Linear Algebra
Assignment: 2

Submitted By:

Faheem Siddiqi (FA20-BCS-018)

Submitted To:

Mr. Umair Umer

Question: 01

Determinent:

The determinant of matrix is a special scalar value that can be computed from square matrix. It is denoted by $|A|$ or $det(A)$

Properties:

Following are some major properties of determinent:

(i)

Determinant of identity matrix is always one i.e. $det(I) = 1$

This can be explain by example:

Consider identity matrix of an order 2×2:

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$det(I) = (1\times1) - (0\times0) = 1$$

(ii)

Consider A be the matrix and its determinant as $det(A)$ than:

$$det(kA) = k^n det(A)$$

This can be explained with the help of an example:

let $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

$\det(A) = 3 - 2 = 1$

Now if we have to find $\det(2A)$. Than multiply A by 2

$\det 2(A) = \begin{bmatrix} 2 & 4 \\ 2 & 6 \end{bmatrix} = 12 - 8 = 4$

According to the property

$\det(2A) = 2^2 \cdot \det(A)$

$= 4(1) = 4.$

(iii)

Consider A as a matrix. If A has rows/colaum equal to zero. Than;

$\det(A) = 0$

This can be explained with the help of example:

let; $A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$

$|A| = 0 - 0 = 0$

(iv)

let A be the matrix than:

$\det(A) = \det(A^t).$

Consider example:

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $|A| = 4 - 6 = -2$ — (i)

$A^t = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ $|A^t| = 4 - 6 = -2$ — (ii)

Since i - ii equations are equal thus property $|A| = |A^t|$ exists.

(V)

Consider a matrix A. If matrix A has 2 identical rows and columns than detrminant of A shall be zero.

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\det |A| = (1\times1) - (1\times1) = 0$$

(VI)

Let A be the square matrix, if we interchange it any of two rows / coloums than its sign for determinant changes i.e

$$|A| = -|A|$$

Let $A = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$ $|A| = 16 - 24 = -8$ — i

Now interchanging R_1 & R_2

$A = \begin{bmatrix} 6 & 8 \\ 2 & 4 \end{bmatrix}$ $|A| = 24 - 16 = 8$ — ii

Observing eq: I & II $-|A| = |A|$

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(VII)

Consider two square matrix A and B than:

$$\det(AB) = \det(A)\det(B)$$

This can be understood by example:

let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

$AB = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$

~~det (AB)~~ =

det (AB) $= 19 \times 50 - 22 \times 43 = 950 - 946 = 4. \quad \text{— (i)}$

det(A) $= (4 \times 1) - (2 \times 3) = -2.$

det(B) $= (8 \times 5) - (6 \times 7) = 40 - 42 = -2$

det(A) × det(B) $= 4 \quad \text{— (ii)}$

As eq i & ii are same thus property is satisfied

(VIII)

If elements of row or coloum are multiplied by non-zero constant determinant is multiplied by same constant. This is also known as **Scalar Multiple Property**.

This can be explained with the help of example:

let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\det(A) = 4 - 6 = -2 \quad \text{— i}$

Multiplying 2 with R_1 of A : A becomes $A' = \begin{bmatrix} 2 & 4 \\ 3 & 4 \end{bmatrix}$

$|A'| = 8 - 12 = -4. \quad \text{— ii}$

From i & ii it is clear that $|A| = 2|A|$ as we multiplied 2 to A row.

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(ix)

If the elements of determinant above or below , the diagonal become zero than determinant equal product of diagonal elements. This is known as **triangular property.**

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ 0 & b_2 & b_3 \\ 0 & 0 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & 0 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 b_2 c_3$$

Consider matrix A ;

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 9 \end{bmatrix} \text{ than } \det(A) = 1 \times 5 \times 9 = 45 \quad -(i)$$

Finding $|A|$ by normal method to check if property exist ;

$$|A| = 1\begin{vmatrix} 5 & 6 \\ 0 & 9 \end{vmatrix} - 2\begin{vmatrix} 0 & 6 \\ 0 & 9 \end{vmatrix} + 3\begin{vmatrix} 0 & 5 \\ 0 & 0 \end{vmatrix}$$

$$= 1(45-0) - 2(0-0) + 3(0*0) = 45 \quad -(ii)$$

Since i & ii are same thus property satisfies.

(x)

Consider a matrix A ;

$$\begin{bmatrix} a_1+b_1 & c_1 & d_1 \\ a_2+b_2 & c_2 & d_2 \\ a_3+b_3 & c_3 & d_3 \end{bmatrix}$$

than $|A| = \begin{vmatrix} a_1 & c_1 & d_1 \\ a_2 & c_2 & d_2 \\ a_3 & c_3 & d_3 \end{vmatrix} + \begin{vmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \end{vmatrix}$

This is known as **Sum property.**

This can be explain with example let $A = \begin{vmatrix} 1+4 & 2 \\ 1+6 & 3 \end{vmatrix}$

$|A| = (1+4)3 - 2(1+6) = 15-14 = 1$ — i

Now using property we can also find determinat by,

$\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 4 & 2 \\ 6 & 3 \end{vmatrix} = 3-2 + 12-12 = 1$ — ii

As eq i & ii are same thus this property exists.