Synopsis of Robust Principal Component Analysis

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***Abstract*—This is a study report on a paper written on a curious phenomenon.The paper supposes we have a data matrix, which is the superposition of a low-rank component and a sparse component. Can we recover each component individually? It proves that under some suitable assumptions, it is possible to recover both the low-rank and the sparse components exactly by solving a very convenient convex program called Principal Component Pursuit; among all feasible decompositions, simply minimize a weighted combination of the nuclear norm and of the 1 norm.We discuss an algorithm for solving this optimization problem, and present applications in the area of video surveillance, where our methodology allows for the detection of objects in a cluttered background, and in the area of face recognition, where it offers a principled way of removing shadows and specularities in images of faces.**

I. Introduction

Suppose we are given a data matrix M such that M can be decomposed as $M = L_0 + S_0$, where L_0 has low rank and S_0 is sparse and both matrix are of arbitrary magnitude. We do not know the low-dimensional column and row space of L_0, similarly the non zero entries of S_0 are not known. There are many prior attempts to solve or atleast alleviate the above mentioned problem.

A. Classical Principle Component Analysis

To solve the dimensionality and scale issue we must leverage on the fact that such data matrix are intrinsically lower in dimension, thus are indirectly sparse in some sense. Perhaps the simplest assumption is that the data in matrix all lie near some lower dimensional subspace, hence we can stack all the data points as column vector of a matrix M, and this column vector can be represented mathematically,

$$M = L_0 + N_0$$

where L_0 is essentially low rank and N_0 is a small perturbation matrix.Classical Principal Component seeks the best rank-k estimate of L_0 by solving

$$minimize \quad ||M - L||$$

$$subject\ to \quad rank(L) \leq k.$$

Throughout this article $||M||$ denotes l^2 norm. In classical PCA it is assumed that N_0 is small.

B. Robust Principle Component Analysis

PCA is unequivocally the best statistical tool for data analysis and dimensionality reduction. However its brittleness to small corrupted data in data matrix puts its validity in jeopardy, as this small corruption could render the estimated $\hat{L}$ arbitrarily far from true L_0. Such problems are ubiquitous in modern applications such as image processing, web data analysis and many more.The problems mentioned above are the idealized version of robust PCA, where we recover low rank matrix from highly corrupted data matrix M such that $M = L_0 + S_0$ where unlike classical PCA S_0 can have arbitrarily large magnitude. and their support is assumed to be sparse.

II. Applications

The applications of the above mentioned Robust PCA along with convex optimization and other multiplier algorithms are ***video surveillance, Face recognition, Latent Semantic Indexing*** and many more field.

A. Video Surveillance

If we stack video as a matrix, then the low rank matrix corresponds to the stationary background and sparse matrix captures moving objects from the video.

B. Face Recognition

It is well known that images of a convex, Lambertian surface un- der varying illuminations span a low-dimensional subspace. This fact has been a main reason why low-dimensional models are mostly effective for imagery data.However, realistic face images often suffer from self-shadowing, specularities, or saturations in brightness, which make this a difficult task and subsequently compromise the recognition performance.

C. Ranking and Collaborative Filtering

The problem of anticipating user tastes is gaining increasing importance in online commerce and advertisement. Companies now routinely collect user rankings for various products.This problem is typically cast as a low-rank matrix completion problem.

III. Understanding the Architechture

A. Using the theorem 1.1

Suppose L_0 is n x n, obeys equations (2) and (3). Fix any n x n matrix Σ of signs. Suppose that the support set Ω of S_0 is uniformly distributed among all sets of cardinality m, and that sgn($[S_0]_{ij}$) = Σ_{ij} for all (i, j) $\in \Omega$. Then, there is a numerical constant c such that with probability at least 1 - cn^{-10} (over the choice of support of S_0), Principal Component Pursuit (1) with λ = 1/$\sqrt{n}$ is exact, that is, $\hat{L}$ = L_0 and $\hat{S}$ = S_0, provided that

$$rank(L_0) \leq \rho_r n \mu^{-1} (logn)^{-2} \, and \, m \leq \rho_s n^2 \qquad (4)$$

In this equation, ρ_r and ρ_s are positive numerical constants. In the general rectangular case, where L_0 is $n_1 x n_2$, PCP with λ = $\frac{1}{\sqrt{n_{(1)}}}$ succeeds with probability at least $1 - cn_{(1)}^{-10}$, provided that rank(L_0) $\leq \rho_r n_{(2)} \mu^{-1} (logn_{(1)})^{-2}$ and $m \leq \rho_s n_1 n_2$.
Matrix L_0 whose principal components are spreaded can be recovered with probability almost onefrom arbitary and completely unknown corruption patterns. It also works for higher ranks like n/$log(n)^2$ when μ is not large. Minimizing

$$||L||_* + \frac{1}{\sqrt{n_{(1)}}} ||S||_1$$

where,

$$n_{(1)} = max(n_1, n_2)$$

under the assumption of theorem, this always gives correct answer. Here we chose λ = $\frac{1}{\sqrt{n_{(1)}}}$ but it is not clear why that has happened. It has been due to mathematical analysis why we are taking that value.

IV. Algorithms

In this section we discuss Principle Component Pursuit *(PCP)* algorithms to successfully retrieve low rank matrix and sparse matrix from a corrupted given data matrix, also to buttress its applicability to large scale problems we rely on convex optimization program. For the experiments performed in this section, we have used *Alternating Direction Method (ADM)* which is a special case of more general Augmented Lagrange multiplier (ALM).

The implementation of the algorithm in Matlab programming language can be found in the same folder as the pdf under the code folder.link for the Github repository is: Here an image of moon is corrupted and processed using RPCA algorith im MATLAB.Low rank matrix and Sparse matrix are extracted using Augmented langrage multiplier.The results are displayed clearly in figure 1 and figure 2 The implementation of the algorithm in Matlab programming language can be found in the same folder as the pdf under the code folder. link for the Github repository is:

A. PsuedoCode

V. Algorithm

The authors have proposed **Augmented Lagrange multiplier** algorithm to compute PCP which works on augmented Lagrangian.

Augmented Lagrange multiplier [1] initialize:S_0 = Y_0 = 0, $\mu > 0$ not converged compute L_{k+1} = $D_{1/\mu}(MS_k + \mu^1 Y_k)$; compute S_{k+1} = $S_{\lambda/\mu}(ML_{k+1} + \mu^1 Y_k)$; compute Y_{k+1} = $Y_{k+\mu}(ML_{k+1} + S_{k+1})$; **return** L,S

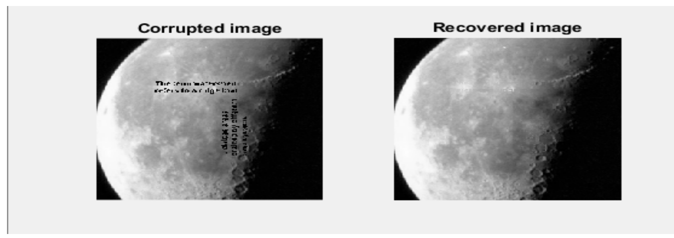

Fig. 1. Corrupted and Recovered , Corrupted values:1138

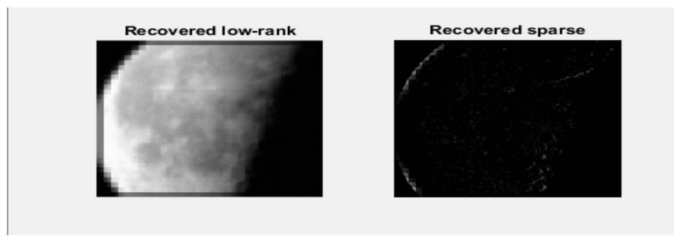

Fig. 2. Recovered Low-Rank and Recovered Sparse, Corrupted values:1138

VI. Conclusion and further proceedings

This article delivers some encouraging news: one can disentangle the low-rank and sparse components exactly by convex programming, and this provably works under quite broad conditions. Further, our analysis has revealed rather close relationships between matrix completion and matrix recovery (from sparse errors) and our results even generalize to the case when there are both incomplete and corrupted entries (i.e., Theorem 1.2). In addition, Principal Component Pursuit does not have any free param- eter and can be solved by simple optimization algorithms with remarkable efficiency and accuracy. More importantly, our results may point to a very wide spectrum of new theoretical and algorithmic issues together with new practical applications that can now be studied systematically. Our study so far is limited to the low-rank component being exactly low-rank, and the sparse component being exactly sparse. It would be interesting to investigate when either or both these assumptions are relaxed.

References

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