

Linear Algebra

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Assignment #2

Properties of Determinant :

Determinant:-

It is scalar value that is calculated from the elements of a scalar matrix.

In matrix the vertical lines are column and the horizontal lines are rows

"n" order of determinant has n number of rows and columns

Determinant of 2x2 matrix:-

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Determinant of 3x3 matrix.

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \Rightarrow \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$= a(ei-fh) - b(di-fy) - c(dh-eg)$$

Properties of Determinant.

Change in value of determinant will not occur if rows and columns are interchanged but sign will be changed.

→ If two rows or columns in determinant have same value, determinant will be zero.

→ If any variable say "k" is multiplied by rows or columns then its value is also multiplied by k.

→ If some or all elements of rows or column are expressed as the sum of two or more terms, then determinant can be expressed in terms of two or more determinant.

1. Reflection Property :

value of determinant is unchanged by interchanging rows and columns. e.g.

$$M = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -1 \end{vmatrix} \rightarrow |M| = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 6 & 1 \\ -3 & 0 & 5 \\ 5 & 4 & -7 \end{vmatrix}$$

Case - 01

$$\begin{aligned} |M| &= 2(0-20) - (-3)(-4^2 - 4) + 5(30-0) \\ &= -40 - 138 + 150 \\ &= -28 \end{aligned}$$

Case - 02

$$\begin{aligned} &= 2(-20) - 6(2)(-25) + 1(-1) \\ &= -40 + 120 - 12 \\ &= -28 \end{aligned}$$

2. Switching property:

If any of two rows or columns are interchanged

sign of determinant is changed. e.g.

$$A = \begin{vmatrix} 4 & 5 & 6 \\ 7 & 3 & 2 \\ 6 & 4 & 3 \end{vmatrix} \Rightarrow \begin{vmatrix} 4 & 5 & 6 \\ 7 & 3 & 2 \\ 6 & 4 & 3 \end{vmatrix}; \begin{vmatrix} 4 & 5 & 6 \\ 6 & 4 & 3 \\ 7 & 3 & 2 \end{vmatrix} \Rightarrow \text{Case 02}$$

Case 1:

$$\begin{aligned} |A| &= 4(9-8) - 5(21-12) + 6(28-18) \\ &= 4 - 45 + 60 \\ |A| &= 19 \end{aligned}$$

Case 2:

$$\begin{aligned} |A| &= 4(8-9) - 5(12-21) + 6(18-28) \\ &= 4(-1) - 5(-9) + 6(-10) \\ &= -4 + 45 - 60 \\ |A| &= -19 \end{aligned}$$

Hence in case 2 sign of determinant is changed.

3. All zero property:

If all the elements of any row or column are in matrix zero then its determinant is zero.

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 0 & 0 & 0 \\ 6 & 7 & 8 \end{bmatrix} \Rightarrow |A| = 0$$

4. Repetition of proportionality.

If two rows or column in matrix are same then determinant will be zero.

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 3 & 4 & 5 \\ 1 & 1 & 1 \end{bmatrix} \Rightarrow |A| = 0$$

5. Scalar Multiplication.

If elements of a row or column of a determinant are multiplied by any non-zero constant then determinant also gets multiplied by same constant.

$$\begin{vmatrix} 12 & 9 & 6 \\ 13 & 4 & 5 \\ 6 & 8 & 9 \end{vmatrix} = 3 \begin{vmatrix} 3(4) & 3(3) & 3(2) \\ 13 & 4 & 5 \\ 6 & 8 & 9 \end{vmatrix} = 3 \times \begin{vmatrix} 4 & 3 & 2 \\ 13 & 4 & 5 \\ 6 & 8 & 9 \end{vmatrix}$$

Here $k=3$ which is a constant.

6. Sum Property:

If elements of row or column of a determinant are expressed as sum of two or more terms then determinant can be expressed as sum of two or more determinants.

e.g

$$\begin{vmatrix} a & b & c \\ a+3y & b+7y & c+2z \\ x & y & z \end{vmatrix} = \begin{vmatrix} a & b & c \\ a & b & c \\ x & y & z \end{vmatrix} + \begin{vmatrix} a & b & c \\ 3y & 7y & 2z \\ x & y & z \end{vmatrix}$$

7. Invariance Property.

Suppose any scalar multiplied of corresponding elements of other two rows or columns are added to every element of any row or column of a determinant. In this case value of determinant remain same

e.g. $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 + kb_1 & a_2 + kb_2 & a_3 + kb_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

8. Factor Property.

If determinant becomes zero when we insert $x=\alpha$ then $(x-\alpha)$ is factor of Δ .

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \rightarrow \Delta_1 = \begin{vmatrix} x_{11} & a_{12} & a_{13} \\ x_{21} & a_{22} & a_{23} \\ x_{31} & a_{32} & a_{33} \end{vmatrix}$$

Here x_{ij} denotes the cofactor of then element a_{ij} of A matrix.

9. Triangle property:

If the elements in determinant below & above the main diagonal are zero then determinant is product of diagonal elements.

e.g.

$$A = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 5 & 6 \end{vmatrix}, |A| = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 5 & 6 \end{vmatrix}$$

$$\text{So } |A| = 0$$

Same if

$$A = \begin{vmatrix} 8 & 7 & 6 \\ 0 & 3 & 5 \\ 0 & 0 & 4 \end{vmatrix} \Rightarrow |A| = \begin{vmatrix} 8 & 7 & 6 \\ 0 & 3 & 5 \\ 0 & 0 & 4 \end{vmatrix}$$

$$|A| = 0$$

10. Determinant of co-factor matrix.

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \quad \Delta_{11} = \begin{vmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{vmatrix}$$

Here Δ_{ij} represents the cofactors of elements of a_{ij} in Δ .