CA4

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Question 1

Suppose X Np(μ,Σ). Show that the maximum likelihood estimator of Σ is biased, and give the bias:

1. The following shows that the MLE of Σ is biased because of the added $\frac{n-1}{n}$ term

$$E[\hat{\Sigma}_{MLE}] = E[\frac{n-1}{n}S] = \frac{n-1}{n}E[S] = \frac{n-1}{n}\Sigma$$

2. The bias of the MLE of Σ

$$Bias(\hat{\Sigma}_{MLE}) = E[\hat{\Sigma}_{MLE}] - \Sigma$$

$$Bias(\hat{\Sigma}_{MLE}) = \frac{n-1}{n}\Sigma - \Sigma$$

$$Bias(\hat{\Sigma}_{MLE}) = -\frac{1}{n}\Sigma$$

Question 2

Consider again the Egyptian skull data from CA1, given in CA1.csv . Examine the variables in period 2 for marginal and multivariate normality by creating the necessary QQ-plot(s) and chi-square plot(s). Apply any statistical test to the univariate hypotheses and report a measure of the p-value. For the multivariate test, interpret the observed squared generalized distances.

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Attaching package: 'dplyr'
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The following objects are masked from 'package:stats':

    filter, lag
```

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The following objects are masked from 'package:base':

    intersect, setdiff, setequal, union
```

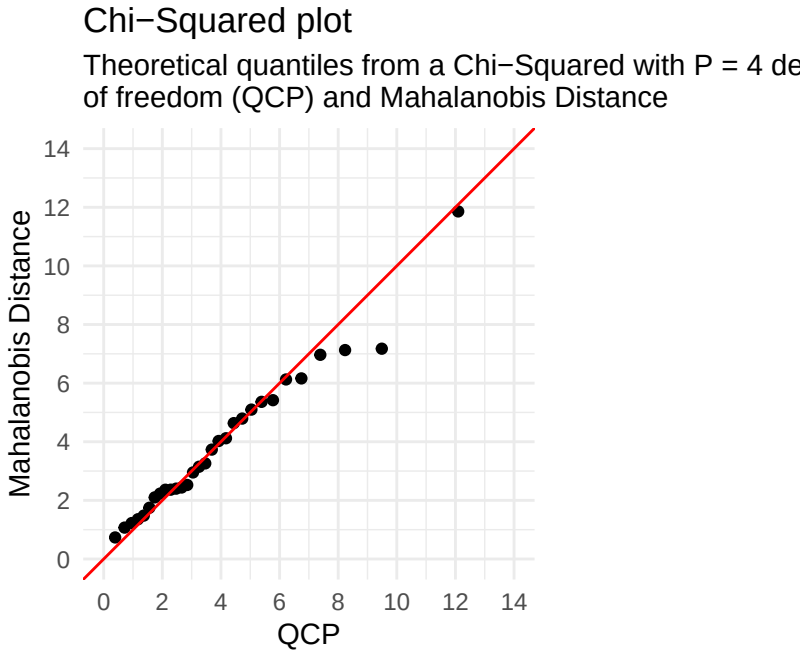


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[1] "Proportion of points where D^2 < critical value 53 2"
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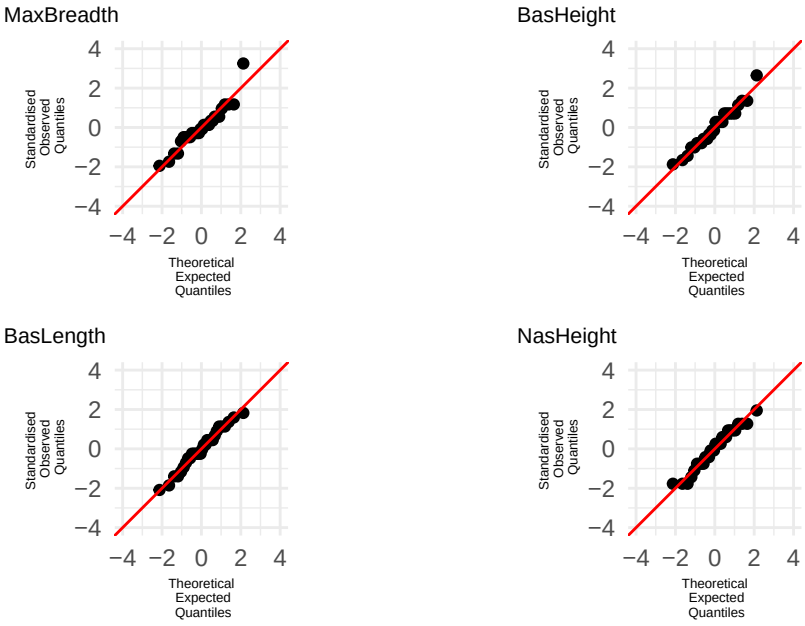

Table 1: Shapiro Wilk Test

	P Value
MaxBreadth	0.0402877
BasHeight	0.5798685
BasLength	0.7494967
NasHeight	0.3762163