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Question # 08

The following reaction between potassium permagnate ($KMnO_4$) and maganese sulfate in water produces manganese dioxide, potassium sulfate and sulphuric acid.

$$KMnO_4 + MnSO_4 \xrightarrow{+H_2O} MnO_2 + K_2SO_4 + H_2SO_4.$$

[For each compound, construct a vector that lists the number of atoms of Potassium(K), magnese, oxygen, sulphur and hydrogen.]

Solution:-

We have the following equation

$$x_1 KMnO_4 + x_2 MnSO_4 + x_3 H_2O \longrightarrow x_4 MnO_2 + x_5 K_2SO_4 + x_6 H_2SO_4$$

To balance the above chemical equation we have to find the value of x_1, x_2, x_3, x_4, x_5 and x_6.

The following vectors list the number of atoms of potassium(K), Magnese (Mn), Oxygen(O), sulphur(S) and Hydrogen (H).

$$KMnO_4: \begin{bmatrix} 1 \\ 1 \\ 4 \\ 0 \\ 0 \end{bmatrix}, \quad MnSO_4: \begin{bmatrix} 0 \\ 1 \\ 4 \\ 1 \\ 0 \end{bmatrix}, \quad H_2O: \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 2 \end{bmatrix},$$

$$MnO_2: \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \quad K_2SO_4: \begin{bmatrix} 2 \\ 0 \\ 4 \\ 1 \\ 0 \end{bmatrix}, \quad H_2SO_4: \begin{bmatrix} 0 \\ 0 \\ 4 \\ 1 \\ 2 \end{bmatrix} \begin{matrix} \rightarrow K \\ \rightarrow Mn \\ \rightarrow O \\ \rightarrow S \\ \rightarrow H. \end{matrix}$$

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The balanced chemical equation must hold;

$$x_1 \begin{bmatrix} 1 \\ 1 \\ 4 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 4 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 2 \end{bmatrix} = x_4 \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} +$$

$$x_5 \begin{bmatrix} 2 \\ 0 \\ 4 \\ 1 \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} 0 \\ 0 \\ 4 \\ 1 \\ 2 \end{bmatrix}.$$

that is,

$$x_1 \begin{bmatrix} 1 \\ 1 \\ 4 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 4 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 2 \end{bmatrix} - x_4 \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} -$$

$$x_5 \begin{bmatrix} 2 \\ 0 \\ 4 \\ 1 \\ 0 \end{bmatrix} - x_6 \begin{bmatrix} 0 \\ 0 \\ 4 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

So,

$$\begin{bmatrix} x_1 \\ x_1 \\ 4x_1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ x_2 \\ 4x_2 \\ x_2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ x_3 \\ 0 \\ 2x_3 \end{bmatrix} - \begin{bmatrix} 0 \\ x_4 \\ 2x_4 \\ 0 \\ 0 \end{bmatrix} -$$

$$\begin{bmatrix} 2x_5 \\ 0 \\ 4x_5 \\ x_5 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 4x_6 \\ x_6 \\ 2x_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Thus,

$$\begin{bmatrix} x_1 + (-2x_5) \\ x_1 + x_2 - x_4 \\ 4x_1 + 4x_2 + x_3 + 4x_5 + 4x_6 - 2x_4 \\ x_2 - x_5 - x_6 \\ 2x_3 - 2x_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Thus,
we have the following equations,

$$x_1 - 2x_5 = 0$$
$$x_1 + x_2 - x_4 = 0$$
$$4x_1 + 4x_2 + x_3 - 2x_4 - 4x_5 - 4x_6 = 0$$
$$x_2 - x_5 - x_6 = 0$$
$$2x_3 - 2x_6 = 0.$$

Let A be the matrix of the cofficients of system and row reduce the agumented matrix [A 0] be echelon form:

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 1 & 1 & 0 & -1 & 0 & 0 & 0 \\ 4 & 4 & 1 & -2 & -4 & -4 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 2 & 0 & 0 & -2 & 0 \end{bmatrix}$$

$R_3 = r_3 - 4r_4$ and R_3 divided by 2

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 1 & 1 & 0 & -1 & 0 & 0 & 0 \\ 4 & 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \end{bmatrix}$$

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$R_2 = r_2 - r_1$

$R_3 = r_3 - 4r_1$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 1 & -2 & 8 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \end{bmatrix}$$

$R_4 = r_4 - r_2$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 1 & -2 & 8 & 0 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \end{bmatrix}$$

Now, we can write as,

$x_1 - 2x_5 = 0$

$x_2 - x_4 + 2x_5 = 0$

$x_3 - 2x_4 + 8x_5 = 0$

$x_4 - 3x_5 - x_6 = 0$

$x_3 - x_6 = 0$

Thus;

$x_1 = 2x_5$ — (I)

$x_2 = x_4 - 2x_5$ — (II)

$x_3 = 2x_4 - 8x_5$ — (III)

$x_4 = 3x_5 + x_6$ — (IV)

$x_3 = x_6$ — (V)

Subsitute eq (IV) in eq (II)

$x_2 = 3x_5 + x_6 - 2x_5$

$x_2 = x_5 + x_6$

Now subsitute eq (iv) and (v) in (ii).

$x_6 - 2(3x_5 + x_6) + 8x_5 = 0$

$x_6 - 6x_5 - 2x_6 + 8x_5 = 0$

$-x_6 + 2x_5 = 0$

$-x_6 + 2x_5 = 0$

$2x_5 = x_6$

$x_5 = \frac{1}{2}(x_6)$ —— (A)

So, the general Solution is

$x_1 = x_6, \quad x_2 = \frac{3}{2}x_6, \quad x_3 = x_6,$

$x_4 = \frac{5}{2}x_6$ and $x_5 = \frac{1}{2}x_6$.

Here x_6 is a free variable.

As, the cooficients in the chemical reaction must be integers. So let $x_6 = 2$. Thus,

$x_1 = 2, \quad x_2 = 3, \quad x_3 = 2, \quad x_4 = 5,$

$x_5 = 1$ and $x_6 = 2$.

Thus The chemical reaction will be

$$2KMnO_4 + 3MnSO_4 + 2H_2O \longrightarrow 5MnO_2 + K_2SO_4 + 2H_2SO_4$$

Question # 09

If possible, use exact arithematic or rational format for calculations in

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balancing the following chemical reaction:

$PbN_6 + CrMn_2O_8 \rightarrow Pb_3O_4 + Cr_2O_3 + MnO_2 + NO.$

Solution:-

Equation involves five type of atoms. So, we need vector in $\mathbb{R}^5$ that lists the number of atoms in a molecule, For example

$$\therefore V = \begin{bmatrix} \text{no. of } Pb \\ \text{no. of } N \\ \text{no. of } Cr \\ \text{no. of } Mn \\ \text{no. of } O \end{bmatrix}$$

So, we have,

$$PbN_6 = \begin{bmatrix} 1 \\ 6 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad CrMn_2O_8 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \\ 8 \end{bmatrix},$$

$$Pb_3O_4 = \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \\ 4 \end{bmatrix}, \quad Cr_2O_3 = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 6 \\ 3 \end{bmatrix},$$

$$MnO_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}, \quad NO = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Let x_1, x_2, x_3, x_4, x_5 and x_6 be the cofficients in the given set.

$$x_1\begin{bmatrix}1\\6\\0\\0\\0\end{bmatrix}+x_2\begin{bmatrix}0\\0\\1\\2\\8\end{bmatrix}\mp x_3\begin{bmatrix}3\\0\\0\\0\\4\end{bmatrix}+x_4\begin{bmatrix}0\\0\\2\\0\\3\end{bmatrix}+$$

$$x_5\begin{bmatrix}0\\0\\0\\1\\2\end{bmatrix}+x_6\begin{bmatrix}0\\1\\0\\0\\1\end{bmatrix}$$

$$x_1\begin{bmatrix}1\\6\\0\\0\\0\end{bmatrix}+x_2\begin{bmatrix}0\\0\\1\\2\\8\end{bmatrix}-x_3\begin{bmatrix}3\\0\\0\\0\\4\end{bmatrix}-x_4\begin{bmatrix}0\\0\\2\\0\\3\end{bmatrix}-x_5\begin{bmatrix}0\\0\\0\\1\\2\end{bmatrix}$$

$$-x_6\begin{bmatrix}0\\1\\0\\0\\1\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}x_1\\6x_1\\0\\0\\0\end{bmatrix}+\begin{bmatrix}0\\0\\x_2\\2x_2\\8x_2\end{bmatrix}-\begin{bmatrix}3x_3\\0\\0\\0\\4x_3\end{bmatrix}-\begin{bmatrix}0\\0\\2x_4\\0\\3x_4\end{bmatrix}-\begin{bmatrix}0\\0\\0\\x_5\\2x_5\end{bmatrix}$$

$$-\begin{bmatrix}0\\x_6\\0\\0\\x_6\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}$$

$x_1 - 3x_3 = 0$

$6x_1 - x_6 = 0$

$x_2 - 2x_4 = 0$

$2x_2 - x_5 = 0$

$8x_2 - 4x_3 - 3x_4 - 2x_5 - x_6 = 0$

Thus in [A O] Matrix form to reduced it to reduced echelon form.

$$= \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 \\ 0 & 8 & 4 & 3 & -2 & -1 & 0 \end{bmatrix}$$

$R_2 = R_2 - 6R_1$

$$= \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 18 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 \\ 0 & 8 & 4 & 3 & -2 & -1 & 0 \end{bmatrix}$$

$R_3 \leftrightarrow R_2$.

$$= \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 18 & 0 & 0 & -1 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 \\ 0 & 8 & 4 & 3 & -2 & -1 & 0 \end{bmatrix}$$

$R_3 = \frac{R_3}{18}$ and $R_4 = R_4 - 2R_2$

$$= \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 1 & 0 & 4 & -1 & 0 & 0 \\ 0 & 8 & 4 & 3 & -2 & -1 & 0 \end{bmatrix}$$

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$R_3 = R_1 - R_2$ and $R_6 = R_6 - 8R_2$.

$$= \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 2 & -1 & 1/18 & 0 \\ 0 & 0 & 0 & 19 & -2 & -1 & 0 \end{bmatrix}$$

$R_1 = R_1 + 3R_3$ and $R_4 = R_4/2$ and $R_5 = R_5 - 19R_4$.

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1/6 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 1 & -1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -17 & -1 & 0 \end{bmatrix}$$

$R_2 = R_2 + 2R_4$.

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1/6 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 1 & -1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -17 & -1 & 0 \end{bmatrix}$$

$R_5 = -\frac{R_5}{17}$.

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1/6 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 1 & -1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1/17 & 0 \end{bmatrix}$$

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$R_4 = R_4 + \frac{1}{2}R_5$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1/6 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1/17 & 0 \end{bmatrix}$$

Hence,

x_6 is the free variable.

$$x_1 = \frac{x_6}{6}, \quad x_2 = 0, \quad x_3 = \frac{x_6}{18},$$

$$x_4 = 0, \quad x_5 = -\frac{x_6}{17}.$$

Question # 10

The chemical reaction below can be used in some industrial processe, such as the production of arsene (AsH_3). we exact arithematic or rational format of calculations to balance this equation

$$MnS + As_2Cr_{10}O_{35} + H_2SO_4 \rightarrow HMnO_4 + AsH_3 + CrS_3O_{12} + H_2O.$$

Solution:-

In vector form.

$$V = \begin{bmatrix} \text{no. of } Mn \\ \text{no. of } S \\ \text{no. of } As \\ \text{no. of } Cr \\ \text{no. of } O \\ \text{no. of } H \end{bmatrix}.$$

The Augmented Matrix is

$$= \left[\begin{array}{ccccccc:c} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & -3 & 0 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 35 & 4 & -4 & 0 & -12 & -1 & 0 \\ 0 & 0 & 2 & -1 & -3 & 0 & -2 & 0 \end{array}\right]$$

$R_2 \leftrightarrow R_3$ and $R_2 = R_2(1/2)$
and $R_4 = = R_4 + (-10)(R_2)$ and
$R_5 = R_5 + (-35)(R_2)$

$$= \left[\begin{array}{ccccccc:c} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & -1 & 0 & 0 \\ 0 & 0 & 4 & -4 & 35/2 & -12 & -1 & 0 \\ 0 & 0 & 2 & -1 & -3 & 0 & -2 & 0 \end{array}\right]$$

$R_4 \leftrightarrow R_6$ and $R_4 = \frac{+1(R_4)}{-1}$

~~and~~

$$= \left[\begin{array}{ccccccc:c} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\ 0 & 0 & -2 & 1 & 3 & 0 & 2 & 0 \\ 0 & 0 & 4 & -4 & 35/2 & -12 & -1 & 0 \\ 0 & 0 & 0 & 0 & 5 & -1 & 0 & 0 \end{array}\right]$$

$R_5 \leftrightarrow R_6$ and $R_5/5 = R_5$.

$$\left[\begin{array}{ccccccc:c} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\ 0 & 0 & -2 & 1 & 3 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1/5 & 0 & 0 \\ 0 & 0 & 4 & 4 & 35/2 & -12 & -1 & 0 \end{array}\right]$$

$R_4 = R_4 + 2R_3$ and $R_6 = R_6 - 4R_3$

$$= \left[\begin{array}{ccccccc|c}
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & -1/2 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\
0 & 0 & 0 & 3 & 3 & -6 & 2 & 0 \\
0 & 0 & 0 & 0 & 1 & -1/5 & 0 & 0 \\
0 & 0 & 0 & -8 & 35/2 & 0 & -1 & 0
\end{array}\right]$$

$R_4 = R_4 \times 1/3$ and $R_6 = R_6 + 8R_4$
and $R_6 = \frac{-51}{2}(R_5) + R_6$

$$= \left[\begin{array}{ccccccc|c}
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & -1/2 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & -2 & 2/3 & 0 \\
0 & 0 & 0 & 0 & 1 & -1/5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -109/10 & 13/3 & 0
\end{array}\right]$$

$R_6 = R_6 \times \left(\frac{-10}{109}\right)$ and $R_5 = R_5 + \left(\frac{1}{5}\right) R_6$

$$= \left[\begin{array}{ccccccc|c}
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & -1/2 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & -2 & 2/3 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & -26/327 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -130/327 & 0
\end{array}\right]$$

$R_4 = R_4 + 2R_6$ and $R_3 = R_3 + 3R_6$

$$= \left[\begin{array}{ccccccc|c}
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & -1/2 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & -130/109 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & -14/109 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & -26/109 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -130/327 & 0
\end{array}\right]$$

$R_4 = (-1) R_5$ and $R_2 = R_2 + \frac{1}{2}(R_5)$

So,

$$\left[\begin{array}{ccccccc|c}
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & -13/327 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & -130/109 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & -16/327 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & -26/327 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -130/327 & 0
\end{array}\right]$$

Now,

$R_1 = R_1 + R_4$

and

$R_3 = R_3 + (-1)(R_4)$

$$= \left[\begin{array}{ccccccc|c}
1 & 0 & 0 & 0 & 0 & 0 & -16/327 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & -13/327 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & -374/327 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & -16/327 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & -26/327 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -130/327 & 0
\end{array}\right]$$

Thus,

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General Solution

$\Rightarrow x_1 = \frac{16}{327}(x_7)$

$\Rightarrow x_2 = \frac{13}{327}(x_7)$

$\Rightarrow x_3 = \frac{374}{327}(x_7)$

$\Rightarrow x_4 = \frac{16}{327}(x_7)$

$\Rightarrow x_5 = \frac{26}{327}(x_7)$

$\Rightarrow x_6 = \frac{130}{327}(x_7)$

Taking $x_7 = 327$

$x_1 = 16$
$x_2 = 13$
$x_3 = 374$
$x_4 = 16$
$x_5 = 26$
$x_6 = 130$

Thus Balanced eq. is

$16MnS + 13As_2Cr_{10}O_{35} + 374H_2SO_4 \rightarrow$

$16HMnO_4 + 26AsH_3 + 130CrS_3O_{12} +$

$327H_2O.$

$\longleftrightarrow$

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Question # 11

Find the general flow pattern of the network shown in the figure. Assuming that the flows are all nonnegative, what is the largest possible value for x_3?

Solution:-

Firstly we will determine the general solution of the system. Let us mark the intersection and unknown flows in the branches, as shown in the figure. At each intersection flow in is equal to the flow out.

Intersection	Flow In		Flow out.
A	x_1+x_3	=	20
B	x_2	=	x_3+x_4
C	0	=	x_1+x_2+80

Thus the total input is equal to the total output.

Thus

$$x_1+x_2+x_3+80+20+x_4=0$$
$$x_4=-100.$$

Thus,

$$x_1+x_3=20$$
$$x_2-x_3-x_4=0$$
$$x_2+x_3=-80$$
$$x_4=-100.$$

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Thus The augmented matrix form is,

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & -1 & 0 \\ 1 & 1 & 0 & 0 & -80 \\ 0 & 0 & 0 & 1 & -100 \end{bmatrix}$$

$R_3 = R_3 - R_1$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 & -100 \\ 0 & 0 & 0 & 1 & -100 \end{bmatrix}$$

$R_2 = R_2 - R_3$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 20 \\ 0 & 0 & 0 & -1 & 100 \\ 0 & 1 & -1 & 0 & -100 \\ 0 & 0 & 0 & 1 & -100 \end{bmatrix}$$

$R_4 = R_2 + R_4$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 20 \\ 0 & 0 & 0 & -1 & 100 \\ 0 & 1 & -1 & 0 & -100 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

So,

$$x_1 + x_3 = 20$$
$$x_4 = -100$$
$$x_2 - x_3 = -100$$

The general flow pattern of the network is described by.

$$x_1 = 20 - x_3$$
$$x_2 = x_3 - 100$$
x_3 is free
$$x_4 = 100$$

A negative flow in corresponds to a flow in the direction opposite to that shown in the figure

Thus,
The largest possible value for x_3 is 20 because x_1 cannot be negative.

Thus,
$$x_1 = 20 - x_3$$
$$x_2 = x_3 - 100$$
x_3 is free
$$x_4 = -100.$$

Question # 12

a) Find a general traffic pattern in the freeway network shown in the figure. (Flow rates are in cars/minute).

b) Describe the general traffic pattern when the road whose flow is x_4 closed.

(a)

Solution:-

First we have to write the equations that describe the flow, and then determine the general solution of the system.

Intersection	Flow In		Flow Out
A	x_1	$=$	$x_3 + x_4 + 40$
B	200	$=$	$x_1 + x_2$
C	$x_2 + x_3$	$=$	$100 + x_5$
D	$x_4 + x_5$	$=$	60

Thus,

$$x_1 - x_3 - x_4 = 40$$
$$x_1 + x_2 = 200$$
$$x_2 + x_3 - x_5 = 100$$
$$x_4 + x_5 = 60$$

The Augmented matrix of the system is,

$$= \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 1 & 1 & 0 & 0 & 0 & 200 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix}$$

$R_2 = R_2 - R_1$

$$= \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix}$$

$R_3 = R_3 - R_2$

$$= \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 0 & 0 & -1 & -1 & -60 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix}$$

$R_4 = R_4 + R_3$

$$= \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 0 & 0 & -1 & -1 & -60 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Thus,

$$x_1 - x_3 - x_4 = 40$$
$$x_2 + x_3 + x_4 = 160$$
$$-x_4 - x_5 = -60$$

Thus, The general traffic pattern is the freeway network is described by,

$$x_1 = 40 + x_3 + x_4$$
$$x_2 = 160 - x_3 - x_4$$

x_3 is free.

x_4 is free

$x_5 = 60 - x_4$

Thus Result is,

$x_1 = 40 + x_3 + x_4$

$x_2 = 160 - x_3 - x_4$

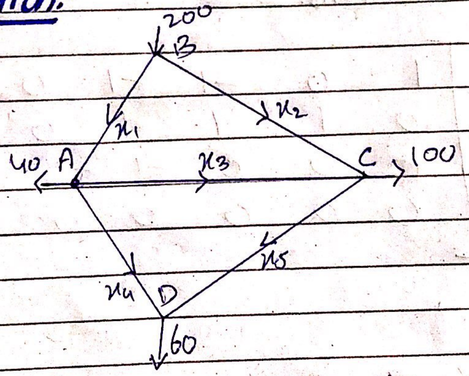
x_3 is free

x_4 is free

$x_5 = 60 - x_4$.

(b)

Solution:-



Now Let us check the flow in and flow out equations at each intersection.

Thus

Intersection	Flow in	Flow Out
A	x_1	$= x_3 + x_4 + 40$
B	200	$= x_1 + x_2$
C	$x_2 + x_3$	$= 100 + x_5$
D	$x_4 + x_5$	$= 60$.
Total Flow	200	$= 200$.

Now In equation form,

$$x_1 - x_3 - x_4 = 40.$$
$$x_1 + x_2 = 200$$
$$x_2 + x_3 - x_5 = 100$$
$$x_4 + x_5 = 60$$

Now In [A O] Form.

$$= \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 1 & 1 & 0 & 0 & 0 & 200 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix}$$

$R_2 = R_2 - R_1$

$$= \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix}$$

$R_3 = R_3 - R_2$

$$= \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 0 & 0 & -1 & -1 & -60 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix}$$

$R_4 = R_4 - R_3.$

$$= \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 0 & 0 & -1 & -1 & -60 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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Multiplying (-1) with R_3 i.e $R_3 = -1(R_3)$

$$= \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 0 & 0 & 1 & 1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 = R_2 - R_3$ and $R_1 = R_1 + R_3$.

$$= \begin{bmatrix} 1 & 0 & -1 & 0 & 1 & 100 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Thus

$x_1 - x_3 + x_5 = 100$

$x_2 + x_3 - x_5 = 100$

$x_4 + x_5 = 60.$

Thus,

$x_1 = 100 + x_3 - x_5.$

$x_2 = 100 - x_3 + x_5.$

$x_4 = 60 - x_5.$

x_5 is free.

When $x_4 = 0$, $x_5 = 60.$

Thus, The general solution is,

$x_1 = 40 + x_3.$

$x_2 = 160 - x_3$

x_3 is free

$x_4 = 0$

$x_5 = 60.$

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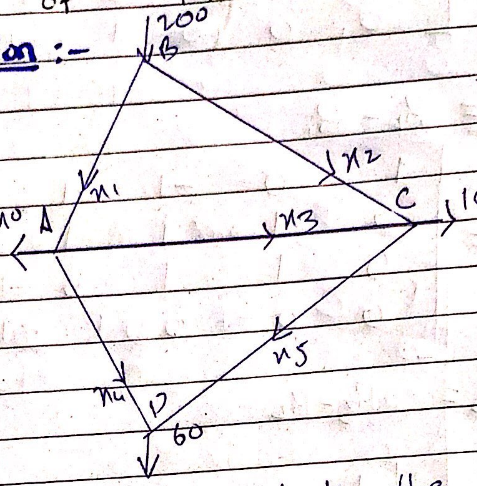
(C)

When $x_4 = 0$, what is minimum value of x_1?

Solution :-



Now, Let us check the flow equations at each intersection point.

Intersection	Flow In = Flow out
A	$x_1 = x_3 + x_4 + 40$
B	$200 = x_1 + x_2$
C	$x_2 + x_3 = 100 + x_5$
D	$x_4 + x_5 = 60$
Total	$200 = 200$

Now In equation form.

$$x_1 - x_3 - x_4 = 40$$
$$x_1 + x_2 = 200$$
$$x_2 + x_3 - x_5 = 100$$
$$x_4 + x_5 = 60$$

Thus In matrix form,

Thus,

$$= \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 1 & 1 & 0 & 0 & 0 & 200 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix}$$

$R_2 = R_2 - R_1$

$$= \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix}$$

$R_3 = R_3 - R_2$

$$= \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 0 & 0 & -1 & -1 & -60 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix}$$

$R_4 = R_4 + R_3.$

$$= \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 0 & 0 & -1 & -1 & -60 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 = 1(-R_3)$ and ~~$R_2 = R$~~

$$= \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 0 & 0 & +1 & +1 & +60 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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$R_2 = R_2 - R_3$ and $R_1 = R_1 + R_3$

$$= \begin{bmatrix} 1 & 0 & -1 & 0 & 1 & 100 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Thus,

$x_1 - x_3 + x_5 = 100$

$x_2 + x_3 - x_5 = 100$

$x_4 + x_5 = 60$

Thus,

$x_1 = 100 + x_3 - x_5$

$x_2 = 100 - x_3 + x_5$

$x_4 = 60 - x_5$.

x_3 is Free

x_5 is Free.

The General Solution is,

$x_1 = 40 + x_3$

$x_2 = 100 - x_3 + x_5$.

x_3 is free

$x_4 = 60 - x_5$.

x_5 is free.

$x_1 = 40 + x_3$

$x_2 = 160 - x_3$

x_3 is free

$x_4 = 0$

$x_5 = 60$

$x_4 = 0$ when $x_5 = 60$.

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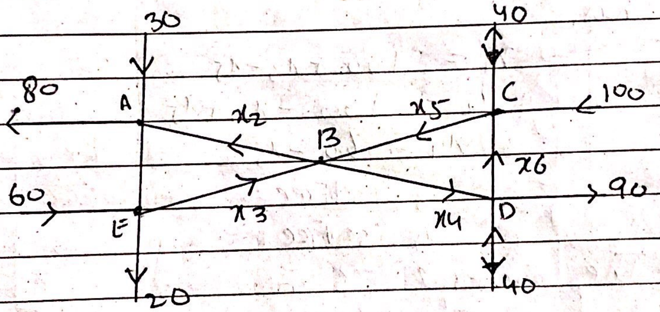
Thus,
The minimum value of x_1 is 40 cars/minute, because x_3 cannot be negative.

Question # 13
(a)

Find the general flow pattern in the network shown in the figure.



Solution:-
The flow In & flow out at each Point is,

Intersection	Flow In		Flow out.
A	$30 + x_2$	$=$	$x_1 + 80$
B	$x_3 + x_5$	$=$	$x_4 + x_2$
C	$100 + x_6$	$=$	$x_3 + 40$
D	$x_4 + 40$	$=$	$x_6 + 90$
E	$x_1 + 60$	$=$	$x_3 + 20$.

So, Lets check the total flow In and total Flow out and It must be equal.

$60+40+100+30 = 80+20+90+40$

Now,

Rewrite the eqn in standard form.

$x_1 - x_2 = -50$

$x_2 - x_3 + x_4 - x_5 = 0$

$x_5 - x_6 = 60$

$x_4 - x_6 = 60$

$x_1 - x_3 = -40$

Now,

In matrix form.

$$= \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \\ 1 & 0 & \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 1 & 0 & -1 & 0 & 0 & 0 & -40 \end{bmatrix}$$

$R_5 = R_5 - R_1$

$$= \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 1 & -1 & 0 & 0 & 0 & 10 \end{bmatrix}$$

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$R_5 = R_5 - R_5$ and $R_3 \leftrightarrow R_1$

$$= \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & -1 & 1 & 0 & 10 \end{bmatrix}$$

$R_5 = R_5 + R_3$

Now, $R_5 = R_5 - R_4$ & $R_2 = R_2 + R_4$

$$= \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \end{bmatrix}$$

$R_2 = R_2 - R_3$

$$= \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & 0 & -1 & 60 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 = R_1 + R_2$

$$= \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \left[\begin{array}{cccccc|c} 1 & 0 & -1 & 0 & 0 & 0 & -40 \\ 0 & 1 & -1 & 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

The general solution is

$$x_1 = -40 + x_3$$
$$x_2 = 10 + x_3$$
$$x_4 = 50 + x_6$$
$$x_5 = 60 + x_6$$

x_3 and x_6 are free variables.

(b)

Assume that the flow must be in the directions indicated, find minimum flow in branches denoted by x_2, x_3, x_4 and x_5.

Sol The minimal flow should be chosen such that all the variables remain non negative. We see that the minimal value of x_3 is 40 to provide non-negative x_1. It follows then, that $x_2 = 50$. To obtain minimal flow for x_4 and x_5 we can choose $x_6 = 0$

So,

$$x_4 = 50 \text{ and } x_5 = 60.$$

Thus,

$x_2 = 50$

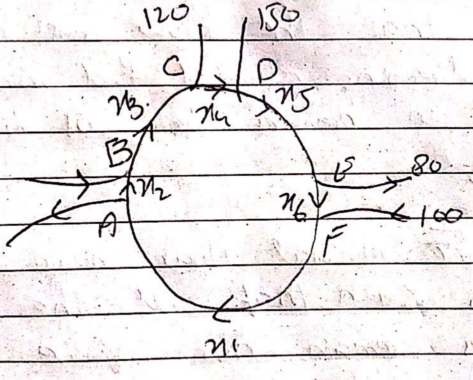
$x_3 = 40$

$x_4 = 50$

$x_5 = 60$

Sol **Question # 14**

Intersections in England are often constructed as one-way "roundabouts", such that shown in Figure. Assume that traffic must travel in the direction shown. Find the general solution of the network flow. Find the smallest possible value for x_6.



Solution

The flow set at each intersection point is given by,

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	Flow In		Flow out
A	x_1	=	$100 + x_2$
B	$x_2 + 50$	=	x_3
C	x_3	=	$120 + x_4$
D	$x_4 + 150$	=	x_5
E	x_5	=	$80 + x_6$
F.	$x_6 + 100$	=	x_1

The rearranged eqs. are

$x_1 - x_2 = 100$

$x_2 - x_3 = -50$

$x_3 - x_4 = 120$

$x_4 - x_5 = -150$

$x_5 - x_6 = 80$

$-x_1 + x_6 = -100.$

The augmented matrix of the system is.

$$= \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 100 \\ 0 & 1 & -1 & 0 & 0 & 0 & -50 \\ 0 & 0 & 1 & -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ -1 & 0 & 0 & 0 & 0 & 1 & -100 \end{bmatrix}$$

$R_5 = R_4 + R_5$, $R_4 = R_3 + R_4$, $R_3 = R_2 + R_3$

& $R_2 = R_1 + R_2$

$$= \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 100 \\ 1 & 0 & -1 & 0 & 0 & 0 & 50 \\ 0 & 1 & 0 & -1 & 0 & 0 & 70 \\ 0 & 0 & 1 & 0 & -1 & 0 & -30 \\ 0 & 0 & 0 & 1 & 0 & -1 & -70 \\ -1 & 0 & 0 & 0 & 0 & 1 & -100 \end{bmatrix}$$

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$R_2 = R_6 + R_2$ and $R_6 = R_1 + R_6$

$$= \left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & 0 & 100 \\ 0 & 0 & -1 & 0 & 0 & 1 & -50 \\ 0 & 1 & 0 & -1 & 0 & 0 & 70 \\ 0 & 0 & 1 & 0 & -1 & 0 & -30 \\ 0 & 0 & 0 & 1 & 0 & -1 & -70 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

So,

$$x_1 - x_2 = 100$$
$$-x_3 + x_6 = -50$$
$$x_2 - x_4 = 70$$
$$x_3 - x_5 = -30$$
$$x_4 - x_6 = -70$$
$$-x_2 + x_6 = 0$$

The general solution is

$$x_1 = 100 + x_6$$
$$x_2 = x_6$$
$$x_3 = 50 + x_6$$
$$x_4 = x_6 - 70$$
$$x_5 = 80 + x_6$$

x_6 is free

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