PHYS 222 Projects Notes

Fahed Abu Shaer

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1 Khalil: Ergodicity

In his presentation, Khalil talked about ergodicity in dynamical system and the idea of randomness. Dynamical system can be deterministic or stochasitic. He ran a simulation that involved random walkers and saw that over time there is a high change for the walkers to fill up the whole space. However, these walkers are indistinguishable. The only way to distinguish them is when randomness is broken by giving the walker a bias. He talked about two types of averages: the ensemble average and the time average. He discussed with us the Poly's Urn Model where we have an urn with two balls of different colors. You take one of the balls out, and put it back with an additional ball with the same color. This increases the probability of the color to be chosen which make it dominate the system. I liked how he tied it to the idea of "the rich gets richer" and capitalism. It shows that our models and physics can be used to understand social stuff. He ended his presentation with the Baker's Map that illustrates the forbidden areas in the phase space.

2 Jad: Scalar Curvature in Discrete Gravity

In his project, Jad tried to find a way to calculate the curvature of a tennis ball using discrete gravity. He introduced the notion of discrete gravity. He started with general relativity and defined the continuous manifold. Then, he split the manifold into elementary cells with Planck volume. This is a way to quantize the ball. So after quatizing the ball, he defined curvature in term of chaninges in the connections and is derived through shifting a vector in elementary cells through shifts and rotation. If the vector conserved its direction then the object is flat, and if it didn't then there is some curvature. At the end, after a lot of math, he found the curvature of the ball using discrete gravity and compared it to the value gotten from general relativity, and it turned out to be the same.

3 Roua: Communicability in Brain Network

Roua imagined the brain as a complex connected network. The brain network is characterized by the neuron being the nodes, the synapses being the connections or edges. This visualization was very efficient and redundant because it helps us work the dynamical interactions between the neurons and model scenarios where the neuron are malfunctioning. So, in the brain network, the transmissions between the neurons/nodes, take the shortest path; however, there are also relative short paths. So, starting from a time series, the Lorenz time series, Roua wanted to model the loss of connectivity between the neurons. So, she generated multiple small world networks because the small world network resembles the brain the most. Then, she started attacking the network by removing nodes with the highest centrality and then randomly. She concluded that the communicability is lost faster when you remove based on centrality.

4 Issar: Taken's Embeddings

In her project, Issar tried to construct a whole dynamical system using one solution series. She defined the dimensionality and time delay. She took the first minimum of the mutual information as a function of the time delay. Then, she looked at close neighbouring points and filtered through the fake neighbours. After the fake neighbours diverging to zero, she took the first three delayed versions and shifter $x(t)$ three times v_1, v_2, v_3. The system that she repreduced was topologically equivalent to the Lorenz system that she was trying to recover. Then, she attemped to the same thing but using singular value decomposition and reconstruct the system by plotting the first three entries of the matrices. Lastly, she computed the Lyapunov exponent of each of the new system she got, and she compared them. It turned out that they all had close λ which means that they are topologically equivalent. She was able to recover the original system, which was the Lorenz attractor, only using one part of the time series.

5 Garo: Laplacian Coarse-Gaining of Complex Networks

Garo, in his presentation, dealt with complex networks and tried to find a way to simplify the networks to make it easier to study. He studied the Ising model, a model that formulates ferromagnetism and spins of particles. He used statistical mechanics to use normalization to simplify the dipole moment and bound strength of the interacting particles. He then he connected it via an analogy to a network, and transformed us to work with networks. He expressed the network's Laplacian which showed up in the Hamiltonian of the system, and tried to combine nodes in a way that makes sense. These combinations take us from a N network to a network with $\frac{N}{2}$ nodes. If we do this for multiple levels, we can get a simpler network while retaining the original network's properties, but it is easier to study.

6 Semaan: Solving the 1D Heat Equation Using Finite Difference and Chebychev Polynomials

In his project, Semaan tried to solve the heat equation in one dimensions, $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \beta \frac{\partial u}{\partial x}$. He defined for us the Chebychev polynomials and their generating function being dependent on theta, $T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$. He observed that the polynomials shows a clustering behavior and there is more resolution around the boundaries. So, he decomposed his partial differential equation where he solved the left hand side using finite differences and the right hand side using the Chebychev polynomials by substituting the derivatives by a matrix multiplication. Using the Chebychev polynomials, he was able to solve the system and take care of the boundary terms where they don't do weird behaviors like they do when other methods are used.