Honours Multivariate Analysis
Continuous Assessment 3

Instructions:

- You will be divided into groups for this assessment. Only 1 submission per group is required.
- Your **.pdf** report may be compiled using any software you like (Rmarkdown, LaTeX, MSWord, etc.), as long as the presentation is neat.
- Do NOT paste R output verbatim, this will be penalised. If you want to include R output, typeset it properly or present it in a table.
- To help the reader easily assimilate the information, round values to a small number of decimal places (unless there is a reason for expressing a more exact value).

1. Let $\boldsymbol{X} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ with $|\boldsymbol{\Sigma}| > 0$. Using the spectral decomposition of the covariance matrix, prove that
$$(\boldsymbol{X} - \boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{X} - \boldsymbol{\mu}) \sim \chi_p^2$$

2. Consider $\boldsymbol{X} \sim N_5\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}\right)$, where $\boldsymbol{\mu} = \begin{bmatrix} 5 & 0 & -2 & 6 & 2 \end{bmatrix}'$ and $\boldsymbol{\Sigma} = \begin{bmatrix} 8 & 3 & -1 & 0 & 5 \\ 3 & 12 & 2 & 2 & -2 \\ -1 & 2 & 9 & 0 & 1 \\ 0 & 2 & 0 & 8 & 2 \\ 5 & -2 & 1 & 2 & 10 \end{bmatrix}$.

   Now define $\boldsymbol{X}_1 = \begin{bmatrix} X_1 \\ X_2 \\ X_4 \end{bmatrix}$ and $\boldsymbol{X}_2 = \begin{bmatrix} X_3 \\ X_5 \end{bmatrix}$. Find the conditional (joint) distribution of $\boldsymbol{X}_1 | \boldsymbol{X}_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$.