Q1

Tuesday, September 16, 2025 7:19 PM

Problem 1 (CO5): Pumping Lemma (5 points)

Let $\Sigma = \{0, 1\}$. Consider the following language.

$$L = \{w \in \Sigma^* : w = 0^a 1^b 1^c 0^d, \text{ where } a + b = c + d \text{ and } a, b, c, d \geq 0\}$$

Use the pumping lemma to **demonstrate** that L is not regular language.

let, $0^a 1^b 1^c 0^d$; $a+b = c+d$, $a, b, c, d \geq 0$

$X = 0^{a-j}$

$Y = 0^j$

$Z = 1^b 1^c 0^d$

here,

$p = a - j + j + b$

$= a + b = c + d$

now, $XYYZ$

$= 0^{a-j} 0^j 0^j 1^b 1^c 0^d$

$= 0^{a+j} 1^b 1^c 0^d$

here, if $j > 0$

$a + b + j \neq c + d$

$\therefore XYYZ \notin L$

$\therefore$ non-regular

Q2

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Problem 2 (CO3): Designing Context-Free Grammars (10 points)

Let $\Sigma = \{0, 1\}$. Consider the following languages. Recall that for a string w, $|w|$ denotes the length of w.

$$L_1 = \{w \in \Sigma^* : w \text{ is an even length palindrome}\}$$
$$L_2 = \{w \in \Sigma^* : \text{length of } w \text{ is even}\}$$
$$L_3 = \{x11y : x, y \in L_2, |x| = |y|\}$$
$$L_4 = L_1 \cap L_3$$

Now solve the following problems.

(a) **Give** a context-free grammar for the language L_1. (3 points)

(b) **Give** a context-free grammar for the language L_3. (4 points)

(c) **Give** a context-free grammar for the language L_4.(3 point)

a) $S \rightarrow 0S0 \mid 1S1 \mid \epsilon$

b) $S \rightarrow XSX \mid 11$

$X \rightarrow 00 \mid 01 \mid 10 \mid 11$

c) $S \rightarrow 00S00 \mid 11S11 \mid 01S10 \mid 10S01 \mid 11$

Q4

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Problem 4 (CO3): Constructing Pushdown Automata (10 points)

Let $\Sigma = \{0, 1\}$. Consider the following languages.

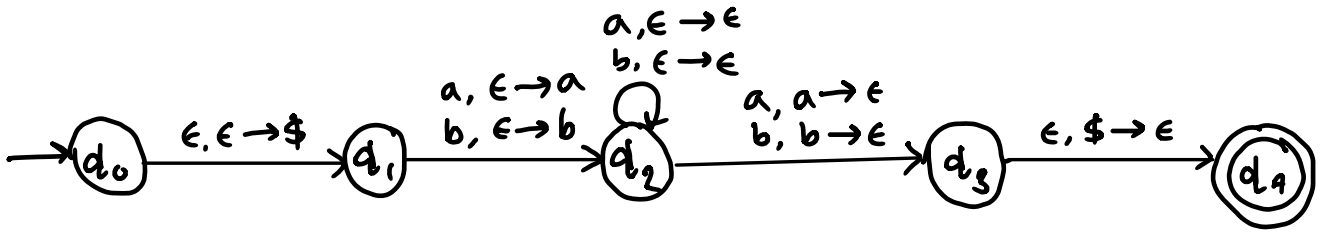
$$L_1 = \{w \mid w \text{ starts and ends with the same character}\}$$

$$L_2 = \{w \mid \text{the number of 0s in } w \text{ is not same as the number of 1s}\}$$

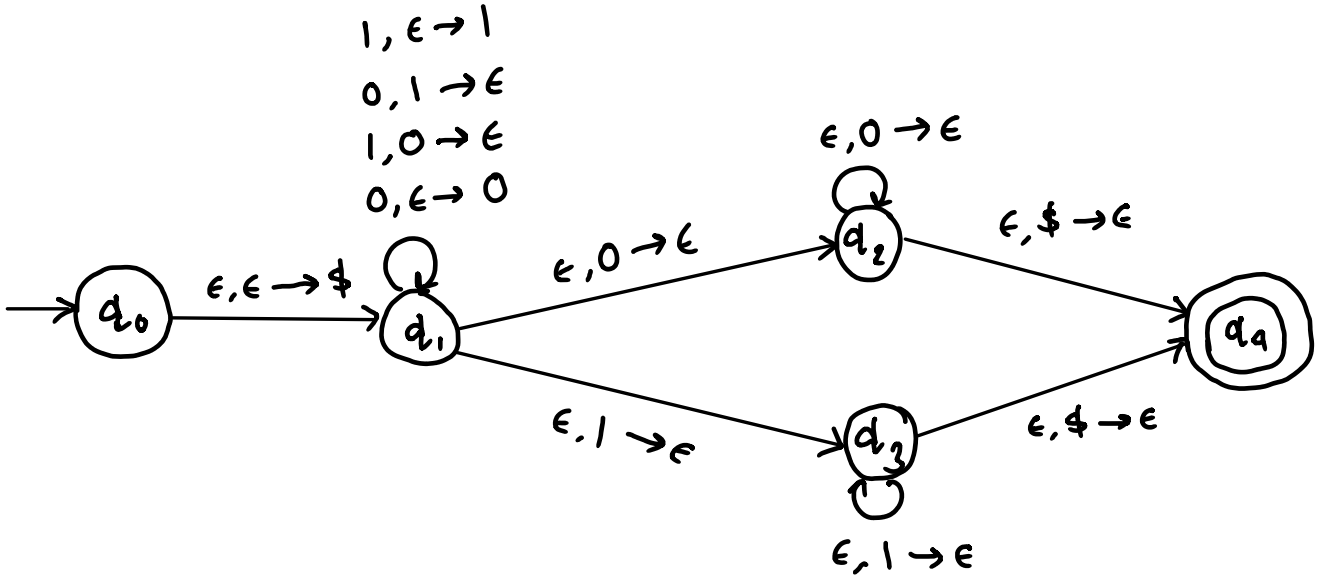
(a) **Give** the state diagram of a pushdown automaton that recognizes L_1. (4 points)

(b) **Give** the state diagram of a pushdown automaton that recognizes L_2. (6 points)

a)



b)



Q5

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Problem 5 (CO3): Derivations, Parse Trees and Ambiguity (10 points)

Take a look at the grammar below and solve the following problems.

$$A \to 1A \mid 1C \mid 0B \mid 00A$$
$$B \to 0A \mid 1B \mid 00B$$
$$C \to 0C0 \mid 0C1 \mid 1C0 \mid 1C1 \mid \varepsilon$$

(a) **Give** a leftmost derivation for the string 01011001. (3 points)

(b) **Sketch** the parse tree corresponding to the derivation you gave in (a). (2 points)

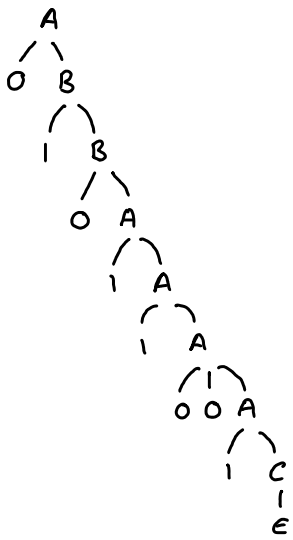
(c) **Demonstrate** that the given grammar is ambiguous by showing two more parse trees (apart from the one you already found in (b)) for the same string. (3 points)

(d) **Find** a string w of length six such that w has exactly one parse tree in the grammar above. (1 point)

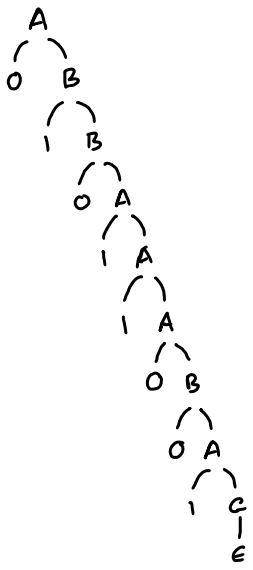
(e) **Desgin** an unambiguous Context Free Grammar for the language represented by the given ambiguous grammar. (1 point)

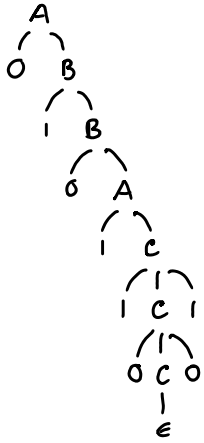
a) A
→ 0B
→ 01B
→ 010A
→ 0101A
→ 01011A
→ 010110A
→ 0101100 1C
→ 01011001

b)



c)





d) 011101

e) not in syllabus