Ивев А.Д 3824 М1ПМОм

Д/з №3

№9

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|---|---|---|---|---|---|---|---|---|
| $x_1$ | 0 | 1 | 0 | 2 | 2 | 2 | 4 | 3 |
| $x_2$ | -1 | 0 | 0 | 0 | 1 | 0 | 1 | 2 |
| $y$ | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |

1) $\hat{P}r\{Y=0\} = \frac{5}{8}$ $\quad \hat{P}r\{Y=1\} = \frac{3}{8}$

$$\hat{\mu}_0 = \frac{1}{5}\begin{pmatrix}5\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix} \qquad \hat{\mu}_1 = \frac{1}{3}\begin{pmatrix}9\\3\end{pmatrix} = \begin{pmatrix}3\\1\end{pmatrix}$$

$$\hat{\Sigma}_0 = \frac{1}{4}\left(\begin{pmatrix}1&1\\1&1\end{pmatrix} + \begin{pmatrix}1&0\\0&0\end{pmatrix} + \begin{pmatrix}1&0\\0&0\end{pmatrix} + \begin{pmatrix}1&1\\1&1\end{pmatrix}\right) = \frac{1}{4}\begin{pmatrix}4&2\\2&2\end{pmatrix}$$

$$\hat{\Sigma}_1 = \frac{1}{2}\left(\begin{pmatrix}1&1\\1&1\end{pmatrix} + \begin{pmatrix}1&0\\0&0\end{pmatrix} + \begin{pmatrix}0&0\\0&1\end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix}2&1\\1&2\end{pmatrix}$$

$$\hat{\Sigma} = \frac{1}{6}\left(4\hat{\Sigma}_0 + 2\hat{\Sigma}_1\right) = \frac{1}{6}\begin{pmatrix}6&3\\3&4\end{pmatrix}$$

$$\hat{\Sigma}_0^{-1} = \begin{pmatrix}2&-2\\-2&4\end{pmatrix} \qquad \hat{\Sigma}_1^{-1} = \frac{1}{3}\begin{pmatrix}4&-2\\-2&4\end{pmatrix} \qquad \hat{\Sigma}^{-1} = \frac{1}{5}\begin{pmatrix}8&-6\\-6&12\end{pmatrix}$$

$$\delta_i(x) = x^T\hat{\Sigma}^{-1}\hat{\mu}_i - \frac{1}{2}\hat{\mu}_i^T\hat{\Sigma}^{-1}\hat{\mu}_i + \ln \hat{P}r\{Y=i\}$$

$$\Rightarrow \delta_0(x) = \frac{1}{5}(x_1\ x_2)\begin{pmatrix}8&-6\\-6&12\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} - \frac{1}{10}(1\ 0)\begin{pmatrix}8&-6\\-6&12\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} + \ln\frac{5}{8} =$$

$$= \frac{8}{5}x_1 - \frac{6}{5}x_2 - \frac{8}{10} + \ln\frac{5}{8}$$

$$\delta_1(x) = \frac{1}{5}(x_1\ x_2)\begin{pmatrix}8&-6\\-6&12\end{pmatrix}\begin{pmatrix}3\\1\end{pmatrix} - \frac{1}{10}(3\ 1)\begin{pmatrix}8&-6\\-6&12\end{pmatrix}\begin{pmatrix}3\\1\end{pmatrix} + \ln\frac{3}{8} =$$

$$= \frac{18}{5}x_1 - \frac{6}{5}x_2 - \frac{24}{5} + \ln\frac{3}{8}$$

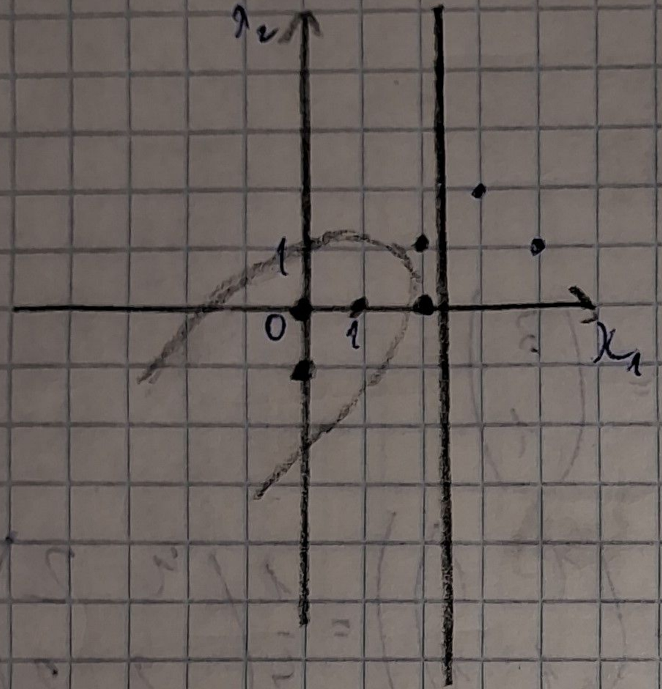
$$\delta_0(x) = \delta_1(x) \Rightarrow 2x_1 - 4 + \ln\frac{3}{8} - \ln\frac{5}{8} = 0 \Rightarrow x_1 = 2 + \frac{1}{2}\ln\left(\frac{5}{3}\right)$$

2) $\tilde{\delta}_i(x) = -\frac{1}{2}\ln\det\hat{\Sigma}_i - \frac{1}{2}(x-\hat{\mu}_i)^T\hat{\Sigma}_i^{-1}(x-\mu_i) + \ln \hat{P}r\{Y=i\}$

$$\tilde{\delta}_0(x) = \ln\frac{5}{16} + \left(-x_1^2 - 2x_2^2 + 2x_1x_2 + 2x_1 - 2x_2 - 1\right) \cancel{+ \ln\left(\frac{5}{8}\right)}$$

$$\tilde{\delta}_1(x) = \ln\frac{3\sqrt{3}}{16} + \left(-\frac{2}{3}x_1^2 - \frac{2}{3}x_2^2 + \frac{2}{3}x_1x_2 + \frac{10}{3}x_1 - \frac{2}{3}x_2 - \frac{14}{3}\right)$$

$$\tilde{\delta}_0(x) = \tilde{\delta}_1(x) \Rightarrow \ln\left(\frac{3\sqrt{3}}{5}\right) + \left(\frac{1}{3}x_1^2 + \frac{4}{3}x_2^2 - \frac{4}{3}x_1x_2 + \frac{4}{3}x_1 + \frac{4}{3}x_2 - \frac{11}{3}\right) = 0$$



N15

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|---|---|---|---|---|---|---|---|---|---|---|
| $x_1$ | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| $x_2$ | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $y$ | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |

$\hat{P}_V\{Y=0\} = \frac{1}{2}$ $\quad \hat{P}_V\{Y=1\} = \frac{1}{2}$

$\hat{P}_V\{X_1=1 \mid Y=0\} = \frac{2}{5}$ $\quad \hat{P}_V\{X_1=1 \mid Y=1\} = \frac{3}{5}$

$\hat{P}_V\{X_2=1 \mid Y=0\} = \frac{3}{5}$ $\quad \hat{P}_V\{X_2=1 \mid Y=1\} = 1$

$$\Rightarrow \hat{P}_V\{Y=0 \mid X_1=1, X_2=1\} = \frac{\frac{2}{5}\cdot\frac{3}{5}\cdot\frac{1}{2}}{\frac{2}{5}\cdot\frac{3}{5}\cdot\frac{1}{2}+\frac{3}{5}\cdot 1\cdot\frac{1}{2}} = \frac{\frac{3}{25}}{\frac{3}{25}+\frac{3}{10}} = \frac{2}{7}$$

$$\hat{P}_V\{Y=1 \mid X_1=1, X_2=1\} = \frac{\frac{3}{10}}{\frac{3}{25}+\frac{3}{10}} = \frac{5}{7}$$

N16

| | | | | |
|---|---|---|---|---|
| $x_1$ | 4 | 0 | -2 | 2 |
| $x_2$ | 3 | 1 | -3 | -1 |

$$X = \begin{pmatrix} 4 & 3 \\ 0 & 1 \\ -2 & -3 \\ 2 & -1 \end{pmatrix} \quad \bar{x} = (1, 0) \Rightarrow X_c = \begin{pmatrix} 3 & 3 \\ -1 & 1 \\ -3 & -3 \\ 1 & -1 \end{pmatrix}$$

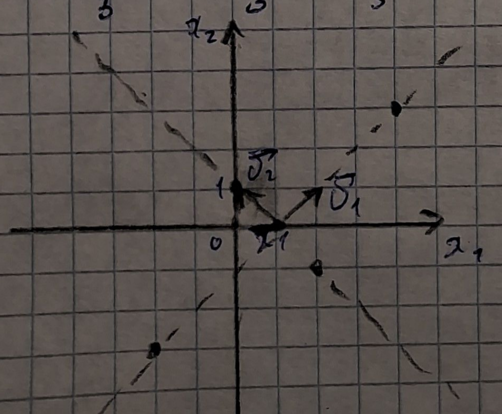
$$C = X_c^T X_c = \begin{pmatrix} 20 & 18 \\ 18 & 20 \end{pmatrix}$$

$$\det(C - \lambda I) = \begin{vmatrix} 20-\lambda & 18 \\ 18 & 20-\lambda \end{vmatrix} = (20-\lambda)^2 - 18^2 = \lambda^2 - 40\lambda + 76 = 0$$

$$\Rightarrow \lambda_1 = 2, \quad \lambda_2 = 38 \qquad \begin{pmatrix} 18 & 18 \\ 18 & 18 \end{pmatrix} \Rightarrow \delta_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\frac{1}{3}\cdot 2 = \frac{2}{3} \qquad \frac{1}{3}\cdot 38 = \frac{38}{3}$$

$$\begin{pmatrix} -18 & 18 \\ 18 & -18 \end{pmatrix} \Rightarrow \delta_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$



N19 $\quad q_k(s_1,\dots,s_K) = e^{s_k} \Big/ \sum\limits_{l=1}^{K} e^{s_l}$

$$R^{(i)} = -\sum_{k=1}^{K} I(y^{(i)}=k)\ln q_k(s_1,\dots,s_K)$$

1) $$\frac{\partial q_k}{\partial s_l} = \frac{\partial \dfrac{e^{s_k}}{\sum_{p=1}^{K} e^{s_p}}}{\partial s_l} = \frac{-e^{s_l}e^{s_k} + I(k=l)e^{s_k}\sum_{p=1}^{K} e^{s_p}}{\left(\sum_{p=1}^{K} e^{s_p}\right)^2} = q_k\left(\frac{I(k=l)\sum_{p=1}^{K} e^{s_p} - e^{s_l}}{\sum_{p=1}^{K} e^{s_p}}\right) =$$

$$= q_k\left(I(k=l) - q_l\right)$$

2) $$\frac{\partial R^{(i)}}{\partial q_k} = \cancel{-\sum} - \frac{\partial \sum_{p=1}^{K} I(y^{(i)}=p)\ln q_p(s_1,\dots,s_K)}{\partial q_k} =$$

$$= -I(y^{(i)}=k)\cdot\frac{\partial \ln q_k(s_1,\dots,s_K)}{\partial q_k} = -\frac{I(y^{(i)}=k)}{q_k}$$

3) $$\frac{\partial R^{(i)}}{\partial s_l} = \sum_{k=1}^{K}\frac{\partial R^{(i)}}{\partial q_k}\cdot\frac{\partial q_k}{\partial s_l} = \sum_{k=1}^{K}\left(-\frac{I(y^{(i)}=k)}{q_k}\right)\cdot q_k\left(I(k=l) - q_l\right) =$$

$$= +\sum_{k=1}^{K}\left(q_l I(y^{(i)}=k) - I(y^{(i)}=k)I(k=l)\right) = q_l - I(y^{(i)}=l)$$