Д/з №1 Швев Александр

№35 $L(y', y) = (y'-y)^2 \quad f^*(x) = \arg\min_c E((Y-c)^2 | X=x)$

$$E((Y-c)^2|X=x) = E(Y^2|X=x) - 2cE(Y|X=x) + c^2$$

$$\min_c E((Y-c)^2|X=x) = E(Y^2|X=x) + \min_c(-2cE(Y|X=x) + c^2)$$

$$\frac{d}{dc}(-2cE(Y|X=x) + c^2) = -2E(Y|X=x) + 2c = 0 \Rightarrow$$

$$\Rightarrow c = E(Y|X=x) \Rightarrow f^*(x) = \arg\min_c E((Y-c)^2|X=x) = E(Y|X=x)$$

$$R(f^*) = E(L(f^*(x), Y)) = E((E(Y|X=x) - Y)^2) =$$

$$= E_x(E((E(Y|X=x) - Y)^2|X=x)) = E_x(D(Y|X=x))$$

№36 $L(y', y) = |y'-y| \quad f^*(x) = \text{median}(Y|X=x)$?

$$f^*(x) = \arg\min_c E(|Y-c| \,|\, X=x) \cancel{= \arg\min_c \int_{-\infty}^{+\infty} |Y-c| p(Y|X=x)}$$

, ~~если~~ так как E обладает свойством монотонности ⇒ и при $X \geq 0$ $E(X|H) \geq 0$ $\Rightarrow \arg\min_c E(|Y-c| \,|\, X=x) =$

$$= \arg\min_c |Y-c| \Rightarrow f^*(x) = \text{median}(Y|X=x) = F^{-1}_{Y|X=x}\left(\frac{1}{2}\right)$$

$$R(f^*) = E(L(f^*(x), Y)) = E(|\text{median}(Y|X=x) - Y|)$$

№37 $f^*(x) = \text{mode}(Y|X=x) \quad L(y', y) - ?$

$$f^*(x) = \arg\min_c E(L(Y, c)|X=x)$$

$$\Rightarrow \cancel{\frac{d}{dc}(E(L(Y,c)|X=x))}\Big|_{f^*(x)} = 0$$

$$\Rightarrow \exists\, L(y', y) = \begin{cases} 0, & y = y' \\ 1, & y \neq y' \end{cases} \Rightarrow \arg\min_c E(L(Y,c)|X=x) =$$

$$= \text{mode}(Y|X=x), \text{ т.к. } L(Y, \text{mode}(Y|X=x)) = \begin{cases} 0, & Y = \text{mode}(Y|X=x) \\ 1, & Y \neq \text{mode}(Y|X=x) \end{cases}$$

, т.е. $= 0$ на большинстве знач.

№38 классификация $\{0,1\}$ $f^*(x) = \arg\max_{y\in\{0,1\}} l_y \Pr(y|x)$

$L(y',y)$: $L(0,0) = L(1,1) = 0$; $L(1,0) = l_1$, $L(0,1) = l_0$

$f^*(x) = \arg\min_c \left(\sum_{y=0}^{1} L(c,y)\Pr(y|x)\right)$

$\sum_{y=0}^{1} L(c,y)\Pr(y|x) = L(c,0)\Pr(0|x) + L(c,1)\Pr(1|x) =$

$\Rightarrow$, если $c=0$ $l_0 \Pr(1|x)$ $\Rightarrow f^*(x) = \begin{cases} 0, & l_0\Pr(1|x) < l_1\Pr(0,x) \\ 1, & l_0\Pr(1|x) \geq l_1\Pr(0,x) \end{cases}$

, если $c=1$ $l_1\Pr(0|x)$

№39 $Y = \{1,\dots,k\}$ классификация

$L(y',y) = l_{y'y}$ (матрица L с элем $l_{y'y}$) $y',y\in\{1,\dots,k\}$)

$f^*(x) = \arg\min_c \left(\sum_{y=1}^{k} L(c,y)P(y|x)\right) = \arg\min_c \left(\sum_{y=1}^{k} l_{cy} P(y|x)\right)$