

There are a total of five problems. You have to solve all the problems.

Problem 1 (CO1): DFA and Regular Languages (15 points)

Let  $\Sigma = \{a, b\}$ . Consider the following languages over  $\Sigma$ .

$$\begin{aligned} L_1 &= \{w : \text{length of } w \text{ is three more than multiple of four}\} \\ L_2 &= \{w : \text{every even position letter in } w \text{ is the same as the first letter of } w\} \\ L_3 &= \{w : \text{every } 2k+1 \text{ position in } w \text{ is } a, \text{ where } k \geq 0\} \\ L_4 &= \{w : \text{every } 2k+1 \text{ position in } w \text{ is } b, \text{ where } k \geq 0\} \end{aligned}$$

Now solve the following problems.

- (a) Give the state diagram for a DFA that recognizes  $L_1$ . (3 points)
- (b) Give the state diagram for a DFA that recognizes  $L_2$ . (3 points)
- (c) Give the state diagram for a DFA that recognizes  $L_3$ . (3 points)
- (d) If you were to use the “cross product” construction to obtain a DFA for the language  $L_2 \cap (L_3 \cup L_4)$ , how many states would it have? (1 point)
- (e) Find all four-letter strings in  $L_2 \cap (L_3 \cup L_4)$ . (1 point)
- (f) Give the state diagram for a DFA that recognizes  $L_2 \cap (L_3 \cup L_4)$  using only four states. (2 points)
- (g) Find a four-letter string in  $\overline{L_3} \circ L_4$ . [Recall:  $\overline{L}$  denotes the complement of the language  $L$  i.e.,  $\overline{L} = \Sigma^* - L$ ] (1 point)
- (h) Is  $\overline{L_3} \circ L_4 = \overline{L_3}$ ? Give justification for your answer. (1 point)

There are a total of five problems. You have to solve the first four. Problem 5 is optional.

Problem 1 (CO1): DFA and Regular Languages (15 points)

Let  $\Sigma = \{0, 1\}$ . Consider the following languages over  $\Sigma$ . Note that we define  $0^m$  to be the string  $\overbrace{000 \dots 000}^{m \text{ times}}$ .  $1^n$  is defined analogously.

$$L_1 = \{w : w \text{ does not contain } 01 \text{ as a substring}\}$$

$$L_2 = \{0^m : m \text{ is even}\}$$

$$L_3 = \{1^n : n \geq 0\}$$

$$L_4 = L_2 \circ L_3$$

Now solve the following problems.

- (a) **Give** the state diagram for a DFA that recognizes  $L_1$ . (4 points)
- (b) **Give** the state diagram for a DFA that recognizes  $L_2$ . (4 points)
- (c) **Find** all the four and five-letter strings in  $L_4$ . (1 point)
- (d) **Give** the state diagram for a DFA that recognizes  $L_4$ . (2 points)
- (e) If you were to use the “cross product” construction shown in class to obtain a DFA for the language  $L_1 \cap L_4$ , how many states would it have? (1 point)
- (f) **Find** all five-letter strings in  $L_1 \cap L_4$ . (1 point)
- (g) **Give** the state diagram for a DFA that recognizes  $L_1 \cap L_4$  using only five states. (2 points)

Problem 1 (CO1): DFA and Regular Languages (15 points)

Let  $\Sigma = \{0, 1\}$ . Consider the following languages over  $\Sigma$ .

$$L_1 = \{w \text{ starts with } 10\}$$

$$L_2 = \{w \text{ doesn't contain } 11\}$$

$$L_3 = \{w \text{ doesn't contain } 00\}$$

$$L_4 = \{w = 10\}$$

$$L_5 = L_2 \cap L_3$$

Now solve the following problems.

- (a) **Give** the state diagram for a DFA that recognizes  $L_1$ . (3 points)
- (b) **Give** the state diagram for a DFA that recognizes  $L_2$ . (3 points)
- (c) If you were to use the “cross product” construction shown in class to obtain a DFA for the language  $L_5$ , how many states would it have? (1 point)
- (d) **Find** all four-letter strings in  $L_5$ . (1 point)
- (e) **Give** the state diagram for a DFA that recognizes  $L_5$  using only four states. (2 points)
- (f) **Find** one six-letter string in  $L_4^*$ . (1 point)
- (g) **Give** the state diagram for a DFA that recognizes  $L_4^*$ . (2 points)
- (h) Is  $L_4^*$  and  $L_1 \cap L_5$  same? **Give** justification for your answer. (2 points)

Spring 23 set 2

Problem 1 (CO1): DFA and Regular Languages (10 points)

Let  $\Sigma = \{0, 1\}$ . Consider the following languages over  $\Sigma$ .

$$L_1 = \{w : w = 1^m 0^n, \text{ where } m, n \geq 0\}$$

$$L_2 = \{w : 1 \text{ does not appear at any even position in } w\}$$

$$L_3 = L_1 \cap L_2$$

Now solve the following problems.

- (a) **Give** the state diagram for a DFA that recognizes  $L_1$ . (3 points)
- (b) **Give** the state diagram for a DFA that recognizes  $L_2$ . (3 points)
- (c) If you were to use the “cross product” construction shown in class to obtain a DFA for the language  $L_3$ , how many states would it have? (1 point)
- (d) **Find** all four-letter strings in  $L_3$ . (1 point)
- (e) **Give** the state diagram for a DFA that recognizes  $L_3$  using only three states. (2 points)

Spring 22

**Problem 2: Constructing a DFA (10 points)**

Consider the following language.

$$L = \{w \in \{0, 1\}^* : w = 0^m 1^n \text{ where } m \text{ and } n \text{ are either both even or both odd}\}$$

(a) Write down the strings  $w \in L$  such that the length of  $w$  is six. (2 points)

(b) Consider the following pair of languages.

$$L_1 = \{w \in \{0, 1\}^* : w = 0^m 1^n \text{ where } m \text{ and } n \text{ are both even}\},$$

$$L_2 = \{w \in \{0, 1\}^* : w = 0^m 1^n \text{ where } m \text{ and } n \text{ are both odd}\}.$$

Notice that  $L = L_1 \cup L_2$ . So, one way of designing a DFA for  $L$  would be to construct DFA for  $L_1$  and  $L_2$  and combine them using the “cross-product” construction shown in class.

Construct a DFA for  $L_1$ . (5 points)

(c) If you were to construct a DFA for  $L$  using the method described in (b), how many states would it have? Your answer should only be a number. (1 point)

(d) However, there is a DFA for  $L$  using at most seven states. Find that DFA. (2 points)

## Spring 24 set1

**Problem 1 (CO1): DFA and Regular Languages (15 points)**

Let  $\Sigma = \{0, 1\}$ . Consider the following languages over  $\Sigma$ .

$$L_1 = \{w \text{ starts with } 01\}$$

$$L_2 = \{w \text{ doesn't contain } 00\}$$

$$L_3 = \{w \text{ doesn't contain } 11\}$$

$$L_4 = \{w = 01\}$$

Now solve the following problems.

(a) **Give** the state diagram for a DFA that recognizes  $L_1$ . (3 points)

(b) **Give** the state diagram for a DFA that recognizes  $L_2$ . (3 points)

(c) If you were to use the “cross product” construction shown in class to obtain a DFA for the language  $L_2 \cap L_3$ , how many states would it have? (1 point)

(d) **Find** all four-letter strings in  $L_2 \cap L_3$ . (1 point)

(e) **Give** the state diagram for a DFA that recognizes  $L_2 \cap L_3$  using only four states. (2 points)

(f) **Find** one six-letter string in  $L_4^*$ . (1 point)

(g) **Give** the state diagram for a DFA that recognizes  $L_4^*$ . (2 points)

(h) Is  $L_4^*$  and  $L_1 \cap L_2 \cap L_3$  same? **Give** justification for your answer. (2 points)

## Spring 24 set2

Problem 1 (CO1): DFA and Regular Languages (10 points)

Let  $\Sigma = \{0, 1\}$ . Consider the following languages over  $\Sigma$ .

$$L_1 = \{w : w = 1^m 0^n, \text{ where } m, n \geq 0\}$$
$$L_2 = \{w : 1 \text{ does not appear at any even position in } w\}$$
$$L_3 = L_1 \cap L_2$$

Now solve the following problems.

- (a) **Give** the state diagram for a DFA that recognizes  $L_1$ . (3 points)
- (b) **Give** the state diagram for a DFA that recognizes  $L_2$ . (3 points)
- (c) If you were to use the “cross product” construction shown in class to obtain a DFA for the language  $L_3$ , how many states would it have? (1 point)
- (d) **Find** all four-letter strings in  $L_3$ . (1 point)
- (e) **Give** the state diagram for a DFA that recognizes  $L_3$  using only three states. (2 points)

Fall 24 Set B

Problem 1 (CO1): DFA and Regular Languages (15 points)

Let  $\Sigma = \{0, 1\}$ . Consider the following languages over  $\Sigma$ .

$$L_1 = \{w : \text{length of } w \text{ is exactly three}\}$$
$$L_2 = \{w : \text{every even position in } w \text{ is } 1\}$$
$$L_3 = \{w : 10 \text{ appears even number of times in } w \text{ as a substring}\}$$
$$L_4 = L_1 \cap L_2 \cap L_3$$
$$L_5 = \{w : 1^m 0^n, \text{ where } m, n \geq 0\}$$

Now solve the following problems.

- (a) **Give** the state diagram for a DFA that recognizes  $L_1$ . (3 points)
- (b) **Give** the state diagram for a DFA that recognizes  $L_2$ . (3 points)
- (c) **Give** the state diagram for a DFA that recognizes  $L_3$ . (3 points)
- (d) If you were to use the “cross product” construction shown in class to obtain a DFA for the language  $L_4$ , how many states would it have? (1 point)
- (e) **Find** all the strings in  $L_4$ . (1 point)
- (f) **Give** the state diagram for a DFA that recognizes  $L_4$  using only five states. (2 points)
- (g) Is  $L_4$  a subset of  $L_5$ ? **Give** justification for your answer. (2 points)

Fall 24 Set A

Problem 1 (CO1): DFA and Regular Languages (15 points)

Let  $\Sigma = \{0, 1\}$ . Consider the following languages over  $\Sigma$ .

$$L_1 = \{w : \text{length of } w \text{ is exactly three}\}$$

$$L_2 = \{w : \text{every even position in } w \text{ is } 0\}$$

$$L_3 = \{w : 01 \text{ appears even number of times in } w \text{ as a substring}\}$$

$$L_4 = L_1 \cap L_2 \cap L_3$$

$$L_5 = \{w : 0^m 1^n, \text{ where } m, n \geq 0\}$$

Now solve the following problems.

- (a) **Give** the state diagram for a DFA that recognizes  $L_1$ . (3 points)
- (b) **Give** the state diagram for a DFA that recognizes  $L_2$ . (3 points)
- (c) **Give** the state diagram for a DFA that recognizes  $L_3$ . (3 points)
- (d) If you were to use the “cross product” construction shown in class to obtain a DFA for the language  $L_4$ , how many states would it have? (1 point)
- (e) **Find** all the strings in  $L_4$ . (1 point)
- (f) **Give** the state diagram for a DFA that recognizes  $L_4$  using only five states. (2 points)
- (g) Is  $L_4$  a subset of  $L_5$ ? **Give** justification for your answer. (2 points)

Fall 23 Set K

## DFA Set A

### Problem 1 (CO1): DFA and Regular Languages (15 points)

We define the last two digits of your Student ID to be AB [e.g: If your Student ID is 2102895, then A = 9, B = 5]

Given,  $\Sigma = \{A, B, \#\}$ . Consider the following languages over  $\Sigma$ .

$$L_1 = \{w : w \text{ starts with } A\}$$

$$L_2 = \{w : w \text{ contains } AB\# \text{ as a substring}\}$$

$$L_3 = L_1 \circ L_2$$

Now solve the following problems. For questions (a)-(f), you must use your specific  $\Sigma$  to answer.

- If  $\Sigma = \{A, B, \#\}$ , then **define**  $\Sigma$  according to your Student ID. (1 point)
- Give the state diagram for a DFA that recognizes  $L_1$ . (3 points)
- Give the state diagram for a DFA that recognizes  $L_2$ . (3 points)
- Find all the four-letter strings in  $L_1 \cap L_2$ . (2 points)
- If you were to use the "cross product" construction shown in class to obtain a DFA for the language  $L_1 \cap L_2$ , how many states would it have? (1 point)
- Prove**  $L_3$  is a regular language by giving the state diagram for a DFA or an NFA that recognizes  $L_3$ . (2 points)

Now, let  $\Sigma = \{0, 1\}$ . Consider the following diagram of the NFA to answer the questions (g)-(h) defined for  $\Sigma$ .



- Choose the language recognized by this NFA? (1 point)
  - $\{w : w \text{ has a length equal to or more than three.}\}$
  - $\{w : w = (010)^n, n \geq 0\}$
  - $\{w : w \text{ contains } 010 \text{ as a subsequence}\}$
  - $\{w : w \text{ contains } 010 \text{ as a substring}\}$
- Select the paths that accepts 010110 in the given NFA? There can be more than one path that accepts the string. (2 points)
  - $a \rightarrow b \rightarrow b \rightarrow b \rightarrow b \rightarrow c \rightarrow d$
  - $a \rightarrow b \rightarrow c \rightarrow d \rightarrow d \rightarrow d \rightarrow d$
  - $a \rightarrow b \rightarrow b \rightarrow b \rightarrow b \rightarrow b \rightarrow b$
  - $a \rightarrow a \rightarrow b \rightarrow b \rightarrow c \rightarrow c \rightarrow d$
  - $a \rightarrow a \rightarrow a \rightarrow b \rightarrow c \rightarrow c \rightarrow d$

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**Problem 1 (CO1): DFA and Regular Languages (10 points)**

Let  $\Sigma = \{0, 1\}$ . Consider the following languages over  $\Sigma$ .

$$L_1 = \{w : w \text{ starts with either } 01 \text{ or } 10\}$$

$$L_2 = \{w : w \text{ does not start with } 11\}$$

$$L_3 = \{w : \text{the length of } w \text{ is at least two}\}$$

Now solve the following problems.

- (a) Give the state diagram for a DFA that recognizes  $L_1$ . (3 points)
  - (b) Give the state diagram for a DFA that recognizes  $L_2$ . (3 points)
  - (c) Give the state diagram for a DFA that recognizes  $L_3$ . (2 points)
  - (d) Give the state diagram for a DFA that recognizes  $\overline{L_1} \cap L_2 \cap L_3$  using only four states. Here  $\overline{L}$  denotes the complement of the language  $L$  i.e.,  $\overline{L} = \Sigma^* - L$ . (2 points)
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**Fall 22 Set 1**

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**Problem 1 (CO1): DFA and Regular Languages (10 points)**

Let  $\Sigma = \{0, 1\}$ . Consider the following languages over  $\Sigma$ .

$$L_1 = \{w : \text{the length of } w \text{ is at most three}\}$$

$$L_2 = \{w : w \text{ starts and ends with different letters}\}$$

$$L_3 = \{w : \text{the length of } w \text{ is at least two}\}$$

Now solve the following problems.

- (a) Give the state diagram for a DFA that recognizes  $L_1$ . (2 points)
  - (b) Give the state diagram for a DFA that recognizes  $L_2$ . (3 points)
  - (c) Give the state diagram for a DFA that recognizes  $L_3$ . (2 points)
  - (d) Find a shortest string in  $\overline{L_1} \cap L_3$ . Here  $\overline{L}$  denotes the complement of the language  $L$  i.e.,  $\overline{L} = \Sigma^* - L$ . (1 point)
  - (e) If you were to use the “cross product” construction shown in class to obtain a DFA for the language  $L_2 \cap L_3$ , how many states would it have? (1 point)
  - (f) How many states does the smallest DFA for  $L_2 \cap L_3$  have? (1 point)
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**Summer 22**



**Problem 1: Finite Automata and the Regular Operations (10 points)**

Let  $\Sigma = \{0, 1, \#\}$ . Consider the following two languages.

$$L_1 = \{w \in \Sigma^* : w \text{ does not contain } \# \text{ and the number of 0s in } w \text{ is not a multiple of } 3\}$$

$$L_2 = \{w \in \Sigma^* : \text{the substring between any two successive occurrences of } \# \text{ in } w \text{ is in } L_1\}$$

Now solve the following problems.

- (a) Write down a string  $w \in L_2$  such that the length of  $w$  is ten. (1 point)
- (b) Give the state diagram for a DFA that recognizes  $L_1$ . (4 points)
- (c) Give the state diagram for a DFA that recognizes  $L_2$ . (3 points)
- (d) If you use the “cross product” construction shown in class to obtain a DFA for  $L_1 \cap L_2$ , how many states will it have? (1 point)
- (e) Give an upper bound on the number of states in the smallest DFA that recognizes  $L_1 \cap L_2$ . (1 point)

Summer 23

**Problem 1 (CO1): DFA and Regular Languages (15 points)**

Let  $\Sigma = \{0, 1\}$ . Consider the following languages over  $\Sigma$ . Note that we define  $0^m$  to be the string  $\overbrace{000 \dots 000}^{m \text{ times}}$ .  $1^n$  is defined analogously.

$$L_1 = \{w : w \text{ does not contain } 01 \text{ as a substring}\}$$

$$L_2 = \{0^m : m \text{ is even}\}$$

$$L_3 = \{1^n : n \geq 0\}$$

$$L_4 = L_2 \circ L_3$$

Now solve the following problems.

- (a) Give the state diagram for a DFA that recognizes  $L_1$ . (4 points)
- (b) Give the state diagram for a DFA that recognizes  $L_2$ . (4 points)
- (c) Find all the four and five-letter strings in  $L_4$ . (1 point)
- (d) Give the state diagram for a DFA that recognizes  $L_4$ . (2 points)
- (e) If you were to use the “cross product” construction shown in class to obtain a DFA for the language  $L_1 \cap L_4$ , how many states would it have? (1 point)
- (f) Find all five-letter strings in  $L_1 \cap L_4$ . (1 point)
- (g) Give the state diagram for a DFA that recognizes  $L_1 \cap L_4$  using only five states. (2 points)

RE

## Spring 25Set A

Automata and Computability

DURATION: 90 MINUTES



### Problem 4 (CO1): Regular Expressions (10 points)

Let  $\Sigma = \{0, 1\}$ . Consider the following languages over  $\Sigma$ .

$$\begin{aligned} L_1 &= \{w : \text{length of } w \text{ is exactly } 4\} \\ L_2 &= \{w : \text{the third last digit of } w \text{ is } 0\} \\ L_3 &= \{w : w \text{ contains at most two } 11\} \\ L_4 &= \overline{L_1^*} \cap \overline{L_2} \end{aligned}$$

Now solve the following problems.

- (a) Give a regular expression for the language  $L_1$ . (1 point)
- (b) Give a regular expression for the language  $L_1^*$ . (1 point)
- (c) Give a regular expression for the language  $\overline{L_1^*}$ . [Recall:  $\overline{L}$  denotes the complement of the language  $L$  i.e.,  $\overline{L} = \Sigma^* - L$ ] (2 points)
- (d) Give a regular expression for the language  $L_2$ . (2 points)
- (e) Give a regular expression for the language  $\overline{L_3}$ . (2 points)
- (f) Give a regular expression for the language  $L_4$ . (2 points)

## Summe 23Set 2

### Problem 2 (CO1): Regular Expressions (15 points)

Let  $\Sigma = \{a, b\}$ . Give regular expressions generating each of the following languages over  $\Sigma$ .

- (a)  $\{w : \text{the first and last letters of } w \text{ are } a \text{ and } b \text{ respectively}\}$  (3 points)
- (b)  $\{w : \text{the length of } w \text{ is odd}\}$  (3 points)
- (c)  $\{w : \text{every } a \text{ in } w \text{ is followed by an even number of } b\text{'s}\}$  (3 points)
- (d)  $\{w : w \text{ does not contain } ab\}$  (3 points)
- (e)  $\{w : ab \text{ appears in } w \text{ exactly once}\}$  (3 points)  
(Hint: If  $w = xaby$ , what can you say about  $x$  and  $y$ ?)

## Spring 24 Set 1

Problem 2 (CO1): Regular Expressions (15 points)

Let  $\Sigma = \{0, 1\}$ . Consider the following languages over  $\Sigma$ .

$$L_1 = \{w \text{ contains exactly two } 1\}$$

$$L_2 = \{w \text{ doesn't start with } 0\}$$

$$L_3 = \{\text{every third position in } w \text{ is } 1\}$$

$$L_4 = \{\text{every } 1 \text{ in } w \text{ is followed by at least two } 0\}$$

$$L_5 = L_3 \cap L_4$$

Now solve the following problems.

- (a) Give a regular expression for the language  $L_1$ . (3 points)
- (b) Give a regular expression for the language  $L_2$ . (3 points)
- (c) Give a regular expression for the language  $L_3$ . (3 points)
- (d) Write a five-letter string that belongs to  $L_5$ . (1 point)
- (e) Give a regular expression for the language  $L_5$ . (2 points)
- (f) Give a regular expression for the language  $\bar{L}_4$ . Here  $\bar{L}$  denotes the complement of the language  $L$  i.e.,  $\bar{L} = \Sigma^* - L$ . (3 points)

Spring 23 Set 2

Problem 2 (CO1): Regular Expressions (10 points)

Consider the following languages over  $\Sigma = \{0, 1\}$ .

$$L_1 = \{w : w \text{ does not contain } 00\}$$

$$L_2 = \{w : \text{every } 0 \text{ in } w \text{ is preceded by at least one } 1\}$$

$$L_3 = \{w : \text{the number of times } 0 \text{ appears in } w \text{ is even}\}$$

Now solve the following problems.

- (a) Give a regular expression for the language  $L_1$ . (2 points)
- (b) Your friend claims that  $L_1 = L_2$ . **Prove** him wrong by writing down a five-letter string in  $L_1 \setminus L_2$ . Recall that  $L_1 \setminus L_2$  contains all strings that are in  $L_1$  but not in  $L_2$ . (2 points)
- (c) Give a regular expression for the language  $L_1 \setminus L_2$ . (2 points)
- (d) Give a regular expression for the language  $L_3$ . (2 points)
- (e) Give a regular expression for the language  $L_2 \setminus L_3$ . (2 points)

Spring 24 Set 1

(h) Is  $L_4^*$  and  $L_1 \cap L_2 \cap L_3$  same? Give justification for your answer. (2 points)

**Problem 2 (CO1): Regular Expressions (15 points)**

Let  $\Sigma = \{0, 1\}$ . Consider the following languages over  $\Sigma$ .

$$L_1 = \{w \text{ contains exactly two } 1\}$$

$$L_2 = \{w \text{ doesn't start with } 0\}$$

$$L_3 = \{w \text{ every third position in } w \text{ is } 1\}$$

$$L_4 = \{w \text{ every } 1 \text{ in } w \text{ is followed by at least two } 0\}$$

$$L_5 = L_3 \cap L_4$$

Now solve the following problems.

- (a) Give a regular expression for the language  $L_1$ . (3 points)
- (b) Give a regular expression for the language  $L_2$ . (3 points)
- (c) Give a regular expression for the language  $L_3$ . (3 points)
- (d) Write a five-letter string that belongs to  $L_5$ . (1 point)
- (e) Give a regular expression for the language  $L_5$ . (2 points)
- (f) Give a regular expression for the language  $\bar{L}_4$ . Here  $\bar{L}$  denotes the complement of the language  $L$  i.e.,  $\bar{L} = \Sigma^* - L$ . (3 points)

Spring 22

**Problem 1: Regular Expressions (10 points)**

Write down regular expressions for each of the following languages. Assume that  $\Sigma = \{0, 1\}$ .

- (a) The language containing strings where 0s and 1s alternate. (3 points)
- (b) The language containing strings in which the number of 1s is divisible by 4. (3 points)
- (c) The language containing strings in which the number of 0s between every pair of consecutive 1s is even. (4 points)

Fall 24 set 1

**Problem 3 (CO1): Regular Expressions (15 points)**

Let  $\Sigma = \{0, 1\}$ . Consider the following languages over  $\Sigma$ .

$$L_1 = \{w \text{ does not contain consecutive } 1\}$$

$$L_2 = \{w \text{ starts with } 0\}$$

$$L_3 = \{w \text{ starts and ends with the same character}\}$$

$$L_4 = L_2 \setminus L_3$$

Now solve the following problems.

- (a) Give a regular expression for the language  $L_1$ . (3 points)
- (b) Give a regular expression for the language  $\overline{L_2}$ . [Recall:  $\overline{L_2}$  denotes the complement of the language  $L_2$  i.e.,  $\overline{L_2} = \Sigma^* - L_2$ ] (3 points)
- (c) Give a regular expression for the language  $L_3$ . (3 points)
- (d) Write four four-letter strings in  $L_4$ . (2 point)
- (e) Give a regular expression for the language  $L_4$ . [Recall:  $L_2 \setminus L_3$  contains all strings that are in  $L_2$  but not in  $L_3$ ] (2 points)
- (f) Give a regular expression for the language  $\overline{L_4}$ . (2 points)

## Fall 22 set 2

**Problem 2 (CO1): Regular Expressions (10 points)**

Let  $\Sigma = \{0, 1\}$ . Consider the following pair of languages over  $\Sigma$ .

$$L_1 = \{w : w \text{ contains } 11 \text{ as a substring}\}$$

$$L_2 = \{w : w \text{ contains } 10 \text{ as a substring}\}$$

Now solve the following problems.

- (a) Write down a regular expression for the language  $L_1$ . (2 points)
- (b) Write down a regular expression for the language  $L_2$ . (2 points)
- (c) Your friend wants a regular expression for the language  $\overline{L_1 \cap L_2}$  where  $\overline{L}$  denotes the complement of the language  $L$  i.e.,  $\overline{L} = \Sigma^* - L$ . He wants your help. You tell him to make use of the fact  $\overline{L_1 \cap L_2} = \overline{L_1} \cup \overline{L_2}$ .
  - (i) Write down a regular expression for the language  $\overline{L_1}$ . (2 points)
  - (ii) Write down a regular expression for the language  $\overline{L_2}$ . (2 points)
  - (iii) Using the fact above, write down a regular expression for the language  $\overline{L_1 \cap L_2}$ . (2 points)

## Fall 23 set k

Problem 2 (CO1): Regular Expressions (15 points)

Let  $\Sigma = \{0, 1\}$ . Give regular expressions for each of the languages (a)-(f) over  $\Sigma$ .

- (a)  $\{w : w \text{ contains } 11 \text{ or } 101 \text{ as a substring.}\}$  (2 points)
- (b)  $\{w : w \text{ contains exactly four } 1\text{s.}\}$  (2 points)
- (c)  $\{w : \text{The length of } w \text{ is two more than multiple of five.}\}$  (2 points)
- (d)  $\{w : w \text{ consists of any combination of } 01 \text{ and } 110.\}$  (2 points)
- (e)  $\{w : w \text{ doesn't end with } 01\}$  (2 points)
- (f)  $\{w : \text{Number of } 01 \text{ substring is more than number of } 10 \text{ substrings in } w\}$  (2 points)
- (g) You write a regular expression  $0(0+1)^*1^*0^*0$ . Your friends write another regular expression  $01^*0^*(0+1)^*0$ . Are they the same? **Write** Yes or No only. (1 point)
- (h) You write a regular expression  $(1+01)^*$ . Your friends write another regular expression  $1^*(011^*)^*$ . Are they the same? **Give** justification for your answer. (2 points)

RE Set B

Problem 2 (CO1): Regular Expressions (15 points)

Let  $\Sigma = \{0, 1\}$ . Give regular expressions for each of the languages (a)-(f) over  $\Sigma$ .

- (a)  $\{w : w \text{ starts with } 00 \text{ or } 010.\}$  (2 points)
- (b)  $\{w : w \text{ contains at least three } 1\text{s.}\}$  (2 points)
- (c)  $\{w : \text{The length of } w \text{ is three more than multiple of five.}\}$  (2 points)
- (d)  $\{w : w \text{ consists of any combination of } 10 \text{ and } 001.\}$  (2 points)
- (e)  $\{w : w \text{ doesn't end with } 11\}$  (2 points)
- (f)  $\{w : \text{Number of } 01 \text{ substring is less than number of } 10 \text{ substrings in } w\}$  (2 points)
- (g) You write a regular expression  $11^*(0+1)^*0^*1$ . Your friends write another regular expression  $10^*1^*(0+1)^*1$ . Are they the same? **Write** Yes or No only. (1 point)
- (h) You write a regular expression  $(0+10)^*$ . Your friends write another regular expression  $0^*(100^*)^*$ . Are they the same? **Give** justification for your answer. (2 points)

**Problem 2: Regular Expressions (10 points)**

Mike and Willy recently learned how to write regular expressions. Mike wrote the regular expression  $10^*1^*$  for a language  $L_1$  on the board and Willy wrote the regular expression  $1^*01^*$  for another language  $L_2$  below that.

- (a) Write down a string that is present in the language  $L_1$  but not in the language  $L_2$ . (2 points)
- (b) Write down a string that is not present in the language  $L_1$  but present in the language  $L_2$ . (2 points)
- (c) Write down a string that is neither present in the language  $L_1$  nor in the language  $L_2$ . (2 points)
- (d) Mike and Willy asked their friend Dustin to write a regular expression for the language  $L_1 \cap L_2$ . Dustin came up with  $1^*0^*1^*$ . Is Dustin's regular expression correct? If you think it's not correct, then write down a correct regular expression for  $L_1 \cap L_2$ . (4 points)

Summer 23 Set 1

**Problem 2 (CO1): Regular Expressions (15 points)**

Let  $\Sigma = \{0, 1\}$ . **Give** regular expressions generating each of the following languages over  $\Sigma$ .

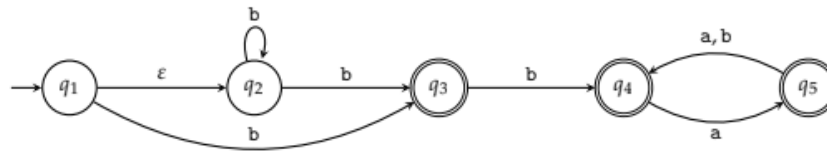
- (a)  $\{w : w \text{ starts with a 1 and ends in a 0}\}$  (3 points)
  - (b)  $\{w : \text{the length of } w \text{ is even}\}$  (3 points)
  - (c)  $\{w : \text{every 1 in } w \text{ is followed by an even number of 0s}\}$  (3 points)
  - (d)  $\{w : w \text{ does not contain } 10\}$  (3 points)
  - (e)  $\{w : 10 \text{ appears in } w \text{ exactly once}\}$  (3 points)
- (Hint: If  $w = x10y$ , what can you say about  $x$  and  $y$ ?)

NFA

Spring 25 Set A

Problem 5 (CO2): Subset Construction Method (5 points)

Consider the following NFA:



Now answer the following questions. [Note: You do not need to convert the given NFA into its equivalent DFA to answer the questions.]

- (a) If you convert the given NFA into an equivalent DFA using the subset construction method, what is the maximum number of states that the DFA can have? (1 point)
- (b) what is the maximum number of accepting states that the equivalent DFA can have? (1 point)
- (c) **Write** the  $\epsilon$ -closure of state  $q_1$  in the given NFA. (1 point)
- (d) **Write** the subset of states of the given NFA that will be the starting state in its equivalent DFA. (1 point)
- (e) What is  $\delta(\{q_1, q_3\}, b)$  in the given NFA? [Recall:  $\delta(\{q\}, a)$  is the set of states in which the NFA transitions when it is in state  $q$  and receives input  $a$ .] (1 point)