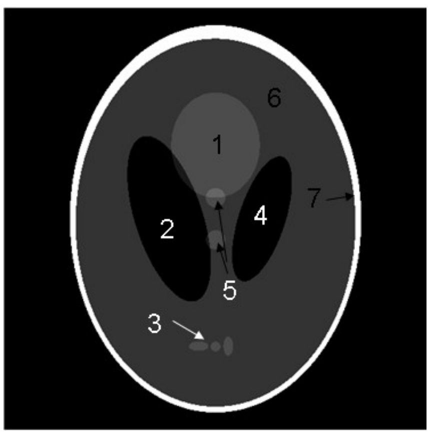
Generation of simple magnetic resonance imaging (MRI) using a Shepp-Logan Digital Phantom

Use Python
You can use functions and libraries for generating and presenting graphs and images.

Background: Among the most famous digital phantoms are based on a 2D one originally introduced by Shepp and Logan in 1974 (L. A. Shepp and B. F. Logan, "The Fourier reconstruction of a head section," IEEE Transactions on Nuclear Science, vol. NS-21, pp. 21-43, 1974) and later extended to 3D by Shepp (L. A. Shepp, "Computerized tomography and nuclear magnetic resonance," J Comput Assist Tomogr, vol. 4, pp. 94-107, 1980.). Those phantoms were originally made to study reconstruction algorithms in Computed Tomography (CT) and then extended to MRI.



This figure shows a standard Shepp Logan phantom generated in Matlab with the phantom function. In Matlab the code we used was:

```
>> ph = phantom(1024);
>> figure, imshow(ph);
```

The purpose of this assignment is to familiarize yourself with the source code of a 2D Shepp-Logan digital phantom and make it appropriate for Magnetic Resonance Imaging.

You have three tasks

1. Modify the Shepp-Logan phantom: Start with the source code that generates this phantom in Matlab or Python. Modify the default parameters that define the ellipses so that:

- There are no overlapping structures, as example: make structure (1) smaller, move structure (2) lower.
- make the three structures in (3) to be circles and not overlapping,
- Add the display of the phantom.

Report the code as you modified it and an outcome (like figure above – without the numbers!

Modified Shepp-Logan Phantom

2. A question: How many different compartments you have? Assume that tissue in (4) is the same with tissue in (2)

ANSWER:
Assuming the structures 2 & 4 are the same, similarly structures 1,3 & 5 display the same intensities and hence are considered the same.
The final number of components is 4.

3. Create a simple virtual MRI scanner: Let's further modify your code to create a simple patient-mimicking digital phantom for use in generating MRI. We will assume that your virtual scanner simulates a simple MRI machine that collects only *one type of MRI images* that the signal intensity (SI) acquired from a voxel is given by the formula:

$$SI = A * (1 - \exp(-TR / T1)) * \exp(-TE / T2)$$

A = a number that depends on the amount of water in this voxel
T1 = is a property of tissue in the voxel (longitudinal relaxation time)
T2 = is a property of the tissue in the voxel (transverse relaxation time)
TR = is a parameter that the operator of the system selects (the repetition time)
TE = is a parameter that the operator of the system selects (the echo time)

Therefore, we must assign different T1 and T2 values to the different tissues!

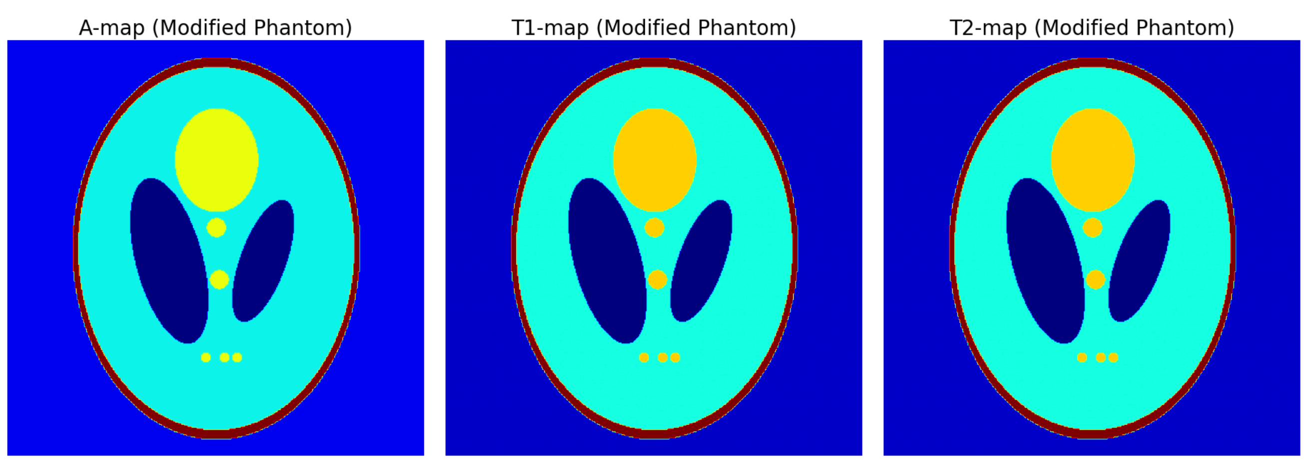
If N is the number of different compartments, then use these formulas
$T1 = 50 + (j-1)*250$ j = 1 to N
$T2 = 10 + (j-1)*50$ j = 1 to N
Where j is a compartment form the figure.

Assign an A to each compartment between 0.0 and 1.0 …

Question: which compartments are those with A = 0.0 and 1.0?
So, do you want to update the answer to the previous question of "how many compartments has your phantom?"

ANSWER: Same as before 4.

3.1 Physical Properties Maps: Generate and present the A-map, T1-map and T2-map of your phantom.
ANSWER:



3.2 Effect of User-selected Acquisition parameters:

In a table like this one below report the corresponding values, T1, T2,

ANSWER:

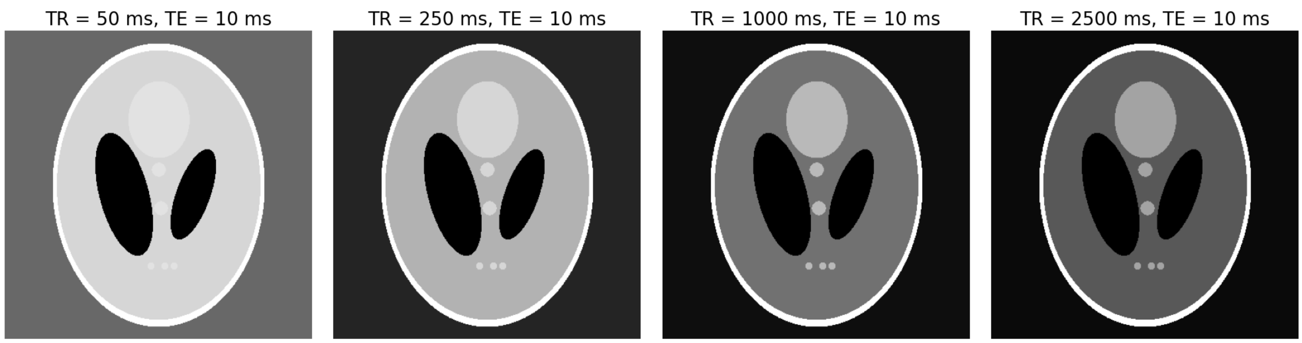
Structure index	***A***	***T1***	***T2***	***SI1***	***SI2***	***SI3***	***SI4***
1	0.1	50	10	0.0233	0.0365	0.0368	0.0368
2	0.3667	300	60	0.0476	0.1755	0.2993	0.3103
…	0.6333	550	110	0.0503	0.2112	0.4844	0.5722
N	1.0	800	1600	0.0569	0.2521	0.6703	0.8981

Calculate the SI above for the four cases of acquisition parameters.

	TR	TE
SI1	50	10
SI2	250	10
SI3	1000	10
SI4	2500	10

3.3 *Report the four generated MRI* next to each other. What is the effect of the changing the TR?

ANSWER:

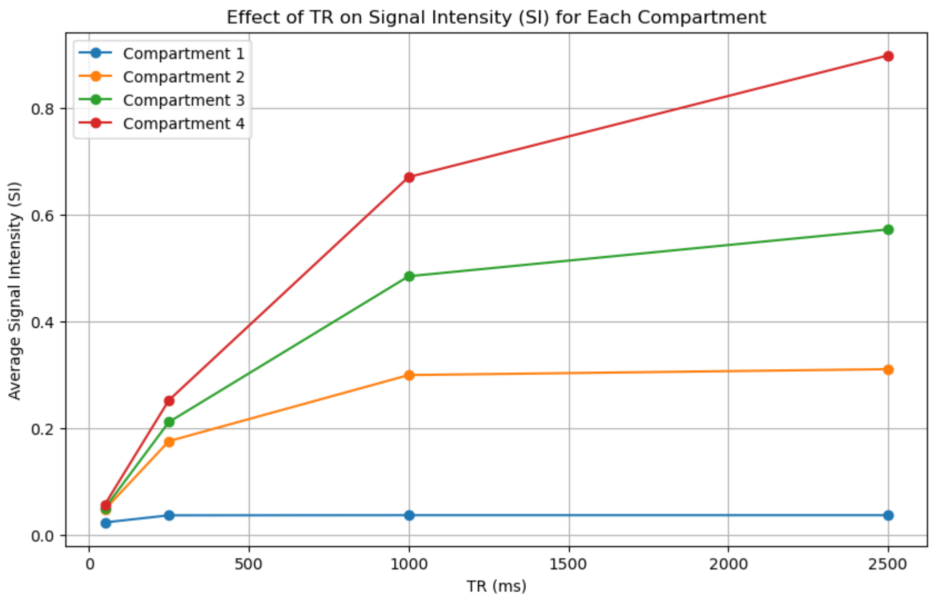


Observations on the effect of changing TR:

- As TR increases, the brightness of the MRI images also increases.
- The contrast between different compartments is more pronounced at higher TR values.
- At shorter TR values (e.g., TR = 50 ms), the image is darker overall, and the contrast between some structures is less distinct.
- At longer TR values (e.g., TR = 2500 ms), the image is much brighter, and different structures are more distinguishable.

In MRI, the choice of TR affects the longitudinal magnetization recovery. A short TR allows little time for the longitudinal magnetization to recover before the next pulse, resulting in a lower signal. In contrast, a longer TR allows for more recovery of the longitudinal magnetization, resulting in a stronger signal and brighter images. The effect of TR on image contrast is especially notable between tissues with different T1 relaxation times.

3.4 *Effect of Acquisition Parameters on contrast:* Plot SI versus TR and TE
What is the role of each operator selectable parameter to the SI?
When you change the TR which physical parameter's contribution to contrast is explored?
Similarly, when you change the TR which physical parameter's contribution to contrast is explored?
ANSWER:



The plot showing the effect of TR (Repetition Time) on the average Signal Intensity (SI) for each compartment of the modified Shepp-Logan Phantom.

Observations:

- For all compartments, as TR increases, the SI also generally increases.
- The rate of increase in SI with TR is more pronounced for some compartments than for others, indicating differences in their T1 relaxation times.

Role of Operator-Selectable Parameters on SI:

- TR (Repetition Time): TR affects the time allowed for the longitudinal magnetization (parallel to the magnetic field) to recover before the next excitation pulse. A shorter TR results in less recovery, leading to a lower SI, while a longer TR allows more recovery, resulting in a higher SI. The difference in SI between tissues at different TR values can result in contrast, especially when tissues have different T1 relaxation times.

- TE (Echo Time): TE is the time between the excitation pulse and the peak of the echo signal. It affects the time allowed for the transverse magnetization (perpendicular to the magnetic field) to decay. A shorter TE captures the signal before much decay has occurred, resulting in a higher SI, while a longer TE allows more decay, leading to a lower SI. The contrast between tissues at different TE values is determined by their T2 relaxation times. However, in our simulations, TE was kept constant at 10ms, so we didn't observe variations due to TE.

Physical Parameter's Contribution to Contrast: When you change the TR:

- The primary physical parameter's contribution to contrast being explored is the T1 relaxation time. Differences in T1 times between tissues lead to differences in the recovery of longitudinal magnetization, which results in contrast in the images.

- When you change the TE: The primary physical parameter's contribution to contrast being explored is the T2 relaxation time (or T2 in the presence of magnetic field inhomogeneities). Differences in T2 times between tissues lead to differences in the decay of transverse magnetization, affecting image contrast. However, in our case, since TE was constant, we did not explore this effect.