-1)}{[(x-1)^2+y^2]^2} - \frac{-2y(x+1)}{[(x+1)^2+y^2]}$$

$\because u_{xx} + u_{yy} = 0$

$\therefore$ is the solution of Laplace's equation.

homework 4.

$$\frac{\partial^2 E(x,t)}{\partial t^2} = c^2 \frac{\partial^2 E(x,t)}{\partial x^2}$$

$$u(x,t) = A\cos(x+ct) + B\sin(x-ct)$$

$$u_x(x,t) = -A\sin(x+ct) + B\cos(x-ct)$$

$$u_{xx}(x,t) = -A\cos(x+ct) - B\sin(x-ct)$$

$$u_{y_t}(x,t) = -Ac\sin(x+ct) - Bc\cos(x-ct)$$

$$u_{tt}(x,t) = -Ac^2 \cancel{si}\cos(x+ct) - Bc^2\sin(x-ct)$$

$$c^2 u_{xx}(x,t) = \cancel{u_{yy}}\ u_{tt}(x,t)$$

$\therefore$ the solution of is above equation.

homework 5.

①. $u(x,t) = e^{-t}(2\sin x + 3\cos x)$

$u_x(x,t) = e^{-t}(2\cos x - 3\sin x)$

$u_{xx}(x,t) = e^{-t}(-2\sin x - 3\cos x)$

$u_t(x,t) = -(2\sin x + 3\cos x)e^{-t}$

$\therefore u_t(x,t) = u_{xx}(x,t)$. $k=1$.

$\therefore u(x,t)$ satisfy the heat equation.

② $u(x,t) = Ae^{-a^2t}\cos ax$.

$u_x(x,t) = -Ae^{-a^2t}\sin ax$.

$u_{xx}(x,t) = -Ae^{-a^2t}a^2\cos ax$

$u_t(x,t) = A\cos ax \cdot e^{-a^2t} \cdot (-a^2) = -Aa^2\cos ax\, e^{-a^2t}$

$u_{xx}(x,t) = u_t(x,t)$

$\therefore u(x,t)$ satisfy the heat equation.

③ $u = \sin(x-at) + \ln(x+at)$

$u_x = \cos(x-at) + \frac{1}{x+at}$

$u_{xx} = -\sin(x-at) + \frac{-1}{(x+at)^2}$

$u_t = -a\cos(x-at) + \frac{a}{x+at}$

$u_{xx} \neq u_t$

$\therefore$ doesn't satisfy the heat equation.

11.4.

homework 1.

$Z = x^2 - xy + 3y^2$

$(3,-1) \rightarrow (2.96, -0.95)$. $\because Z(3,-1) = 9+3+3 = 15$.

$Z_x = 2x - y = 2\times3 - (-1) = 7$. $Z(2.96, -0.95) = 14.2811$

$Z_y = -x + 6y = (-3) + 6\times(-1) = -9$.

$dx = -(3-2.96) = -0.04$. $dy = -[-1-(-0.95)] = 0.05$.

$\cancel{\Delta Z = 0.632}$.

$\Delta Z \approx L(x,y) - f(a,b) = 14.2811 - 15 = -0.7189$.

$dZ = f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 7\times(-0.04) + (-9)\times 0.05$

$= -0.73$

$\Delta Z < dZ$ $\Delta Z - dZ = -0.0111$

homework 2.

k $3 \rightarrow 3.1$ r $0.7 \rightarrow 0.8$ $(a,b) \rightarrow (3, 0.7)$ $(x,y) \rightarrow (3.1, 0.8)$

$$T_r = \frac{mg(2r^2+R^2) - mgr(4r)}{(2r^2+R^2)^2} = \frac{mgR^2 - 2mgr^2}{(2r^2+R^2)^2} = \frac{8.02mg}{99.6004}$$

$$T_R = \frac{-mgr(2R)}{(2r^2+R^2)^2} = \frac{-2mgRr}{(2r^2+R^2)^2} = \frac{-4.2mg}{99.6004}$$

$\Delta Z = T_r \times 0.1 + T_R \times 0.1 = 3.83\times10^{-3} > 0$

Increase

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homework 3.

$PV = 8.31T$.

V: $12L \to 12.3L$. T: $310K \to 305K$.

$P = \frac{8.31T}{V}$ $\therefore P_V = \frac{-8.31T}{V^2}$ $P_V(12, 310) = \frac{-8.31\times310}{12^2}$

$\therefore P_T = \frac{8.31}{V}$ $dV = 0.3L$ $dT = -5K$

$\Delta Z = P_V dV + P_T dT = -\frac{8.31\times310}{144}\times0.3 + \frac{8.31}{12}\times(-5) = -8.83$