EE209AS Final Project Report: Thermal Modeling and Optimization based on Utility

Peizhe Zhang (504512136), Pan Xin (904402353)
Department of Electrical Engineering
University of California, Los Angeles
Los Angeles, United States
zjuzpz@gmail.com, xinpan1117@ucla.edu

Abstract*—HVAC systems in most houses are the top energy consumption source, and existing thermostats either do not consider the dynamic price or do not take into account the comfort user feel. In this report, we first model the existing traditional/rigid thermostat, and then propose a novel optimized model which incorporates the effect of both dynamic price and the comfort index. Simulation results confirm that both the rigid and optimized model cost much less than the traditional model. Although the cost of optimized model is a slightly more than the rigid model, the total reduced utility caused by the cost and uncomfortableness of the users is smaller than the rigid model. And both the rigid and optimized models exhibit some power shift effects which is good for the smart power grid.

***Keywords*—Thermostat, Modeling, Optimization, HVAC, Comfort, Utility, Smart Grid.**

I. Introduction

The rapidly growing world energy use has already raised concerns over supply difficulties, exhaustion of energy resources and heavy environmental impacts. And a significant portion of the energy is consumed by house usage, while heating and cooling takes up nearly 40%. Efficiently management of the heating and cooling then becomes important in saving energy and cost.

Most American homes now have thermostats to control *HVAC* (Heating, venting, and air conditioning) systems. However, there are many different thermostats which can be roughly categorized into two types. For traditional ones, users can set a target temperature or a range, their only job is to keep the temperature stable and never consider the cost. The other is the so called “Smart Thermostat” such as Nest, they are beautifully designed and they can automatically turn on/off and even learn your habits. But it has no response for demand and does not consider the effect of dynamic price.

Apart from the user’s side, the power grid is also being challenged. Domestic energy consumption in a day is not a constant, and they typical reach the peak in the afternoon, and glide down in night. However, for most of time, the power usage is far from the peak value and this causes a huge waste in terms of capacity.

In this project, we built models for traditional/rigid thermostats and proposed an optimized model incorporating the effects of both dynamic price and comfort. Simulation results confirm that the optimized model achieves maximum utility – creates more comfort for users if room temperatures are constrained in the range.

This report will firstly describe modeling for the traditional/rigid and optimized thermostats. Then, illustrate with details how these models work, and finally analysis their simulation results.

II. Thermostats Modeling

When modeling the thermodynamic process of different types of thermostats, discretization is required. And all models share some principle laws which are described in the following parts.

First, the amount of electricity Q_n required to change the temperature of the building in period n is:

$$Q_n = k_1 \left| X_{tar,n} - T_{ini,n} \right| = k_1 \Delta T \tag{1}$$

where k_1 is the building's projected heat capacity—the amount of electricity required by HVAC to change the building's temperature by 1 ${}^oF$, $T_{ini,n}$ here is the initial room temperature in period n.

By Newton's law of cooling, the rate of heat flux from the ambience to the internal space of the building at time s ($s \in [t_n, t_{n+1}]$) is:

$$\begin{aligned}\overrightarrow{q_s} &= \frac{dQ}{ds} = kA(T_{out,n} - T_{in,s}) \\ &= k_2(T_{out,n} - T_{in,s})\end{aligned} \tag{2}$$

where k is the building material's conductance and A is the cross-sectional area of the conducting surface. And $T_{out,n}$ is the ambient temperature in period n, $T_{in,s}$ is room temperature at time s. $k_2 = kA$ denotes the projected conductance rate of the room. A large k_2 indicates that the building is easy to lose or gain heat from the outside when there is difference of the indoor and outdoor temperature.

When HVAC is off, the room temperature is only influenced by the outside temperature. And the rate of change of room temperature can be obtained as:

$$k_1 \frac{dT_{in,s}}{ds} = k_2(T_{out.n} - T_{in,s}) \tag{3}$$

Then we can get the $T_{in,s}$ at time s:

$$\begin{aligned}T_{in,s} &= f^T(T_{out,n}, X_{tar,n}, s - t_{stop,n}) \\ &= T_{out,n} + (X_{tar,n} - T_{out,n})e^{-\frac{k_2}{k_1}(s - t_{stop,n})}\end{aligned} \tag{4}$$

where $X_{tar,n}$ is the target room temperature for HVAC at the beginning of period n, and treated as the room temperature at the end of the previous HVAC run.

A. Traditional Thermostats

Thermostats in nowadays US families are generally traditional, they allow user to set a target temperature or a temperature range. HVAC is activated every time the room temperature deviates from the target temperature (or falls out of the target range), the traditional thermostat gives the same response regardless of energy prices.

The cost of heating and cooling is described as:

$$C_n = F^C(T_{ini,n}, X_{tar,n}) = -p_n Q = -p_n k_1 |T_{ini,n} - X_{tar,n}| \tag{5}$$

where p_n is the price of electricity in the period n. In this paper, finally, it is the total utility will be compared among the three models. Because the cost has a negative effect on the total utility, there is a minus sign in the expression.

As a simple example, when modeling traditional thermostat, assuming the user's preferred temperature is 76 ${}^oF$, when the indoor temperature is less than 75 ${}^oF$, the HVAC system will heat the room to 77 ${}^oF$, and when the indoor temperature is over 77 ${}^oF$, the system will cool the room to 75 ${}^oF$. And this kind of policy will make sure that the room temperature will not fall out of the target temperature range just after the HVAC system off.

B. Rigid Thermostats

A rigid thermostat runs an optimization routine in the background that, given dynamic prices, finds the lowest-cost path to keep room temperatures within a rigid range set by the user.

The difference between rigid model and traditional model is that the rigid model will take the dynamic electricity price into consideration. In the other words, the rigid model will see the temperature and dynamic price of electricity in the next several hours and determine the best target temperature at now. Also there is a range of target temperature of rigid model, the rigid model will choose a target temperature value in this range to make sure that 1) The room temperature will not fall out of the target temperature range from now to the next time HVAC system run and 2) The cost is minimum.

Denotes the total cost from period n to the end of the the planning horizons as $J_{rigid,n}$, we can model the thermostat's decisions as a dynamic programming problem, and then the Bellman equation for the rigid model is:

$$J_{rigid,n}(T_{ini,n}) = \max_{Xtar,n} F^C(T_{ini,n}, X_{tar,n}) + \alpha J_{rigid,n+1}(T_{ini,n+1})$$

$$T_{ini,n+1} = [f^T(T_{out,n}, X_{tar,n}, h) + \frac{1}{2}] \tag{6}$$

$$X_{tar,n}, T_{ini,n+1} \in [I_{\min,n}, I_{\max,n}]$$

Where $J_{rigid,N+1}(T_{ini,N+1}) \equiv 0$, α is the discount factor, which is set to be 1 in all of this paper. By solving this equation, the following expression can be gotten:

$$J_{rigid,0}(T_{ini,0}) = \sum_{i=0}^{N} - p_i k_1 | T_{ini,i} - X_{tar,i} |$$
$$= \sum_{i=0}^{N} - p_i k_1 | T_{out,i-1} + (X_{tar,i-1} - T_{out,i-1}) e^{-\frac{k_2}{k_1}\Delta t} - X_{tar,i} | \tag{7}$$

From equation (10), the total cost in one period is a function of the target temperature of thermostat both at now ($X_{tar,\, i-1}$) and at the next time period ($X_{tar,\, i}$) and of course, the outdoor temperature ($T_{out,\, i-1}$). To minimize the total cost, the thermostat needs to know the dynamic price of electricity and the outdoor temperature in the next several periods.

C. Optimized Thermostats

The problem of rigid model is that even it can help the users to save considerable cost, it does not take the users' comfortableness into consideration.

The optimized model, in the other hand, will consider the users' comfortableness. In the reference paper, the inputs of the optimized model are just some users' preference parameters. However, it is very confused for users, thus in our model, the input of optimized model is also set to be a range of temperature likes rigid model. Also the users need to input two parameters a and b to indicate basically, how much money they are willing to pay to get the room temperature is close to the preferred temperature.

The optimized thermostat trades off comfort for cost saving based on users' preferences.

To model the comfort of the user as a parameter is difficult. Thus, in this paper, it is the uncomfortableness to be modeled as a parameter. Obviously, when the room temperature is deviated from the users' preferred temperature, the users will feel uncomfortable.

The uncomfortableness gain per unit time is described in the following negative-valued concave function:

$$\begin{aligned} d(s) &= f^D\left(T_{out,n}, T_{pfr,n}, X_{tar,n}, s - t_n\right) \\ &= a\left|f^T(T_{out,n}, X_{tar,n}, s - t_{stop,n}) - T_{pfr,n}\right|^2 \\ &\quad + b\left|f^T(T_{out,n}, X_{tar,n}, s - t_{stop,n}) - T_{pfr,n}\right| \\ &= a\left|T_{in,s} - T_{pfr,n}\right|^2 + b\left|T_{in,s} - T_{pfr,n}\right| \\ &= a\Delta T^2 + b\Delta T \end{aligned} \tag{8}$$

where a and b are coefficients of the second-order and first-order deviation terms. As can be seen, $T_{pfr,n}$ is the preferred temperature, and users usually become increasingly uncomfortable when the room temperature deviates further from $T_{pfr,n}$. Like mentioned as cost, the uncomfortableness also have negative effect on total utility, so there is a minus in the expression.

By integration, the comfort gain in a period can be obtained as:

$$\begin{aligned} D_n &= F^D\left(T_{out,n}, T_{pfr,n}, X_{tar,n}\right) = \int_{t_n}^{t_n+h} f^D\left(T_{out,n}, T_{pfr,n}, X_{tar,n}, s - t_n\right) ds \\ &= \int_{t_n}^{t_n+h} a\left|T_{in,s} - T_{pfr,n}\right|^2 + b\left|T_{in,s} - T_{pfr,n}\right| ds \\ &= \int_{t_n}^{t_n+h} a\left|f^T(T_{out,n}, X_{tar,n}, s - t_{stop,n}) - T_{pfr,n}\right|^2 ds \\ &\quad + \int_{t_n}^{t_n+h} b\left|f^T(T_{out,n}, X_{tar,n}, s - t_{stop,n}) - T_{pfr,n}\right| ds \\ &= \int_{t_n}^{t_n+h} (a\Delta T^2 + b\Delta T) ds \end{aligned} \tag{9}$$

Denotes the total reduced utility from period n to the end of the planning horizons as $V_{opt,n}$, we can also get the dynamic programming problem equation:

$$V_{opt,n}(T_{ini,n}) = \max_{Xtar,n} F^{C}(T_{ini,n}, X_{tar,n}) + F^{D}(T_{out,n}, T_{pfr,n}, X_{tar,n})$$
$$+\alpha V_{opt,n+1}(T_{ini,n+1})$$
$$s.t. \rightarrow T_{ini,n+1} = [\, f^{T}(T_{out,n}, X_{tar,n}, h) + \frac{1}{2}] \tag{10}$$
$$X_{tar,n}, T_{ini,n+1} \in [I_{\min,n}, I_{\max,n}]$$

The optimized model is differ to the rigid model that having a term of F^D. In the optimized model, the thermostat make a decision to get a maximum total utility that is to minimize the sum of cost and uncomfortableness of users.

From the above, when it comes to cost only, the rigid model should perform the best and the optimized model should perform better than the traditional model. And the cost difference between the rigid model and optimized model should not be too large.

However, if we consider both the cost and the uncomfortableness caused by the deviation of room temperature to preferred temperature, the optimized should perform best and the rigid model should perform better than the traditional model.

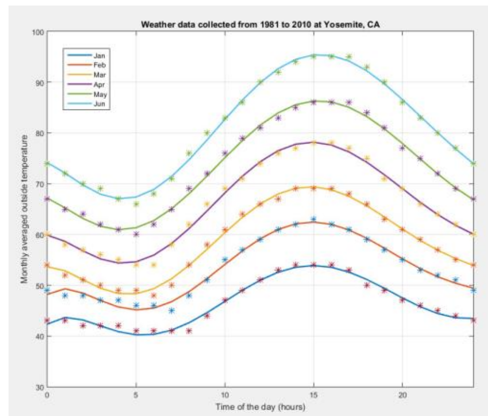
III. Simulation Results

In order to verify the thermostats with optimized and rigid model can benefit the small grid users, we first collect the temperature data from 1981 to 2010 at Yosemite, CA as shown in figure 1 and 2.

By implementing three models, we can find the total cost and reduced utility difference among the three models. Because there is no currently real-time-price data for electricity (It is because now most of companies charge users with different electricity price only for the different total electricity they have used other than for the different time), we generate simulated dynamic prices by the following sine function.

$$p(t_i) = p_b - p_s \sin(\frac{2\pi}{N}) \tag{11}$$

Where p_b and p_s are the baseline and scale of the prices. We set they are equal to 0.2\$/kWh and 0.11\$/kWh, respectively, such that the electricity prices vary from 0.09\$/kWh in the midnight to 0.31\$/kWh in the afternoon.



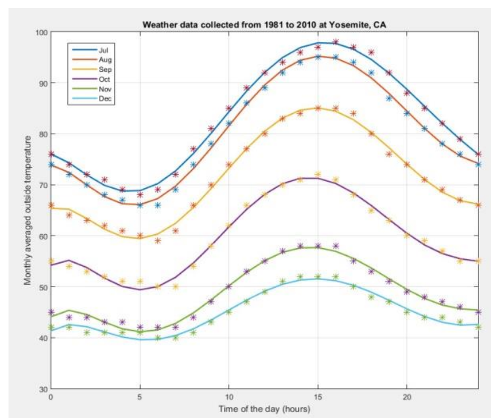


Fig 1, 2. The monthly averaged temperature data from 1981 to 2010 in a day at Yosemite, CA

First we investigate the cost with different models in every month. Here, we set a = -1.5 * 10^{-2}\$/($^{o}F \times hr$)and b = -1 * 10^{-2}\$/($^{o}F \times hr$). These parameters indicate that the users are willing to pay extra \$0.025 per hour to lower the room temperature from 74 ^{o}F to 73 ^{o}For an extra \$0.085 per hour to lower the room temperature from 76 ^{o}F to 75 ^{o}F when the preferred room temperature is 73 ^{o}F.

In this simulation, the *k1* and *k2* are set to 1 and 0.1 which means the HVAC system requires 1 unit of electricity to change the room temperature by $1\,^{o}F$ and the room temperature will rise from $73\,^{o}F$ to $74\,^{o}F$ in one hour without HVAC system when the outside temperature is $83\,^{o}F$. Actually, the specific numbers of them are not important, because the point we want to verify is the optimized and rigid model can benefit users with smart grid over the traditional model.

We further set the users' preferred temperature to $70\,^{o}F$. In order to make the simulation more convinced, we set both the three models have the range of target temperature, although in general, the traditional thermostat model just have one target temperature other than a range. For all of them, we set the target temperature range is $67\,^{o}F$ to $73\,^{o}F$. Furthermore, the unit period of time is set to one hour in all simulations. In one hand, it will relieve the calculation budget significantly. In the other hand, it also ensures reasonable accuracy since all the indoor temperature, outdoor temperature and dynamic electricity price will not change a lot within one hour.

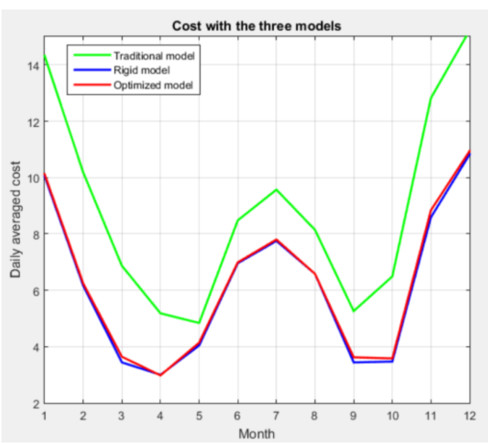


Fig 3. Cost with different models

The figure 3 presents the different cost in a single day among different months with the three models. From the figure, in all months, the traditional model cost much more than the optimized and rigid model. The optimized model costs a little bit more than the rigid model, since it take user's comfortableness into consideration when setting target temperature.

Over the entire year, if we set the cost of traditional thermostat to 1, the normalized cost of rigid and optimized are 0.692 and 0.703 respectively. In the other words, both rigid and optimized thermostat will reduce 30% costs of the users in a year. A minor point we want to indicate is because here we set the preferred temperature to $70\,^{o}F$, which is much higher than the daily temperature at winter, the cost in winter is much higher the in summer. However, even if the users set different target temperature or temperature range for the thermostat, the optimized and rigid model will also reduce about 30% of costs of the users.

Then we investigate the total utility of the three models. Here, because both the cost and the uncomfortableness caused by the deviation of preferred temperature will reduce the total utility, we can just calculate the total reduced utility by the cost and the uncomfortableness with the three models as mentioned before.

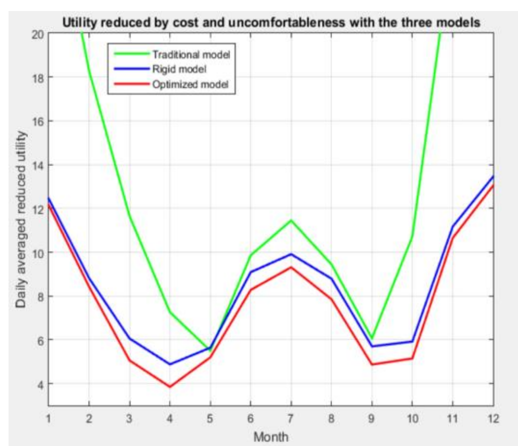


Fig 4. Reduced utility with different models

Figure 4 shows the simulation result where obviously, the optimized model performs best, and the rigid model performs better than the traditional model. As mentioned before, the cost of optimized model is a little bit more than that of the rigid model. However, it will bring a better temperature environment to the users, thus the total reduced utility caused by cost and uncomfortableness of optimized model is less than that of the rigid and traditional model. Similarly, if we set the total reduced utility of traditional model to 1, the normalized number for rigid and optimized model are 0.593 and 0.546, respectively. One point need to be emphasized here is if we set different value of a and b, we will have different results, and for smaller a and b, that is the absolute value of a and b is larger and indicating the users are willing to pay more money to get a better room temperature, the optimized thermostat will perform even better.

In order to demonstrate that the optimized and rigid model can benefit users in all types of buildings, we did the simulations for different $k2$. In both models, the attributes of the a building is denoted as a single value $k2$, which indicate how fast the building loses or gains heat from the outside when there is difference between the indoor temperature and outdoor temperature. Typically, the $k2$ won't be too large, since the normal building has a certain of insolation of heat.

In the simulation, because the unit of time is one hour as mentioned before, we cannot set $k2$ too large. This is because if we set $k2$ too large, the room temperature will fall out of the given temperature range in a unit period of time regardless of the target temperature. In that case, all three models will generate the same target temperature sets because the thermostats need to make sure the indoor temperature is in the given temperature range all the time. For example, if the given temperature is [66, 74], and the outside temperature is 80 ^oF, both traditional, rigid and optimized models will set the target temperature to 66 ^oF to make sure the room temperature won's exceed 74 ^oF before the next period of time starting if the $k2$ is very large. The constraint of $k2$ will not affect the final result since first, in reality, the $k2$ will not be too large, and second, for large $k2$, we can set a smaller period of unit time. For example, we can set a unit of time to 1 minute or even 1 second, and it will not affect the final result for the reason that the total energy consumption is only the function of the temperature difference between the indoor temperature and the target temperature.

The following figures show the simulation results of the total reduced utility when $k2$ varying from 0.1 to 0.2 in January and August.

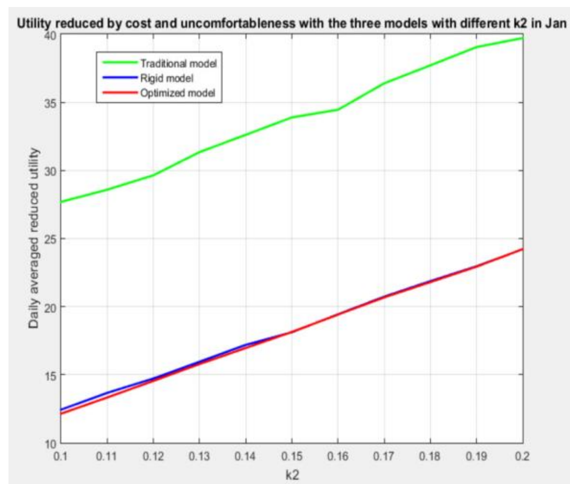


Fig 5. Reduced utility with different models in January

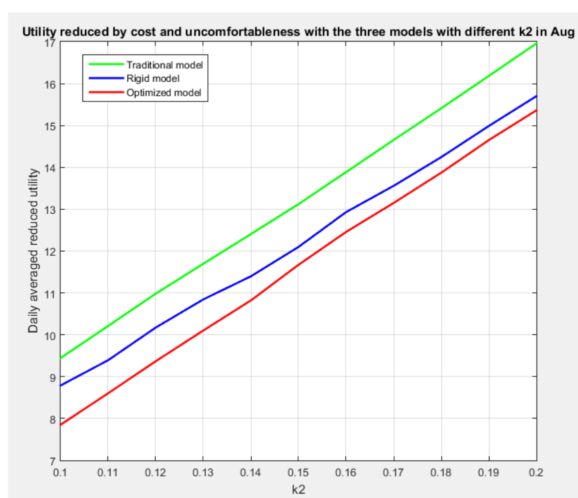


Fig 6. Reduced utility with different models in August

Figure 5 and 6 present that even with different *k2* that indicates different losing and gaining rate of heat when there is difference between the indoor temperature and outdoor temperature, the optimized model and rigid model performs better than the traditional model, and the improvement is a weak function of *k2*.

The figure also indicates that, in the winter, the performance of the optimized thermostat is close to the rigid model, and it is because the preferred temperature (70 ^oF) is far from the averaged outside temperature (50 ^oF at the daytime and 40 ^oF at the night). In the summer, the optimized model performs much better than traditional and rigid model.

IV. Conclusion

In this report, we model the three types of thermostats, traditional, rigid and optimized. The traditional model activates each time the temperature falls out of the target temperature or a range, and does not consider the dynamic price, so the cost of this model is highest, and total utility is lowest. For the rigid model which takes into account the dynamic price, it can precool and preheat the room, and allow the temperature to be floating in a relative range while keeping the total cost low. For the optimized model, it considers both the effect of dynamic price and the comfort. The cost of optimized model is just a slight larger than the rigid model, however, the total utility of the users using optimized model is much larger than the rigid and traditional model.

Acknowledgment

We would like to thank Yang Liang, Zuo-Jun Shen and etc. their paper “*Thermostats for the smart grid: Models, benchmarks, and insights*" gives us the direction to do the project and write this report. We also want to thank Prof. Lei He and the TA Xiao who gave us the opportunity to do the project and also gave us fruitful discussion and invaluable comments and suggestions.

References

[1] National Centers for Environmental Information (NCEI), formerly the National Climatic Data Center (NCDC). http://www.ncdc.noaa.gov

[2] Levine, David I., Yong Liang, and Zuo-Jun Shen. "Thermostats for the smart grid: Models, benchmarks, and insights." The Energy Journal 33.4 (2012).

[3] Tzivanidis, C., K. A. Antonopoulos, and F. Gioti. "Numerical simulation of cooling energy consumption in connection with thermostat operation mode and comfort requirements for the Athens buildings." Applied Energy 88.8 (2011): 2871-2884.

[4] Bowen, Brian Brian Richard. Climate control: smart thermostats, demand response, and energy efficiency in Austin, Texas. Diss. Massachusetts Institute of Technology, 2015.

[5] Saha, Ankita, Murat Kuzlu, and Manisa Pipattanasomporn. "Demonstration of a home energy management system with smart thermostat control." Innovative Smart Grid Technologies (ISGT), 2013 IEEE PES. IEEE, 2013.