ASSIGNMEN 02

NAME : FAHAD ISHAQ

REGISTRATION NO : FA20-BCS-017

CIASS : BCS-4

SECTION : A

SUBMITTED TO : SIR UMAIR UMER

SUBMISSION DATE :

PROPERTIES OF DETERMINANT :

Determinant :

→ It is a scalar value that is calculated from the elements of a square matrix.

→ In matrix the vertical lines are columns and the horizontal lines are rows.

→ "n" order of determinant has "n" number of rows & columns.

Determinant of 2x2 matrix :

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Determinant of 3x3 matrix :

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \Rightarrow \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a(ei-fh) - b(di-fg) - c(dh-eg)$$

Properties of Determinant :

→ change in value of determinant will not occur if rows and columns are interchanged but sign will be changed.

→ If two rows or columns in determinant have same value, determinant will be zero.

→ If any variable say "k" is multiplied by rows or columns then its value is also multiplied by "k".

→ If some or all elements of row or column are expressed as the sum of two or more terms, then determinant can be expressed in terms of two or more determinant

1: REFLECTION PROPERTY :

value of determinant is unchanged by interchanging rows and columns. e.g.

$$M = \begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix} \rightarrow |M| = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix} = \begin{vmatrix} 2 & 6 & 1 \\ -3 & 0 & 5 \\ 5 & 4 & -7 \end{vmatrix}$$

Case 01

$$|M| = 2(0-20) - (-3)(-42-4) + 5(30-0)$$

$$|M| = -40 - 138 + 150$$

$$= -28$$

Case 02

$$= 2(-20) - 6(2)(-25) + 1(-12)$$

$$= -40 + 24 - 12$$

$$= -28$$

2: SWITCHING PROPERTY:

If any of two rows or columns are interchanged sign of determinant is changed. eg.

case 02

$$A = \begin{bmatrix} 4 & 5 & 6 \\ 7 & 3 & 2 \\ 6 & 4 & 3 \end{bmatrix} \Rightarrow \begin{vmatrix} 4 & 5 & 6 \\ 7 & 3 & 2 \\ 6 & 4 & 3 \end{vmatrix} \; ; \quad \begin{vmatrix} 4 & 5 & 6 \\ 6 & 4 & 3 \\ 7 & 3 & 2 \end{vmatrix}$$

Interchanging R_2 & R_3.

case 01:

$|A| = 4(9-8) - 5(\overset{21}{\cancel{12}} - \overset{12}{\cancel{21}}) + 6(28-18)$;

$|A| = 4 + 45 + 60$

$|A| = 1\cancel{0}9$

So,

$|A| = 4(8-9) - 5(12-21) + 6(18-28)$

$= 4(-1) - 5(-9) + 6(-10)$

$= -4 + 45 - 60$

$= -(19)$

$= -19$ Ans.

Hence, the sign of determinant is change in case 02.

3: ALL-ZERO PROPERTY:

If All the elements of any row or column are in matrix zero then its determinant is zero.

e.g

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 0 & 0 & 0 \\ 6 & 7 & 8 \end{bmatrix} = |A| = 0$$

4: REPETION OR PROPORTIONALITY:

If two rows or columns in matrix are (zero) same then determinant of that matrix will zero. eg

$$A = \begin{vmatrix} 3 & 4 & 5 \\ 3 & 4 & 5 \\ 1 & 1 & 1 \end{vmatrix} = |A| = 0.$$

5- SCALAR MULTIPLICATION:

If elements of a row or column of a determinant are multiplied by any non-zero constant then determinant also gets multiplied by same constant.

e.g

$$\begin{vmatrix} 12 & 9 & 6 \\ 13 & 4 & 5 \\ 6 & 8 & 9 \end{vmatrix} = \begin{vmatrix} 3(4) & 3(3) & 3(2) \\ 13 & 4 & 5 \\ 6 & 8 & 9 \end{vmatrix} = 3 \times \begin{vmatrix} 4 & 3 & 2 \\ 13 & 4 & 5 \\ 6 & 8 & 9 \end{vmatrix}$$

so, Here $K = 3$ "which is constant value"

6- SUM PROPERTY:

If elements of rows or column of a determinant are expressed as sum of two or more terms then determinant can be expressed as a sum of two or more determinants.

e.g

$$\begin{vmatrix} a & b & c \\ a+3a & b+7y & c+2z \\ x & y & z \end{vmatrix} = \begin{vmatrix} a & b & c \\ a & b & c \\ x & y & z \end{vmatrix} + \begin{vmatrix} a & b & c \\ 3a & 7y & 2z \\ x & y & z \end{vmatrix}$$

7- Invariance PROPERTY:

Suppose any any scalar multiplies of corresponding elements of other two rows or columns are added to every element of any row or column of a determinant. In this case, value of determinant remains same.

e.g

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1+kb_1+ & a_2+kb_2 & a_3+kb_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

8- Factor Property

If determinant Δ becomes zero when we insert $x = \alpha$ then $(x-\alpha)$ is factor of Δ.

e.g.

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \Rightarrow \Delta 1 = \begin{vmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{vmatrix}$$

Here x_{ij} denotes the cofactor of then element a_{ij} of A matrix

9- Triangle Property

If the elements below & above the main diagonal in determinant are zero then determinant is the product of diagonal elements.

e.g

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 5 & 6 \end{bmatrix}, \quad |A| = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 5 & 6 \end{vmatrix}$$

So,

$$|A| = 0.$$

Same if

$$A = \begin{bmatrix} 8 & 7 & 6 \\ 0 & 3 & 5 \\ 0 & 0 & 4 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 8 & 7 & 6 \\ 0 & 3 & 5 \\ 0 & 0 & 6 \end{vmatrix}$$

$$|A| = 0.$$

10- Determinant of Cofactor matrix:

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \quad \& \quad \Delta 1 = \begin{vmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{vmatrix}$$

Here

(Δ^x) $\Delta 1$ represents the cofactors of the elements of a_{ij} in Δ.