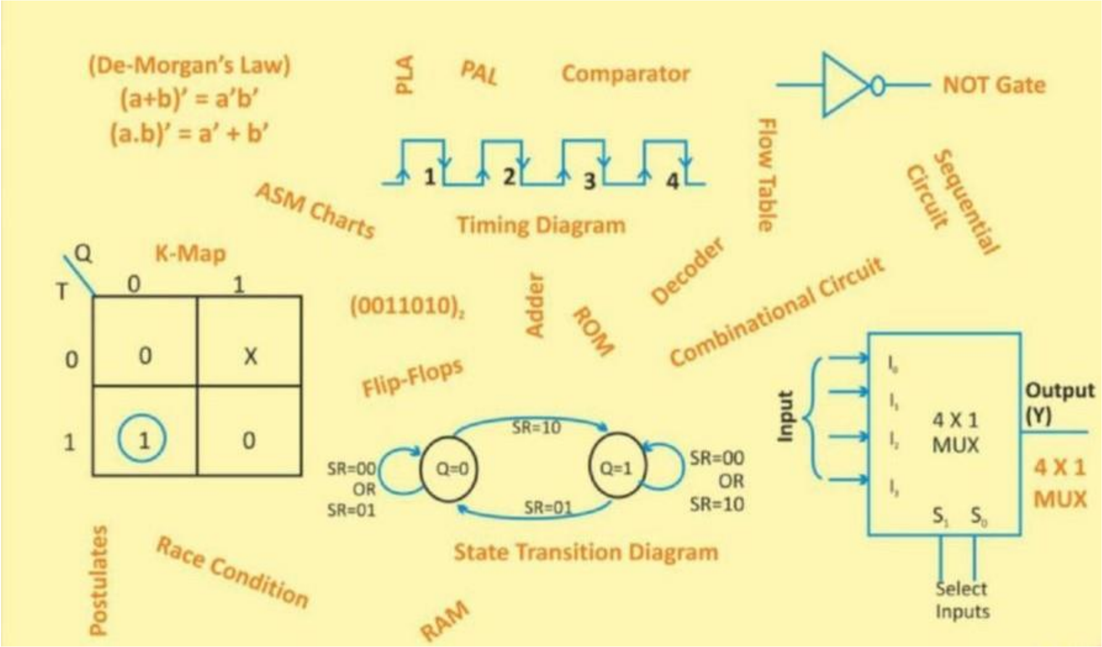
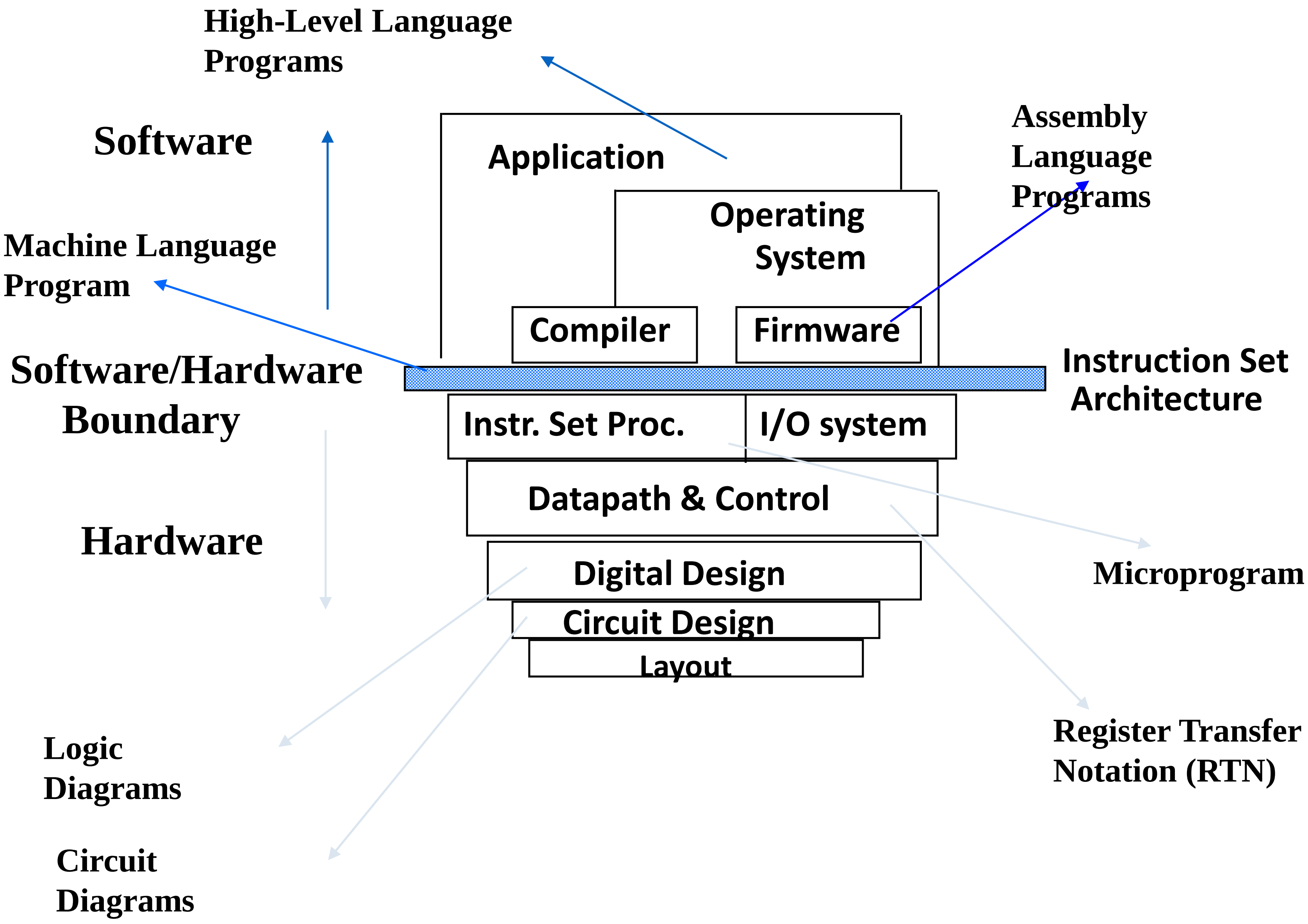
CSE 231: Digital Logic Design



Introduction

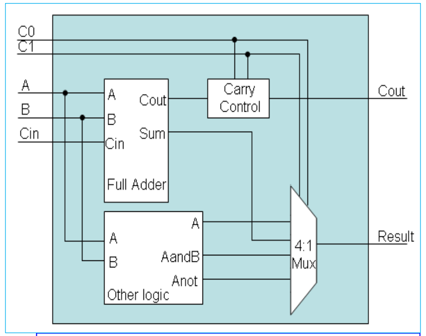
Hierarchy of Computer Architecture



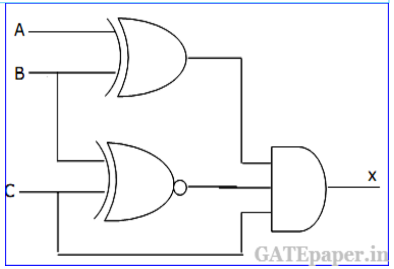
Where does it fit?

Design to fabrication → EEE 435

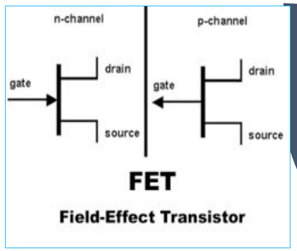
Digital Systems Design (Ex. Computer/Mobile Processor) → CSE 332



Digital Logic Circuits (ALU, Memory) → CSE 231



Logical components using transistors → EEE 111



Chapter 1
Digital Systems and Binary Numbers

Outline

1.1 Digital Systems

1.2 Binary Numbers

1.3 Number-base Conversions

1.4 Octal and Hexadecimal Numbers

1.5 Complements

1.6 Signed Binary Numbers

1.7 Binary Codes

1.8 Binary Storage and Registers (Optional)

1.9 Binary Logic (Optional)

Digital Systems and Binary Numbers

- Digital age and information age
- Digital computers
  - General purposes
  - Many scientific, industrial and commercial applications
- Digital systems
  - Telephone switching exchanges
  - Digital camera
  - Electronic calculators, PDA's, Digital TV
- Discrete information-processing systems
  - Manipulate discrete elements of information
  - For example, {1, 2, 3, …} and {A, B, C, …}…

Analog and Digital Signal

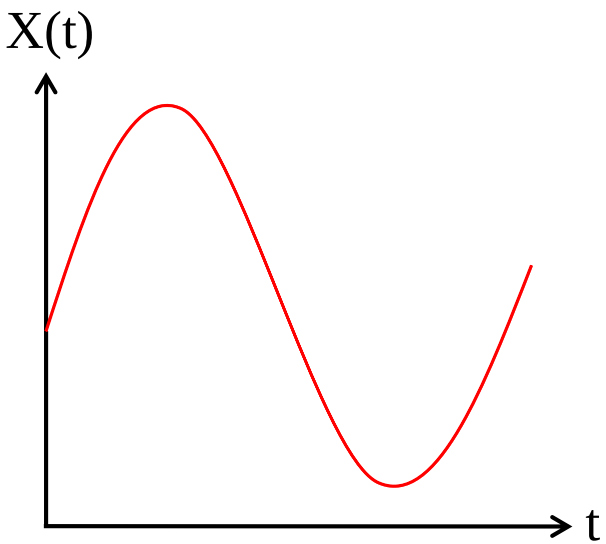
Analog system

The physical quantities or signals may vary continuously over a specified range.

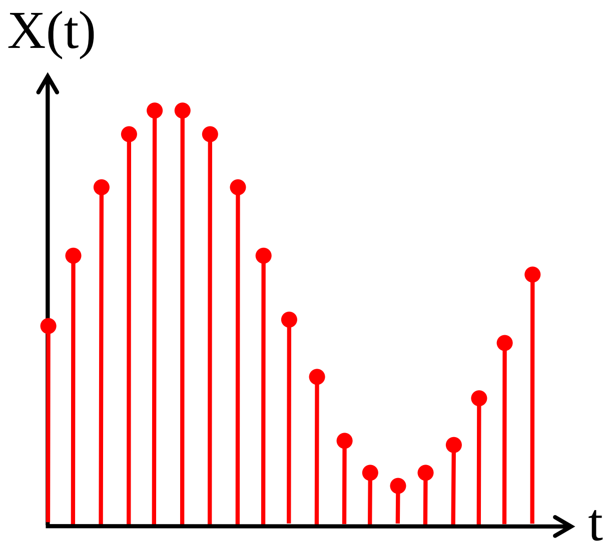
Digital system

The physical quantities or signals can assume only discrete values.

Greater accuracy



Analog signal



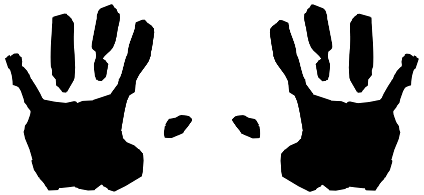
Digital signal

Binary Digital Signal

- An information variable represented by physical quantity.
- For digital systems, the variable takes on discrete values.
  - Two level, or binary values are the most prevalent values.
- Binary values are represented abstractly by:
  - Digits 0 and 1
  - Words (symbols) False (F) and True (T)
  - Words (symbols) Low (L) and High (H)
  - And words On and Off
- Binary values are represented by values or ranges of values of physical quantities.

Decimal Number System

- Base (also called radix) = 10
  - 10 digits { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 }
- Digit Position
  - Integer & fraction
- Digit Weight
  - Weight = (*Base*) *Position*
- Magnitude
  - Sum of "*Digit* x *Weight*"
- Formal Notation



2	1	0		-1	-2
5	1	2	.	7	4
100	10	1		0.1	0.01
			.		
500	10	2		0.7	0.04

$$d_2 {*} B^2 + d_1 {*} B^1 + d_0 {*} B^0 + d_{-1} {*} B^{-1} + d_{-2} {*} B^{-2}$$

$$(512.74)_{10}$$

Octal Number System

- Base = 8
  - 8 digits { 0, 1, 2, 3, 4, 5, 6, 7 }
- Weights
  - Weight = (*Base*) $^{\textit{Position}}$
- Magnitude
  - Sum of “*Digit* x *Weight*”
- Formal Notation

64	8	1		1/8	1/64
5	**1**	**2**		**7**	**4**
2	1	0		-1	-2

$$5 * 8^{2} + 1 * 8^{1} + 2 * 8^{0} + 7 * 8^{-1} + 4 * 8^{-2}$$

$$=(330.9375)_{10}$$

$$(512.74)_{8}$$

Binary Number System

- Base = 2
  - 2 digits { 0, 1 }, called *b*inary dig*its* or "*bits*"
- Weights
  - Weight = (*Base*) *Position*
- Magnitude
  - Sum of "*Bit* x *Weight*"
- Formal Notation
- Groups of bits

4	2	1	1/2	1/4
1	**0**	**1**	**0**	**1**
2	1	0	-1	-2

$$1 * 2^{2} + 0 * 2^{1} + 1 * 2^{0} + 0 * 2^{-1} + 1 * 2^{-2}$$

$$= (5.25)_{10}$$

$$(101.01)_{2}$$

8 bits = *Byte*

1 0 1 1

1 1 0 0 0 1 0 1

Hexadecimal Number System

- Base = 16
  - 16 digits { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F }
- Weights
  - Weight = (*Base*) ^{Position}
- Magnitude
  - Sum of "*Digit* x *Weight*"
- Formal Notation

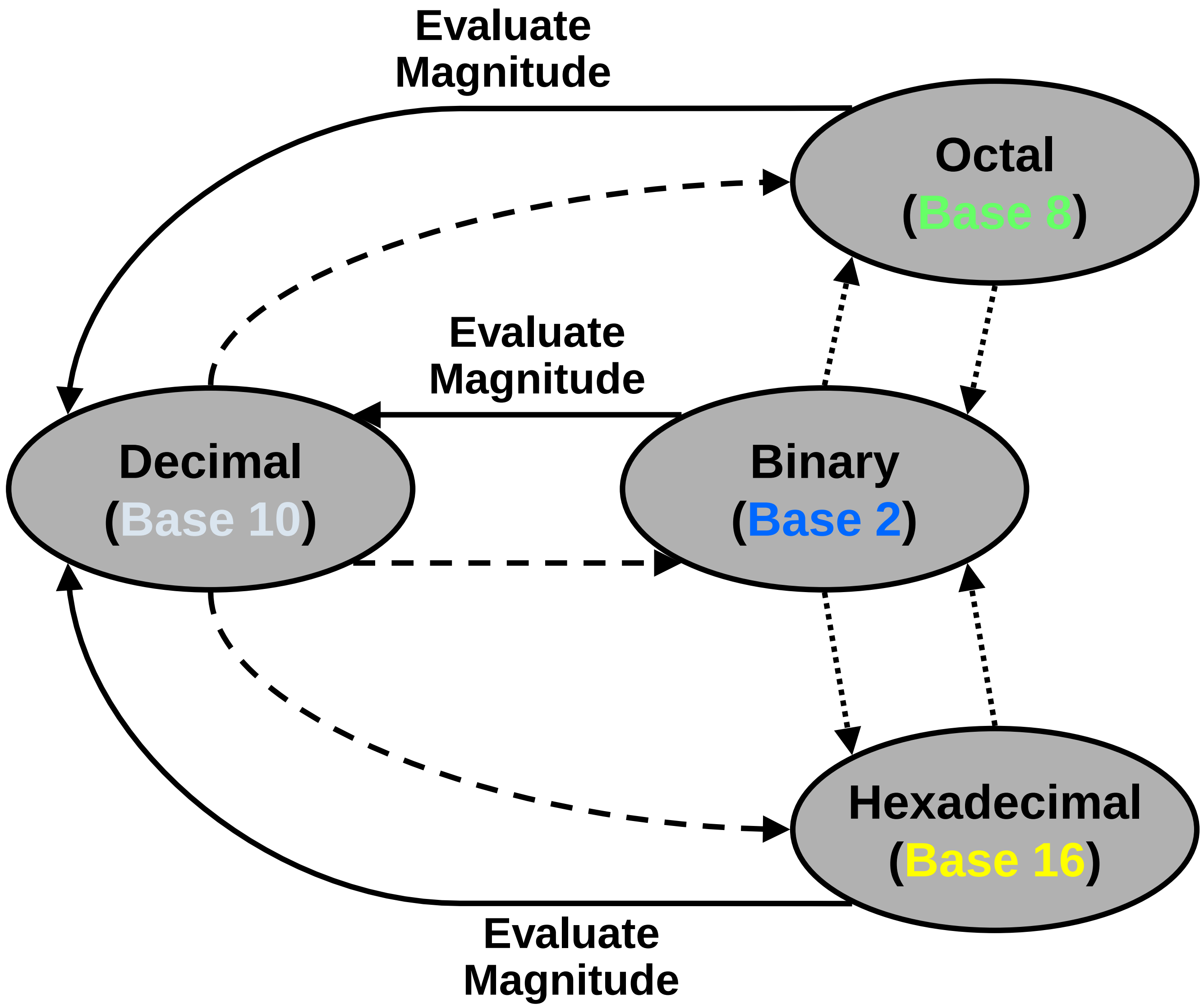
256	16	1		1/16	1/256
1	**E**	**5**		**7**	**A**
2	1	0		-1	-2

$$1 * 16^2 + 14 * 16^1 + 5 * 16^0 + 7 * 16^{-1} + 10 * 16^{-2}$$

$$= (485.4765625)_{10}$$

$$(1E5.7A)_{16}$$

Number Base Conversions



Decimal (*Integer*) to Binary Conversion

- Divide the number by the 'Base' (=2)
- Take the remainder (either 0 or 1) as a coefficient
- Take the quotient and repeat the division

Example: $(13)_{10}$

	Quotient	Remainder	Coefficient
13 / 2 =	6	1	$a_0 = 1$
6 / 2 =	3	0	$a_1 = 0$
3 / 2 =	1	1	$a_2 = 1$
1 / 2 =	0	1	$a_3 = 1$

Answer: $(13)_{10} = (\underset{\text{MSB}}{a_3}\, a_2\, a_1\, \underset{\text{LSB}}{a_0})_2 = (1101)_2$

Decimal (*Fraction*) to Binary Conversion

- Multiply the number by the ‘Base’ (=2)
- Take the integer (either 0 or 1) as a coefficient
- Take the resultant fraction and repeat the division

Example: $(0.625)_{10}$

		Integer		Fraction	Coefficient
0.625	* 2 =	1	.	25	$a_{-1} = 1$
0.25	* 2 =	0	.	5	$a_{-2} = 0$
0.5	* 2 =	1	.	0	$a_{-3} = 1$

Answer: $(0.625)_{10} = (0.a_{-1}\, a_{-2}\, a_{-3})_2 = (0.101)_2$

MSB LSB

Decimal to Octal Conversion

Example: $(175)_{10}$

	Quotient	Remainder	Coefficient
175 / 8 =	21	7	$a_0 = 7$
21 / 8 =	2	5	$a_1 = 5$
2 / 8 =	0	2	$a_2 = 2$

Answer: $(175)_{10} = (a_2\, a_1\, a_0)_8 = (257)_8$

Example: $(0.3125)_{10}$

	Integer		Fraction	Coefficient
0.3125 * 8 =	2	.	5	$a_{-1} = 2$
0.5 * 8 =	4	.	0	$a_{-2} = 4$

Answer: $(0.3125)_{10} = (0.a_{-1}\, a_{-2}\, a_{-3})_8 = (0.24)_8$

Binary – Octal Conversion

$8 = 2^3$

Each group of 3 bits represents an octal digit

Example:

Assume Zeros

$$(\ 1\,0\,1\,1\,0\,.\,0\,1\ )_2$$

$$(\ 2\quad 6\ .\ 2\ )_8$$

Octal	Binary
0	0 0 0
1	0 0 1
2	0 1 0
3	0 1 1
4	1 0 0
5	1 0 1
6	1 1 0
7	1 1 1

Works both ways (Binary to Octal & Octal to Binary)

Binary – Hexadecimal Conversion

$16 = 2^4$

Each group of 4 bits represents a hexadecimal digit

Example:

Assume Zeros

$(10110.01)_2$

$(16.4)_{16}$

Hex	Binary
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 0 1 1
4	0 1 0 0
5	0 1 0 1
6	0 1 1 0
7	0 1 1 1
8	1 0 0 0
9	1 0 0 1
A	1 0 1 0
B	1 0 1 1
C	1 1 0 0
D	1 1 0 1
E	1 1 1 0
F	1 1 1 1

Works both ways (Binary to Hex & Hex to Binary)

Octal – Hexadecimal Conversion

Convert to Binary as an intermediate step

Example:

$$(2 \quad 6 \quad . \quad 2)_8$$

Assume Zeros

Assume Zeros

$$(0\,1\,0\,1\,1\,0\,.\,0\,1\,0)_2$$

$$(1 \quad 6 \quad . \quad 4)_{16}$$

Works both ways (Octal to Hex & Hex to Octal)

Decimal, Binary, Octal and Hexadecimal

Decimal	Binary	Octal	Hex
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Addition

Decimal Addition

$$
\begin{array}{cccc}
 & 1 & 1 & \\
 & & 5 & 5 \\
+ & & 5 & 5 \\
\hline
 & 1 & 1 & 0
\end{array}
$$

Carry

= Ten ≥ Base

➔ Subtract a Base

Binary Addition

$$
\begin{array}{cccccccccc}
 & 1 & 1 & 1 & 1 & 1 & 1 & & & \\
 & & 1 & 1 & 1 & 1 & 0 & 1 & & = 61 \\
+ & & & 1 & 0 & 1 & 1 & 1 & & = 23 \\
\hline
 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & & = 84
\end{array}
$$

$\geq (2)_{10}$

Binary Subtraction

$$
\begin{array}{cccccccccc}
 & & 1 & & & 2 & & & & \\
 & 0 & \cancel{2} & 2 & 0 & 0 & 2 & & & = (10)_2 \\
 & \cancel{1} & 0 & 0 & \cancel{1} & \cancel{1} & 0 & 1 & & = 77 \\
- & & & 1 & 0 & 1 & 1 & 1 & & = 23 \\
\hline
 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & & = 54
\end{array}
$$

Binary Multiplication

				1	0	1	1	1
x					1	0	1	0
				0	0	0	0	0
			1	0	1	1	1	
		0	0	0	0	0		
	1	0	1	1	1			
	1	1	1	0	0	1	1	0

Complements

There are two types of complements for each base-*r* system: the radix complement and diminished radix complement.

- **Diminished Radix Complement - (r-1)'s Complement**
  - Given a number N in base r having n digits, the $(r-1)$'s complement *of* N is defined as:
  - $(r^n - 1) - N$
- **Example for 6-digit <u>decimal</u> numbers**:
  - 9's complement is $(r^n - 1) - N = (10^6 - 1) - N = 999999 - N$
  - 9's complement of 546700 is 999999–546700 = 453299
- **Example for 7-digit <u>binary</u> numbers:**
  - 1's complement is $(r^n - 1) - N = (2^7 - 1) - N = 1111111 - N$
  - 1's complement of 1011000 is 1111111–1011000 = 0100111
- **Observation:**
  - Subtraction from $(r^n - 1)$ will never require a borrow
  - Diminished radix complement can be computed digit-by-digit
  - For binary: 1 – 0 = 1 and 1 – 1 = 0

Complements

1’s Complement (*Diminished Radix* Complement)

All ‘0’s become ‘1’s

All ‘1’s become ‘0’s

Example $(10110000)_2$

⇨ $(01001111)_2$

If you add a number and its 1’s complement

$$
\begin{array}{r}
1\,0\,1\,1\,0\,0\,0\,0 \\
+\ 0\,1\,0\,0\,1\,1\,1\,1 \\
\hline
1\,1\,1\,1\,1\,1\,1\,1
\end{array}
$$

Complements

Radix Complement

The r's complement of an n-digit number N in base r is defined as $r^n - N$ for $N \neq 0$ and as 0 for $N = 0$. Comparing with the $(r - 1)$'s complement, we note that the r's complement is obtained by adding 1 to the $(r - 1)$'s complement, since $r^n - N = [(r^n - 1) - N] + 1$.

Example: Base-10

The 10's complement of 012398 is 987602
The 10's complement of 246700 is 753300

Example: Base-2

The 2's complement of 1101100 is 0010100
The 2's complement of 0110111 is 1001001

Complements

- 2's Complement (*Radix* Complement)

  Take 1's complement then add 1

  Toggle all bits to the left of the first '1' from the right

  Example:

  Number:

  1's Comp.:

	1 0 1 1 0 0 0 0	1 0 1 1 0 0 0 0
	0 1 0 0 1 1 1 1	
+	1	
	0 1 0 1 0 0 0 0	0 1 0 1 0 0 0 0

Complements

Subtraction with Complements

The subtraction of two n-digit unsigned numbers $M - N$ in base r can be done as follows:

1. Add the minuend M to the r's complement of the subtrahend N. Mathematically, $M + (r^n - N) = M - N + r^n$.
2. If $M \geqq N$, the sum will produce and end carry r^n, which can be discarded; what is left is the result $M - N$.
3. If $M < N$, the sum does not produce an end carry and is equal to $r^n - (N - M)$, which is the r's complement of $(N - M)$. To obtain the answer in a familiar form, take the r's complement of the sum and place a negative sign in front.

Complements

Example 1.7

Given the two binary numbers X = 1010100 and Y = 1000011, perform the subtraction (a) $X - Y$; and (b) $Y - X$, by using 2's complement.

(a)

$$
\begin{aligned}
X &= 1010100 \\
\text{2's complement of } Y &= \underline{+0111101} \\
\text{Sum} &= 10010001 \\
\text{Discard end carry } 2^7 &= \underline{-10000000} \\
\text{Answer. } X - Y &= 0010001
\end{aligned}
$$

(b)

$$
\begin{aligned}
Y &= 1000011 \\
\text{2's complement of } X &= +\underline{0101100} \\
\text{Sum} &= 1101111
\end{aligned}
$$

There is no end carry. Therefore, the answer is $Y - X = -$ (2's complement of 1101111) $= -$ 0010001.

Complements

Subtraction of unsigned numbers can also be done by means of the (r – 1)'s complement. Remember that the (r – 1) 's complement is one less then the r's complement.

Example 1.8

Repeat Example 1.7, but this time using 1's complement.

$$
\begin{aligned}
&\text{(a)}\quad X - Y = 1010100 - 1000011 \\
&\qquad\qquad X = \quad 1010100 \\
&\text{1's complement of } Y = \pm\, 0111100 \\
&\qquad\text{Sum} = \quad 10010000 \\
&\text{End-around carry} = \quad \underline{+ \qquad\quad 1} \\
&\text{Answer. } X - Y = \quad 0010001
\end{aligned}
$$

$$
\begin{aligned}
&\text{(b)}\, Y - X = 1000011 - 1010100 \\
&\qquad\qquad Y = \quad 1000011 \\
&\text{1's complement of } X = \underline{+\,0101011} \\
&\qquad\text{Sum} = \quad 1101110
\end{aligned}
$$

There is no end carry, Therefore, the answer is Y – X = – (1's complement of 1101110) = – 0010001.

1.6 Signed Binary Numbers

To represent negative integers, we need a notation for negative values.

It is customary to represent the sign with a bit placed in the leftmost position of the number since binary digits.

The convention is to make the sign bit 0 for positive and 1 for negative.

Example:

Signed-magnitude representation:	10001001
Signed-1's-complement representation:	11110110
Signed-2's-complement representation:	11110111

Table 1.3 lists all possible four-bit signed binary numbers in the three representations.

Signed Binary Numbers

Table 1.3
Signed Binary Numbers

Decimal	Signed-2's Complement	Signed-1's Complement	Signed Magnitude
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
−0	—	1111	1000
−1	1111	1110	1001
−2	1110	1101	1010
−3	1101	1100	1011
−4	1100	1011	1100
−5	1011	1010	1101
−6	1010	1001	1110
−7	1001	1000	1111
−8	1000	—	—

Signed Binary Numbers

Arithmetic addition

The addition of two numbers in the signed-magnitude system follows the rules of ordinary arithmetic. **If the signs are the same**, we add the two magnitudes and give the sum the common sign. **If the signs are different**, we subtract the smaller magnitude from the larger and give the difference the sign if the larger magnitude.

The addition of two signed binary numbers with negative numbers represented in signed-2's-complement form is obtained from the addition of the two numbers, including their sign bits.

A carry out of the sign-bit position is discarded.

Example:

+ 6	00000110	− 6	11111010
+13	00001101	+13	00001101
+ 19	00010011	+ 7	00000111
+ 6	00000110	− 6	11111010
−13	11110011	−13	11110011
− 7	11111001	− 19	11101101

Binary Codes

BCD Code

- In this **code** each decimal digit is represented by a 4-bit binary number. **BCD** is a way to express each of the decimal digits with a binary **code**
- A number with k decimal digits will require 4k bits in BCD.
- Decimal 396 is represented in BCD with 12bits as 0011 1001 0110, with each group of 4 bits representing one decimal digit.
- The binary combinations 1010 through 1111 are not used and have no meaning in BCD.

Table 1.4
Binary-Coded Decimal (BCD)

Decimal Symbol	BCD Digit
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

Binary Code

- Example:

  Consider decimal 185 and its corresponding value in BCD and binary:

  $$(185)_{10} = (0001\ 1000\ 0101)_{BCD} = (10111001)_2$$

- BCD addition

$$\begin{array}{rrrrrr}
4 & 0100 & 4 & 0100 & 8 & 1000 \\
\underline{+5} & \underline{+0101} & \underline{+8} & \underline{+1000} & \underline{+9} & \underline{+1001} \\
9 & 1001 & 12 & 1100 & 17 & 10001 \\
 & & & \underline{+0110} & & \underline{+0110} \\
 & & & 10010 & & 10111
\end{array}$$

Binary Code

Example:

Consider the addition of 184 + 576 = 760 in BCD:

BCD	1	1		
	0001	1000	0100	184
	+0101	0111	0110	+576
Binary sum	0111	10000	1010	
Add 6		0110	0110	
BCD sum	0111	0110	0000	760

Binary Codes

Other Decimal Codes – weighted representation of decimal numbers in binary.

Table 1.5
Four Different Binary Codes for the Decimal Digits

Decimal Digit	BCD 8421	2421	Excess-3	8, 4, −2, −1
0	0000	0000	0011	0000
1	0001	0001	0100	0111
2	0010	0010	0101	0110
3	0011	0011	0110	0101
4	0100	0100	0111	0100
5	0101	1011	1000	1011
6	0110	1100	1001	1010
7	0111	1101	1010	1001
8	1000	1110	1011	1000
9	1001	1111	1100	1111
	1010	0101	0000	0001
Unused	1011	0110	0001	0010
bit	1100	0111	0010	0011
combi-	1101	1000	1101	1100
nations	1110	1001	1110	1101
	1111	1010	1111	1110

Binary Codes)

Gray Code

The advantage is that only bit in the code group changes in going from one number to the next.

- Error detection.
- Representation of analog data.
- Low power design.

Table 1.6
Gray Code

Gray Code	Decimal Equivalent
0000	0
0001	1
0011	2
0010	3
0110	4
0111	5
0101	6
0100	7
1100	8
1101	9
1111	10
1110	11
1010	12
1011	13
1001	14
1000	15

Binary Codes

American Standard Code for Information Interchange (ASCII) Character Code

Table 1.7
American Standard Code for Information Interchange (ASCII)

	$b_7b_6b_5$							
$b_4b_3b_2b_1$	**000**	**001**	**010**	**011**	**100**	**101**	**110**	**111**
0000	NUL	DLE	SP	0	@	P	`	p
0001	SOH	DC1	!	1	A	Q	a	q
0010	STX	DC2	“	2	B	R	b	r
0011	ETX	DC3	#	3	C	S	c	s
0100	EOT	DC4	$	4	D	T	d	t
0101	ENQ	NAK	%	5	E	U	e	u
0110	ACK	SYN	&	6	F	V	f	v
0111	BEL	ETB	‘	7	G	W	g	w
1000	BS	CAN	(	8	H	X	h	x
1001	HT	EM	)	9	I	Y	i	y
1010	LF	SUB	*	:	J	Z	j	z
1011	VT	ESC	+	;	K	[	k	{
1100	FF	FS	,	<	L	\	l	|
1101	CR	GS	–	=	M	]	m	}
1110	SO	RS	.	>	N	∧	n	~
1111	SI	US	/	?	O	–	o	DEL

Binary Codes

ASCII Character Code

Control characters

NUL	Null	DLE	Data-link escape
SOH	Start of heading	DC1	Device control 1
STX	Start of text	DC2	Device control 2
ETX	End of text	DC3	Device control 3
EOT	End of transmission	DC4	Device control 4
ENQ	Enquiry	NAK	Negative acknowledge
ACK	Acknowledge	SYN	Synchronous idle
BEL	Bell	ETB	End-of-transmission block
BS	Backspace	CAN	Cancel
HT	Horizontal tab	EM	End of medium
LF	Line feed	SUB	Substitute
VT	Vertical tab	ESC	Escape
FF	Form feed	FS	File separator
CR	Carriage return	GS	Group separator
SO	Shift out	RS	Record separator
SI	Shift in	US	Unit separator
SP	Space	DEL	Delete

ASCII Character Codes

- American Standard Code for Information Interchange (Refer to Table 1.7)
- A popular code used to represent information sent as character-based data.
- It uses 7-bits to represent:
  - 94 Graphic printing characters.
  - 34 Non-printing characters.
- Some non-printing characters are used for text format (e.g. BS = Backspace, CR = carriage return).
- Other non-printing characters are used for record marking and flow control (e.g. STX and ETX start and end text areas).

ASCII Properties

ASCII has some interesting properties:

- Digits 0 to 9 span Hexadecimal values 30_{16} to 39_{16}
- Upper case A-Z span 41_{16} to $5A_{16}$
- Lower case a-z span 61_{16} to $7A_{16}$
  - Lower to upper case translation (and vice versa) occurs by flipping bit 6.

Boolean Algebra

Set of Elements: {0,1}

Set of Operations: {., + , ¬ }

x	y	x . y
0	0	0
0	1	0
1	0	0
1	1	1

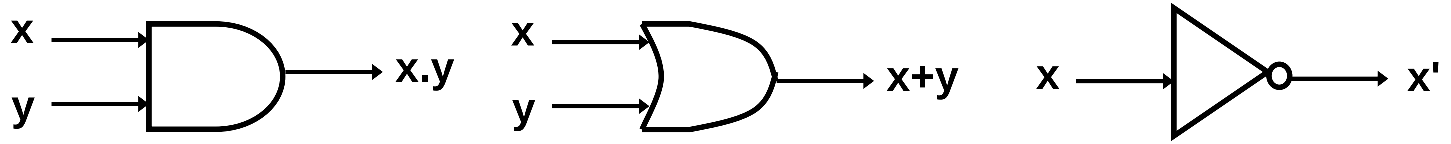
AND

x	y	x + y
0	0	0
0	1	1
1	0	1
1	1	1

OR

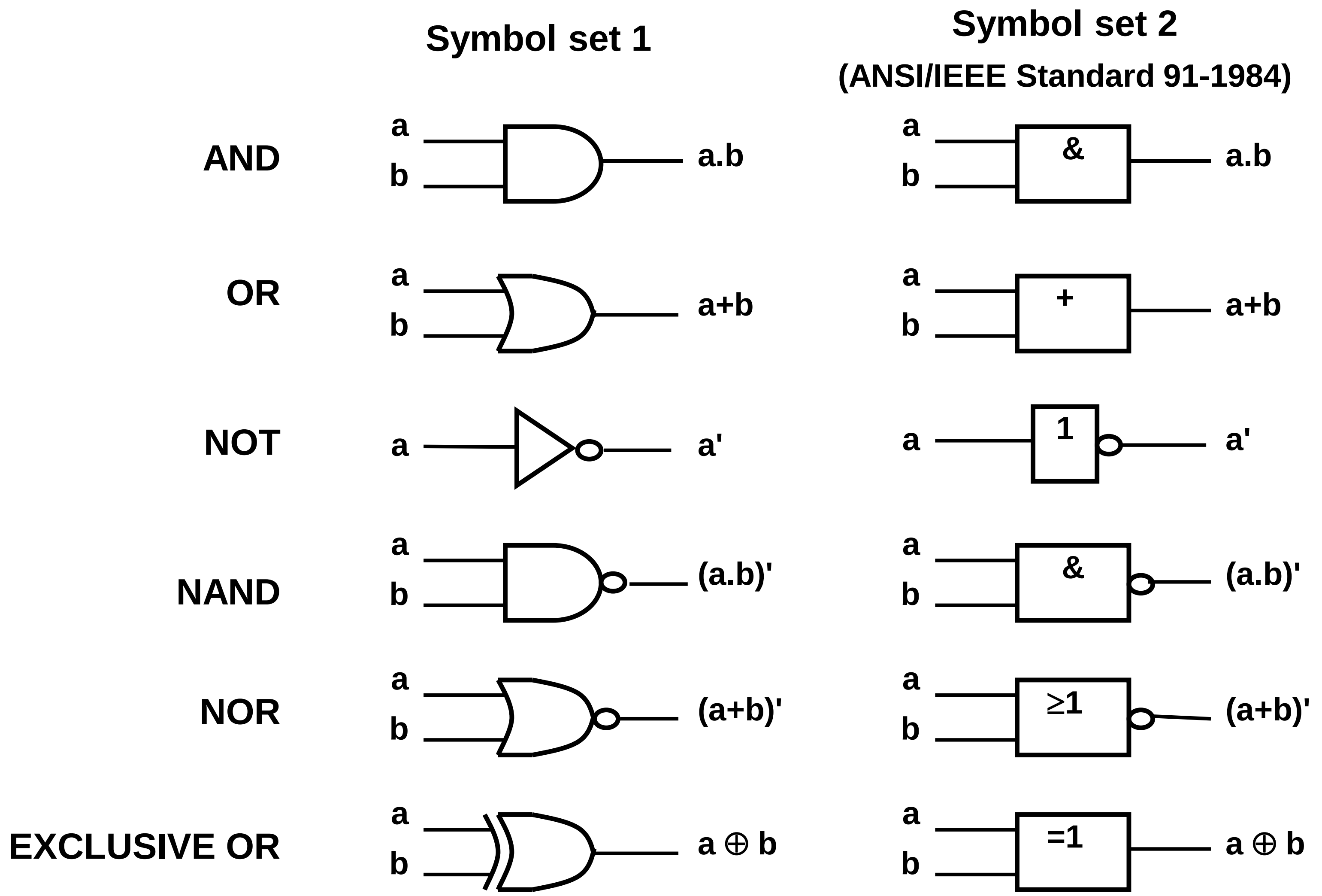
x	¬x
0	1
1	0

NOT



Signals: High = 5V = 1; Low = 0V = 0

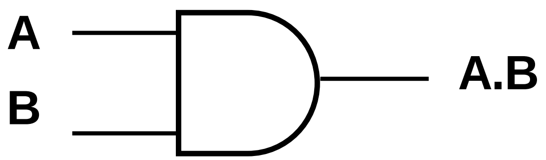
Logic gates



Symbol set 1
Symbol set 2
(ANSI/IEEE Standard 91-1984)
AND
a
b
a.b
&
a
b
a.b
OR
a
b
a+b
+
a
b
a+b
NOT
a
a'
1
a
a'
NAND
a
b
(a.b)'
&
a
b
(a.b)'
NOR
a
b
(a+b)'
≥1
a
b
(a+b)'
EXCLUSIVE OR
a
b
a ⊕ b
=1
a
b
a ⊕ b

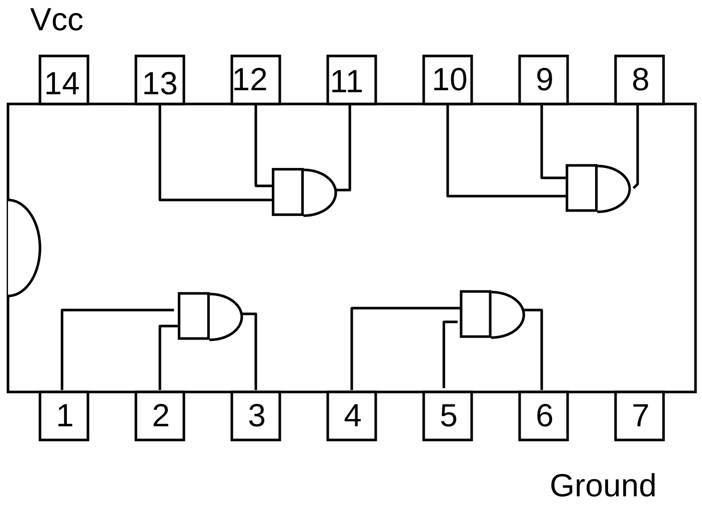
Logic Gates: The AND Gate

The AND Gate



A	B	A . B
0	0	0
0	1	0
1	0	0
1	1	1

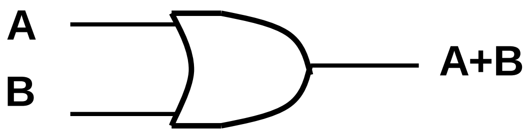
Truth table



Top View of a TTL 74LS family 74LS08 Quad 2-input AND Gate IC Package

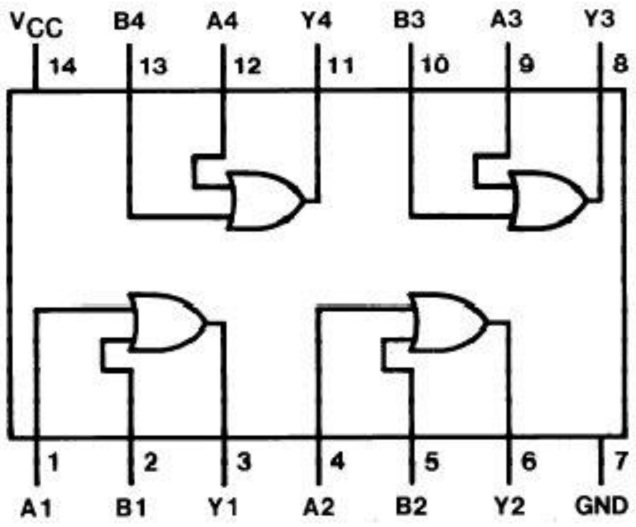
Logic Gates: The OR Gate

The OR Gate



A	B	A + B
0	0	0
0	1	1
1	0	1
1	1	1

Truth table

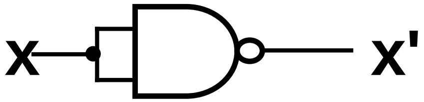


Top View of a TTL 74LS family 74LS08 Quad 2-input OR Gate IC Package

Logic Gates: The NAND Gate

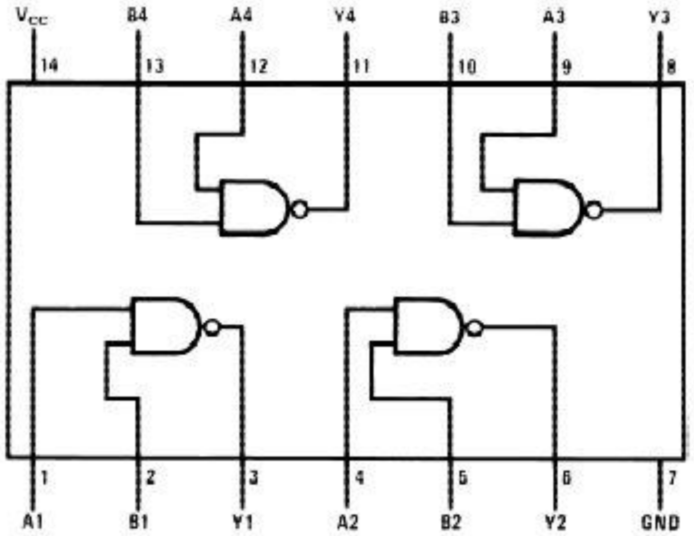
The NAND Gate

- NAND gate is self-sufficient (can build any logic circuit with it).
- Can be used to implement AND/OR/NOT.
- Implementing an inverter using NAND gate:



A	B	(A.B)'
0	0	1
0	1	1
1	0	1
1	1	0

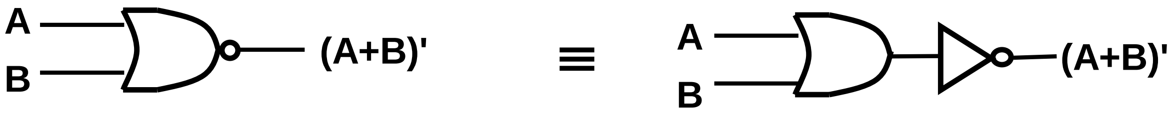
Truth table



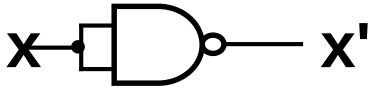
Top View of a TTL 74LS family 74LS00 Quad 2-input NAND Gate IC Package

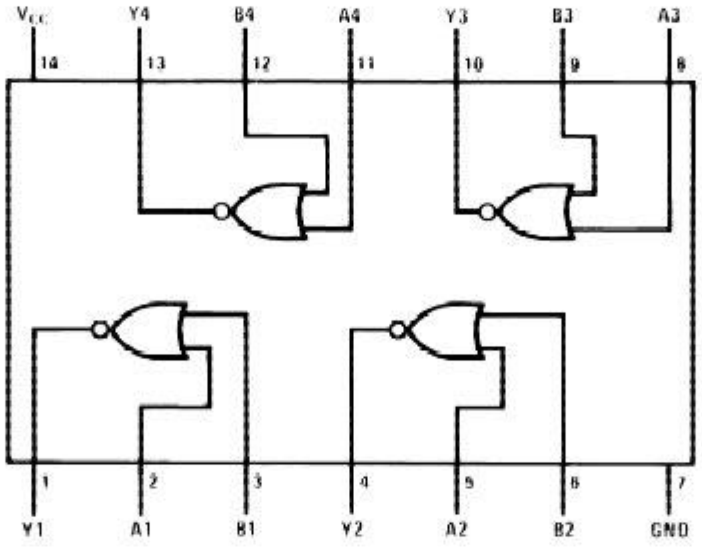
Logic Gates: The NOR Gate

The NOR Gate



- NOR gate is also self-sufficient (can build any logic circuit with it).
- Can be used to implement AND/OR/NOT.
- Implementing an inverter using NOR gate:





A	B	(A+B)'
0	0	1
0	1	0
1	0	0
1	1	0

Truth table

Top View of a TTL 74LS family 74LS02 Quad 2-input NOR Gate IC Package

Thank you

Mustafa Kemal Uyguroğlu