# PHAS1240 Longer task: Calculating e/m using a weighted fit

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## 1. Weighted least-squares fit

The fit we did in session 5 assumed that each of the data points were equally valid.

Frequently though, we find that experimental data do not have equal errors, and some data points may have significantly larger errors than others.

We can take the relative sizes of the error bars into account by weighting them proportionally. The weight we will give each data point is

$$w_i = \frac{1}{(\Delta y_i)^2}. \tag{1}$$

If these weights are included in the least-squares derivation, we end up with new expressions for the slope $m$:

$$m = \frac{\sum_i w_i \sum_i w_i x_i y_i - \sum_i w_i x_i \sum_i w_i y_i}{\sum_i w_i \sum_i w_i x_i^2 - \left(\sum_i w_i x_i\right)^2} \tag{2}$$

$$= \frac{\sum_i w_i \sum_i w_i x_i y_i - \sum_i w_i x_i \sum_i w_i y_i}{\delta}, \tag{3}$$

and the intercept $c$:

$$c = \frac{\sum_i w_i x_i^2 \sum_i w_i y_i - \sum_i w_i x_i \sum_i w_i x_i y_i}{\delta}; \tag{4}$$

where we have used an abbreviation for the denominator

$$\delta = \sum_i w_i \sum_i w_i x_i^2 - \left(\sum_i w_i x_i\right)^2, \tag{5}$$

The uncertainties in these quantities are given by

$$\Delta m = \left(\frac{\sum_i w_i}{\delta}\right)^{\frac{1}{2}} \tag{6}$$

and

$$\Delta c = \left( \frac{\sum_i x_i^2 w_i}{\delta} \right)^{\frac{1}{2}} . \tag{7}$$

## 2. Task: calculate e/m

Download the data file "eovermdata.csv" from the assignment page on Moodle.

This data is from a student experiment to calculate $e/m$, the charge-to-mass ratio of the electron. This is a version of an experiment first performed in 1897 by J.J. Thomson (see for example http://en.wikipedia.org/wiki/J._J._Thomson).

In this particular student experiment, a cathode ray tube was used to accelerate electrons through a voltage $V$. The electron beam was then bent into a circular path using a magnetic field $B = 1.28 \pm 0.01 \times 10^{-3}$ Tesla. The students used a ruler to measure the radius $r$ of the circular orbit of the electron beam. This was quite difficult to measure accurately, and so the errors are quite large. However, for two of the readings, the student taking the reading was distracted by a seagull flying past the window and misread the ruler. The student's lab partner noticed this and recorded a much larger error for these data points.

Your task is to use this data to calculate the experimental value of $e/m$ using

1. an unweighted least-squares fit (as in session 5)
2. a weighted least-squares fit that takes the errors in the readings into account.

### 2.1 What you need to do

For this task, you need to prepare an **IPython Notebook**, clearly explaining what you are doing at each step, both in terms of code and your analysis of the problem. Your final notebook should read as a clear, self-contained document, although you can reference equations both from this script and the Laboratory Data Analysis booklet.

- The force experienced by the electrons is given by:

  $$F_r = evB = \frac{mv^2}{r} \tag{8}$$

  The speed $v$ of the electrons is given by:

  $$\frac{1}{2}mv^2 = eV. \tag{9}$$

  Rearrange these equations to give an expression involving $e/m$ in the form of a straight line (hint: use $\sqrt{V}$ as the $x$-axis quantity and $r$ as the $y$-axis quantity).

- Write code (you can use your corrected code from session 5 as a starting point) to:
  - Load the data from the file

  - Fit an unweighted straight line, outputting the slope, intercept and uncertainties at full precision.
  - Fit a weighted straight line, outputting the slope, intercept and uncertainties at full precision.
  - Plot the data, errorbars, and both straight line fits on a suitably labelled and formatted graph.
    * *Hint:* We covered error bars in the script for session 4.
  - You don't need to include the equations for the fit on the plot (unless you want), but you should make sure that the fitted equations and their parameters are clearly stated in either a code cell output or a text cell.
  - Use your results to calculate values of $e/m$ for both the weighted and unweighted fits.
- The accepted value of $e/m$ is $(1.75882002 \pm 0.00000001) \times 10^{11}$ C/kg (CODATA Fundamental Constants 2014). Discuss briefly how the results of both your weighted and unweighted calculations compare to this.

### 2.2 Important things to note

- Your code should import the data from the same directory as the notebook.
- Do not rename the data file or make any changes to the data file itself.
- The grading of this assignment is anonymised. ***Do not include your name in the filename or anywhere in the assignment itself.*** Instead, please include your student ID (the 8-digit number on your ID card, for most of you this will start with 15) in the filename. Make sure you type the number correctly. If you have an e-sticker from Student Disability Services please copy and paste the wording (but not your name!) into a clearly labelled text cell at the top of your assignment.

### 2.3 Assessment

Your work will be graded using a rubric based on the following assessment criteria:

- Whether the values you have calculated are correct (45% of available marks);
- The quality of your plot(s), code comments, and coding style (35% of available marks).
- The quality and coherence of the text commentary and discussion (20% of available marks).

The grading will be taking into account that you have significantly more time for this task than the in-session assignments, and you should bear this in mind, particularly when writing your text commentary.

## References

For the calculation of errors, see either Chapter 6 of Hughes and Hase "Measurements and their uncertainties", OUP 2010 (e-book available via the UCL library catalogue), or the "Experimental methods and data analysis" booklet from Lab 1.

If you want to include equations in your IPython notebook text cells (recommended!), you may find one of the following online LaTeX equation editors useful (you can pick the maths symbols you want from a palette and then copy and paste the LaTeX code into your notebook. See the notebook for Session 3 if you need a refresher).

http://www.sciweavers.org/free-online-latex-equation-editor

http://www.codecogs.com/latex/eqneditor.php