MULTIRESOLUTION SEQUENTIAL EDGE LINKING

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ABSTRACT

In this paper we describe a multiresolution approach to edge detection using a sequential search algorithm. The use of a multiresolution image pyramid allows integration of global edge information contained in lower resolutions to guide the sequential search at higher resolutions. As a consequence, dependence on *a priori* knowledge about the image edges is greatly reduced. Estimating the sequential search parameters from lower resolution images provides for a more accurate and less costly search of edge paths in the image.

1. INTRODUCTION

An important operation in image processing and computer vision is the detection of edges [1]. Generally, edge detection may be thought of as a two step process. In the first step the edges are enhanced, usually based on estimating spatial derivatives of the image [1, 2]. The second step is determining whether a particular pixel is an edge.

In this paper we describe a multiresolution approach to edge detection using a sequential search algorithm known as Sequential Edge Linking (SEL) [3, 4, 5]. The use of a multiresolution image pyramid allows integration of global edge information contained in lower resolutions to guide the sequential search of the image at higher resolutions. As a consequence, dependence on *a priori* knowledge is greatly reduced.

SEL has a number of advantages over other edge detection schemes, such as lower false-alarm rates while maintaining full connectivity of the edges [3]. Edge decisions are made by using global information derived from past decisions on the existence of an edge pixel.

The SEL algorithm can be decomposed into two parts: enhancement followed by detection and linking. An enhancement operator such as a Gaussian weighted gradient operator using a relatively large window can be used [2]. The detection and linking phase is based on a sequential search that ranks potential edges and edge paths according to a log-likelihood statistic.

A crucial concept in any sequential searching algorithm is that of a path measure of goodness, known as the path metric. In the SEL path metric, both the strength of an edge and the past path of the edge are used to determine the search direction [5]. The strength of an edge is measured by the likelihood ratio of the conditional probability that a edge pixel exists; this term dominates when the signal-to-noise ratio is high. The past path of an edge is modeled by a Markov random chain; this term dominates in situations where there is a low signal-to-noise ratio. The sequential searching algorithm begins at a known edge pixel and sequentially examines paths, or sequences of possible edge pixels, in the image. At each iteration of the algorithm, one of the paths explored so far is extended by one node and the metric for this new path is calculated. Decisions are made based on these metrics as to which path should be extended in the next iteration.

One of the most important facets of SEL is the ability to model and determine the amount of correlation between the pixels of an image. In [3] the image is modeled as a homogeneous Gaussian field having an autocorrelation function of a one-dimensional p^{th}-order wide-sense Markov sequence, with city block (two-dimensional) distance taking the place of the (one-dimensional) integer lags. With this model, pixel correlation may be modeled using an autoregressive-moving average (ARMA) process. As a consequence, an uncorrelated (and thus independent) innovations sequence can be obtained from the path observations.

SEL relies on two stochastic models: a random field for the the enhanced image, and a Markov random chain to model edge paths. Consequently, parameters which describe the SEL algorithm are the ARMA model of correlation between pixels in the enhanced image, the conditional probability distribution functions based on the innovations of the ARMA process, and the Markov random chain state probabilities for the edge model. For more details on SEL see [3, 5].

Optimum estimation of the conditional probability distributions of the random field was not addressed in [5], where an estimation technique based on the starting or root edge pixel was used to determine the probability density functions of the pixels. In addition, the Markov chain parameters were derived by assuming the present image was similar to the known edges in similar images.

Multiresolution or pyramidal techniques have become popular in many areas of computer vision and image processing. Example applications include compression [6], segmentation [7], and edge detection [8, 9]. All of these algorithms share the common characteristic of combining in a

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computationally efficient way image data processed at different scales.

2. MULTIRESOLUTION SEQUENTIAL EDGE LINKING

The parameters required for detection of edges using SEL relies on *a priori* knowledge of the edge structure so that we can estimate the Markov chain probabilities and the conditional probability density functions of the pixels in the image. We choose to use a multiresolution pyramid decomposition of the image such that lower resolution images are used to guide the parameter estimation for the higher resolution images. After pyramid decomposition the SEL algorithm is used with a simple probability density function estimate of the pixels and a uniform Markov transition matrix to detect edges in the smallest pyramidal image. The search parameters for the image in the next pyramid is then obtained as described below. At each level of the pyramid the parameters are refined and edges are detected for use in the next level in the pyramid.

2.1. Pyramid Decomposition

An image pyramid may be conceived as having the highest resolution image at the base of a pyramid upon which lower resolution images may be stacked to form the pyramid [6]. For this research, we generate the image pyramid by employing a kernel design described in [10]. The SEL algorithm is not dependent on the exact form of enhancement and many different formulations, including wavelet decomposition [11], may be employed. Gaussian smoothing is often used in conjunction with spatial derivative estimation [2], but for the results presented below we have *not* employed any smoothing.

2.2. Markov Chain Probability Estimation

The SEL path metric strikes a balance between the current likelihood measure and prediction of behavior from past observations along the path. We derive the Markov chain probabilities from the edges of the previous, lower resolution image on the pyramid in order to capture "global" information. The probabilities of the state transitions are estimated as the relative frequency of those transitions in the observed state sequences. Initially, each edge has state transition probabilities which are equiprobable; as the number of pixels in the chain increases, the relative frequencies will become more reflective of the actual state transition matrices of the edge data. We note that since we are estimating the state transition matrix for the higher resolution image from the lower resolution image, the estimates are correct for edges which are straight, but inaccuracies may result for highly curved segments. However, it is more difficult to distinguish noise in highly curved edges and the effect is masked in these cases.

2.3. Conditional Probability Density Estimation

Because of the correlation of image pixels from the use of the discontinuities in the enhanced image, the path that a potential edge takes plays an important part in generating the innovations from the ARMA model used for the random field. In fact, it is the conditional probability density functions of the innovations sequence which must be estimated. The information we have, however, is the path of an edge in the lower resolution image. If we follow a path nominally described in the lower resolution image for the higher resolution image (taking into account the increase in resolution by using each path direction in the chain twice) there is uncertainty as to which pixels correspond to edge pixels in the higher resolution image. In essence, we need to both estimate the conditional probability densities *and* estimate the probability that the pixel itself is or is not an edge pixel. Since the innovations are conditionally independent, i.e. based on whether an edge exists or not, we may model the path sequence using a Hidden Markov Model (HMM) [12] and use the expectation-maximization (EM) algorithm [13] to estimate the conditional probability density functions. It has been shown that this method of estimation is equivalent to finding the solution to the constrained maximum likelihood problem [12]. The innovations are the known signal and the two states of the hidden random variables indicate whether the pixel is part of an edge or not. In general, the estimated path will weave in and out of the true path and this will be reflected in the transition probabilities of the HMM. To reduce the possibility that the EM algorithm converges to a local, but not a global, maximum, nine paths are followed, each starting at a different point in relation to the root node. The parameters for the nine paths are then averaged for the final parameter estimation.

3. RESULTS

The 336×432 8-bit grayscale image shown on the left in Figure 1 has light and dark values of 196 and 128, respectively. This image was corrupted with additive independent identically distributed Gaussian noise with $\sigma^2 = 1600$, and the normalized magnitude of the gradient of the resulting image is shown in Figure 1 on the right. In Figure 2, the edge map of the SEL algorithm is shown on the left. In this case, *no* Gaussian smoothing operator [2] was applied before the gradient operator. The conditional probability density functions were estimated using the technique described in [4], with an estimation parameter of 20. This parameter sets the variance estimate to approximately the same value as the Gaussian noise. The extraneous edges are generated from the method of estimating the root or starting node [4]. Because of the high noise in the image, several of the root nodes were incorrect. On the right in Figure 2 is the the edge map of the SEL algorithm executed on the low resolution image, size 168×216. For illustration purposes the edge map has been doubled in size in both dimensions. The edge information is much stronger, and the SEL algorithm finds the edges with no difficulty. Figure 3 demonstrates the necessity of a good estimate of the conditional probability density functions. To illustrate the extra searching required, *all* of the candidate search points found by the SEL algorithm are shown. On the left, the estimation parameter was set to 10, which translates to a variance which is twice as high as the noise variance. Because the discrimination between edge and non-edge pixels

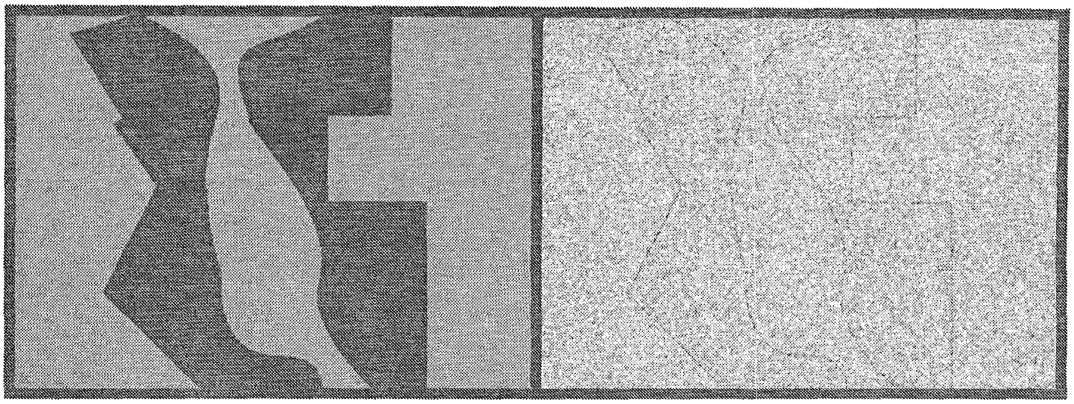

Figure 1: Left: original test image, 336 × 432, grayscale values of 196 and 128. Right: magnitude of gradient of test image corrupted with additive i.i.d. zero mean Gaussian noise, $\sigma^2 = 1600$

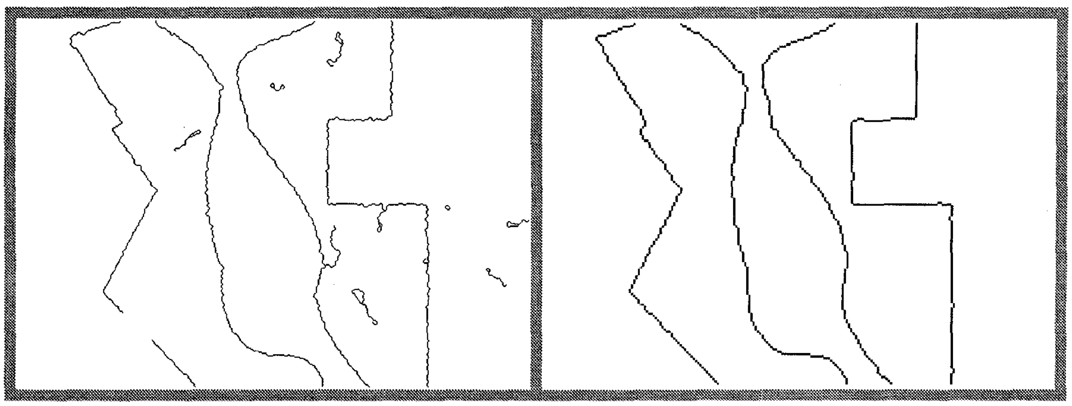

Figure 2: Left: edge map for standard SEL algorithm, estimation parameter = 20. Right: edge map for standard SEL algorithm, run on low resolution 168 × 216 image (The edge map has been doubled in size in both dimensions for display purposes.)

is low, a large number of extraneous nodes are required to be searched. The image on the right has an estimate which is half of the required value. Here not much searching is required, but false paths are generated because the algorithm is relying on values which are in the tails of the conditional probability density functions, and are not very accurate. Finally, in Figure 4 are illustrated the results of executing the multiresolution SEL algorithm for pyramids with levels of two and three, on the left and right respectively. In comparison to the edge map of Figure 2, the edges are much more prone to take a straighter path, especially for the vertical edges where deviations are much more noticeable. In addition, the conditional probability density functions were estimated from the image itself; very little *a priori* information is needed to produce a good edge map. The addition of a third level low resolution image can improve the edge map, but only to the extent that the second level was inaccurate. Clearly, for the edge segment at the upper right of the image, the better estimate helped steer the edge path to the top rather than following a false path to the right.

The software and images described in this paper and a postscript version of this paper are available via anonymous ftp to `skynet.ecn.purdue.edu` in the directory `/pub/dist/delp/icip95-msel`.

4. REFERENCES

[1] V. Torre and T. A. Poggio, "On edge detection," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. PAMI-8, no. 2, pp. 147–163, March 1986.

[2] J. Canny, "A computational approach to edge detection," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. PAMI-8, no. 6, pp. 679–698, November 1986.

[3] P. H. Eichel and E. J. Delp, "Sequential edge detection in correlated random fields," *Proceedings of the IEEE Computer Vision and Pattern Recognition Conference*, June 1985, San Francisco, pp. 14–21.

[4] P. H. Eichel and E. J. Delp, "Sequential detection of linear features in two-dimensional fields," TR-EE 85-

Figure 3: Left: edge map with all nodes searched for standard SEL algorithm, estimation parameter = 10. Right: edge map with all nodes searched for standard SEL algorithm, estimation parameter = 40.

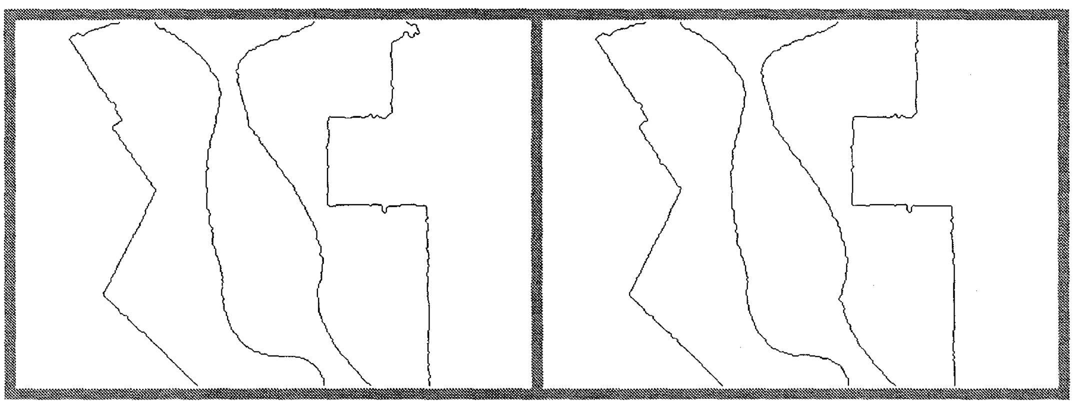

Figure 4: Left: edge map for multiresolution SEL algorithm with a two-level pyramdid. Right: edge map for multiresolution SEL algorithm, with a three-level pyramdid.

13, Purdue University, West Lafayette, Indiana, July 1985.

[5] P. H. Eichel, E. J. Delp, K. Koral, and A. J. Buda, "A method for fully automatic definition of coronary arterial edges from cineangiograms," *IEEE Transactions on Medical Imaging*, vol. MI-7, no. 4, pp. 313–320, December 1988.

[6] P. J. Burt and E. H. Adelson, "The Laplacian pyramid as a compact image code," *IEEE Transactions on Communications*, vol. COM-31, no. 4, pp. 337–345, April 1983.

[7] M. L. Comer and E. J. Delp, "Multiresolution image segmentation," *Proceedings of the 1995 IEEE International Conference on Acoustics, Speech,*, May 9–12, 1995, Detroit, Michigan, pp. 2415–2418.

[8] A. Rosenfeld and M. Thurston, "Edge and curve detection for visual scene analysis," *IEEE Transactions on Computers*, vol. C-20, no. 5, pp. 562–569, May 1971.

[9] F. Bergholm, "Edge focusing," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. PAMI-9, no. 6, pp. 726–741, November 1987.

[10] R. Wilson and A. H. Bhalerao, "Kernal desings for efficient multiresolution edge detection and orientation estimation," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 14, no. 3, pp. 384–390, March 1992.

[11] S. Mallat and S. Zhong, "Characterization of signals from multiscale edges," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 14, no. 7, pp. 710–732, July 1992.

[12] S. E. Leveinson, L. R. Rabiner, and M. M. Sondhi, "An introduciton to the application of the theory of probabilistic functions of a Markov process in automatic speech recognition," *The Bell System Technical Journal*, vol. 62, no. 4, pp. 1035–1074, April 1983.

[13] A. P. Dempster, N. M. Laird, and D. B. Rubin, "Maximum likelihood from incomplete data via the EM algorithm," *Journal of the Royal Statistical Society B*, vol. 39, pp. 1–38, 1977.