EEE 141
ELECTRICAL CIRCUITS

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CIRCUIT THEOREMS

Linearity Property

Superposition

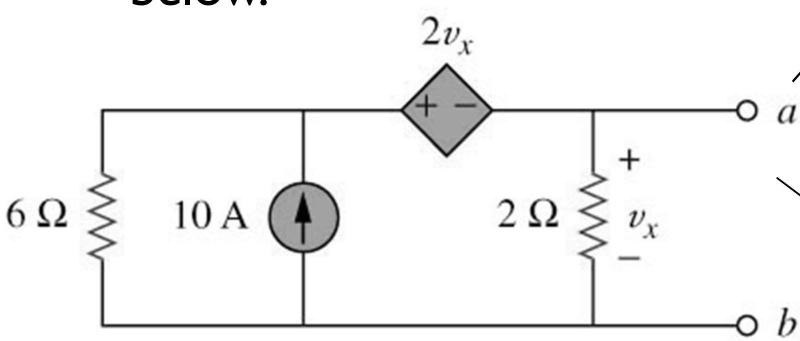
Thevenin's Theorem

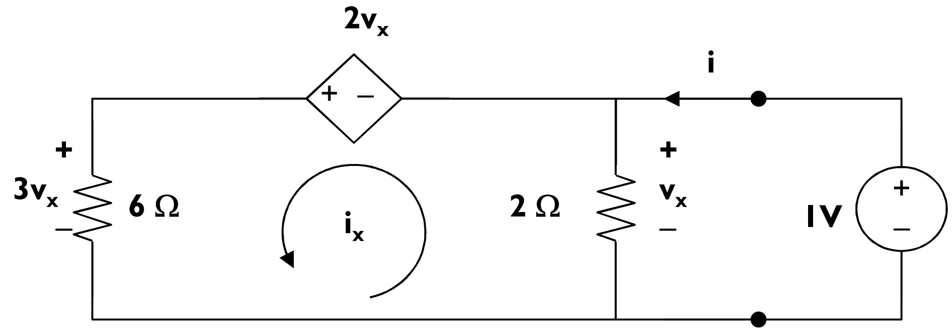
Norton's Theorem

Maximum Power Transfer

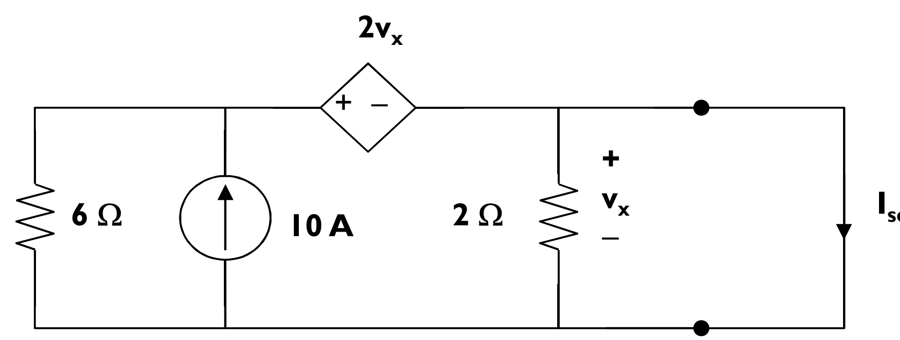
EXAMPLE

Find the Norton equivalent circuit of the circuit shown below.





(a)



(b)

*Refer to in-class illustration, textbook, R_N = 1Ω, I_N = 10A.

EXAMPLE

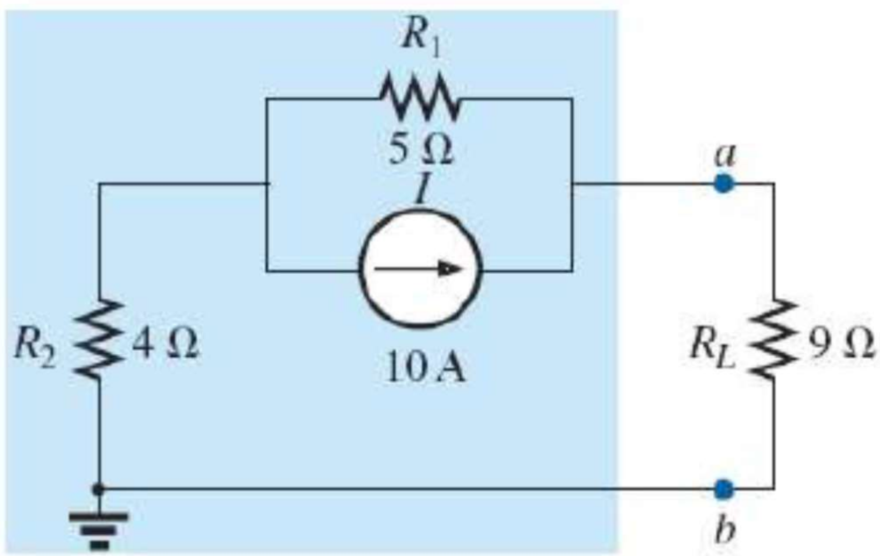


FIG. 9.67 *Example 9.12.*

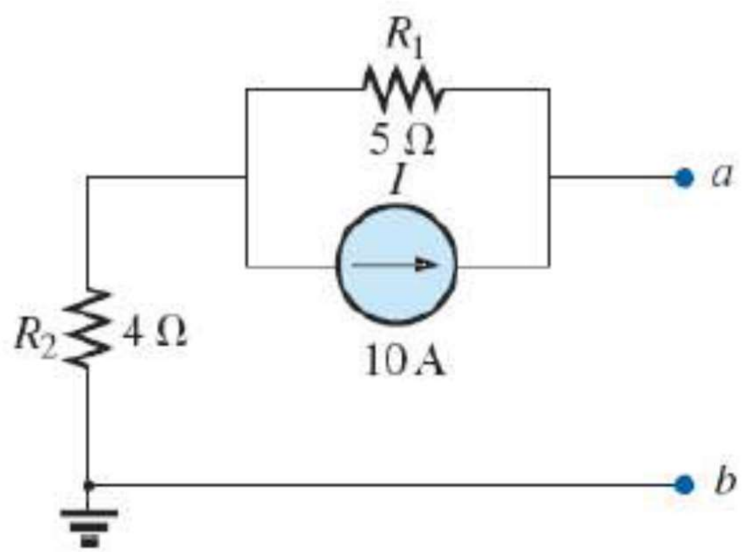


FIG. 9.68 *Identifying the terminals of particular interest for the network in Fig. 9.67.*

EXAMPLE

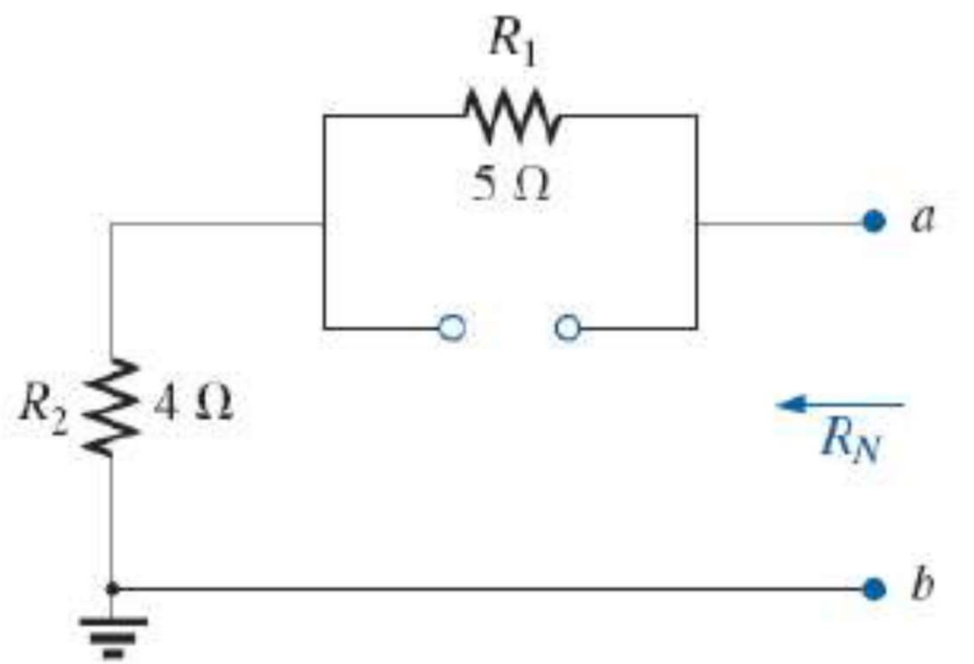


FIG. 9.69 *Determining R_N for the network in Fig. 9.68.*

EXAMPLE

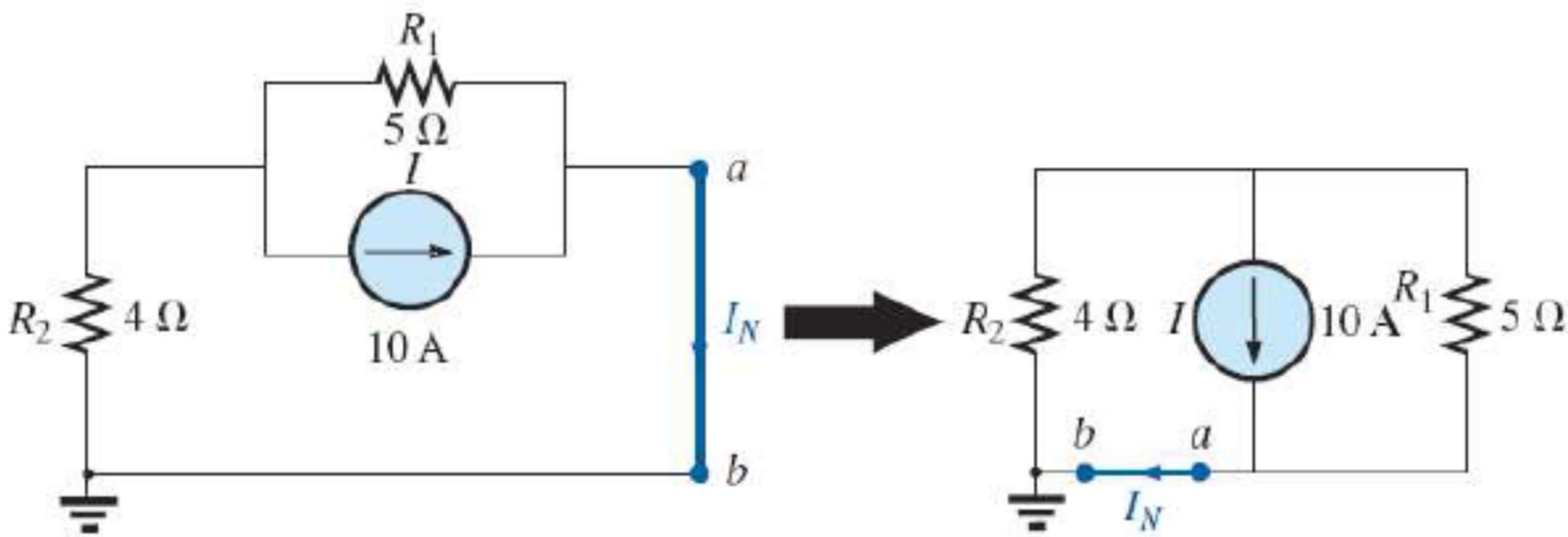


FIG. 9.70 *Determining I_N for the network in Fig. 9.68.*

EXAMPLE

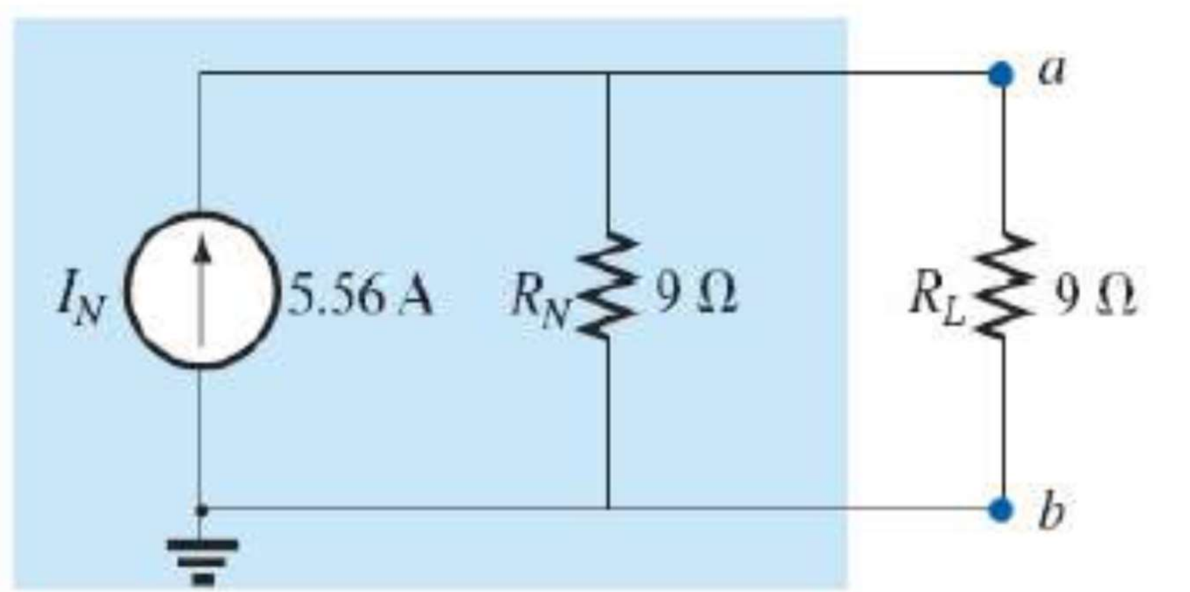


FIG. 9.71 *Substituting the Norton equivalent circuit for the network external to the resistor R_L in Fig. 9.67.*

EXAMPLE

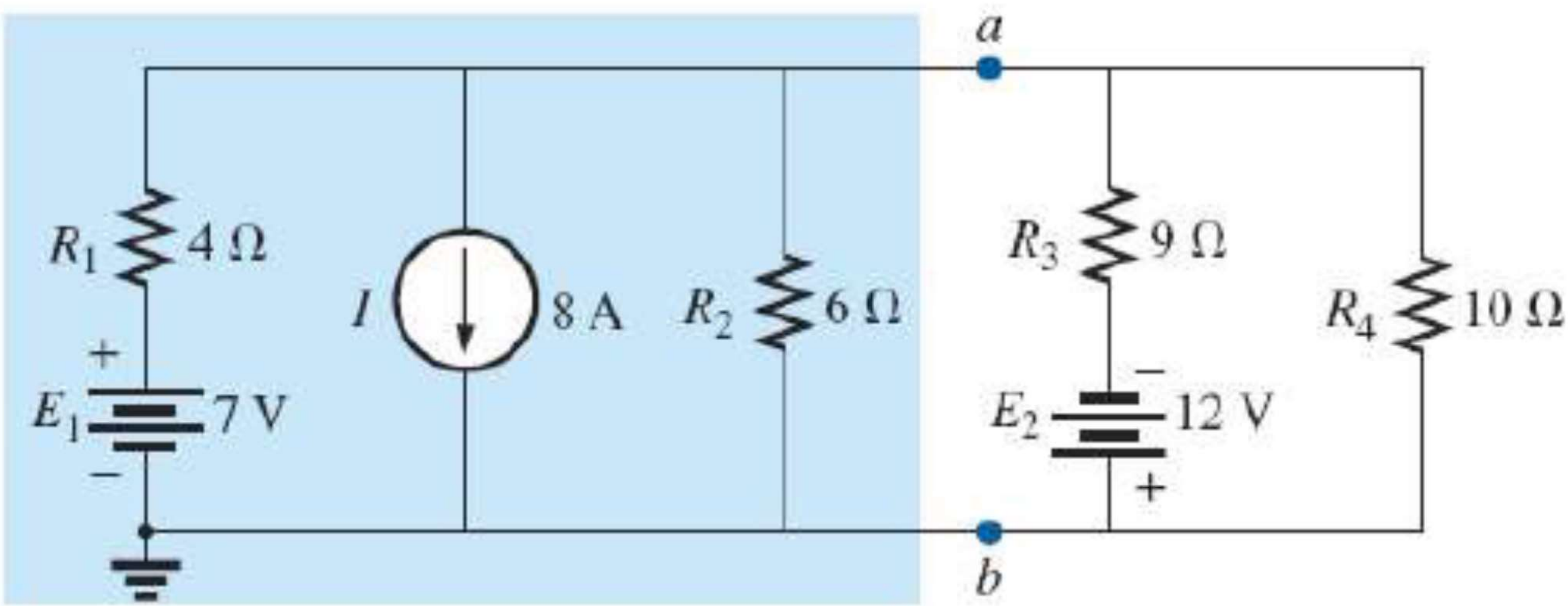


FIG. 9.72 *Example 9.13.*

EXAMPLE

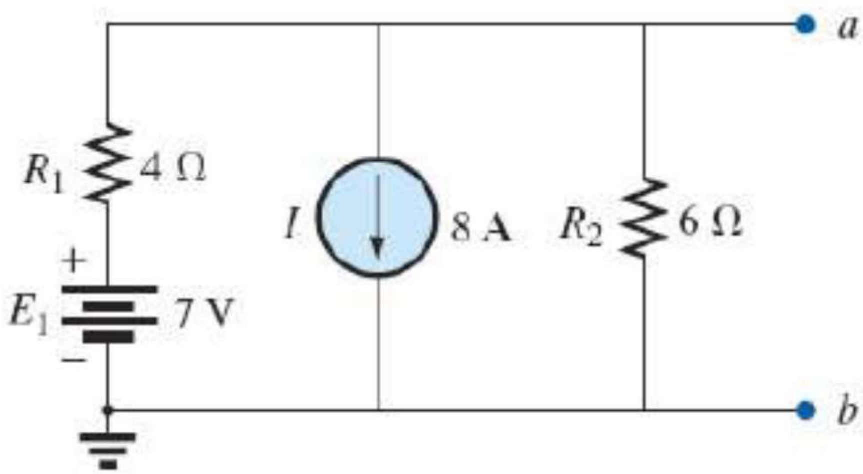


FIG. 9.73 *Identifying the terminals of particular interest for the network in Fig. 9.72.*

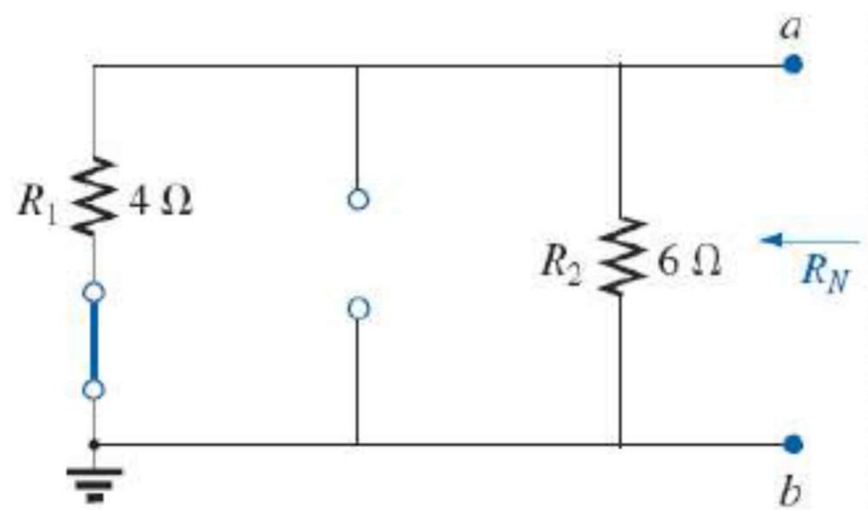


FIG. 9.74 *Determining R_N for the network in Fig. 9.73.*

EXAMPLE

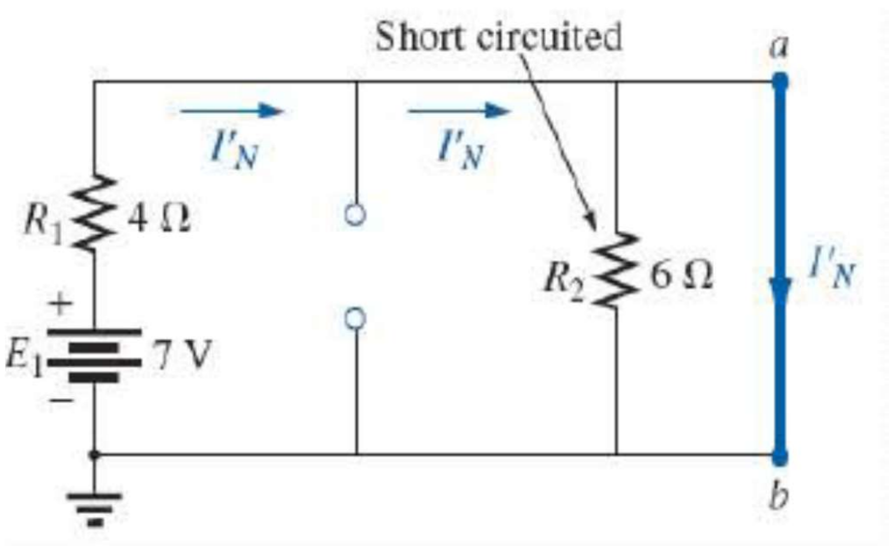


FIG. 9.75 *Determining the contribution to IN from the voltage source E_1.*

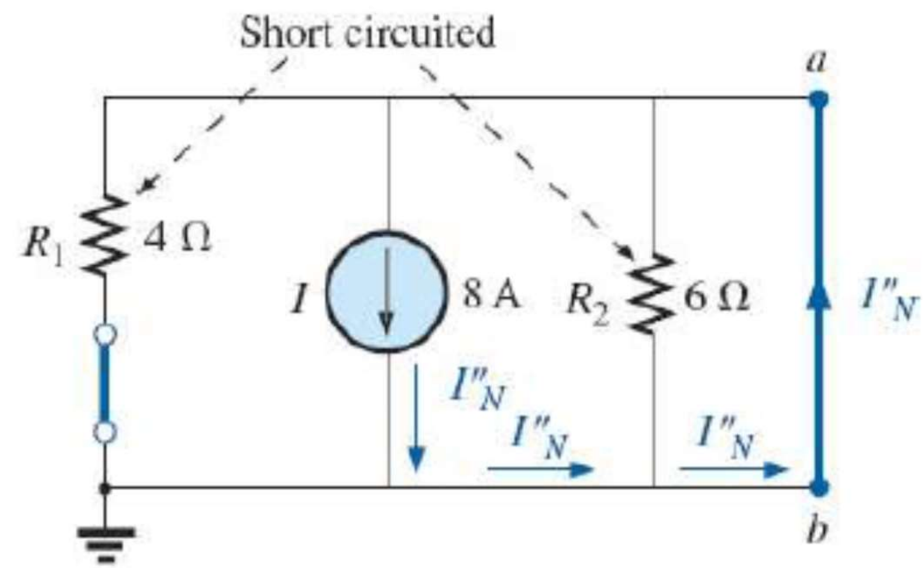


FIG. 9.76 *Determining the contribution to IN from the current source I.*

EXAMPLE

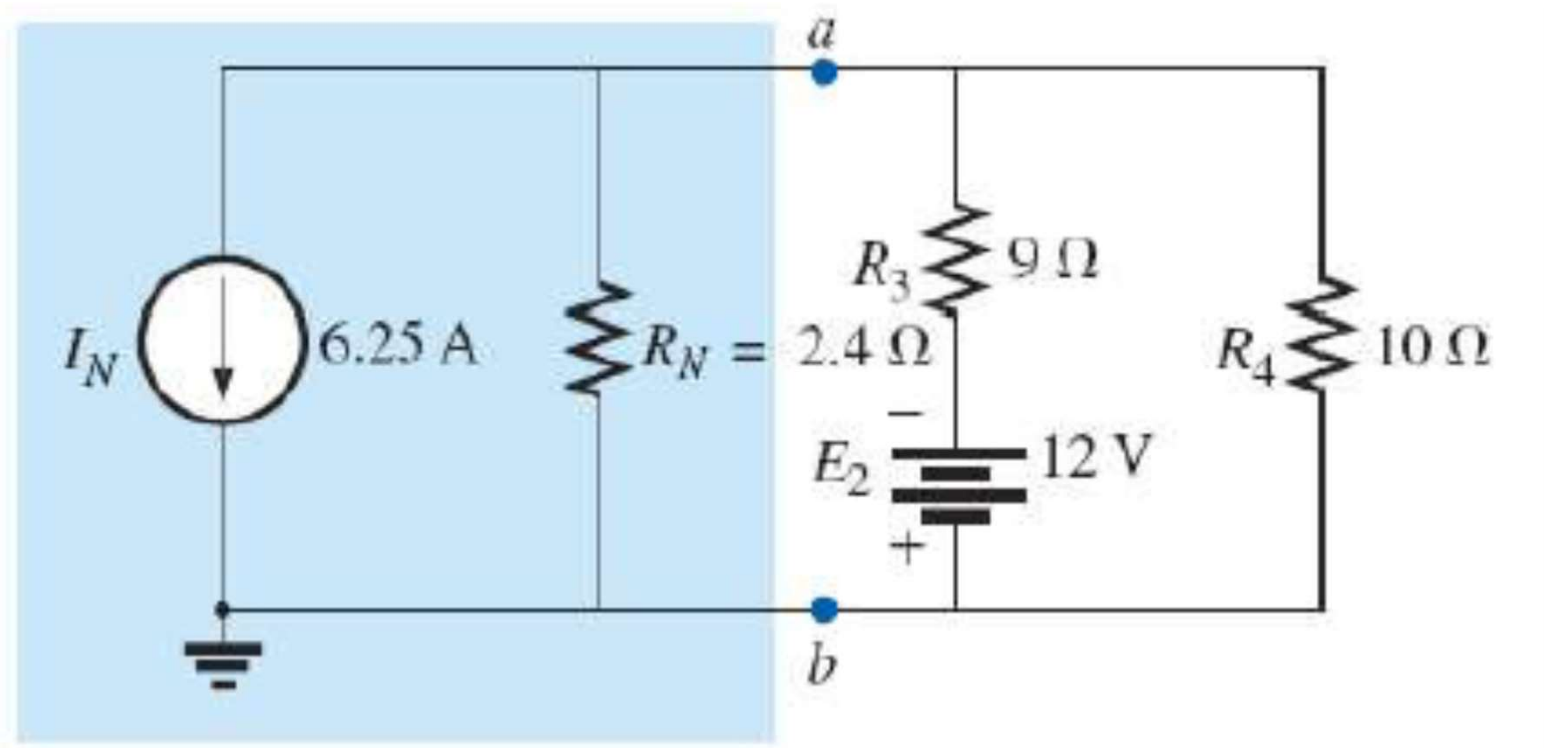


FIG. 9.77 *Substituting the Norton equivalent circuit for the network to the left of terminals a-b in Fig. 9.72.*

MAXIMUM POWER TRANSFER THEOREM

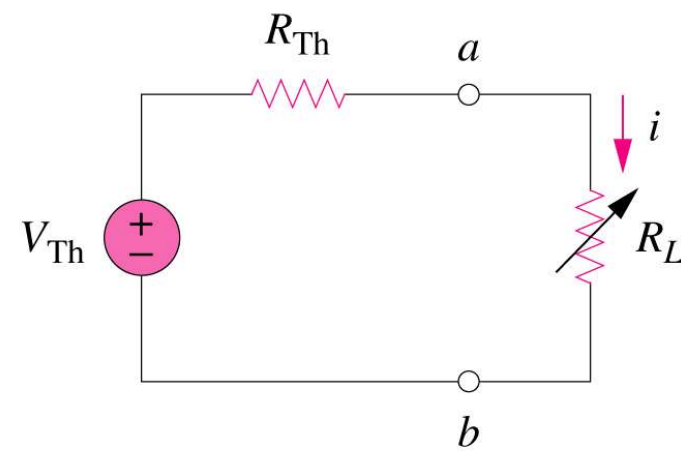
When designing a circuit, it is often important to be able to answer one of the following questions:

– ***What load should be applied to a system to ensure that the load is receiving maximum power from the system?***

Conversely:

– ***For a particular load, what conditions should be imposed on the source to ensure that it will deliver the maximum power available?***

MAXIMUM POWER TRANSFER



If the entire circuit is replaced by its Thevenin equivalent except for the load, the power delivered to the load is:

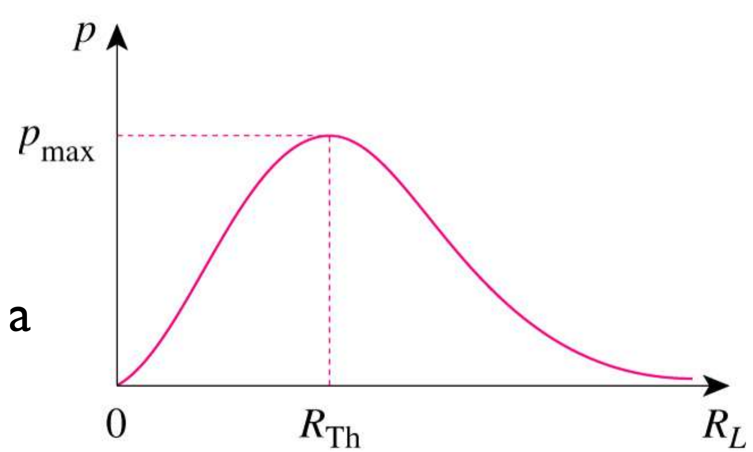
$$P = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$

$$\frac{dp}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th} + R_L)^2 - 2R_L(R_{Th} + R_L)}{(R_{Th} + R_L)^4} \right]$$

$$= V_{Th}^2 \left[\frac{(R_{Th} + R_L - 2R_L)}{(R_{Th} + R_L)^3} \right] = 0$$

For maximum power dissipated in R_L, P_{max}, for a given R_{TH}, and V_{TH},

$$R_L = R_{TH} \quad \Rightarrow \quad P_{max} = \frac{V_{Th}^{\ 2}}{4R_L}$$



The power transfer profile with different R_L

MAXIMUM POWER TRANSFER

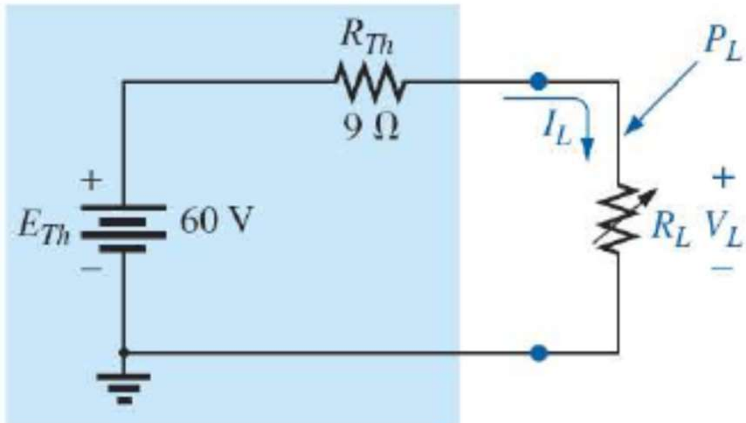


FIG. 9.79 *Thévenin equivalent network to be used to validate the maximum power transfer theorem.*

MAXIMUM POWER TRANSFER

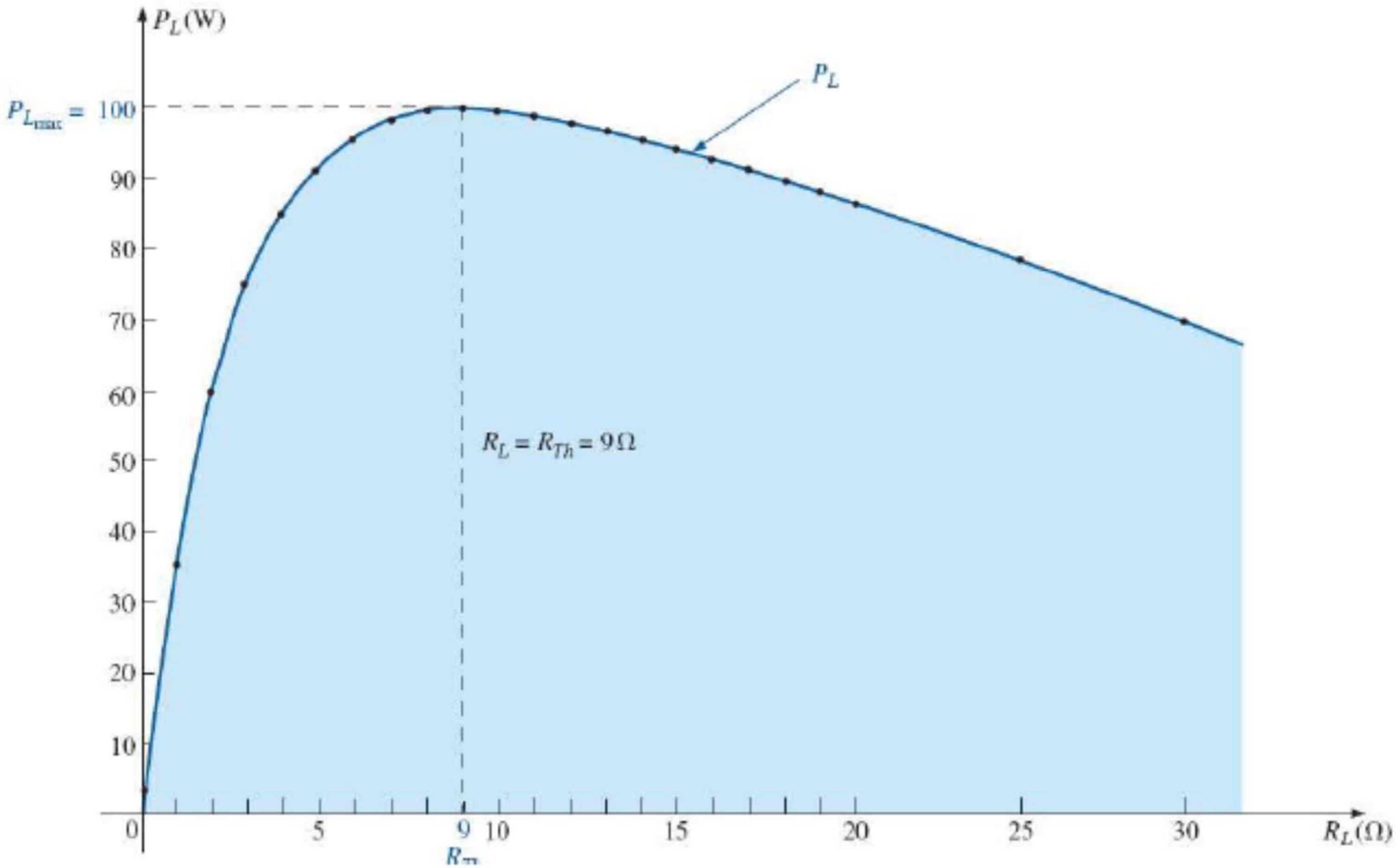


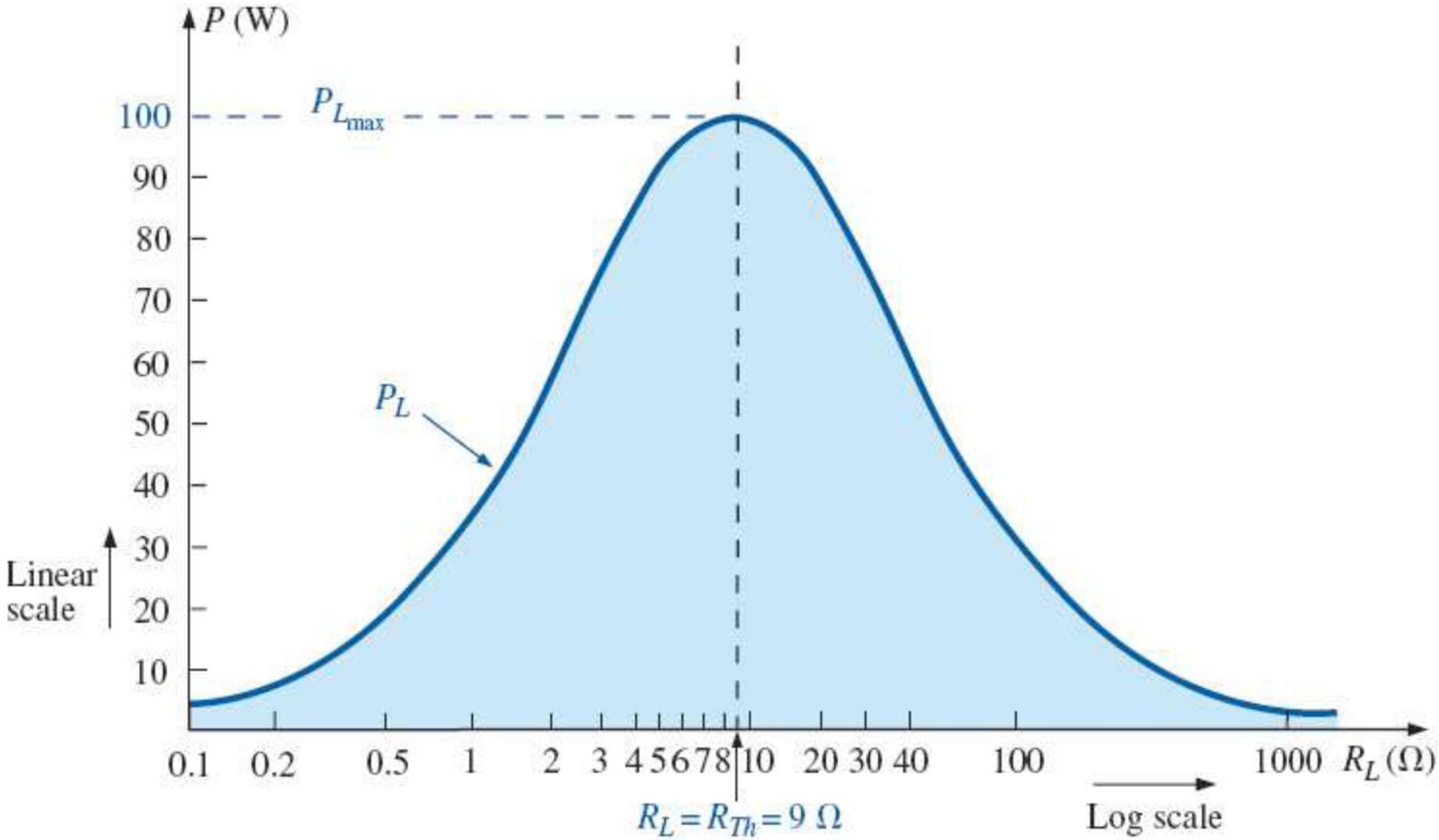
FIG. 9.80 *P_l versus R_l for the network in Fig. 9.79.*

MAXIMUM POWER TRANSFER

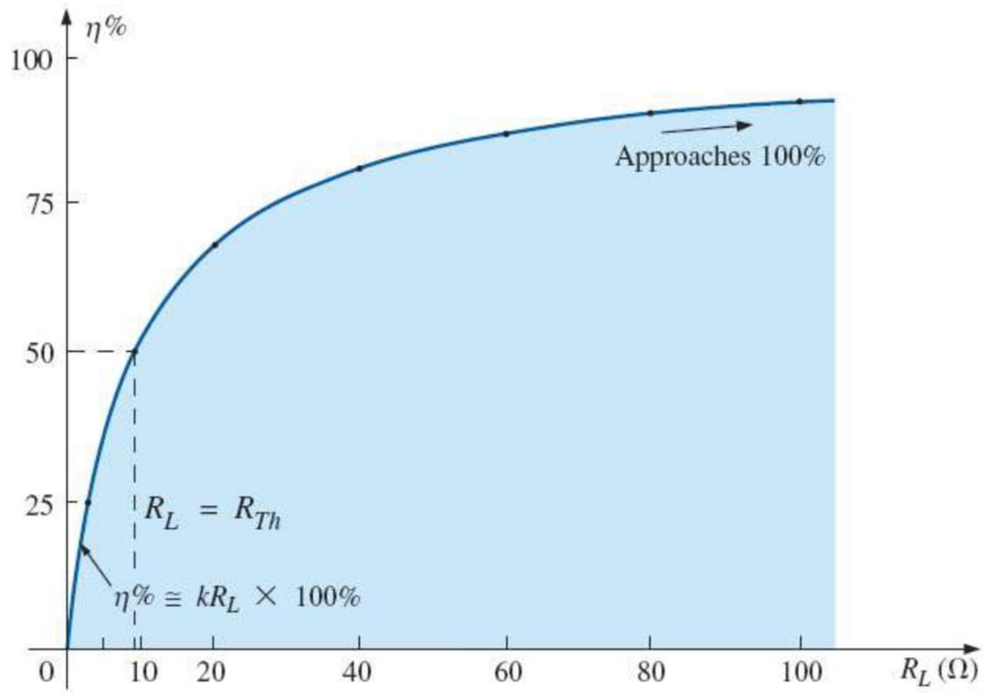
If the load applied is less than the Thévenin resistance, the power to the load will drop off rapidly as it gets smaller.

However, if the applied load is greater than the Thévenin resistance, the power to the load will not drop off as rapidly as it increases.

MAXIMUM POWER TRANSFER



MAXIMUM POWER TRANSFER



$$\eta\% = \frac{P_L}{P_s} \times 100\% = \frac{I_L^2 R_L}{I_L^2 R_T} \times 100\%$$

If efficiency is the overriding factor, then the load should be much larger than the internal resistance of the supply. If maximum power transfer is desired and efficiency less of a concern, then the conditions dictated by the maximum power transfer theorem should be applied.

EXAMPLE

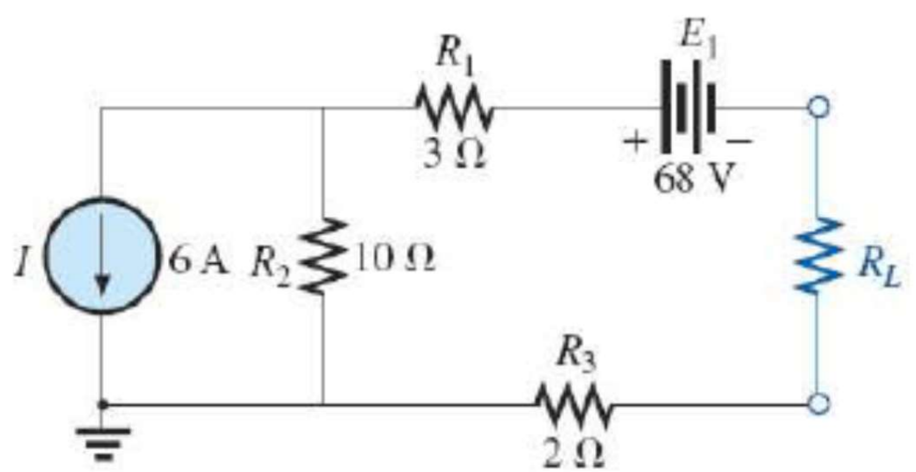


FIG. 9.88 *Example 9.17.*

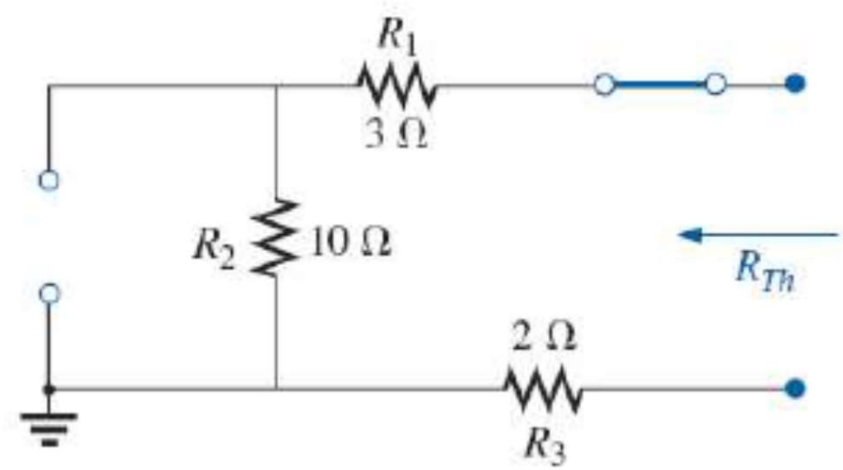


FIG. 9.89 *Determining R_{Th} for the network external to resistor R_L in Fig. 9.88.*

EXAMPLE

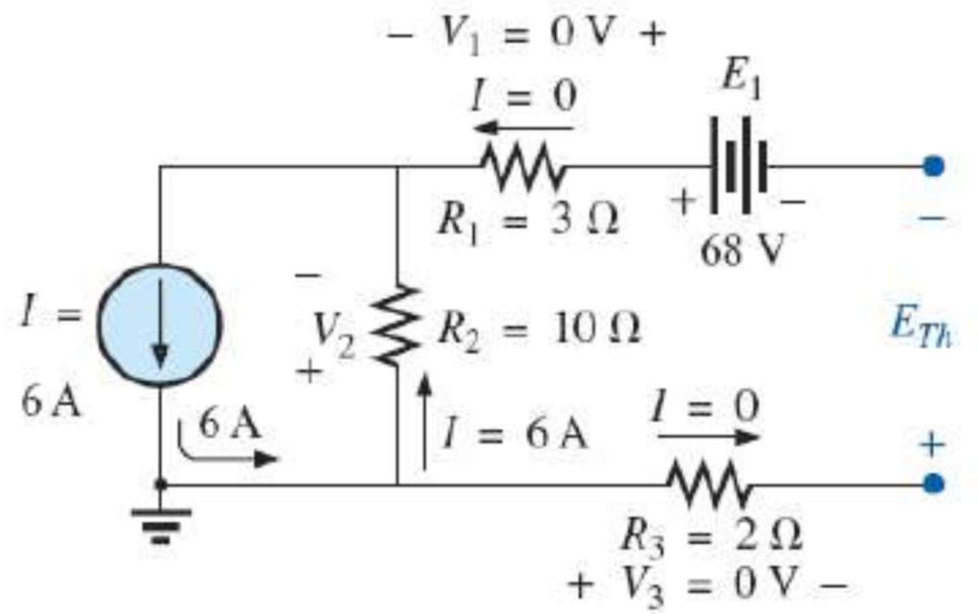


FIG. 9.90 *Determining E_{Th} for the network external to resistor R_L in Fig. 9.88.*

$$V_1 = V_3 = 0\text{ V}$$

and $$V_2 = I_2R_2 = IR_2 = (6\text{ A})(10\ \Omega) = 60\text{ V}$$

Applying Kirchhoff's voltage law,

$$\Sigma_{\circlearrowright} V = -V_2 - E_1 + E_{Th} = 0$$

and $$E_{Th} = V_2 + E_1 = 60\text{ V} + 68\text{ V} = 128\text{ V}$$

Thus, $$P_{L_{\max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(128\text{ V})^2}{4(15\ \Omega)} = \mathbf{273.07\ W}$$

EXAMPLE

Example 4.13

Find the value of R_L for maximum power transfer in the circuit of Fig. 4.50. Find the maximum power.

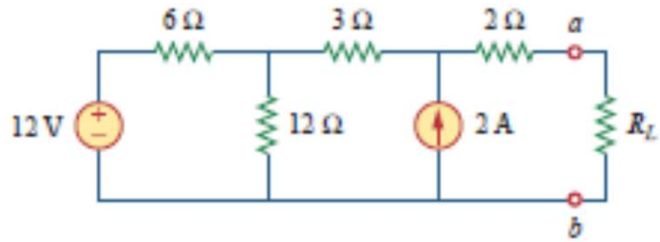


Figure 4.50
For Example 4.13.

Solution:

We need to find the Thevenin resistance R_{Th} and the Thevenin voltage V_{Th} across the terminals *a*-*b*. To get R_{Th}, we use the circuit in Fig. 4.51(a) and obtain

$$R_{Th} = 2 + 3 + 6 \| 12 = 5 + \frac{6 \times 12}{18} = 9\ \Omega$$

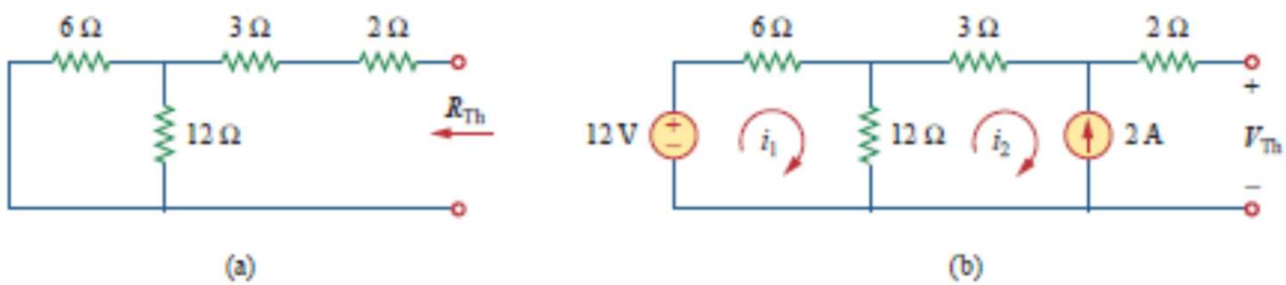


Figure 4.51
For Example 4.13: (a) finding R_{Th}, (b) finding V_{Th}.

EXAMPLE

To get V_{Th}, we consider the circuit in Fig. 4.51(b). Applying mesh analysis gives

$$-12 + 18i_1 - 12i_2 = 0, \qquad i_2 = -2\text{ A}$$

Solving for i_1, we get $i_1 = -2/3$. Applying KVL around the outer loop to get V_{Th} across terminals a-b, we obtain

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{\text{Th}} = 0 \qquad \Rightarrow \qquad V_{\text{Th}} = 22\text{ V}$$

For maximum power transfer,

$$R_L = R_{\text{Th}} = 9\ \Omega$$

and the maximum power is

$$p_{\max} = \frac{V_{\text{Th}}^2}{4R_L} = \frac{22^2}{4 \times 9} = 13.44\text{ W}$$