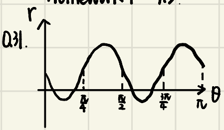
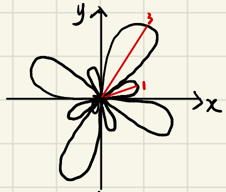
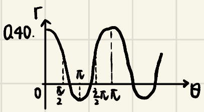
Homework 1 – 9.3

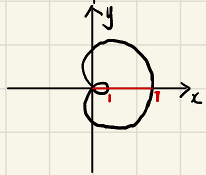
Q31.





Q40.





Homework 2 – 9.3

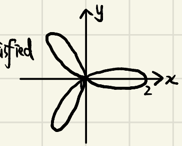
(i) $\theta \to -\theta$: $r = 2\cos(-3\theta)$ $r = 2\cos 3\theta$ $\Rightarrow$ symmetric about the polar axis.

$r \to -r$: $-r = 2\cos 3\theta$ $\Rightarrow$ not satisfied

$\theta \to \theta+\pi$: $r = 2\cos 3(\theta+\pi)$ $r = -2\cos 3\theta$ not satisfied

$\theta \to \theta-\pi$: $r = 2\cos(\theta-\pi)$ $\Rightarrow$ not satisfied

$r \to -r, \theta \to -\theta$: $-r = 2\cos(-3\theta)$ $\Rightarrow$ not satisfied.

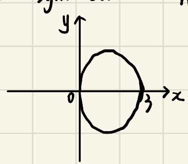


(ii) $\theta \to -\theta$: $r^2 = 9\cos(-\theta)$ $\Rightarrow$ $r^2 = 9\cos\theta$ $\Rightarrow$ symmetric about the polar axis.

$r \to -r$: $(-r)^2 = 9\cos\theta \Rightarrow r^2 = 9\cos\theta$ $\Rightarrow$ symmetric about the pole.

$\theta \to \pi-\theta$: $r^2 = 9\cos(\pi-\theta) \Rightarrow r^2 = -9\cos\theta \Rightarrow$ not satisfied

$r \to -r, \theta \to -\theta$: $(-r)^2 = 9\cos(-\theta) \Rightarrow r^2 = 9\cos\theta \Rightarrow$ symmetric about $\theta = \frac{\pi}{2}$.



Homework 1 – 9.4

$r = 0 \Rightarrow 2\cos\theta - \frac{1}{\cos\theta} = 0 \Rightarrow \cos\theta = \pm\frac{\sqrt{2}}{2}$ $\quad \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$A = 2\int_0^{\frac{\pi}{4}} \frac{1}{2}\left(2\cos\theta - \frac{1}{\cos\theta}\right)^2 d\theta = \int_0^{\frac{\pi}{4}} (4\cos^2\theta - 4 + \sec^2\theta)\,d\theta$

$A = \int_0^{\frac{\pi}{4}} (2 + 2\cos 2\theta - 4 + \sec^2\theta)\,d\theta = [2\theta + \sin 2\theta - 4\theta + \tan\theta]_0^{\frac{\pi}{4}}$

$A = (1 - \frac{\pi}{2} + 1) - (0 - 0 + 0) = 2 - \frac{\pi}{2}$

The area of one loop is $2 - \frac{\pi}{2}$.

Homework 2 – 9.4

$3\cos\theta = 1 + \cos\theta \Rightarrow \cos\theta = \frac{1}{2}$ $\theta = \pm\frac{\pi}{3}$

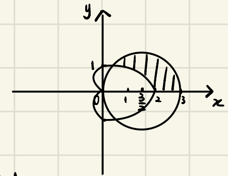
$\cos\theta = \frac{r}{3}$ $\quad x = r\cos\theta = \frac{r^2}{3}$

$3x = x^2 + y^2$ $\quad (x - \frac{3}{2})^2 + y^2 = \frac{9}{4}$

$A = 2 \times \int_0^{\frac{\pi}{3}} \frac{1}{2} \times (9\cos^2\theta - 1 - \cos^2\theta - 2\cos\theta)\,d\theta$

$A = \int_0^{\frac{\pi}{3}} (4(1+\cos 2\theta))\,d\theta - 2\int_0^{\frac{\pi}{3}} \cos\theta\,d\theta - \int_0^{\frac{\pi}{3}} d\theta = \pi$

$\therefore$ the area is π.



Homework 3 – 9.4

$\cos\theta = \frac{r}{\sqrt{3}}$ $\quad x = r\cos\theta$ $\quad x = \frac{r^2}{\sqrt{3}}$ $\quad x^2 + y^2 - \sqrt{3}x = 0$ $\quad (x - \frac{\sqrt{3}}{2})^2 + y^2 = \frac{3}{4}$

$r = \sin\theta$ $\quad y = r\sin\theta$ $\quad y = r^2$ $\quad x^2 + y^2 - y = 0$ $\quad x^2 + (y - \frac{1}{2})^2 = \frac{1}{4}$

$\sqrt{3}\cos\theta = \sin\theta$ $\quad \tan\theta = \sqrt{3}$ $\quad \theta = \frac{\pi}{3}$

$A = \int_0^{\frac{\pi}{3}} \frac{1}{2}\sin^2\theta\,d\theta + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{3}{2}\cos^2\theta\,d\theta$

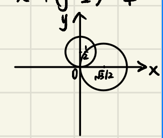
$A = \int_0^{\frac{\pi}{3}} \frac{1 - \cos 2\theta}{4}\,d\theta + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{3(1 + \cos 2\theta)}{4}\,d\theta$

$A = [\frac{1}{4}\theta - \frac{\sin 2\theta}{8}]_0^{\frac{\pi}{3}} + [\frac{3}{4}\theta + \frac{3\sin 2\theta}{8}]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$

$A = (\frac{\pi}{12} - \frac{\sqrt{3}}{16}) + (\frac{3\pi}{8} + 0 - \frac{\pi}{4} - \frac{3\sqrt{3}}{16})$

$A = \frac{5\pi}{24} - \frac{\sqrt{3}}{4}$

$\therefore$ The area is $\frac{5\pi}{24} - \frac{\sqrt{3}}{4}$.



Homework 4 – 9.4

$\cos 3\theta = \sin 3\theta \Rightarrow \tan 3\theta = 1$ $\quad 3\theta = \frac{\pi}{4}$ or $\frac{5\pi}{4}$ $\quad \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3}{4}\pi, \frac{13}{12}\pi, \frac{17}{12}\pi, \frac{7}{4}\pi$

$\cos 3\theta = 0 \Rightarrow \theta = \frac{\pi}{6}$ $\quad \sin 3\theta = 0$ $\quad \theta = 0$

$\Rightarrow$ the intersection points are $(\frac{\sqrt{2}}{2}, \frac{\pi}{12})$, $(-\frac{\sqrt{2}}{2}, \frac{5\pi}{12})$, $(\frac{\sqrt{2}}{2}, \frac{3}{4}\pi)$, $(-\frac{\sqrt{2}}{2}, \frac{13}{12}\pi)$, $(\frac{\sqrt{2}}{2}, \frac{17}{12}\pi)$, $(-\frac{\sqrt{2}}{2}\ \frac{7}{4}\pi)$, $(0,0)$

Homework 5 – 9.4

$L = \int_\alpha^\beta \sqrt{(f(\theta))^2 + (f'(\theta))^2}\,d\theta = \int_0^{\frac{\pi}{2}} \sqrt{\left(\frac{\sqrt{2}}{1+\cos\theta}\right)^2 + \left(\frac{\sqrt{2}\sin\theta}{(1+\cos\theta)^2}\right)^2}\,d\theta$

$L = \int_0^{\frac{\pi}{2}} \sqrt{\frac{2(1+\cos\theta)^2}{(1+\cos\theta)^4} + \frac{2\sin^2\theta}{(1+\cos\theta)^4}}\,d\theta = \int_0^{\frac{\pi}{2}} \sqrt{\frac{2 + 4\cos\theta + 2\cos^2\theta + 2\sin^2\theta}{(1+\cos\theta)^4}}\,d\theta$

$L = \int_0^{\frac{\pi}{2}} \frac{2\sqrt{1 + \cos\theta}}{(1+\cos\theta)^2}\,d\theta$

$L = 2\int_0^{\frac{\pi}{2}} (1+\cos\theta)^{-\frac{3}{2}}\,d\theta = 2\int_0^{\frac{\pi}{2}} [2(\cos\frac{\theta}{2})^2]^{-\frac{3}{2}}\,d\theta = \frac{\sqrt{2}}{2}\int_0^{\frac{\pi}{2}} (\cos\frac{\theta}{2})^{-3}\,d\theta = \frac{\sqrt{2}}{2}\int_0^{\frac{\pi}{2}} \sec^3\frac{\theta}{2}\,d\theta$

$\therefore L = \sqrt{2}\int_0^{\frac{\pi}{2}} \sec^3\frac{\theta}{2}\,d\frac{\theta}{2}$ $\quad$ let $t = \frac{\theta}{2}$. $L = \sqrt{2}\int_0^{\frac{\pi}{4}} \sec^3 t\,dt = \sqrt{2} \cdot \frac{1}{2}[\sec t\tan t + \ln|\sec t + \tan t|]_0^{\frac{\pi}{4}}$

$\therefore L = 1 + \frac{\sqrt{2}}{2}\ln(1+\sqrt{2})$

-1

Homework 6 – 9.4

$\frac{dy}{dx} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta} = \frac{(\cos\theta - 2\sin\theta)\sin\theta + (\sin\theta + 2\cos\theta)\cos\theta}{(\cos\theta - 2\sin\theta)\cos\theta - (\sin\theta + 2\cos\theta)\sin\theta}$

$= \frac{-2\sin^2\theta + 2\cos^2\theta + 2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta - 4\sin\theta\cos\theta}$

when $\theta = \frac{\pi}{2}$, $\frac{dy}{dx} = \frac{-2}{-1} = 2$