Image Interpolation & Superresolution (II)

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Bicubic Convolution Interpolation [1]

- An interpolation function is a special type of approximating function. A fundamental property of interpolation functions is that they must *coincide* with the sampled data at their interpolation nodes.

$$g(x_k) = f(x_k)$$

- For *equally spaced* data, many interpolation functions can be written in the form

$$g(x) = \sum_k c_k \cdot u\left(\frac{x - x_k}{h}\right)$$

One of the options in "imresize" — "cubic" was implemented according to this paper.

where h represents the sampling increment

x_k are the interpolation nodes

c_k are parameters which depend on the sampled data

u is the interpolation kernel (basis function)

g is the interpolation function

Cubic Convolution Interpolation Kernel

- The cubic convolution interpolation kernel is composed of **piecewise cubic polynomials** defined on the subinterval (-2, -1), (-1, 0), (0, 1) and (1, 2). Outside of the interval (-2, 2), the interpolation kernel is *zero*.
- The interpolation kernel must be **symmetric**:

$$u(s) = \begin{cases} A_1|s|^3 + B_1|s|^2 + C_1|s| + D_1 & 0 < |s| < 1 \\ A_2|s|^3 + B_2|s|^2 + C_2|s| + D_2 & 1 < |s| < 2 \\ 0 & 2 < |s| \end{cases}$$

- u(0) = 1 and u(n) = 0 (when n is a nonzero integer)
  - This is because that the difference between the interpolation nodes x_j and x_k is $(j\text{-}k)h$, $g(x_j) = \Sigma c_k u(j\text{-}k) \rightarrow$ when $j = k$, $g(x_k) = f(x_k) \rightarrow c_k = f(x_k)$; $j \neq k$, $u(\mathrm{j}-\mathrm{k}) = 0$.

Cubic Convolution Interpolation Kernel

- The conditions u(0) = 1 and u(1) = u(2) = 0 provide four equations for these coefficients:

$$1 = \mathrm{u}(0) = D_1$$

$$0 = u(1^-) = A_1 + B_1 + C_1 + D_1$$

$$0 = u(1^+) = A_2 + B_2 + C_2 + D_2$$

$$0 = u(2^-) = 8A_2 + 4B_2 + 2C_2 + D_2$$

- Three more equations are obtained from the fact that u' is ***continuous*** at the nodes 0, 1, and 2:

$$-C_1 = u'(0^-) = u'(0^+) = C_1$$

$$3A_1 + 2B_1 + C_1 = u'(1^-) = u'(1^+) = 3A_2 + 2B_2 + C_2$$

$$0 = u'(2^-) = u'(2^+) = 12A_2 + 4B_2 + C_2$$

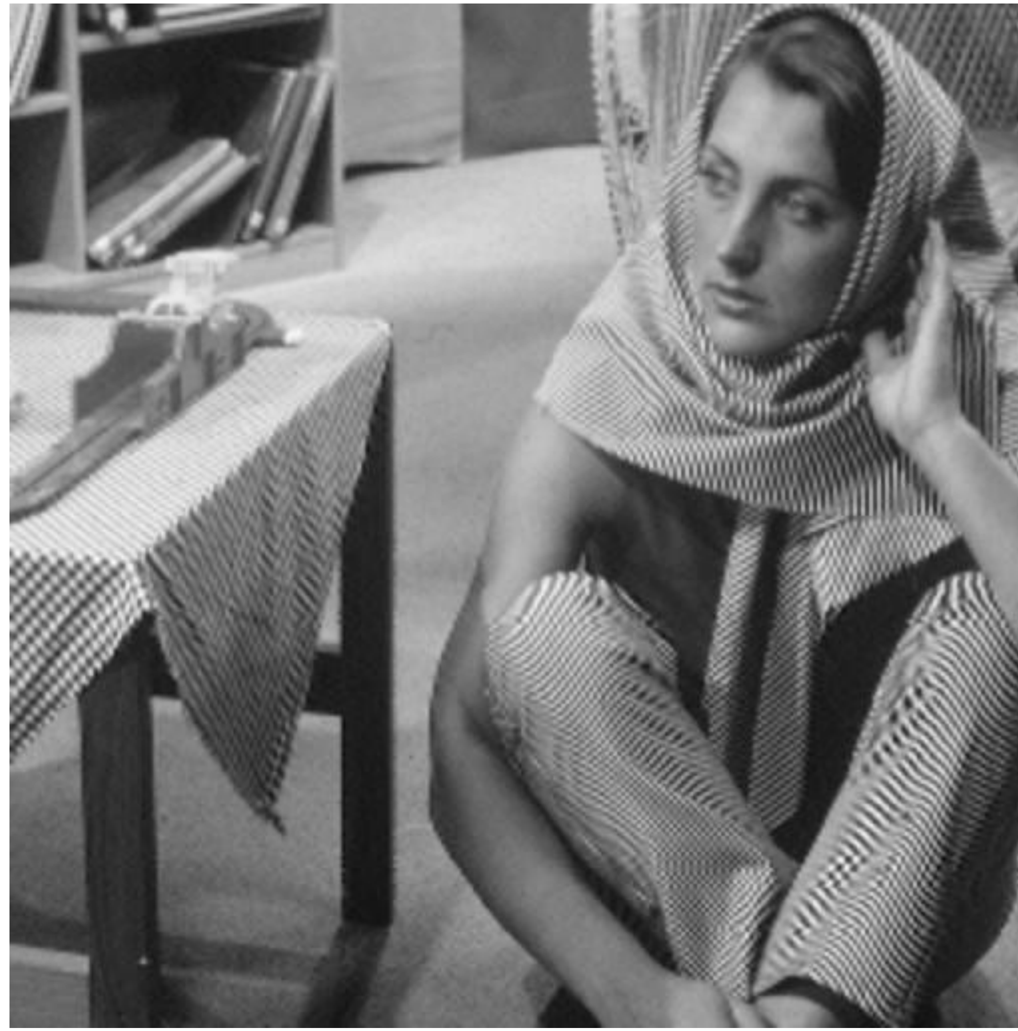
Cubic Convolution Interpolation Kernel

- In addition to being 0 or 1 at the interpolation nodes, the interpolation kernel must be ***continuous*** and have ***a continuous first derivative***.
  - But we have only 7 equations for 8 unknown parameters of the interpolation kernel.
- The idea in this paper is to choose $\boldsymbol{A_2}$ so that the cubic convolution interpolation function and the Taylor series expansion for *f* agree for as many terms as possible.$\rightarrow$ $\boldsymbol{A_2 = -0.5}$

$$\boxed{u(s) = \begin{cases} \frac{3}{2}|s|^3 - \frac{5}{2}|s|^2 + 1 & 0 < |s| < 1 \\ -\frac{1}{2}|s|^3 + \frac{5}{2}|s|^2 - 4|s| + 2 & 1 < |s| < 2 \\ 0 & 2 < |s| \end{cases}}$$

Interpolated Results (I)

Input image 256×256



Output high-resolution image 512×512

Interpolated Results (II)

Input image 256×256

Output high-resolution image 512×512

References

- [1]   R. G. Keys, “Cubic convolution interpolation for digital image processing,” IEEE Trans. On Acoustics, Speech, and Signal Processing, ASSP-29(6): 1153-1160, 1981.

Thank You!

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