Q1

Friday, August 22, 2025 11:57 AM

Question 1 [10 marks]

Use pumping lemma and prove following languages are not regular

I. $L1 = \{w \in \{0, 1\}^* : 0^x1^{y+1}0^z$ where x = y and $x, y, z \geq 0\}$

II. $L2 = \{w \in \{0, 1\}^* : ww^R$, *where w is a string and R denoting reversed string*$\}$

Conditions :

i) for each $i \geq 0$, $XY^iZ \in L$

ii) $|Y| > 0$

iii) $|XY| \leq P$

I) $0^x 1^{y+1} 0^z$; $x = y$; $x, y, z > 0$

$\Rightarrow 0^x 1^{x+1} 0^z$; $x, z > 0$

let,

$\underbrace{0}_{X}\underbrace{0}_{Y}\underbrace{0111100}_{Z}$

$X = 0^1$; $j = 1$

$Y = 0^1$; $k = 1$

$Z = 0^1 1^4 0^2$; $m = 1$, $P+1 = 4$, $r = 2$

Here,

$P = j + k + m$

now,

$XYYZ = 0^j 0^k 0^k 0^m 1^{P+1} 0^r$

$= 0^{j+k+m+k} 1^{P+1} 0^r$

$= 0^{P+k} 1^{P+1} 0^r \notin L$

II) ww^R, w^R is w reversed

let,

$\underbrace{0}_{X}\underbrace{0}_{Y}\underbrace{011000}_{Z}$

$X = 0^1$; $j = 1$

$Y = 0^1$; $k = 1$

$Z = 0^1 1^2 0^3$; $m = 1$, $r = 2$, $P = 3$

Here,

$P = j + k + m$

now,

$XYYZ = 0^j 0^k 0^k 0^m 1^r 0^P$

$= 0^{j+k+m+k} 1^r 0^P$

$= 0^{P+k} 1^r 0^P \notin L$

Q2

Friday, August 22, 2025 2:41 PM

Question 2 [20 marks]

Let $\Sigma = \{0, 1\}$. Consider the following languages. Recall that for a string w, |w| denotes the length of w.

L1 = {w ∈ Σ : w is an even length palindrome}* *L2 = {w ∈ Σ*: length of w is even}*

L3 = {x11y : x, y ∈ L2, |x| = |y|} *L4 = L1 ∩ L3*

Now solve the following problems.

(a) Give a context-free grammar for the language L4.

(b) Convert the following regular expressions into context free grammar.

I. $a^*b + a(b^+ + a^*b)$

II. $(a^*+b)bb(b^* + a)^*$

Let Σ = {0, 1}. Consider the following languages over Σ:

L1 = { w ∈ Σ : w starts and ends with the same symbol }*
L2 = { w ∈ Σ : length of w is odd and w contains "11" as a substring }*
L3 = { w ∈ Σ : w is a palindrome and has odd length }*
L4 = { w ∈ Σ : w has exactly three 1's }*
L5 = L1 ∩ L4

Answer the following:
(c) Give a context-free grammar for L2.
(d) Give a context-free grammar for L2 ∩ L3.
(e) Give a context-free grammar for the language L5.

a) $S \rightarrow 00S00 \mid 01S10 \mid 10S01 \mid 11S11 \mid 11$

b) I) $a^*b + a(b^+ + a^*b)$

$S \rightarrow X \mid aB \mid aX$
$X \rightarrow Ab$
$A \rightarrow aA \mid \epsilon$
$B \rightarrow bB \mid b$

II) $(a^*+b)bb(b^*+a)^*$

$S \rightarrow AXY \mid bXY$
$A \rightarrow aA \mid \epsilon$
$B \rightarrow bB \mid \epsilon$
$X \rightarrow bb$
$Y \rightarrow BY \mid aY \mid \epsilon$

c) $S \rightarrow 0X11X \mid 1X11X \mid X11X0$
$X \rightarrow 00X \mid 01X \mid 10X \mid 11X \mid \epsilon$

d) $S \rightarrow XYX \mid 111$
$X \rightarrow 11$
$Y \rightarrow 00Y00 \mid 01Y10 \mid 10Y01 \mid$
$11Y11 \mid 0 \mid 1$

e) $S \rightarrow 1A1 \mid 0B1B1B1B0$
$A \rightarrow 0A \mid A0 \mid 1$
$B \rightarrow 0B \mid \epsilon$

Q3

Sunday, September 7, 2025 1:32 PM

Question 3 [3+3+3+1 marks = 10]

Take a look at the grammar below and solve the following problems.

$$A \rightarrow 1A \mid 1C \mid 0B \mid 00A$$
$$B \rightarrow 0A \mid 1B \mid 00B$$
$$C \rightarrow 0C0 \mid 0C1 \mid 1C0 \mid 1C1 \mid \varepsilon$$

(a) Give a leftmost derivation for the string 01011001.

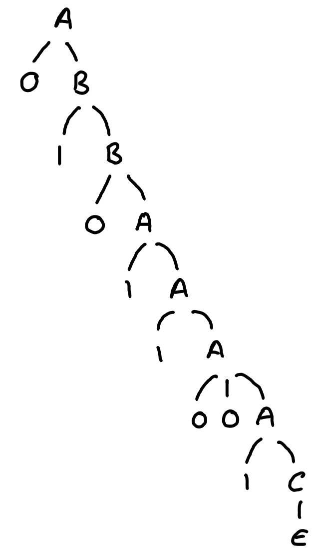
(b) Sketch the parse tree corresponding to the derivation you gave in (a).

(c) Demonstrate that the given grammar is ambiguous by showing two more parse trees (apart from the one you already found in (b)) for the same string.

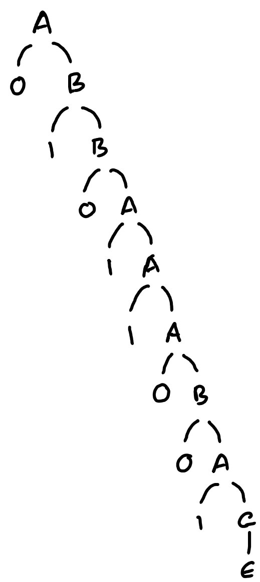
(d) Find a string w of length six such that w has exactly one parse tree in the grammar above.

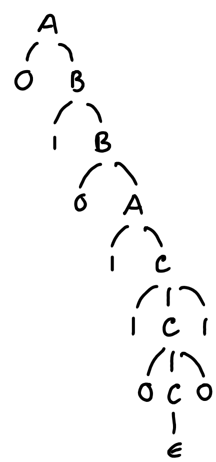
a) A
→ 0B
→ 01B
→ 010A
→ 0101A
→ 01011A
→ 0101100A
→ 01011001C
→ 01011001

b)



c)





d) 011101

Question 4 [10 marks]

Question A: Let $\Sigma = \{a, b\}$. Consider the following languages.

$L1 = \{w \mid w \text{ is a palindrome and the length of } w \text{ is odd}\}$

$L2 = \{w \mid w = x0y : x, y \in \text{any positive length string}, |x| = |y|\}$

(a) Give the state diagram of a pushdown automaton that recognizes L1.
(b) Give the state diagram of a pushdown automaton that recognizes L2.
(c) Give the state diagram of a pushdown automaton that recognizes L1∩L2.

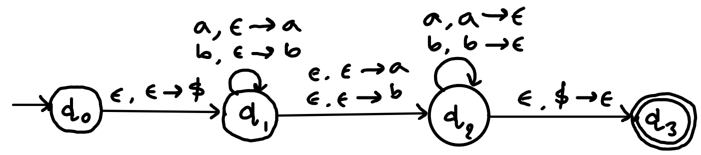
Question B: Let $\Sigma = \{0, 1\}$. Consider the following language.

$L = \{$**x#y**: $x, y \in \Sigma^*$, and the number of occurrences of 0 in x is equal to the number of occurrences of 10 in y$\}$

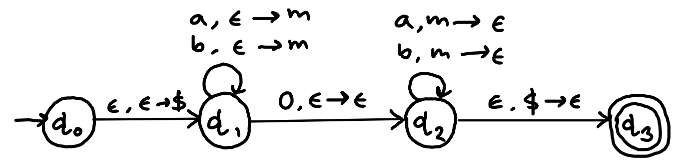
Solve the following problems.
(a) Find all strings $w \in L$ such that w starts with 110110# and has a length of 10.
(b) Give the state diagram of a pushdown automaton that recognizes L.

A.

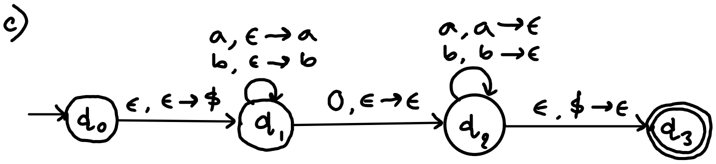
a)



b)



c)



B.

a) None

b)