Color Image Segmentation — Mean Shift

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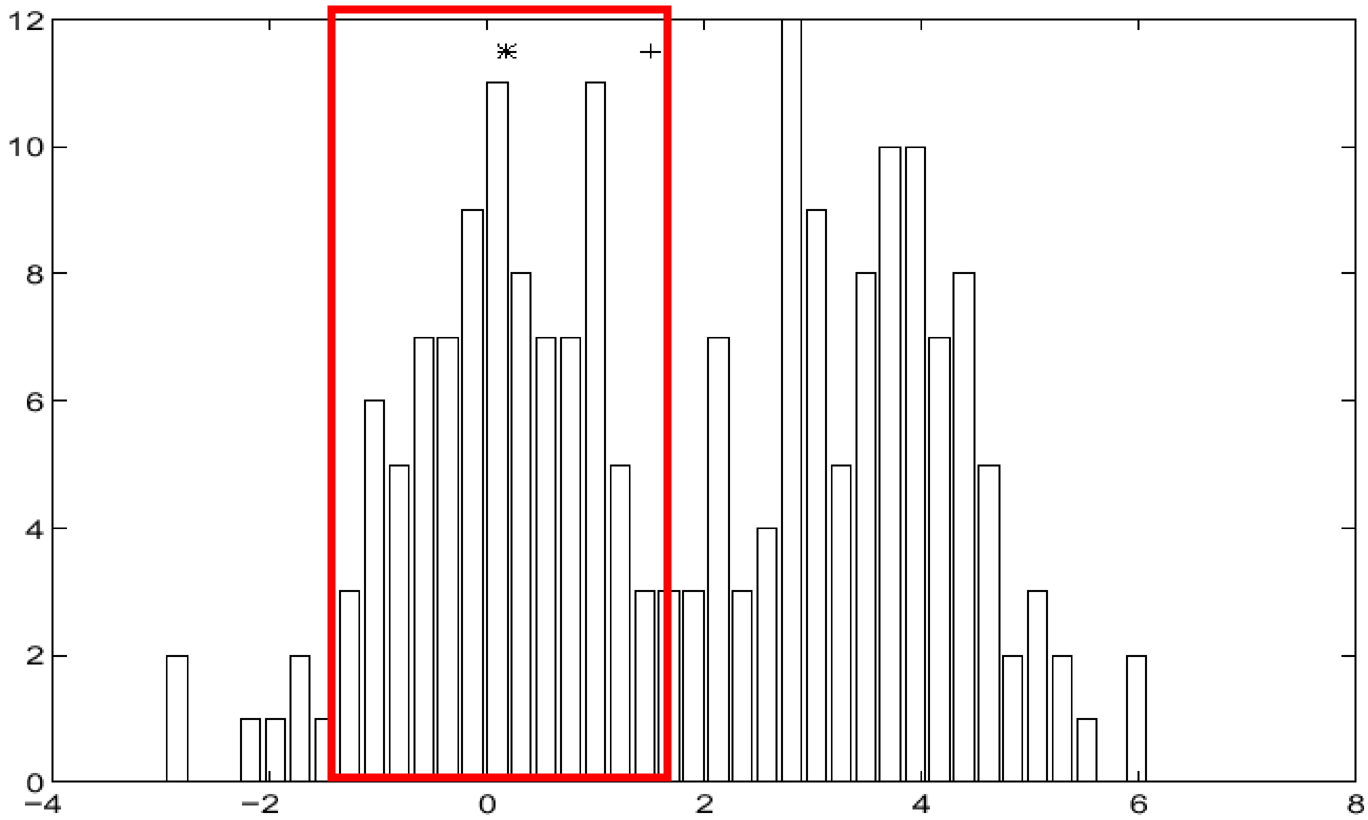
Mean-shift Algorithm [1]



- Iterative Mode Search
  1. Initialize random seed, and fixed window
  2. Calculate center of gravity of the window (the "mean")
  3. Translate the search window to the mean
  4. Repeat Step 2 until convergence

Background

- Feature space analysis is a widely used tool for solving low-level image understanding tasks.
  - **Significant** features correspond to high density regions in the feature space.
  - Feature space analysis is the procedure of recovering the centers of the high density regions.
  - Mean-shift – a simple nonparametric procedure for estimating density gradients.

Traditional Clustering Techniques

- Only reliable if the number of clusters is **small** and **known a prior**.
  - K-means
- Too often assume that the individual clusters obey multivariate normal distributions, i.e. the feature space can be modeled as a **mixture** of Gaussians.
- A strong artifact cluster may appear when several features are mapped into partially overlapping regions.

Spatial Constraints

- In image understanding tasks the data to be analyzed originates in the image domain. That is, the feature vectors satisfy additional spatial constraints.
- The feature space should be isotropic. A space is isotropic if the distance between two points is independent on the location of the point pair.

Mean-shift Algorithm

- Mean-shift was proposed in **1975** by Fukunaga and Hostetler.
- For the moment, assume the probability density function p(**x**) of the *p*-dimensional feature vectors **x** is unimodal.
- A sphere S_x of radius r, centered on **x** contains the feature vectors **y** such that $\| \mathbf{y} - \mathbf{x} \| \leq r$. The expected value of the vector $\mathbf{z} = \mathbf{y} - \mathbf{x}$, given **x** and $S_{\mathbf{x}}$ is

$$\mu = E\left[\mathbf{z} \mid S_x\right] = \int_{S_x} (\mathbf{y} - \mathbf{x})\, p(\mathbf{y} \mid S_x)\, d\mathbf{y} = \int_{S_x} (\mathbf{y} - \mathbf{x}) \frac{p(\mathbf{y})}{p(\mathbf{y} \in S_x)} d\mathbf{y}$$

Mean-shift Algorithm

$$\boxed{\mu = E\left[\mathbf{z} \mid S_x\right] = \int_{S_x} (\mathbf{y}-\mathbf{x})\, p(\mathbf{y} \mid S_x)\, d\mathbf{y} = \int_{S_x} (\mathbf{y}-\mathbf{x}) \frac{p(\mathbf{y})}{p(\mathbf{y} \in S_x)} d\mathbf{y}}$$

- If $S_{\mathbf{x}}$ is sufficiently small, we can approximate

$$p(\mathbf{y} \in S_x) \approx p(\mathbf{x}) V_{S_x} \qquad V_{S_x} = c \cdot r^p$$

- The first order approximation of p(**y**) is

$$p(\mathbf{y}) = p(\mathbf{x}) + (\mathbf{y}-\mathbf{x})^T \nabla p(\mathbf{x})$$

- Where ∇p(**x**) is the gradient of the probability density function in **x**

$$\mu = \int_{S_x} \frac{(\mathbf{y}-\mathbf{x})(\mathbf{y}-\mathbf{x})^T}{V_{S_x}} \frac{\nabla p(\mathbf{x})}{p(\mathbf{x})} d\mathbf{y} = \frac{r^2}{p+2} \frac{\nabla p(\mathbf{x})}{p(\mathbf{x})}$$

$$\mu = \int_{S_x} (\mathbf{y}-\mathbf{x})\, p(\mathbf{y} \mid S_x) d\mathbf{y} = \int_{S_x} (\mathbf{y}-\mathbf{x}) \frac{p(\mathbf{y})}{p(\mathbf{y} \in S_x)} d\mathbf{y}$$

$$\because \quad p(\mathbf{y} \in S_x) \approx p(\mathbf{x}) V_{S_x}$$

$$\because \quad p(\mathbf{y}) = p(\mathbf{x}) + (\mathbf{y}-\mathbf{x})^T \nabla p(\mathbf{x})$$

$$\mu = \int_{S_x} (\mathbf{y}-\mathbf{x}) \frac{p(\mathbf{x}) + (\mathbf{y}-\mathbf{x})^T \nabla p(\mathbf{x})}{p(\mathbf{x}) V_{S_x}} d\mathbf{y}$$

$$= \int_{S_x} (\mathbf{y}-\mathbf{x}) \frac{1}{V_{S_x}} d\mathbf{y} + \int_{S_x} (\mathbf{y}-\mathbf{x})(\mathbf{y}-\mathbf{x})^T \frac{\nabla p(\mathbf{x})}{p(\mathbf{x}) V_{S_x}} d\mathbf{y}$$

$$= \int_{S_x} \frac{(\mathbf{y}-\mathbf{x})(\mathbf{y}-\mathbf{x})^T}{V_{S_x}} \frac{\nabla p(\mathbf{x})}{p(\mathbf{x})} d\mathbf{y} = \frac{r^2}{p+2} \frac{\nabla p(\mathbf{x})}{p(\mathbf{x})}$$

Mean-shift Vector

- The vector of difference between the local mean and the center of the window is proportional to the gradient of the probability density at **x**.

$$\mu = \int_{S_x} (\mathbf{y} - \mathbf{x}) p(\mathbf{y} \mid S_x) d\mathbf{y} = \frac{r^2}{p+2} \frac{\nabla p(\mathbf{x})}{p(\mathbf{x})}$$

- This is beneficial when the highest density region of the probability density function is sought, such region corresponds to large p(**x**) and small ∇p(**x**).
- Low density regions correspond to large mean shifts. The shifts are always in the direction of the probability density maximum.

Mean-shift Algorithm

- At the mode the mean shift is close to zero.
  - Choose the radius *r* of the search window.
  - Choose the initial location of the window.
  - Compute the mean shift vector and translate the search window by that amount.
  - Repeat till convergence.

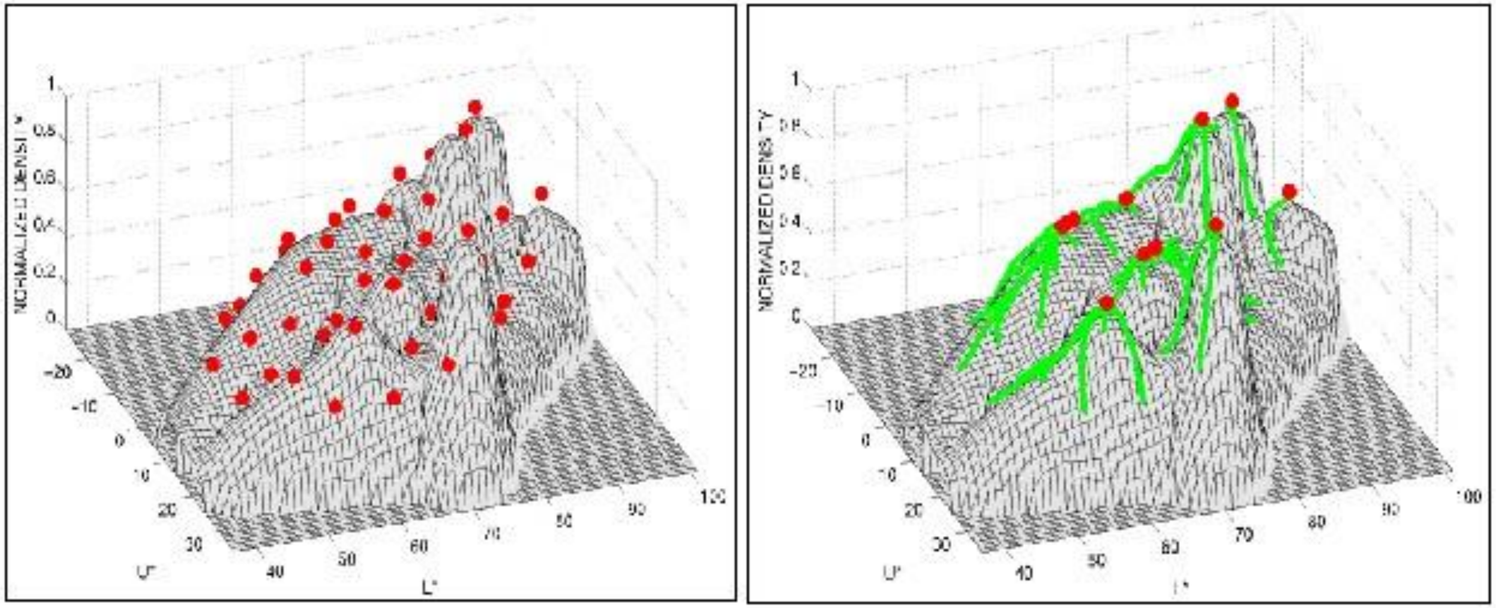
Feature Space Analysis

- Map the image domain into the feature space.
- Define an adequate number of search windows at random locations in the space.
- Find the high density region centers by applying the mean-shift algorithm to each window.
- Validate the extracted centers with image domain constraints to provide the feature palette.
- Allocate, using image domain information, all the feature vectors to the feature palette.

Mean-Shift of Multimodal

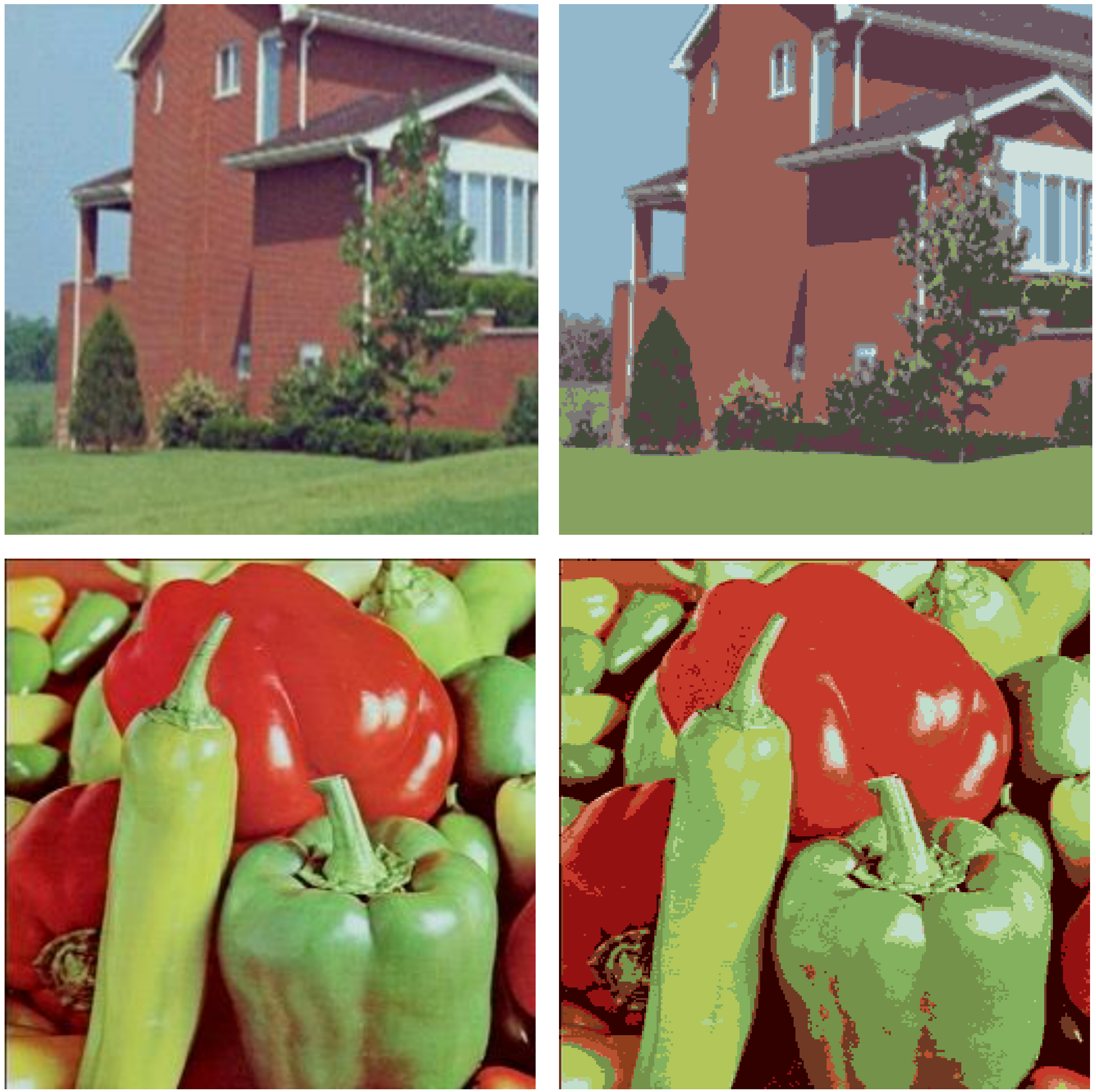
Multimodal Distributions

- Parallel processing of an initial tessellation.
- Pruning of mode candidates.
- Classification based on the basin of attraction.



Mean shift trajectories

Mean-shift for Natural Color Images



References

- [1] D. Comaniciu and P. Meer, “Robust analysis of feature spaces: color image segmentation,” in Proc. Of IEEE CVPR’1997.

Thank You!

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