Homework 1 - 10.1

(i). equation $(x+3)^2+(y-2)^2+(z-5)^2=16$

the intersection of the sphere is a circle

the centre of the circle is (0,2,5).

and radius is $\sqrt{4^2-3^2}=\sqrt{7}$

So the equation of the circle is:

$\cancel{x^2+(y}$ $(y-2)^2+(z-5)^2=7$

the circle is on y-z plane

(ii). it's $\sqrt{2^2+3^2+4^2}=\sqrt{29}$

So the length is $\sqrt{29}$

(iii). determine whether $\vec{PQ}=k\vec{QR}$, which k is a real number.

if $k>0$, then Q lies between P and R:

if $k<0$, then the opposite.

determine $\vec{PR}=m\vec{RQ}$, $m>0$, then R lies between P and Q.

determine $\vec{RP}=n\vec{PQ}$. $n>0$. then P lies between R and Q

Homework 2 - 10.1

$x^2+y^2+z^2+4x-2y+4z+5=0$

$\Rightarrow (x+2)^2+(y-1)^2+(z+2)^2=4$.

the centre of this sphere is: $(-2,1,-2)$.

the another sphere: $x^2+y^2+z^2=4$.

two centre's distance:

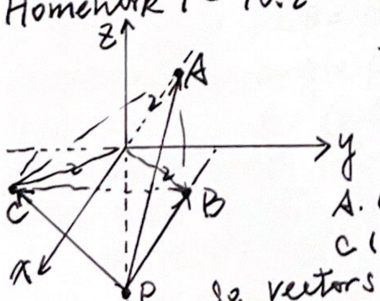
$d=\sqrt{(-2)^2+1^2+(-2)^2}=3 < 2+2$.

they intersect

$V=2\pi\cdot\int_{\frac{3}{2}}^{2} y^2\,dx = 2\pi\int_{\frac{3}{2}}^{2}(4-x^2)\,dx$

$=\frac{11}{12}\pi$

Homework 1 - 10.2



suppose: the load point is P. $(0,0,-2\sqrt{3})$.

anchored points:

A. $(-2,0,0)$. B. $(1,\sqrt{3},0)$.

C $(1,-\sqrt{3},0)$ -0.5

So vectors of forces

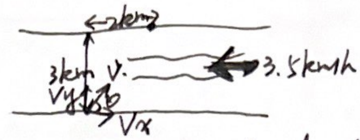
$f_1=\vec{PA}=(-2,0,2\sqrt{3})\cdot\frac{250\sqrt{3}}{9}$

$f_2=\vec{PB}=(1,\sqrt{3},2\sqrt{3})\cdot x$

$f_3=\vec{PC}=(1,-\sqrt{3},2\sqrt{3})\cdot x$

Homework 2 - Ex 10.2

(a).



the true speed of the boat:

$\begin{cases} V_x = V\cos\theta - 3.5 \\ V_y = V\sin\theta \end{cases}$ $V=13\,km/h$

$\Rightarrow \frac{2}{13\cos\theta-3.5}=\frac{3}{13\sin\theta}$

i). $\Rightarrow 39\cos\theta-26\sin\theta=10.5$ $n\in Z$

$\cancel{i)}$ $\theta=6.28n-1.93$ or $6.28n+0.76$. ~~$n\in Z$.~~

~~ii). time is $\frac{3}{13\sin(6.28n-1.93)}$~~

~~or $\frac{3}{13\sin(6.28n+0.76)}$ $n\in Z$.~~

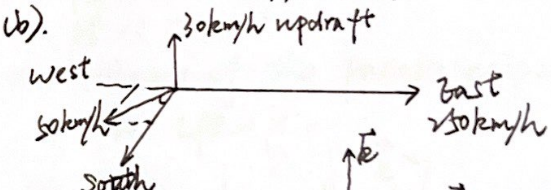
the θ should be in the range $[0,\frac{\pi}{2}]$

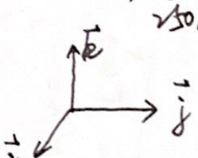
So, $\theta\approx 0.76$ rad.

that is, 0.76 rad to the horizontal line (the river).

ii). time is $\frac{3}{13\sin(0.76)}\approx 0.335\,h$

(b).





the speed: $250\vec{j}+30\vec{k}+(25\sqrt{2}\vec{i}-25\sqrt{2}\vec{j})$

$=25\sqrt{2}\vec{i}+(250-25\sqrt{2})\vec{j}+30\vec{k}$

the magnitute of the speed is:

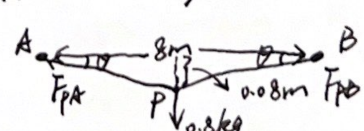
$\sqrt{(25\sqrt{2})^2+(250-25\sqrt{2})^2+30^2}$

$=10\sqrt{659-125\sqrt{2}}\approx 219.60\,km/h$

and the direction of the velocity relative to the ground is:

$(5\sqrt{2}, 50-5\sqrt{2}, 6)$.

Homework 3 - Ex 10.2



$\tan\theta=\frac{0.08}{4}=0.02$

$\Rightarrow \sin\theta=\frac{1}{\sqrt{2501}}$

$\Rightarrow 2F_{PA}\sin\theta=0.8g$

$F_{PA}=F_{PB}=F$

solved for that: $F=196.0\,N$.

So, tension in each is $196.0\,N$.

Homework 1 - Ex 10.3

(a). when $-6b+b^3+2b=0$, they orthogonal.

$\Rightarrow b^2=4 \Rightarrow b=\pm 2$

(b). $\begin{cases} 5x-y=8 \\ x+3y=15 \end{cases}$

$\Rightarrow (\frac{39}{16}, \frac{67}{16})$.

that's the intersection point

$\begin{cases} y=5x-8 \\ y=\frac{15-x}{3} \end{cases}$

$\Rightarrow \frac{dy}{dx}=5$; $\frac{dy}{dx}=-\frac{1}{3}$

So the vectors parallel to two lines: $(1,5)$ and $(1,-\frac{1}{3})$.

angle: $\cos\theta = \dfrac{1-\frac{5}{3}}{\sqrt{26}\cdot\sqrt{1+\frac{1}{9}}} = -\frac{\sqrt{65}}{65}$

$\Rightarrow \theta = \cancel{\arccos\theta}\ \arccos(-\frac{\sqrt{65}}{65})$

$\approx \pm 1.695 + 2k\pi.\ k\in\mathbb{Z}$

So the angle is $(\pm 1.695 + 2k\pi)$ rad. $k\in\mathbb{Z}$

or 82.9°

Homework 2 - Ex 10.3

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\begin{cases} \alpha = \frac{\pi}{4} \\ \beta = \frac{\pi}{3} \end{cases}$

So, $\frac{1}{2}+\frac{1}{4}+\cos^2\gamma = 1$

$\Rightarrow \cos^2\gamma = \frac{1}{4}$

$\gamma = \frac{\pi}{3}$

Therefore, the third direction angle is $\frac{\pi}{3}$

Homework 3 - Ex 10.3

(a). the vector of a diagonal of a cube is $(1,1,1)$.

the length of the vector: $d=\sqrt{1+1+1}=\sqrt{3}$

So, the direction angles:

$\cos\alpha = \frac{1}{\sqrt{3}} = \cos\beta = \cos\gamma = \cos\theta$.

$\cancel{\cos\alpha} \Rightarrow$ the angle $\theta = \arccos\frac{\sqrt{3}}{3}$

$\theta = (0.955 + 2k\pi)$ rad. $k\in\mathbb{Z}$.

(b). $(\vec{u}+\vec{v})$ and $(\vec{u}-\vec{v})$ are orthogonal

$\Rightarrow (\vec{u}+\vec{v})\cdot(\vec{u}-\vec{v})=0$

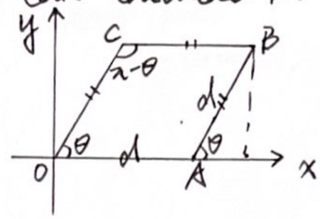
$\Rightarrow |\vec{u}|^2 = |\vec{v}|^2$

So, the length of $\vec{u}$ & $\vec{v}$ must be identical, such that $(\vec{u}+\vec{v}) \perp (\vec{u}-\vec{v})$.

Homework 4 - Ex 10.3

the quadrilateral: suppose the length is d.

we can calculate in a x-y coordinate.



suppose the angle is θ

$O.(0,0)$. $A.(d,0)$.

$B.(d\cdot(1+\cos\theta), d\sin\theta)$

$C.(d\cos\theta, d\sin\theta)$.

So. $\vec{OB} = (d(1+\cos\theta), d\sin\theta)$.

$\vec{AC} = (d\cdot(\cos\theta-1), d\sin\theta)$.

$\vec{OB}\cdot\vec{AC} = d^2\cdot(1+\cos\theta)\cdot(\cos\theta-1) + d^2\sin^2\theta$

$= d^2\cdot(\cos^2\theta-1) + d^2\sin^2\theta$

$= d^2\cdot(\cos^2\theta+\sin^2\theta) - d^2 = 0$.

$\Rightarrow \vec{OB}\cdot\vec{AC}=0$. So they're perpendicular.

Therefore, the diagonals of this quadrilateral is perpendicular.

Homework 1 - Ex 10.4

$P(-2,1,0)$. $Q(2,3,2)$. $R(1,4,-1)$. $S(3,6,1)$.

So, $\vec{PQ} = (4,2,2)$.

$\vec{PR} = \langle 3,3,-1\rangle$

$\vec{PS} = \langle 5,5,1\rangle$

the volume of the parallelepiped:

$\vec{PQ}\cdot(\vec{PR}\times\vec{PS})$

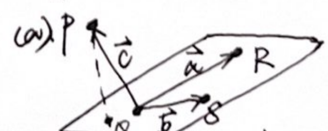
$\vec{PR}\times\vec{PS} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 3 & -1 \\ 5 & 5 & 1 \end{vmatrix}$

$= 8\vec{i} - 8\vec{j} + 0\vec{k} = (8,-8,0)$.

So. volume is $32-16=16$.

Homework 2 - Ex 10.4.

(a)



$\vec{a}\cdot(\vec{b}\times\vec{c})$ is the volume of parallelepiped which consist of $\vec{a}$, $\vec{b}$ and $\vec{c}$. and $\vec{a}\times\vec{b}$ is the bottom of this parallelepiped.

$\underset{\vec{a}\cdot(\vec{b}\times\vec{c})}{V} = \underset{\vec{a}\times\vec{b}}{\text{bottom}} \times \underset{\text{distance from } P \text{ to the plane}}{h}$.

So, $h = \dfrac{V}{\text{bottom}}$

$= \dfrac{|\vec{a}\cdot(\vec{b}\times\vec{c})|}{|\vec{a}\times\vec{b}|}$

and that's exactly the distance!

(b). from part (a).

$$d = \frac{|\vec{a} \cdot (\vec{b} \times \vec{c})|}{|\vec{a} \times \vec{b}|}$$

and $\vec{a} = \vec{QR}$, $\vec{b} = \vec{QS}$, $\vec{c} = \vec{QP}$.

$P(2,1,4)$. $Q(1,0,0)$. $R(0,2,0)$. $S(0,0,3)$.

so, $\vec{a} = \langle -1, 2, 0 \rangle$. $\vec{b} = \langle -1, 0, 3 \rangle$.

$\vec{c} = \langle 1, 1, 4 \rangle$.

$$d = \frac{|\langle -1, 2, 0 \rangle \cdot \langle -3, 7, -1 \rangle|}{|\langle 6, 3, 2 \rangle|} = \frac{17}{7}$$

$\Rightarrow$ the distance is $\frac{17}{7}$

Homework 3 - Ex 10.4

$|\vec{r}| = 18cm = 0.18m$

$\theta = 70° + 10° = 80°$

So, the magnitude of the turque is:

$$|\vec{\tau}| = |\vec{r}| \cdot |\vec{F}| \cdot \sin\theta = 0.18 \times 60 \times \sin 80°$$
$$\approx 10.636 \text{ Nm}$$

Homework 4 - Ex 10.4

(a). $k_i = \frac{V_j \times V_k}{V_i \cdot (V_j \times V_k)}$.

$$k_i \cdot V_j = \frac{V_j \cdot (V_j \times V_k)}{V_i \cdot (V_j \times V_k)}.$$

the numerator of $k_i \cdot V_j$ is.

$V_j \cdot (V_j \times V_k)$

$V_j \times V_k$ is the vector which perpendicular to V_j, so, $k_i \cdot V_j = 0$

$\Rightarrow$ k_i and V_j are perpendicular

(b). $k_i \cdot V_i = \frac{V_i \cdot (V_j \times V_k)}{V_i \cdot (V_j \times V_k)}$

the numerator and denominator are exactly the same,

Therefore, $k_i \cdot V_i = 1$, for $i = 1, 2, 3$.

(c). $\cancel{k_1 \cdot (k_2 \times k_3)} = \frac{\cancel{V_2 \times V_3}\ 1}{\cancel{V_1 \cdot (V_2 \times V_3)}}$.

$k_1 = \frac{V_2 \times V_3}{V_1 \cdot (V_2 \times V_3)}$, which is a vector with direction perpenticular to the plane V_2 and V_3. and the magnitude is the $\frac{1}{|V_1|}$.

the same way, k_2 is vector perpendicular to V_1V_3-plane with length $\frac{1}{|V_2|}$

k_3 is vector perpendicular to V_1V_2-plane with length $\frac{1}{|V_3|}$

$\Rightarrow$ $k_1 \cdot (k_2 \times k_3)$ is exactly the magnitude of the reciprocal of $V_1 \cdot (V_2 \times V_3)$.

And that's:

$$k_1 \cdot (k_2 \times k_3) = \frac{1}{V_1 \cdot (V_2 \times V_3)}$$