COMSATS UNIVERSITY ISLAMABAD

(MTH231-LINEAR ALGEBRA)

ASSIGNMENT 1

Submitted By

Group 3

Faheem Siddiqi

FA20-BCS-018

Maira Anjum

FA20-BCS-033

Question: 01

According to Question:

Output From		Purchased By
Goods	Services	
0.8	0.3	Services
0.2	0.7	Goods

let: p_g as annual output of goods sector
p_s as annual output of service sector

Equation for row one will be:

$$p_s = 0.8 p_g + 0.3 p_s$$

Equation for row two will be

$$p_g = 0.2 p_g + 0.7 p_s.$$

To solve system of equation move all the unknowns to left

$$-0.8 p_g + 0.7 p_s = 0 \quad - i$$

$$0.8 p_g + (-0.7) p_s = 0 \quad - ii$$

Augmented matrix for equation I & II will be

$$\left[\begin{array}{cc:c} -0.8 & 0.7 & 0 \\ 0.8 & -0.7 & 0 \end{array}\right]$$

adding R_2 & R_1

$$\begin{bmatrix} -0.8 & 0.7 & : 0 \\ 0 & 0 & : 0 \end{bmatrix}$$

dividing R_1 by -0.8

$$\begin{bmatrix} 1 & -0.875 & : 0 \\ 0 & 0 & : 0 \end{bmatrix}$$

- Thus general solution is $p_g = 0.875\,p_s$ and where p_s is free
- Equilibrium will remain uneffected with prices until the ratio $p_g = 0.875\,p_s$ satifies.

  Consider $p_s = 1000$. than $p_g = 0.875 \times 1000 \Rightarrow$

  $p_g = 875$.

Question :02

- Consider $p_s = 300$ million dollar

  than $p_c = 0.94\,p_s$, $p_e = 0.85\,p_s$

  $p_c = 282$, $p_e = 255$

- No! there will be no change in equilibrim until and unless the ratio are not satisfied.

Question: 03

Output From			Purchased By
Chemicals	Fuel	Machienery	
0.2	0.8	0.4	Chemicals
0.3	0.1	0.4	Fuel
0.5	0.1	0.2	Machinery

let ;

p_c as annual output for chemical sector

p_f as annual output for fuel sector

p_m as annual output for machinery sector

Equation for row 1 will be ;

$$p_c = 0.2p_c + 0.8p_f + 0.4\,p_m$$

Equation for row 2 will be :

$$p_f = 0.3p_c + 0.1p_f + 0.4p_m$$

Equation for row 3 will be ;

$$p_m = 0.5p_c + 0.1p_f + 0.2p_m$$

To solve system of eqs move all unknows to left ;

$$0.8p_c + -0.8p_f - 0.4\,p_m = 0$$

$$-0.3p_c + 0.9p_f - 0.4\,p_m = 0$$

$$-0.5p_c - 0.1\,p_f + 0.8\,p_m = 0$$

augmented matrix will be

$$\left[\begin{array}{ccc|c} 0.8 & -0.8 & -0.4 & 0 \\ -0.3 & 0.9 & -0.4 & 0 \\ -0.5 & 0.1 & 0.8 & 0 \end{array}\right]$$

Scanned with CamScanner

multiply each row by 10

$$\begin{bmatrix} 8 & -8 & -4 & : 0 \\ -3 & 9 & -4 & : 0 \\ -5 & -1 & 8 & : 0 \end{bmatrix}$$

dividing R_1 by 8

$$\begin{bmatrix} 1 & -1 & -1/2 & : 0 \\ -3 & 9 & -4 & : 0 \\ -5 & -1 & 8 & : 0 \end{bmatrix}$$

$R_2 + (+3R_1)$; $R_3 + (\cdot 5R_1)$

$$\begin{bmatrix} 1 & -1 & -0.5 & : 0 \\ 0 & 6 & -5.5 & : 0 \\ 0 & -6 & 5.5 & : 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0.5 & : 0 \\ 0 & 1 & -0.917 & : 0 \\ 0 & -6 & 5.5 & : 0 \end{bmatrix} \quad R_3 + 6R_2$$

$$\begin{bmatrix} 1 & -1 & 0.5 & : 0 \\ 0 & 1 & -0.917 & : 0 \\ 0 & 0 & 0 & : 0 \end{bmatrix}$$

$$\cancel{\begin{bmatrix} 1 & 0 & -1.417 & : 0 \\ 0 & 1 & \overset{1.48}{-0.917} & : 0 \\ 0 & 0 & 0 & : 0 \\ 0 & 0 & 0 & \end{bmatrix}} \quad R_2 + R_1 \begin{bmatrix} 1 & 0 & -1.417 & : 0 \\ 0 & 1 & -0.917 & : 0 \\ 0 & 0 & 0 & : 0 \end{bmatrix}$$

The general solution is

$$pc = 1.147\ pm$$
$$pf = 0.9417\ pm$$
$$pm = \text{free variable}$$

(c) If $pm = 100$

$$pc = 1.147(100) = 114.7$$
$$pf = 0.9417(100) = 94.17$$
$$pm = 100$$

Question:04

Output				Purchased By
A	E	M	T	
0.65	.30	.30	.20	A
0.10	.10	.15	.10	E
0.25	.35	.15	.30	M
0	.25	.40	.40	T

Denoting total annual outcome of sectors by pa, pe, pm, pt.

Equation for row 1:

$$pa = 0.65\ pa + 0.3pe + 0.30\ pm + 0.20pt.$$

Eq for row: 2,3,4 respectively are;

$$pe = 0.10\ pa + 0.10pe + 0.15pm + 0.10pt$$
$$pm = 0.25\ pa + 0.35\ pe + 0.15pm + 0.30\ pt$$
$$pt = 0.25\ pe + 0.40\ pm + 0.40pt$$

Date: moving unknowns to the left.

Augmented Matrix form;

$$\left[\begin{array}{cccc|c} 0.35 & -0.3 & -0.3 & -0.2 & 0 \\ 0 & 0.81 & -2.4 & -1.6 & 0 \\ 0 & 0 & 1.0 & -1.17 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

dividing r_1 by 0.35

$$\left[\begin{array}{cccc|c} 1 & -0.86 & -0.86 & 0.57 & 0 \\ 0 & 0.81 & -2.4 & -1.6 & 0 \\ 0 & 0 & 1.0 & -1.17 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

dividing r_2 by 0.81

$$\left[\begin{array}{cccc|c} 1 & -0.86 & -0.86 & 0.57 & 0 \\ 0 & 1 & -2.96 & 1.98 & 0 \\ 0 & 0 & 1.0 & -1.17 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_1 + 0.86R_2 \rightarrow R_1$

$$\left[\begin{array}{cccc|c} 1 & 0 & 2.55 & 0.97 & 0 \\ 0 & 1 & -2.96 & 1.98 & 0 \\ 0 & 0 & 1.0 & -1.17 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_2 + 2.96R_3$ & $R_1 - 2.55R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -0.97 & 0 \\ 0 & 1 & 0 & \overset{-1.48}{\cancel{1.98}} & 0 \\ 0 & 0 & 1 & -1.17 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Scanned with CamScanner

general solution

$p_a = 2.03\,p_t$; $p_e = 0.53\,p_t$ $p_m = 1.17\,p_t$

and p_t = free variable.

if $p_t = 100$ than

$p_a = 203$; $p_e = 053$ $p_m = 117$

Question :05

$B_2S_3 \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix}$, $H_2O \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}$, $H_3BO_3 \begin{bmatrix} 1 \\ 0 \\ 3 \\ 3 \end{bmatrix}$, $H_2S \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}$ - boron (B) - Sulphur (S) - Hydrogen (H) - Oxygen (O)

The co-efficient in equation :

$x_1 B_2S_3 + x_2 H_2O \longrightarrow x_3 H_3BO_3 + x_4 H_2S$

$$x_1 \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 \\ 0 \\ 3 \\ 3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}$$

Moving the right terms to the left side; and writing in augmented form;

$$\left[\begin{array}{cccc|c} 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{array}\right]$$

$R_1 \div 2$; $2R_2 - 3R_1$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1/2 & 0 & 0 \\ 0 & 0 & 3/2 & -1 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{array}\right]$$

Exchanging R_2 and R_3

$$\left[\begin{array}{cccc:c} 1 & 0 & 1/2 & 0 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 0 & 3/2 & -1 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{array}\right]$$

dividing R_2 by 2

$$\left[\begin{array}{cccc:c} 1 & 0 & -1/2 & 0 & 0 \\ 0 & 1 & -3/2 & -2 & 0 \\ 0 & 0 & 3/2 & -1 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{array}\right]$$

$R_2 + R_3$; $R_3 \times 2/3$

$$\left[\begin{array}{cccc:c} 1 & 0 & -1/2 & 0 & 0 \\ 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{array}\right]$$

$\frac{1}{2} R_3 + R_1$; $R_2 - R_4$

$$\left[\begin{array}{cccc:c} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 0 & 3 & -2 & 0 \end{array}\right]$$

$3R_3 - R_4$

$$\left[\begin{array}{cccc:c} 1 & 0 & 0 & 1/3 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

Thus the general solution is,

DATE: ___/___/___

$x_1 = 1/3\ x_4$

$x_2 = 2x_4$

$x_3 = 2/3\ x_4$

with x_4 = free

let $x_4 = 3$

than ;

$x_1 = 1 \qquad x_2 = 6 \qquad x_3 = 2$

The balance equation will be:

$B_2S_3 + 6H_2O \rightarrow 2H_3BO_3 + 3H_2S$

Question : 06

$Na_3PO_4 \begin{bmatrix} 3 \\ 1 \\ 4 \\ 0 \\ 0 \end{bmatrix}, Ba(NO_3)_2 \begin{bmatrix} 0 \\ 0 \\ 6 \\ 1 \\ 2 \end{bmatrix}, Ba_3(PO_4)_2 \begin{bmatrix} 0 \\ 2 \\ 8 \\ 3 \\ 6 \end{bmatrix}, NaNO_3 \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \\ 1 \end{bmatrix}$

co-efficient in eq are:

$$x_1 \begin{bmatrix} 3 \\ 1 \\ 4 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 6 \\ 1 \\ 2 \end{bmatrix} \longrightarrow x_3 \begin{bmatrix} 0 \\ 2 \\ 8 \\ 3 \\ 6 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \\ 1 \end{bmatrix}$$

move the right terms towards left and writing eq in augmented matrix form

$$\left[\begin{array}{cccc:c} 3 & 0 & 0 & -1 & 0 \\ 1 & 0 & -2 & 0 & 0 \\ 4 & 6 & -8 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{array}\right]$$

Sawaping r_1 and r_2

$$\left[\begin{array}{cccc:c} 1 & 0 & -2 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 4 & 6 & -8 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & -01 & 0 \end{array}\right]$$

$R_2 - 3R_1$; $R_3 - 4R_1$: $\cancel{R_5 - R_1}$

$$\left[\begin{array}{cccc:c} 1 & 0 & -2 & 0 & 0 \\ 0 & 0 & 6 & -1 & 0 \\ 0 & 6 & 0 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{array}\right]$$

Swaping r_2 and r_3

$$\left[\begin{array}{cccc:c} 1 & 0 & -2 & 0 & 0 \\ 0 & 6 & 0 & -3 & 0 \\ 0 & 0 & 6 & -1 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{array}\right]$$

$R_2 \div 6$. ; $R_3 \div 6$

$$\left[\begin{array}{cccc:c} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & -1/2 & 0 \\ 0 & 0 & 1 & -1/6 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{array}\right.$$

$R_1 + 2R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & -1/2 & 0 \\ 0 & 0 & 1 & -1/6 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{array}\right]$$

$R_4 - R_2 \quad ; \quad R_5 - 2R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & -1/2 & 0 \\ 0 & 0 & 1 & -1/6 & 0 \\ 0 & 0 & -3 & -1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_4 + 3R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & -1/2 & 0 \\ 0 & 0 & 1 & -1/6 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

general eq is :

$x_1 = 1/2\, x_4 \quad ; \quad x_3 = 1/6\, x_4 \,, \quad x_2 = 1/2\, x_4 \,, \quad x_4 = \text{free variable}$

if $x_4 = 6$

than Balance eq will be:

$2Na_3PO_4 + 3Ba(NO_3)_2 \longrightarrow Ba_3(PO_4)_2 + 6NaNO_3$

(Question. 8)

The following reaction between potassium permanganate ($KMnO_4$) and manganese sulfate in water produces manganese dioxide, potassium sulphate and sulphuric acid:

$$KMnO_4 + MnSO_4 + H_2O \longrightarrow MnO_2 + K_2SO_4 + H_2SO_4$$

Construct a vector that lists no of atoms of potassium, manganese, oxygen, sulphur and hydrogen.

Solution:-

Equation involves 5 types of atoms so construct vector in $\mathbb{R}^5$ for reactants and products.

$$KMnO_4: \begin{bmatrix}1\\1\\4\\0\\0\end{bmatrix}, \quad MnSO_4: \begin{bmatrix}0\\1\\4\\1\\0\end{bmatrix}, \quad H_2O: \begin{bmatrix}0\\0\\1\\0\\2\end{bmatrix}, \quad MnO_2: \begin{bmatrix}0\\1\\2\\0\\0\end{bmatrix}, \quad K_2SO_4: \begin{bmatrix}2\\0\\4\\1\\0\end{bmatrix}, \quad H_2SO_4: \begin{array}{c}\begin{bmatrix}0\\0\\4\\1\\2\end{bmatrix}\end{array}\begin{array}{l}\leftarrow \text{Potassium}\\ \leftarrow \text{Manganese}\\ \leftarrow \text{Oxygen}\\ \leftarrow \text{Sulphur}\\ \leftarrow \text{Hydrogen}\end{array}$$

To balance equation use coefficients $x_1, x_2, x_3, x_4, x_5, x_6$

So eq will be

$$x_1 KMnO_4 + x_2 MnSO_4 + x_3 H_2O \longrightarrow x_4 MnO_2 + x_5 K_2SO_4 + x_6 H_2SO_4 \quad \text{— eq (1}$$

or

$$x_1 \begin{bmatrix}1\\1\\4\\0\\0\end{bmatrix} + x_2 \begin{bmatrix}0\\1\\4\\1\\0\end{bmatrix} + x_3 \begin{bmatrix}0\\0\\1\\0\\2\end{bmatrix} = x_4 \begin{bmatrix}0\\1\\2\\0\\0\end{bmatrix} + x_5 \begin{bmatrix}2\\0\\4\\1\\0\end{bmatrix} + x_6 \begin{bmatrix}0\\0\\4\\1\\2\end{bmatrix}$$

System of eq are

1) $x_1 - 2x_5 = 0$

2) $x_1 + x_2 - x_4 = 0$

3) $4x_1 + 4x_2 + x_3 - 2x_4 - 4x_5 - 4x_6 = 0$

4) $x_2 - x_5 - x_6 = 0$

5) $2x_3 - 2x_6 = 0$

Write in form of augmented matrix

$$\left[\begin{array}{cccccc:c} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 1 & 1 & 0 & -1 & 0 & 0 & 0 \\ 4 & 4 & 1 & -2 & -4 & -4 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 2 & 0 & 0 & -2 & 0 \end{array}\right]$$

* Perform row operations to convert into echlean form

$-R_1 + R_2 \rightarrow R_2$

$-4R_1 + R_3 \rightarrow R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 2 & 0 & 0 \\ 0 & 4 & 1 & -2 & 4 & -4 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 2 & 0 & 0 & -2 & 0 \end{bmatrix}$$

$-4R_2 + R_3 \rightarrow R_3$

$-R_2 + R_4 \rightarrow R_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & -4 & -4 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 2 & 0 & 0 & -2 & 0 \end{bmatrix}$$

$-2R_3 + R_5 \rightarrow R_5$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & -4 & -4 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 0 & -4 & 8 & 6 & 0 \end{bmatrix}$$

$4R_4 + R_5 \rightarrow R_5$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & -4 & -4 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 0 & 0 & -4 & 2 & 0 \end{bmatrix}$$

Now convert into reduced echelon form

$R_5 \div -4 \rightarrow R_5$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & -4 & -4 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -0.5 & 0 \end{bmatrix}$$

$3R_5 + R_4 \rightarrow R_4$

$4R_5 + R_3 \rightarrow R_3$

$-2R_5 + R_2 \rightarrow R_2$

$2R_5 + R_1 \rightarrow R_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1.0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & -6 & 0 \\ 0 & 0 & 0 & 1 & 0 & -2.5 & 0 \\ 0 & 0 & 0 & 0 & 1 & -0.5 & 0 \end{bmatrix}$$

$-2R_4 + R_3 \rightarrow R_3$ and $R_4 + R_2 \rightarrow R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1.5 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -2.5 & 0 \\ 0 & 0 & 0 & 0 & 1 & -0.5 & 0 \end{bmatrix}$$

General Solution

$$x_1 - x_6 = 0 \Rightarrow x_1 = x_6$$
$$x_2 - 1.5x_6 = 0 \Rightarrow x_2 = 1.5x_6$$
$$x_3 - x_6 = 0 \Rightarrow x_3 = x_6$$
$$x_4 - 2.5x_6 = 0 \Rightarrow x_4 = 2.5x_6$$
$$x_5 - 0.5x_6 = 0 \Rightarrow x_5 = 0.5x_6$$

Here x_1, x_2, x_3, x_4, x_5 are basic variables and x_6 is free variable.
If we take $x_6 = 2$ then

$$x_1 = x_6 = 2$$
$$x_2 = 1.5(2) = 3$$
$$x_3 = x_6 = 2$$
$$x_4 = 2.5(2) = 5$$
$$x_5 = 0.5(2) = 1$$
$$x_6 = 2$$

So put all these values in eq (1) to get balanced equation

$$x_1 KMnO_4 + x_2 MnSO_4 + x_3 H_2O \rightarrow x_4 MnO_2 + x_5 K_2SO_4 + x_6 H_2SO_4$$

$$2KMnO_4 + 3MnSO_4 + 2H_2O \rightarrow 5MnO_2 + K_2SO_4 + 2H_2SO_4$$

(Question 9)

If possible use exact arithmetic or rational format for calculations in balancing following reaction

$$PbN_6 + CrMn_2O_8 \rightarrow Pb_3O_4 + Cr_2O_3 + MnO_2 + NO$$

Solution:

Equation involves 5 types of atoms so construct vector in $\mathbb{R}^5$.

$PbN_6: \begin{bmatrix}1\\6\\0\\0\\0\end{bmatrix}$, $CrMnO_8: \begin{bmatrix}0\\0\\1\\2\\8\end{bmatrix}$, $Pb_3O_4: \begin{bmatrix}3\\0\\0\\0\\4\end{bmatrix}$, $Cr_2O_3: \begin{bmatrix}0\\0\\2\\0\\3\end{bmatrix}$ $MnO_2: \begin{bmatrix}0\\0\\0\\1\\2\end{bmatrix}$,

$$NO: \begin{bmatrix}0\\1\\0\\0\\1\end{bmatrix} \begin{matrix}\leftarrow \text{Lead}\\ \leftarrow \text{Nitrogen}\\ \leftarrow \text{Chromium}\\ \leftarrow \text{Manganese}\\ \leftarrow \text{Oxygen}\end{matrix}$$

To balance eq use coefficents $x_1, x_2, x_3, x_4, x_5, x_6$ So eq will be

$$x_1 PbN_6 + x_2 CrMnO_8 \longrightarrow x_3 Pb_3O_4 + x_4 Cr_2O_3 + x_5 MnO_2 + x_6 NO$$

or

$$x_1\begin{bmatrix}1\\6\\0\\0\\0\end{bmatrix} + x_2\begin{bmatrix}0\\0\\1\\2\\8\end{bmatrix} + x_3\begin{bmatrix}-3\\0\\0\\0\\-4\end{bmatrix} + x_4\begin{bmatrix}0\\0\\-2\\0\\-3\end{bmatrix} + x_5\begin{bmatrix}0\\0\\0\\-1\\-2\end{bmatrix} + x_6\begin{bmatrix}0\\-1\\0\\0\\-1\end{bmatrix}$$

System of eq are

1) $x_1 - 3x_3 = 0$
2) $6x_1 - x_6 = 0$
3) $x_2 - 2x_4 = 0$
4) $2x_2 - x_5 = 0$
5) $8x_2 - 4x_3 - 3x_4 - 2x_5 - x_6 = 0$

Write in form of augmented matrix

$$\left[\begin{array}{cccccc|c} 1 & 0 & -3 & 0 & 0 & 0 & 0\\ 6 & 0 & 0 & 0 & 0 & -1 & 0\\ 0 & 1 & 0 & -2 & 0 & 0 & 0\\ 0 & 2 & 0 & 0 & -1 & 0 & 0\\ 0 & 8 & -4 & -3 & -2 & -1 & 0 \end{array}\right]$$

$-6R_1 + R_2 \to R_2$

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 18 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 \\ 0 & 8 & -4 & -3 & -2 & -1 & 0 \end{bmatrix}$$

Replace R_2 and R_3

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 18 & 0 & 0 & -1 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 \\ 0 & 8 & -4 & -3 & -2 & -1 & 0 \end{bmatrix}$$

$-2R_2 + R_4 \to R_4$

$-8R_2 + R_5 \to R_5$

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 18 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 4 & -1 & 0 & 0 \\ 0 & 0 & -4 & 13 & -2 & -1 & 0 \end{bmatrix}$$

$R_3 \div 18$

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 4 & -1 & 0 & 0 \\ 0 & 0 & -4 & 13 & 2 & -1 & 0 \end{bmatrix}$$

$4R_3 + R_5 \to R_5$

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 4 & -1 & 0 & 0 \\ 0 & 0 & 0 & 13 & -2 & -11/9 & 0 \end{bmatrix}$$

$R_4 \div 4$

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 1 & -1/4 & 0 & 0 \\ 0 & 0 & 0 & 13 & -2 & -11/9 & 0 \end{bmatrix}$$

$-13R_4 + R_5 \to R_5$

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 1 & -1/4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5/4 & -11/9 & 0 \end{bmatrix}$$

$R_5 \div 5/4$

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 1 & -1/4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -44/45 & 0 \end{bmatrix}$$

$1/4 R_5 + R_4 \to R_4$

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 1 & 0 & -11/45 & 0 \\ 0 & 0 & 0 & 0 & 1 & -44/45 & 0 \end{bmatrix}$$

$2R_4 + R_2 \to R_2$

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -22/45 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 1 & 0 & -11/45 & 0 \\ 0 & 0 & 0 & 0 & 1 & -44/45 & 0 \end{bmatrix}$$

$3R_3 + R_1 \to R_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1/6 & 0 \\ 0 & 1 & 0 & 0 & 0 & -22/45 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 1 & 0 & -11/45 & 0 \\ 0 & 0 & 0 & 0 & 1 & -44/45 & 0 \end{bmatrix}$$

General Solution:

1) $x_1 = (1/6)\, x_6$
2) $x_2 = (22/45)\, x_6$
3) $x_3 = (1/18)\, x_6$
4) $x_4 = (11/45)\, x_6$
5) $x_5 = (44/45)\, x_6$

Here x_1, x_2, x_3, x_4, x_5 are basic variables and x_6 is free.
If we take $x_6 = 90$ then

$$x_1 = \frac{1}{6}(90) = 15$$

$$x_2 = \frac{22}{45}(\overset{2}{90}) = 44$$

$$x_3 = \frac{1}{18}(\overset{5}{90}) = 5$$

$$x_4 = \left(\frac{11}{45}\right)(\overset{2}{90}) = 22$$

$$x_5 = \frac{44}{45}(90) = 88$$

$$x_6 = 90$$

Put all values in eq

$$15\,PbN_6 + 44\,CrMn_2O_8 \longrightarrow 5\,Pb_3O_4 + 22\,Cr_2O_3 + 88\,MnO_2 + 90\,NO$$

(Question. 10)

The chemical reaction below can be used in some industrial processes such as production of arsene. Use exact arithmetic or rational format for calculation

$$MnS + As_2Cr_{10}O_{35} + H_2SO_4 \longrightarrow HMnO_4 + AsH_3 + CrS_3O_{12} + H_2O$$

Solution

Equation involves 6 type of atoms so construct vector in $\mathbb{R}^6$ for reactants and products.

$$MnS: \begin{bmatrix}1\\1\\0\\0\\0\\0\end{bmatrix}, \quad As_2Cr_{10}O_{35}: \begin{bmatrix}0\\0\\2\\10\\35\\0\end{bmatrix}, \quad H_2SO_4: \begin{bmatrix}0\\1\\0\\0\\4\\2\end{bmatrix}, \quad HMnO_4: \begin{bmatrix}1\\0\\0\\0\\4\\1\end{bmatrix}$$

$$AsH_3: \begin{bmatrix}0\\0\\1\\0\\0\\3\end{bmatrix} \quad CrS_3O_{12}: \begin{bmatrix}0\\3\\0\\1\\12\\0\end{bmatrix} \quad H_2O: \begin{bmatrix}0\\0\\0\\0\\1\\2\end{bmatrix} \begin{matrix}\leftarrow \text{manganese}\\ \leftarrow \text{sulphur}\\ \leftarrow \text{Arsenic}\\ \leftarrow \text{Chromium}\\ \leftarrow \text{Oxygen}\\ \leftarrow \text{Hydrogen}\end{matrix}$$

To balance eq use coefficient $x_1, x_2, x_3, x_4, x_5, x_6, x_7$ So

$$x_1 MnS + x_2 As_2Cr_{10}O_{35} + x_3 H_2SO_4 \longrightarrow x_4 HMnO_4 + x_5 AsH_3 + x_6 CrS_3O_{12} + x_7 H_2O$$

or

$$x_1\begin{bmatrix}1\\1\\0\\0\\0\\0\end{bmatrix} + x_2\begin{bmatrix}0\\0\\2\\10\\35\\0\end{bmatrix} + x_3\begin{bmatrix}0\\1\\0\\0\\4\\2\end{bmatrix} + x_4\begin{bmatrix}-1\\0\\0\\0\\-4\\-1\end{bmatrix} + x_5\begin{bmatrix}0\\0\\-1\\0\\0\\-3\end{bmatrix} + x_6\begin{bmatrix}0\\-3\\0\\-1\\-12\\0\end{bmatrix} + x_7\begin{bmatrix}0\\0\\0\\0\\-1\\-2\end{bmatrix}$$

Scanned with CamScanner

System of eq are

1) $x_1 - x_4 = 0$

2) $x_1 + x_3 - 3x_6 = 0$

3) $2x_2 - x_5 = 0$

4) $10x_2 - x_6 = 0$

5) $35x_2 + 4x_3 - 4x_4 - 12x_6 - x_7 = 0$

6) $2x_3 - x_4 - 3x_5 - 2x_7 = 0$

Write in form of augmented matrix

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & -3 & 0 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 35 & 4 & -4 & 0 & -12 & -1 & 0 \\ 0 & 0 & 2 & -1 & -3 & 0 & -2 & 0 \end{array}\right]$$

$-R_1 + R_2 \rightarrow R_2$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 35 & 4 & -4 & 0 & -12 & -1 & 0 \\ 0 & 0 & 2 & -1 & -3 & 0 & -2 & 0 \end{bmatrix}$$

Interchange row R_2 and R_3

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 35 & 4 & -4 & 0 & -12 & -1 & 0 \\ 0 & 0 & 2 & -1 & -3 & 0 & -2 & 0 \end{bmatrix}$$

$R_2 \div 2$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 35 & 4 & -4 & 0 & -12 & -1 & 0 \\ 0 & 0 & 2 & -1 & -3 & 0 & -2 & 0 \end{bmatrix}$$

$-10R_2 + R_4 \longrightarrow R_4$

$-35R_2 + R_5 \longrightarrow R_5$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10/2 & 0 & 0 \\ 0 & 0 & 4 & -4 & 35/2 & -1 & 0 \\ 0 & 0 & 2 & -1 & -3 & -2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & -8 & 35/2 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 10/2 & -1 & 0 & 0 \\ 0 & 0 & 0 & -3 & -3 & 0 & -2 & 0 \end{bmatrix}$$

$R_4 \div -8$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -35/16 & 0 & 1/8 & 0 \\ 0 & 0 & 0 & 0 & 10/2 & -1 & 0 & 0 \\ 0 & 0 & 0 & -3 & -3 & 0 & -2 & 0 \end{bmatrix}$$

$3R_4 + R_6 \to R_6$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -35/16 & 0 & 1/8 & 0 \\ 0 & 0 & 0 & 0 & 10/2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -153/16 & 6 & -13/8 & 0 \end{bmatrix}$$

$R_5 \div \frac{10}{2}$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -35/16 & 0 & 1/8 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1/5 & 0 & 0 \\ 0 & 0 & 0 & 0 & -153/16 & 6 & -13/8 & 0 \end{bmatrix}$$

$\frac{153}{16} R_5 + R_6 \to R_6$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -35/16 & 0 & 1/8 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1/5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{327}{80} & -13/8 & 0 \end{bmatrix}$$

$R_6 \div \frac{327}{80}$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -35/16 & 0 & 1/8 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1/5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -\frac{130}{327} & 0 \end{bmatrix}$$

Scanned with CamScanner

$\frac{1}{5} R_6 + R_5 \longrightarrow R_5$

$3 R_6 + R_3 \longrightarrow R_3$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 390/27 & 0 \\ 0 & 0 & 0 & 1 & -35/16 & 0 & 1/18 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -26/327 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -130/327 & 0 \end{bmatrix}$$

$\frac{35}{16} R_5 + R_4 \longrightarrow R_4$, $\frac{1}{2} R_5 + R_2 \longrightarrow R_2$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -13/327 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & -390/327 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -16/327 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -26/327 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -130/327 & 0 \end{bmatrix}$$

$R_4 + R_1 \longrightarrow R_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -16/327 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -13/327 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & -390/327 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -16/327 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -26/327 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -130/327 & 0 \end{bmatrix}$$

General Solution

$x_1 = \frac{16}{327} x_7$

$x_2 = \frac{13}{327} x_7$

$$x_3 = \frac{390}{327} x_7$$

$$x_4 = \frac{16}{327} x_7$$

$$x_5 = \frac{26}{327} x_7$$

$$x_6 = \frac{130}{327} x_7$$

x_7 is free

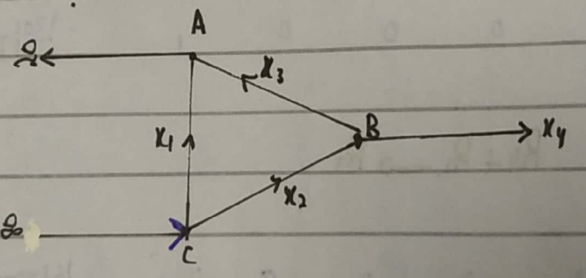
Take $x_7 = 327$

Eq will be

$$16MnS + 13As_2Cr_{10}O_{35} + 390H_2SO_4 \rightarrow 16HMnO_4 + 26AsH_3 + 130CrS_3O_{12} + 327H_2$$

(Question.11)

Find general flow pattern of the network shown in figure. Assuming that the flows are all non-negative what is largest possible value for x_3?



Solution

System of Equations are

$x_1 + x_3 = 20$

$x_2 = x_3 + x_4 \Rightarrow x_2 - x_3 - x_4 = 0$

$80 = x_1 + x_2 \Rightarrow x_1 + x_2 = 80$

* As total flow into the network equals total flow out of network so

Total flow $80 = x_4 + 20 \Rightarrow x_4 = 60$

Now write in form of augmented matrix

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & -1 & 0 \\ 1 & 1 & 0 & 0 & 80 \\ 0 & 0 & 0 & 1 & 60 \end{array}\right]$$

$-R_1 + R_3 \rightarrow R_3$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 60 \\ 0 & 0 & 0 & 1 & 60 \end{bmatrix}$$

$-R_2 + R_3 \rightarrow R_3$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 60 \\ 0 & 0 & 0 & 1 & 60 \end{bmatrix}$$

$-R_3 + R_4 \rightarrow R_4$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 60 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 + R_2 \rightarrow R_2$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & 0 & 60 \\ 0 & 0 & 0 & 1 & 60 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

General Solution:

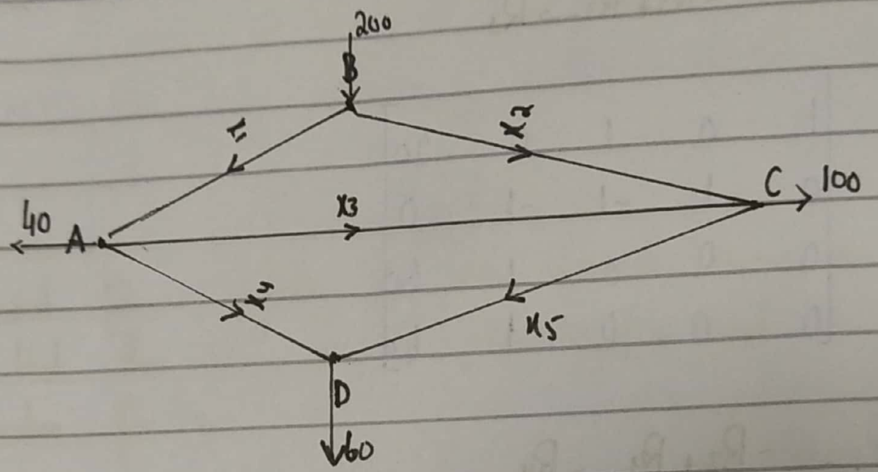
$$\begin{cases} x_1 = 20 - x_3 \\ x_2 = 60 + x_3 \\ x_3 \text{ is free} \\ x_4 = 60 \end{cases}$$

(Question. 12)

a) Find general traffic pattern in freeway network shown in figure.

b) Describe general traffic pattern when the road whose flow is x_4 is closed.

c) When $x_4 = 0$ what is minimum value of x_1?



Solution:

1) $x_1 = x_3 + x_4 + 40 \longrightarrow x_1 - x_3 - x_4 = 40$

2) $200 = x_1 + x_2 \longrightarrow x_1 + x_2 = 200$

3) $x_2 + x_3 = x_5 + 100 \longrightarrow x_2 + x_3 - x_5 = 100$

4) $x_4 + x_5 = 60 \longrightarrow x_4 + x_5 = 60$

5) $200 = 100 + 40 + 60 = 200$

Write in form of augmented matrix

$$\begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 1 & 1 & 0 & 0 & 0 & 200 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix}$$

$-R_1 + R_2 \to R_2$

$$\begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix}$$

$-R_2 + R_3 \to R_3$

$$\begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 0 & 0 & -1 & -1 & -60 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix}$$

$-R_3$

$$\begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 0 & 0 & 1 & 1 & 60 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix}$$

$-R_3 + R_4 \to R_4$

$$\begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 0 & 0 & 1 & 1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$-R_3 + R_2 \to R_2$

$R_3 + R_1 \to R_1$

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 1 & 100 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Scanned with CamScanner

General Solution

$$\begin{cases} x_1 = 100 + x_2 - x_5 \\ x_2 = 100 - x_3 + x_5 \\ x_4 = 60 - x_5 \\ x_3, x_5 \text{ is free} \end{cases}$$

Here x_1, x_2, x_4 are basic variables and x_3, x_5 are free

—(Part b)—

When road is closed it means $x_4 = 0$

$x_4 = 60 - x_5$

For x_4 to be zero x_5 must be 60

$x_4 = 60 - 60$

$x_4 = 0$

So

$$\begin{cases} x_1 = 100 + x_3 - 60 \Rightarrow x_1 = x_3 + 40 \\ x_2 = 160 - x_3 \\ x_4 = 0 \\ x_5 = 60 \\ x_3 \text{ is free} \end{cases}$$

—(Part c)—

If $x_4 = 0$, $x_5 = 60$

so

$x_1 = 40 + x_3$

Minimum value of x_1 is 40 when x_3 will be zero because x_3 cannot be negative

$x_1 = 40 + 0$

$x_1 = 40$

(Question. 13)

a) Find general flow pattern in network shown in figure.
b) Assuming that flow must be in direction indicated, find the minimum flows in branches denoted by x_2, x_3, x_4, x_5.

Solution:-

System of eq are

1) $x_2 + 30 = x_1 + 80 \Rightarrow x_1 - x_2 = -50$
2) $x_3 + x_5 = x_2 + x_4 \Rightarrow x_2 - x_3 + x_4 - x_5 = 0$
3) $x_5 - x_6 = 60$
4) $40 + x_4 = x_6 + 90 \Rightarrow x_4 - x_6 = 50$
5) $60 + x_1 = x_3 + 20 \Rightarrow x_1 - x_3 = -40$

Total flow $30 + 60 + 100 + 40 = 80 + 20 + 40 + 90$

$230 = 230$

Write in form of augmented matrix

$$\left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 1 & 0 & -1 & 0 & 0 & 0 & -40 \end{array}\right]$$

$-R_1 + R_5 \to R_5$

$$\left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 1 & -1 & 0 & 0 & 0 & 10 \end{array}\right]$$

$-R_2 + R_5 \to R_5$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & -1 & 1 & 0 & 10 \end{bmatrix}$$

Interchange R_3 and R_4

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & -1 & 1 & 0 & 10 \end{bmatrix}$$

$R_3 + R_5 \to R_5$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \end{bmatrix}$$

$-R_4 + R_5 \to R_5$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_4 + R_2 \to R_2$ then $-R_3 + R_2 \to R_2$ and $R_2 + R_1 \to R_1$

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & -40 \\ 0 & 1 & -1 & 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

General Solution:

$$\begin{cases} x_1 = -40 + x_3 \\ x_2 = x_3 + 10 \\ x_4 = 50 + x_6 \\ x_5 = x_6 + 60 \\ x_3 \text{ is free}, \; x_6 \text{ is free} \end{cases}$$

(Part b)

1) x_1 cannot be negative it means x_3 should be greater than equal to 40.

$x_3 \geq 40$

2) So As $x_3 \geq 40$

$x_2 = 40 + 10 \qquad \therefore x_3 = 40$

$= 50$

$x_2 \geq 50$

3) As x_6 cannot be negative so minimum value of x_4 will be 50 when $x_6 = 0$

4) Similary minimum value of x_5 will be 60 when $x_6 = 0$

So minimum flows are

$x_2 = 50$

$x_3 = 40$

$x_4 = 50$

$x_5 = 60$

(Question. 14)

Intersection in England are often constructed as one-way 'roundabouts' such as in figure. Assume that traffic must travel in directions shown. Find general solution of network flow. Find smallest possible value for x_6

Solution:

1) $x_1 = 100 + x_2 \Rightarrow x_1 - x_2 = 100$

2) $x_2 + 50 = x_3 \Rightarrow x_2 - x_3 = -50$

3) $x_3 = 120 + x_4 \Rightarrow x_3 - x_4 = 120$

4) $x_4 + 150 = x_5 \Rightarrow x_4 - x_5 = -150$

5) $x_5 = 80 + x_6 \Rightarrow x_5 - x_6 = 80$

7) $x_6 + 100 = x_1 \Rightarrow -x_1 + x_6 = -100$

Total flow $300 = 300$

Write in form of augmented matrix

$$\left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & 0 & 100 \\ 0 & 1 & -1 & 0 & 0 & 0 & -50 \\ 0 & 0 & 1 & -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ -1 & 0 & 0 & 0 & 0 & 1 & -100 \end{array}\right]$$

$R_1 + R_6 \rightarrow R_6$

$$\left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & 0 & 100 \\ 0 & 1 & -1 & 0 & 0 & 0 & -50 \\ 0 & 0 & 1 & -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$R_2 + R_6 \to R_6$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 100 \\ 0 & 1 & -1 & 0 & 0 & 0 & -50 \\ 0 & 0 & 1 & -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ 0 & 0 & -1 & 0 & 0 & 1 & -50 \end{bmatrix}$$

$R_3 + R_6 \to R_6$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 100 \\ 0 & 1 & -1 & 0 & 0 & 0 & -50 \\ 0 & 0 & 1 & -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ 0 & 0 & 0 & -1 & 0 & 1 & 70 \end{bmatrix}$$

$R_4 + R_6 \to R_6$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 100 \\ 0 & 1 & -1 & 0 & 0 & 0 & -50 \\ 0 & 0 & 1 & -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ 0 & 0 & 0 & 0 & -1 & 1 & -80 \end{bmatrix}$$

$R_5 + R_6 \to R_6$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 100 \\ 0 & 1 & -1 & 0 & 0 & -50 \\ 0 & 0 & 1 & -1 & 0 & 120 \\ 0 & 0 & 0 & 1 & 0 & -150 \\ 0 & 0 & 0 & 0 & -1 & 80 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_5 + R_4 \to R_4$

then

$R_4 + R_3 \to R_3$

then

$R_3 + R_2 \to R_2$

then

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 0 & -1 & 100 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 50 \\ 0 & 0 & 0 & 1 & 0 & -1 & -70 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

General Solution:

$$\begin{cases} x_1 = 100 + x_6 \\ x_2 = x_6 \\ x_3 = x_6 + 50 \\ x_4 = x_6 - 70 \\ x_5 = x_6 + 80 \\ x_6 \text{ is free} \end{cases}$$