# Q2

Tuesday, September 16, 2025 2:00 PM

**Problem 2 (CO5): Pumping Lemma (5 points)**

Let $\Sigma = \{0, 1\}$. Consider the following language over $\Sigma$.

$$L = \{w = 0^n 1^{n+1} \text{ where } n \text{ is 2 more than multiple of 4}\}$$

Use the pumping lemma to **demonstrate** that L is not a regular language.

let,

$0^n 1^{n+1}$

$x = 0^j$

$p = j + k + m$

$y = 0^k$

$m = 0^m 1^{p+1}$

now, $xyyz$

$= 0^j 0^k 0^k 0^m 1^{p+1}$

$= 0^{j+k+m+k} 1^{p+1}$

$= 0^{p+k} 1^{p+1}$

if $k > 0$,

$p + k \geq p + 1$

$\therefore xyyz \notin L$

$\therefore$ non-regular

# Q3

Tuesday, September 16, 2025 2:01 PM

**Problem 3 (CO3): Designing Context-Free Grammars (15 points)**

$$L_1 = \{w \in \{a, b\}^* : w \text{ is an even length palindrome}\}$$
$$L_2 = \{w \in \{a, b\}^* : \text{every second letter of } w \text{ is a}\}$$
$$L_3 = \{w \in \{0, 1\}^* : w \text{ contains exactly two 1s}\}$$
$$L_4 = \{w1\#w2 : w1 \in L2, w2 \in \{0, 1\}^* \text{and } |w1| = |w2|\}$$
$$L_5 = \{w1\#w2 : w1 \in L2, w2 \in L3 \text{ and } |w1| = |w2|\}$$

Now solve the following problems.

(a) **Give** a context-free grammar for the language $L_1$. (3 points)

(b) **Give** a context-free grammar for the language $L_2$. (3 points)

(c) **Give** a context-free grammar for the language $L_3$. (3 points)

(d) **Find** all strings $w \in L_4$ such that $w$ ends with #0100 and has a length of 9. (1 point)

(e) **Give** a context-free grammar for the language $L_4$. [Recall: For a string $w$, $|w|$ denotes the length of $w$.] (3 points)

(f) **Give** a context-free grammar for the language $L_5$. (2 points)

a) $S \rightarrow aSa \mid bSb \mid \epsilon$

b) $S \rightarrow aaS \mid baS \mid a \mid b \mid \epsilon$

c) $S \rightarrow X1X1X$

$X \rightarrow 0X \mid \epsilon$

d) aaaa#0100

aaba#0100

baaa#0100

baba#0100

e) $S \rightarrow XSY \mid a\#0 \mid a\#1 \mid b\#0 \mid b\#1 \mid \#$

$X \rightarrow aa \mid ba$

$Y \rightarrow 00 \mid 01 \mid 10 \mid 11$

f) $S \rightarrow PS00 \mid PA01 \mid PA10 \mid PB11$

$A \rightarrow PA00 \mid PB01 \mid PB10 \mid Q\#1$

$B \rightarrow PB00 \mid Q\#0 \mid \#$

$P \rightarrow aa \mid ba$

$Q \rightarrow a \mid b$

aabab#01001

# Q4

Tuesday, September 16, 2025 2:03 PM

**Problem 4 (CO3): Derivations, Parse Trees and Ambiguity (10 points)**

Let $\Sigma = \{0, 1\}$. Consider the following grammar over $\Sigma$.

$$S \to 0S0 \mid 1S1 \mid A$$
$$A \to 00A \mid 01A \mid 10A \mid 11A \mid \varepsilon$$

(a) **Give** a leftmost derivation for the string 01101010. (3 points)

(b) **Draw** the parse tree corresponding to the derivation you gave in (a). (2 points)

(c) **Demonstrate** that the given grammar is ambiguous by showing two more parse trees (apart from the one you already found in (b)) for the given string in (a). (4 points)

(d) How many four-letter strings will have exactly one parse tree in the given grammar? (1 point)

a) $S \to 0S0$

$\to 01S10$

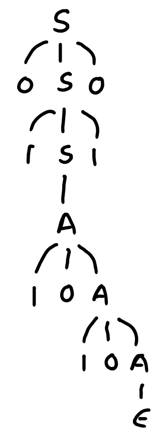
$\to 01A10$

$\to 0110A10$

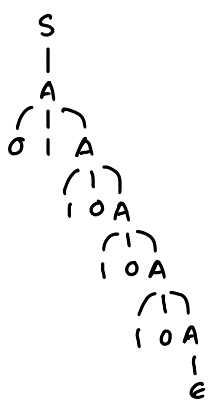
$\to 0110A10$

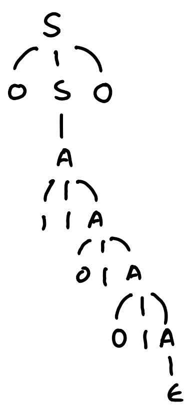
$\to 01101010$

b)



c)





d) 2

0101

1010

# Q5

Tuesday, September 16, 2025 2:33 PM

**Problem 5 (CO3): Designing Pushdown Automata (10 points)**

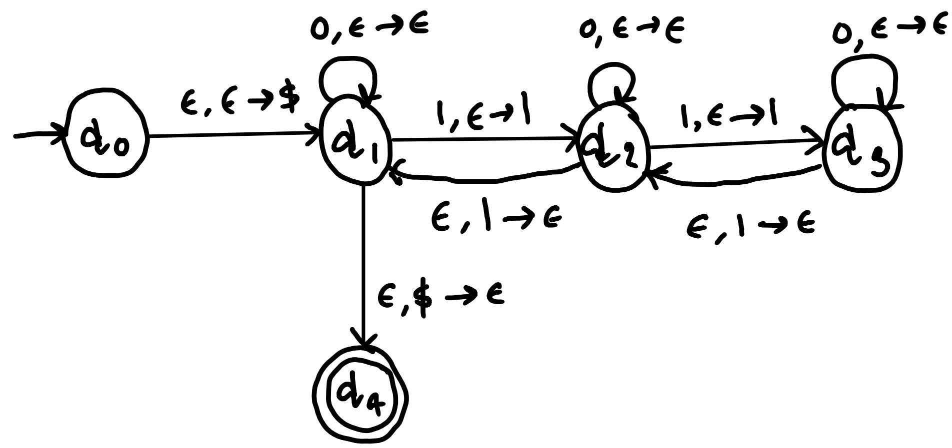
$$L_1 = \{w \in \{0,1\}^* : w \text{ contains at most two 1s}\}$$
$$L_2 = \{w \in \{0,1\}^* : w = 1^{3n}0^{2n} \text{ where } n \geq 0\}$$
$$L_3 = \{w\#x : w, x \in \{0,1\}^* \text{ and } w^R \text{ is a substring of } x\}$$

(a) **Give** the state diagram of a pushdown automaton that recognizes $L_1$. (3 points)

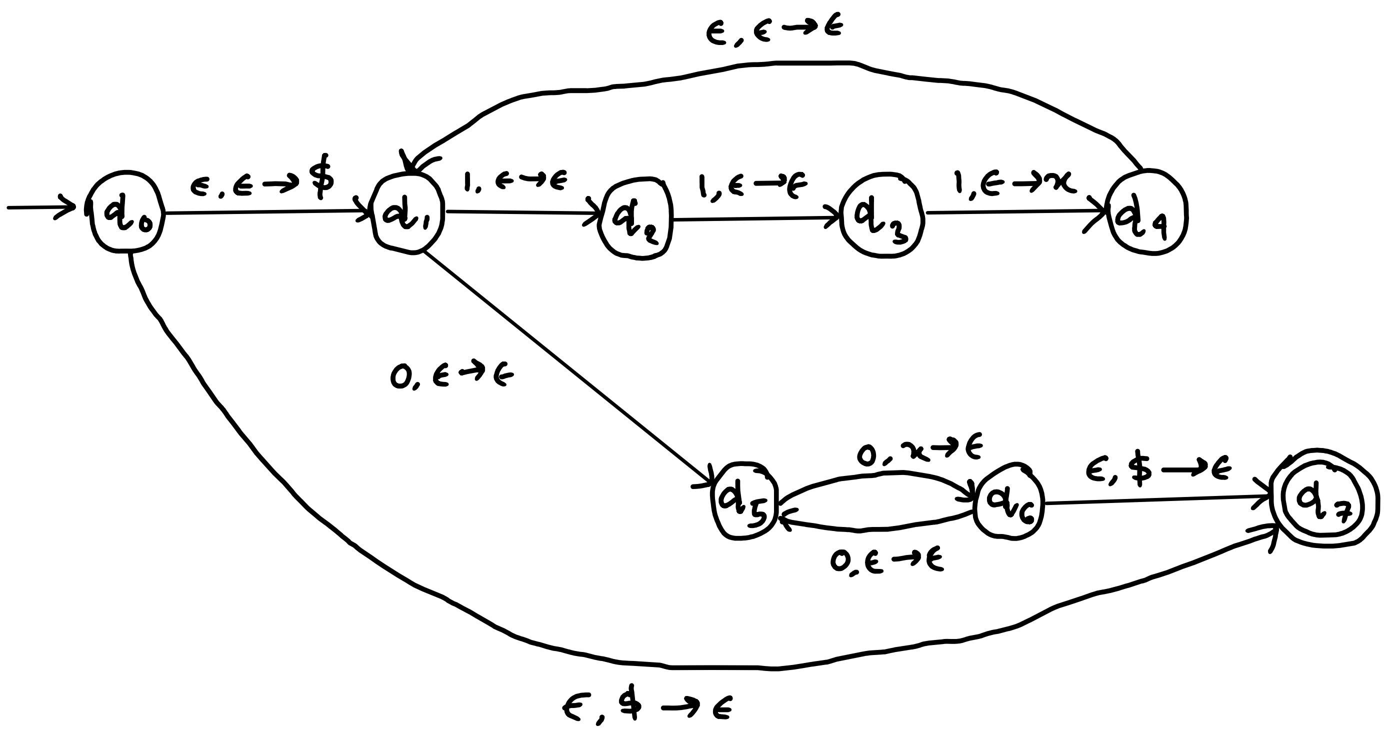
(b) **Give** the state diagram of a pushdown automaton that recognizes $L_2$. (3 points)

(c) **Give** the state diagram of a pushdown automaton that recognizes $L_3$. [Recall: For a string $w$, $w^R$ denotes $w$ in reverse order.] (4 points)

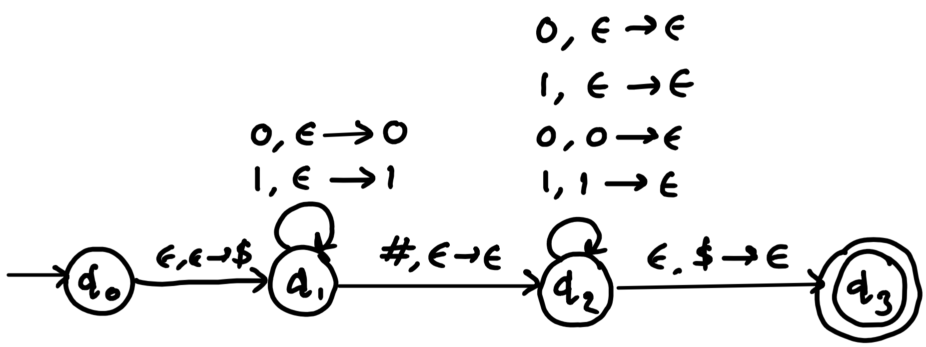
a)




b)



c)



# Q6

Tuesday, September 16, 2025 3:17 PM

**Problem 6 (Bonus): Pumping Lemma (4 points)**

Let $\Sigma = \{0, 1\}$. Consider the following language over $\Sigma$.

$$L = \{w \text{ is not a palindrome.}\}$$

Use the pumping lemma to **demonstrate** that L is not a regular language.

let,

$0^n 1^n$ not a palindrome

$X = 0^j$ $\quad\quad$ $p = j + k + m$

$Y = 0^k$

$Z = 0^m 1^p$

now, $XYYZ$

$= 0^j 0^k 0^k 0^m 1^p$

$= 0^{j+k+m+k} 1^p$

$= 0^{p+k} 1^p \notin 0^n 1^n$

$\therefore$ non-regular