EEE 141
ELECTRICAL CIRCUITS

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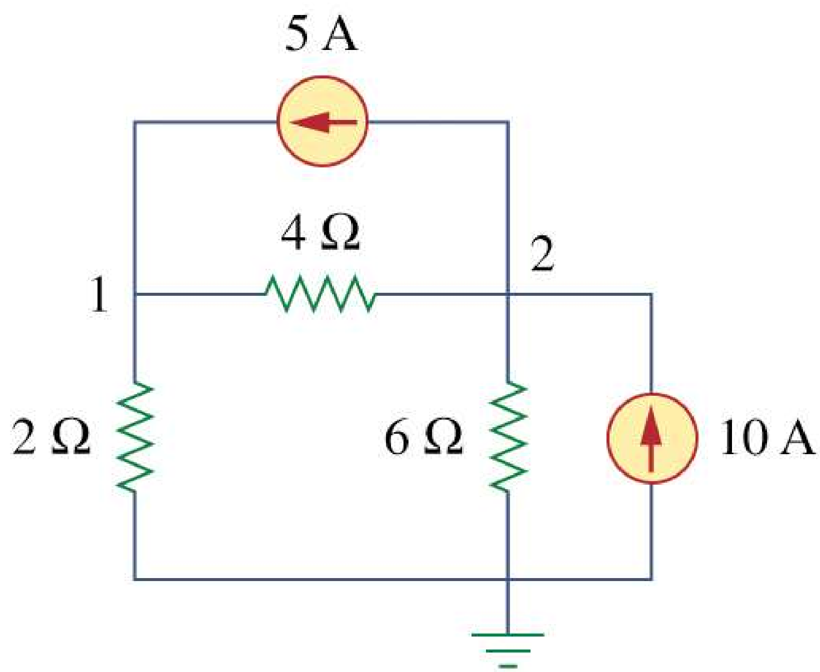
METHODS OF ANALYSIS

- Mesh Analysis
- Nodal Analysis

EXAMPLE

Calculate the node voltage in the circuit shown in Fig(a)



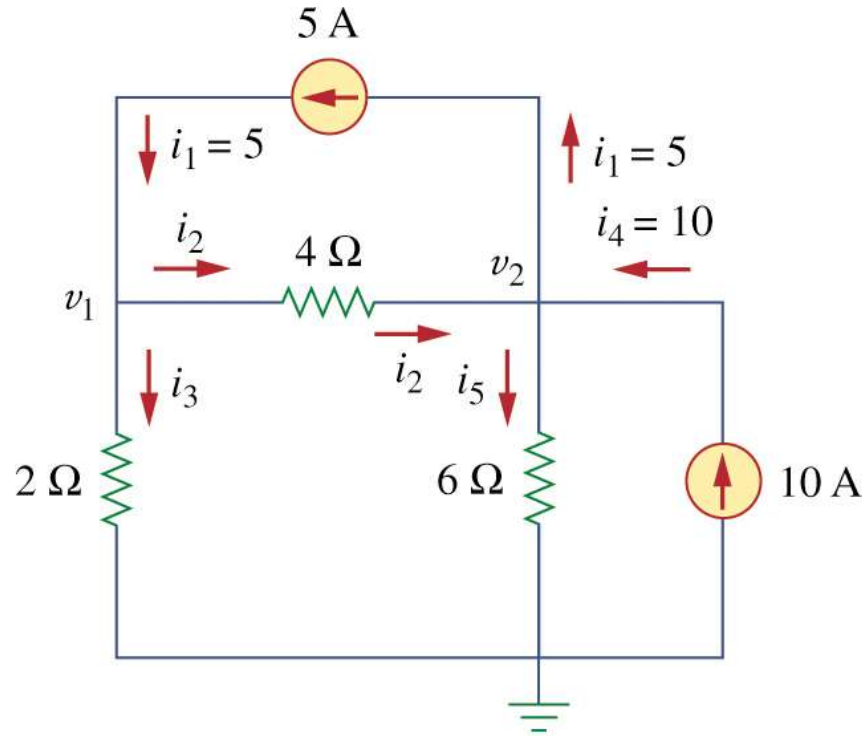
(a)

EXAMPLE

At node 1

$$i_1 = i_2 + i_3$$

$$\Rightarrow 5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2}$$



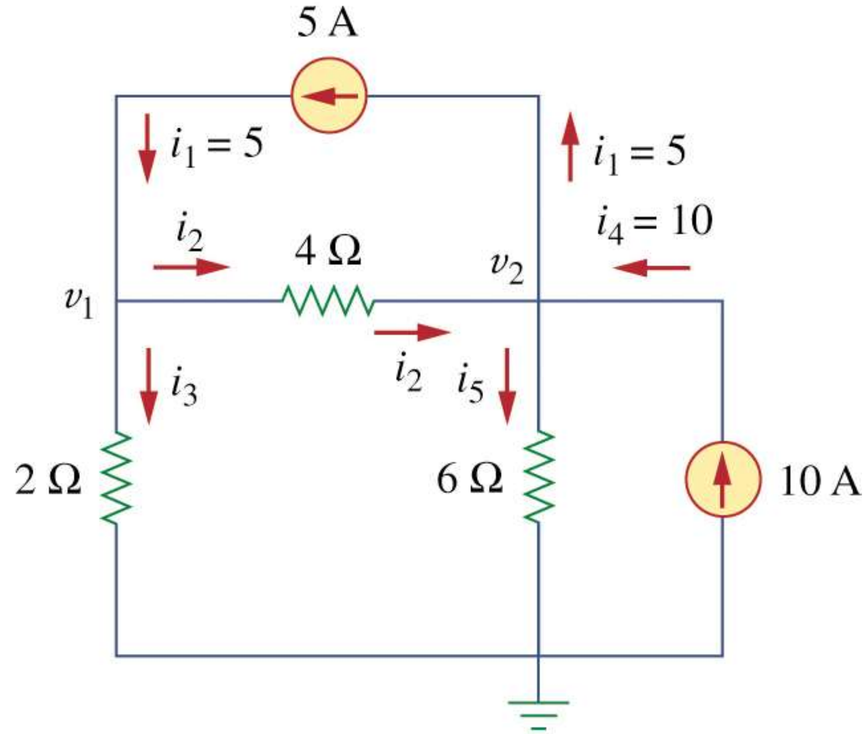
(b)

EXAMPLE

At node 2

$$i_2 + i_4 = i_1 + i_5$$

$$\Rightarrow 5 = \frac{v_2 - v_1}{4} + \frac{v_2 - 0}{6}$$



(b)

EXAMPLE

In matrix form:

$$\begin{bmatrix} \frac{1}{2}+\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{6}+\frac{1}{4} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

EXAMPLE

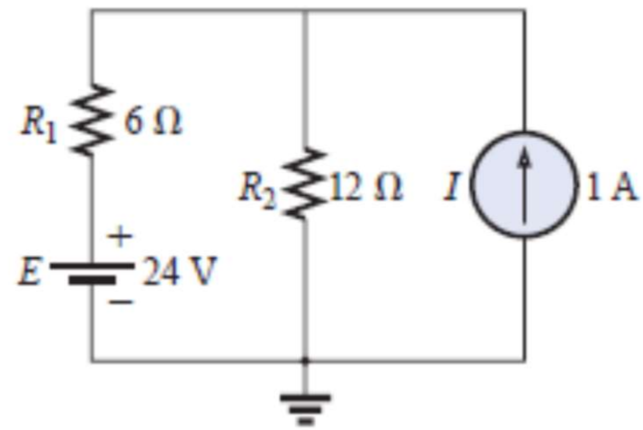


FIG. 8.40
Example 8.19.

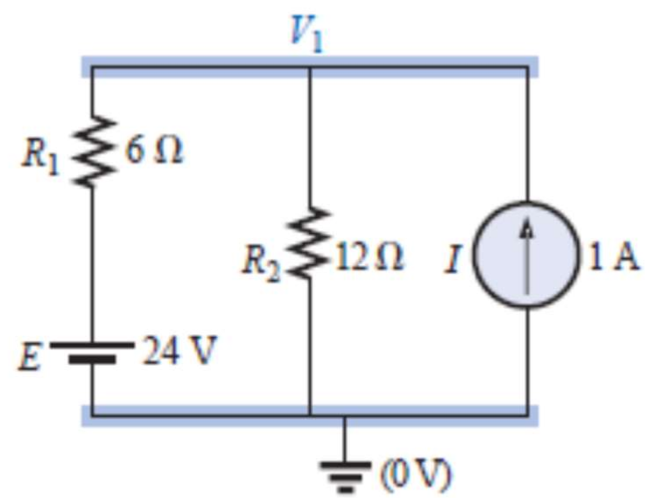


FIG. 8.41
Network of Fig. 8.40 with assigned nodes.

SOLUTION

$$I = I_1 + I_2$$

The current I_2 is related to the nodal voltage V_1 by Ohm's law:

$$I_2 = \frac{V_{R_2}}{R_2} = \frac{V_1}{R_2}$$

The current I_1 is also determined by Ohm's law as follows:

$$I_1 = \frac{V_{R_1}}{R_1}$$

with $$V_{R_1} = V_1 - E$$

Substituting into the Kirchhoff's current law equation:

$$I = \frac{V_1 - E}{R_1} + \frac{V_1}{R_2}$$

and rearranging, we have

$$I = \frac{V_1}{R_1} - \frac{E}{R_1} + \frac{V_1}{R_2} = V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{E}{R_1}$$

or $$V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{E}{R_1} + I$$

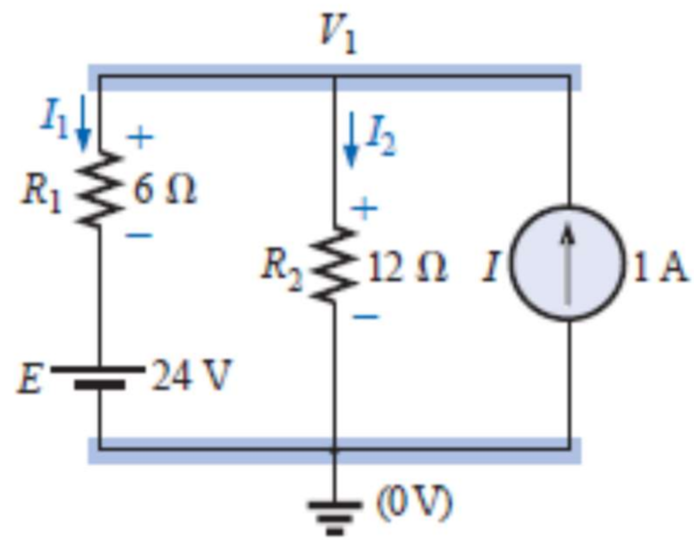


FIG. 8.42
Applying Kirchhoff's current law to the node V_1.

SOLUTION

$$V_1\left(\frac{1}{6\,\Omega} + \frac{1}{12\,\Omega}\right) = \frac{24\text{ V}}{6\,\Omega} + 1\text{ A} = 4\text{ A} + 1\text{ A}$$

$$V_1\left(\frac{1}{4\,\Omega}\right) = 5\text{ A}$$

$$V_1 = \mathbf{20\text{ V}}$$

The currents I_1 and I_2 can then be determined using the preceding equations:

$$I_1 = \frac{V_1 - E}{R_1} = \frac{20\text{ V} - 24\text{ V}}{6\,\Omega} = \frac{-4\text{ V}}{6\,\Omega}$$

$$= \mathbf{-0.667\text{ A}}$$

EXAMPLE

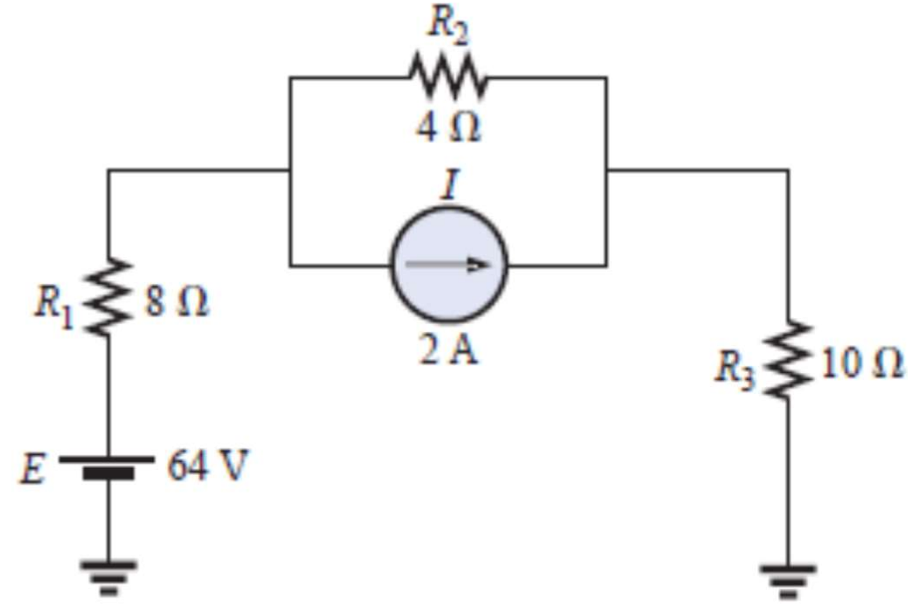


FIG. 8.43
Example 8.20.

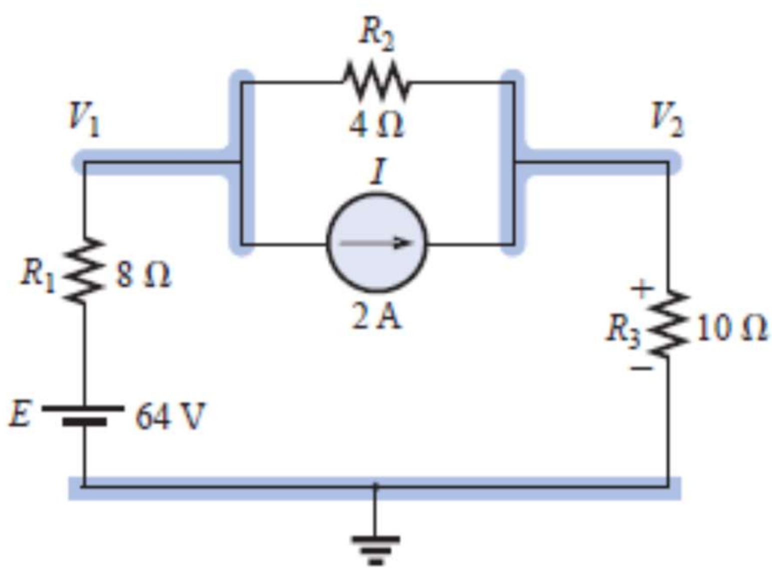


FIG. 8.44
Defining the nodes for the network of Fig. 8.43.

SOLUTION

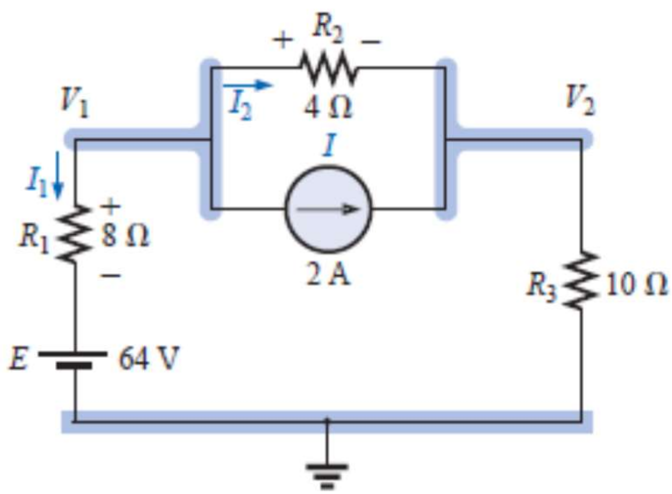


FIG. 8.45
Applying Kirchhoff's current law to node V_1.

Step 3: For node V_1 the currents are defined as shown in Fig. 8.45, and Kirchhoff's current law is applied:

$$0 = I_1 + I_2 + I$$

with
$$I_1 = \frac{V_1 - E}{R_1}$$

and
$$I_2 = \frac{V_{R_2}}{R_2} = \frac{V_1 - V_2}{R_2}$$

so that
$$\frac{V_1 - E}{R_1} + \frac{V_1 - V_2}{R_2} + I = 0$$

or
$$\frac{V_1}{R_1} - \frac{E}{R_1} + \frac{V_1}{R_2} - \frac{V_2}{R_2} + I = 0$$

and
$$V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - V_2\left(\frac{1}{R_2}\right) = -I + \frac{E}{R_1}$$

Substituting values:

$$V_1\left(\frac{1}{8\ \Omega} + \frac{1}{4\ \Omega}\right) - V_2\left(\frac{1}{4\ \Omega}\right) = -2\text{ A} + \frac{64\text{ V}}{8\ \Omega} = 6\text{ A}$$

SOLUTION

$$I = I_2 + I_3$$

$$I = \frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} \qquad I = \frac{V_2}{R_2} - \frac{V_1}{R_2} + \frac{V_2}{R_3}$$

and

$$V_2\left(\frac{1}{R_2} + \frac{1}{R_3}\right) - V_1\left(\frac{1}{R_2}\right) = I$$

Substituting values:

$$V_2\left(\frac{1}{4\,\Omega} + \frac{1}{10\,\Omega}\right) - V_1\left(\frac{1}{4\,\Omega}\right) = 2\text{ A}$$

Step 4: The result is two equations and two unknowns:

$$V_1\left(\frac{1}{8\,\Omega} + \frac{1}{4\,\Omega}\right) - V_2\left(\frac{1}{4\,\Omega}\right) = 6\text{ A}$$

$$-V_1\left(\frac{1}{4\,\Omega}\right) + V_2\left(\frac{1}{4\,\Omega} + \frac{1}{10\,\Omega}\right) = 2\text{ A}$$

which become

$$0.375V_1 - 0.25V_2 = 6$$
$$-0.25V_1 + 0.35V_2 = 2$$

Using determinants,

$$V_1 = \mathbf{37.818\ V}$$
$$V_2 = \mathbf{32.727\ V}$$

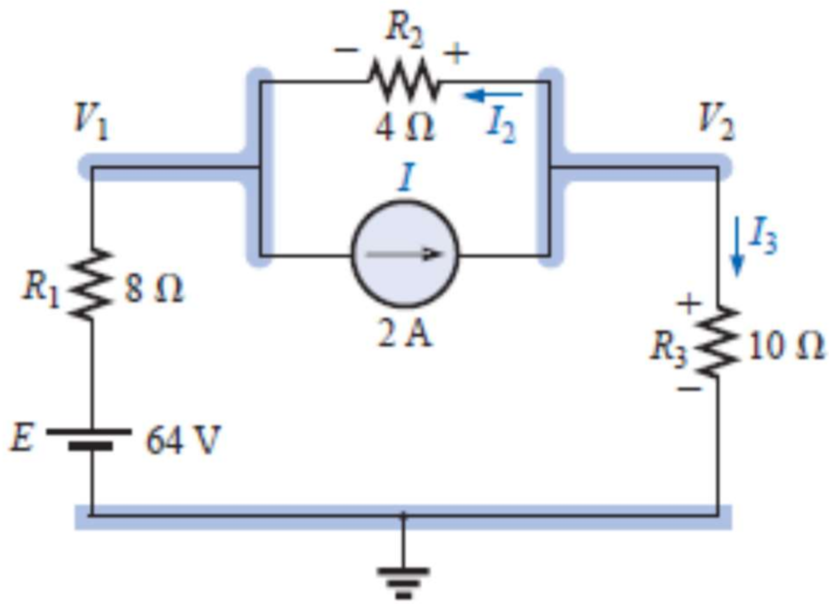


FIG. 8.46
Applying Kirchhoff's current law to node V_2.

SOLUTION

$$I_{R_1} = \frac{E - V_1}{R_1} = \frac{64\,\text{V} - 37.818\,\text{V}}{8\,\Omega} = \mathbf{3.273\,A}$$

The positive value for V_2 results in a current I_{R_3} from node V_2 to ground equal to

$$I_{R_3} = \frac{V_{R_3}}{R_3} = \frac{V_2}{R_3} = \frac{32.727\,\text{V}}{10\,\Omega} = \mathbf{3.273\,A}$$

Since V_1 is greater than V_2, the current I_{R_2} flows from V_1 to V_2 and is equal to

$$I_{R_2} = \frac{V_1 - V_2}{R_2} = \frac{37.818\,\text{V} - 32.727\,\text{V}}{4\,\Omega} = \mathbf{1.273\,A}$$

EXAMPLE

EXAMPLE 8.21 Determine the nodal voltages for the network of Fig. 8.48.

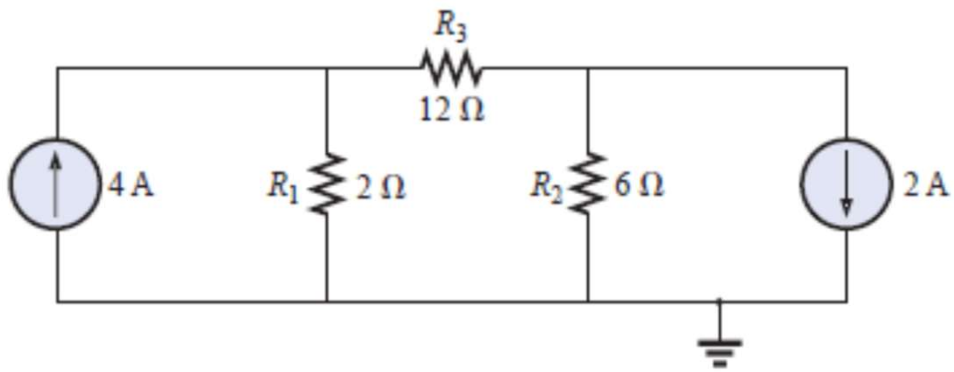
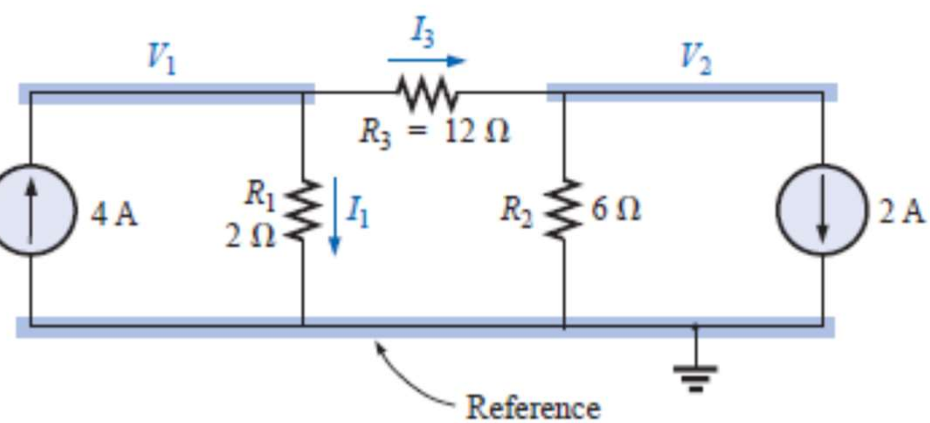


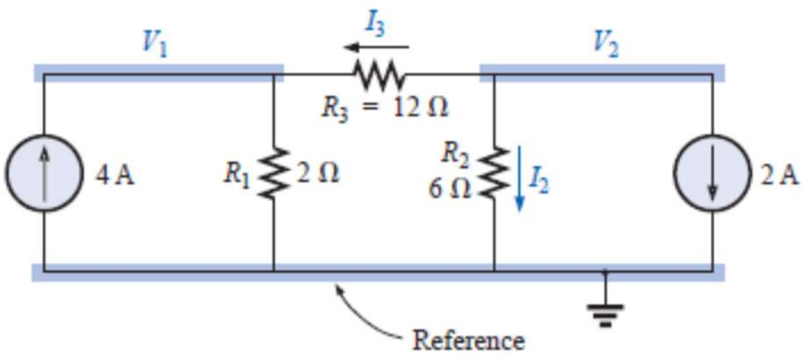
FIG. 8.48
Example 8.21.

Solution:

Steps 1 and 2: As indicated in Fig. 8.49.



SOLUTION



$$\left.\begin{aligned} V_1\left(\frac{1}{2\ \Omega}+\frac{1}{12\ \Omega}\right)-V_2\left(\frac{1}{12\ \Omega}\right)&=+4\text{ A}\\ V_2\left(\frac{1}{12\ \Omega}+\frac{1}{6\ \Omega}\right)-V_1\left(\frac{1}{12\ \Omega}\right)&=-2\text{ A}\end{aligned}\right\}$$

$$\left.\begin{aligned} \frac{7}{12}V_1-\frac{1}{12}V_2&=+4\\ -\frac{1}{12}V_1+\frac{3}{12}V_2&=-2\end{aligned}\right\}\begin{aligned} 7V_1-V_2&=48\\ -1V_1+3V_2&=-24\end{aligned}$$

$$V_1=\frac{\begin{vmatrix}48 & -1\\ -24 & 3\end{vmatrix}}{\begin{vmatrix}7 & -1\\ -1 & 3\end{vmatrix}}=\frac{120}{20}=\mathbf{+6\ V}$$

$$V_2=\frac{\begin{vmatrix}7 & 48\\ -1 & -24\end{vmatrix}}{20}=\frac{-120}{20}=\mathbf{-6\ V}$$

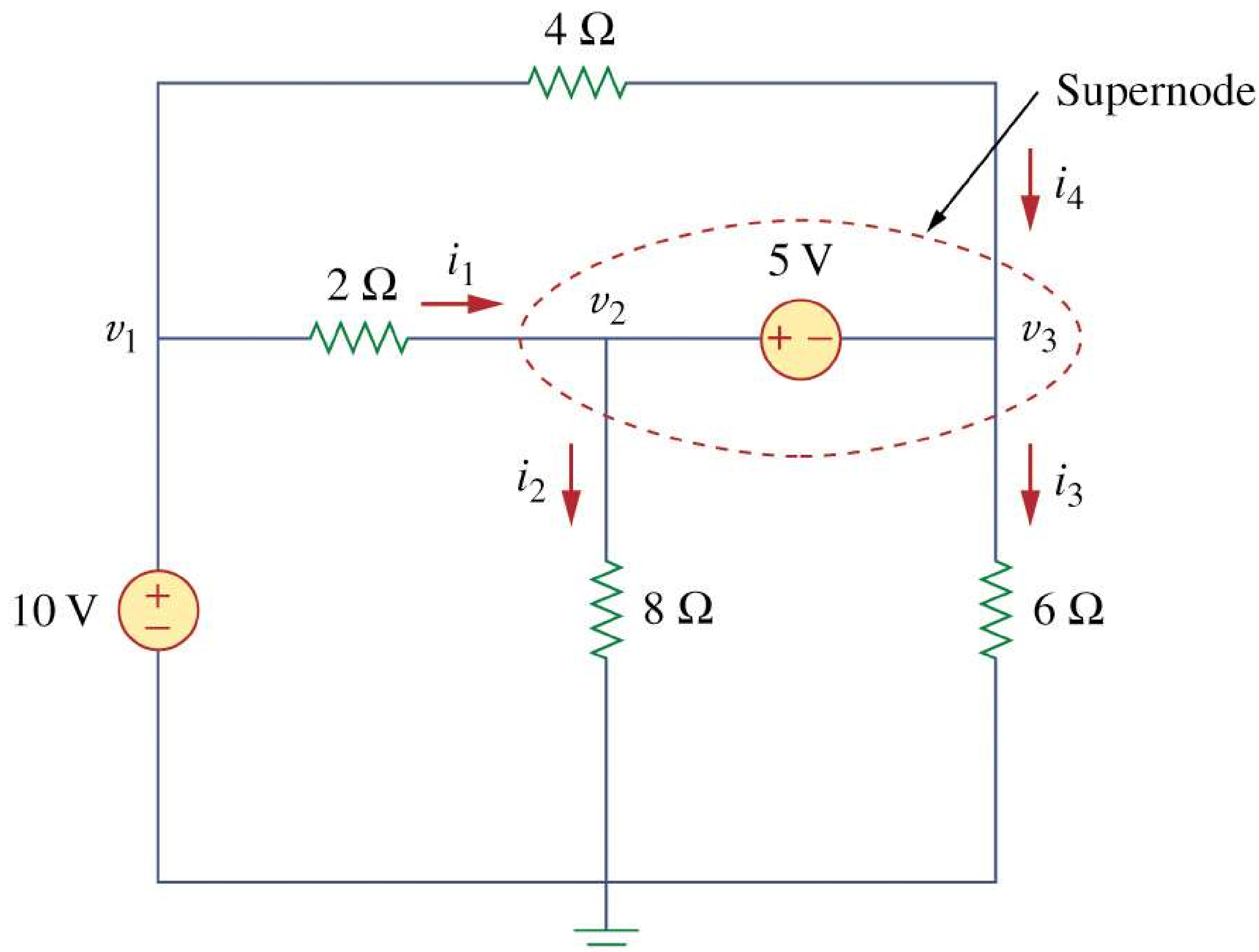
NODAL ANALYSIS WITH VOLTAGE SOURCES

Case 1: The voltage source is connected between a nonreference node and the reference node: The nonreference node voltage is equal to the magnitude of voltage source and the number of unknown nonreference nodes is reduced by one.

Case 2: The voltage source is connected between two nonreferenced nodes: a generalized node (supernode) is formed.

EXAMPLE

A circuit with a supernode.

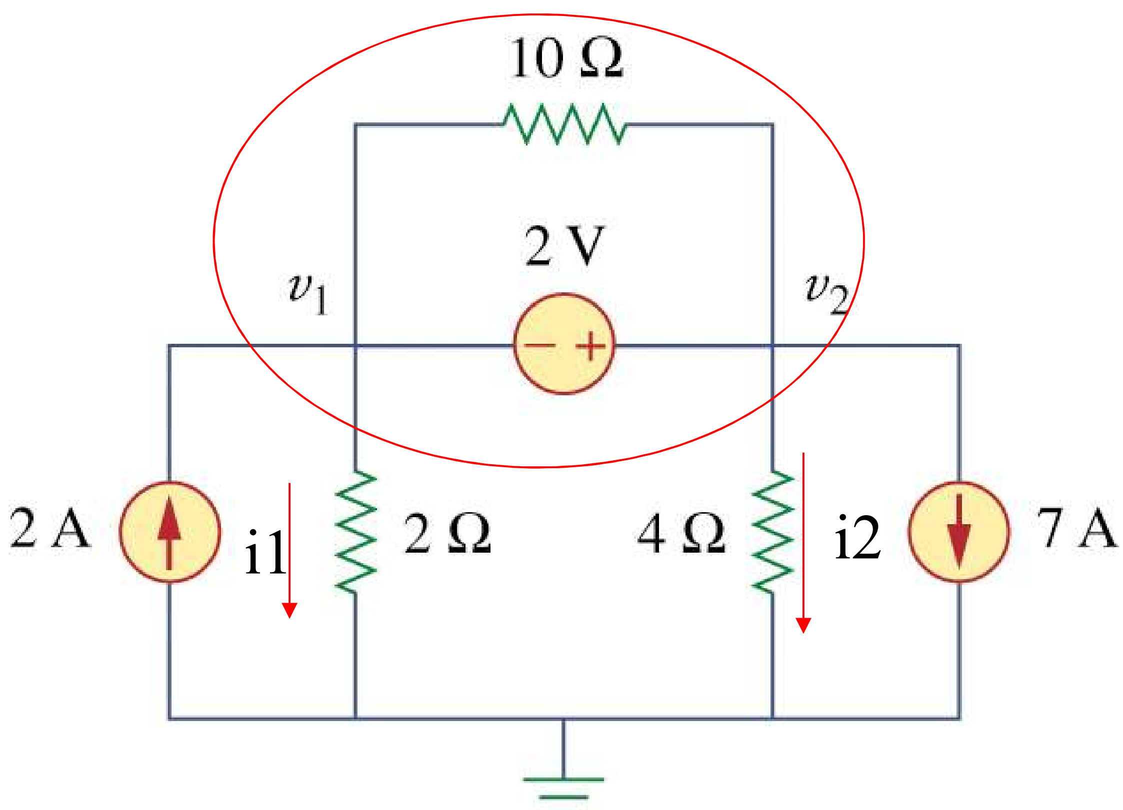


SUPERNODE

- A supernode is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.
- The required two equations for regulating the two nonreference node voltages are obtained by the KCL of the supernode and the relationship of node voltages due to the voltage source.

EXAMPLE

For the circuit shown in Fig., find the node voltages.



$$2-7-i1-i2=0$$

$$2-7-\frac{v_1}{2}-\frac{v_2}{4}=0$$

$$v_1-v_2=-2$$

EXAMPLE

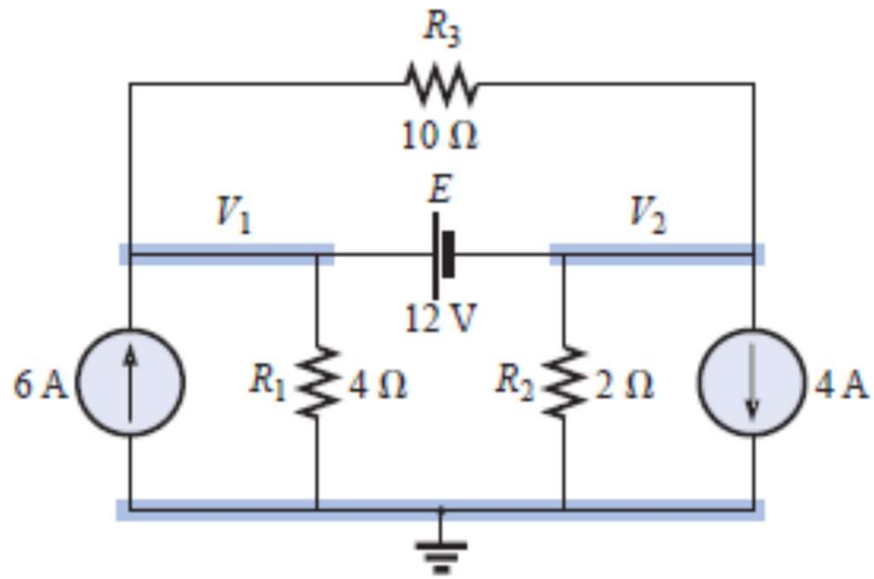


FIG. 8.51
Example 8.22.

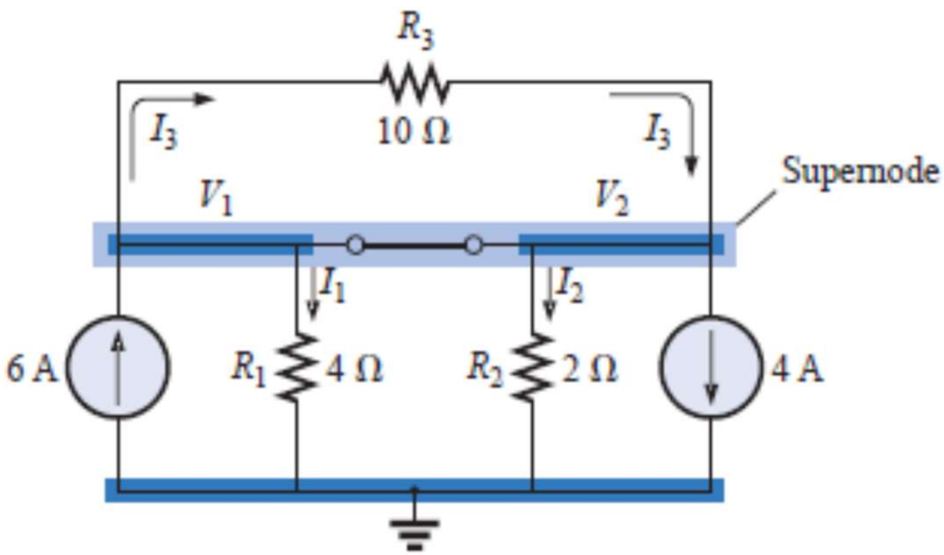


FIG. 8.52
Defining the supernode for the network of Fig. 8.51.

SOLUTION

$$6\text{ A} + I_3 = I_1 + I_2 + 4\text{ A} + I_3$$

$$I_1 + I_2 = 6\text{ A} - 4\text{ A} = 2\text{ A}$$

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} = 2\text{ A}$$

$$\frac{V_1}{4\,\Omega} + \frac{V_2}{2\,\Omega} = 2\text{ A}$$

d nodal voltages to the indepen

$$V_1 - V_2 = E = 12\text{ V}$$

o equations and two unknowns:

$$\begin{aligned} 0.25V_1 + 0.5V_2 &= 2 \\ V_1 - 1V_2 &= 12 \end{aligned}$$

$$V_1 = V_2 + 12$$

$$0.25(V_2 + 12) + 0.5V_2 = 2$$

$$0.75V_2 = 2 - 3 = -1$$

$$V_2 = \frac{-1}{0.75} = \mathbf{-1.333\text{ V}}$$

and $\qquad V_1 = V_2 + 12\text{ V} = -1.333\text{ V} + 12\text{ V} = \mathbf{+10.667\text{ V}}$

The current of the network can then be determined as follows:

$$I_1\downarrow = \frac{V}{R_1} = \frac{10.667\text{ V}}{4\,\Omega} = \mathbf{2.667\text{ A}}$$

$$I_2\uparrow = \frac{V_2}{R_2} = \frac{1.333\text{ V}}{2\,\Omega} = \mathbf{0.667\text{ A}}$$

$$\underset{\rightarrow}{I_3} = \frac{V_1 - V_2}{10\,\Omega} = \frac{10.667\text{ V} - (-1.333\text{ V})}{10\,\Omega} = \frac{12\text{ V}}{10\,\Omega} = \mathbf{1.2\text{ A}}$$

EXAMPLE

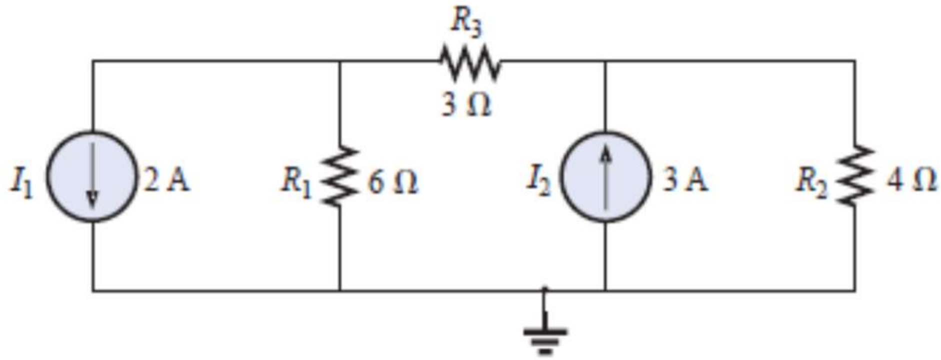
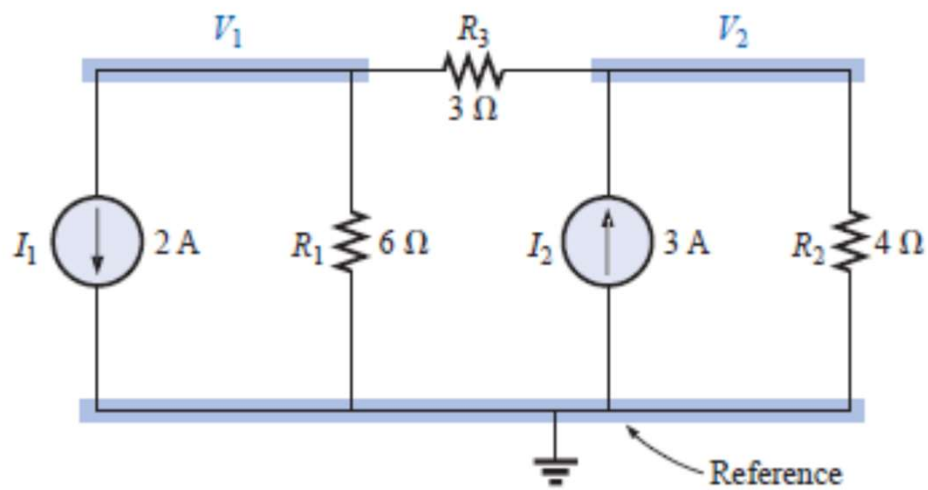


FIG. 8.53
Example 8.23.

Solution:

Step 1: The figure is redrawn with assigned subscripted voltages in Fig. 8.54.



SOLUTION

$$V_1: \quad \underbrace{\left(\frac{1}{6\,\Omega} + \frac{1}{3\,\Omega}\right)}_{\substack{\text{Sum of}\\ \text{conductances}\\ \text{connected}\\ \text{to node 1}}} V_1 - \underbrace{\left(\frac{1}{3\,\Omega}\right)}_{\substack{\text{Mutual}\\ \text{conductance}}} V_2 = \overset{\substack{\text{Drawing current}\\ \text{from node 1}\\ \downarrow}}{-2\text{ A}}$$

$$V_2: \quad \underbrace{\left(\frac{1}{4\,\Omega} + \frac{1}{3\,\Omega}\right)}_{\substack{\text{Sum of}\\ \text{conductances}\\ \text{connected}\\ \text{to node 2}}} V_2 - \underbrace{\left(\frac{1}{3\,\Omega}\right)}_{\substack{\text{Mutual}\\ \text{conductance}}} V_1 = \overset{\substack{\text{Supplying current}\\ \text{to node 2}\\ \downarrow}}{+3\text{ A}}$$

$$\frac{1}{2}V_1 - \frac{1}{3}V_2 = -2$$

$$-\frac{1}{3}V_1 + \frac{7}{12}V_2 = 3$$

EXAMPLE

EXAMPLE 8.24 Find the voltage across the 3-Ω resistor of Fig. 8.55 by nodal analysis.

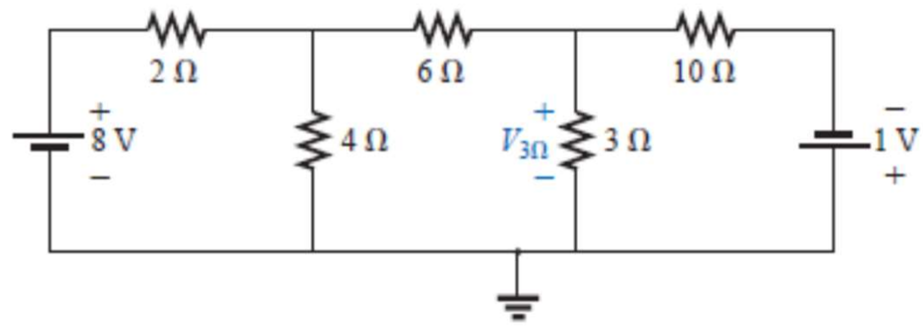


FIG. 8.55
Example 8.24.

Solution: Converting sources and choosing nodes (Fig. 8.56), we have

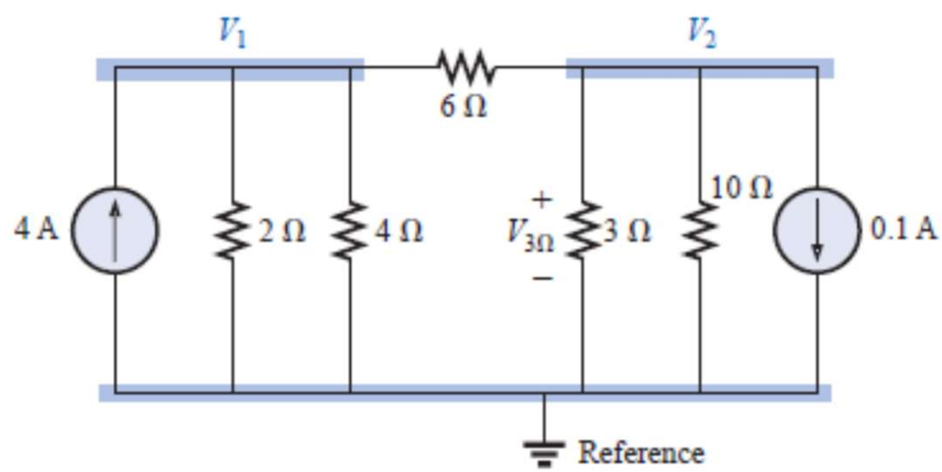


FIG. 8.56
Defining the nodes for the network of Fig. 8.55.

SOLUTION

$$\left.\begin{array}{l}\left(\frac{1}{2\,\Omega}+\frac{1}{4\,\Omega}+\frac{1}{6\,\Omega}\right)V_1-\left(\frac{1}{6\,\Omega}\right)V_2=+4\text{ A}\\ \left(\frac{1}{10\,\Omega}+\frac{1}{3\,\Omega}+\frac{1}{6\,\Omega}\right)V_2-\left(\frac{1}{6\,\Omega}\right)V_1=-0.1\text{ A}\end{array}\right\}$$

$$\frac{11}{12}V_1-\frac{1}{6}V_2=4$$

$$-\frac{1}{6}V_1+\frac{3}{5}V_2=-0.1$$

resulting in

$$\begin{aligned}11V_1-\ \ 2V_2&=+48\\ -5V_1+18V_2&=-3\end{aligned}$$

and

$$V_2=V_{3\Omega}=\frac{\begin{vmatrix}11 & 48\\ -5 & -3\end{vmatrix}}{\begin{vmatrix}11 & -2\\ -5 & 18\end{vmatrix}}=\frac{-33+240}{198-10}=\frac{207}{188}=\mathbf{1.101\ V}$$