EEE 141
ELECTRICAL CIRCUITS

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MAGNETIC CIRCUIT

Magnetic Flux

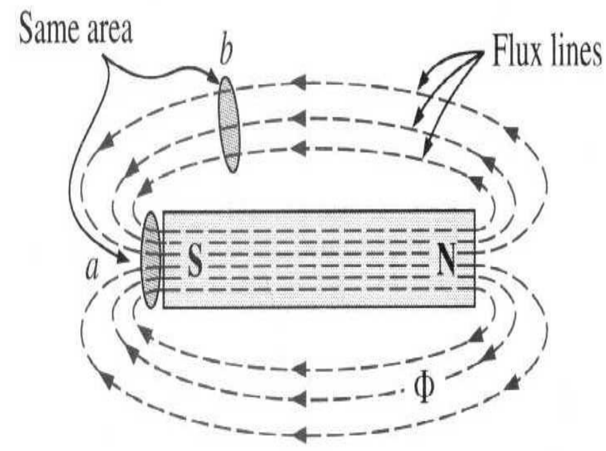
Magnetic Effects of Electric Current

Magnetic Circuit

B-H Curves

Calculations

MAGNETIC FIELDS



- In the region surrounding a permanent magnet there exists a magnetic field, which can be represented by **magnetic flux lines** similar to electric flux lines
- Magnetic flux lines differ from electric flux lines in that they don’t have an origin or termination point
- Magnetic flux lines radiate from the north pole to the south pole through the magnetic bar
- The strength of a magnetic field in a given region is directly related to the density of flux lines in that region

MAGNETIC FIELDS

- If like poles are brought together, the magnets will repel, and the flux distribution will be as shown

- If a nonmagnetic material, such as glass or copper, is placed in the flux paths surrounding a permanent magnet, there will be an almost unnoticeable change in the flux distribution

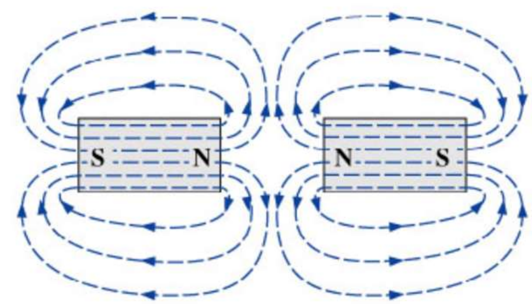
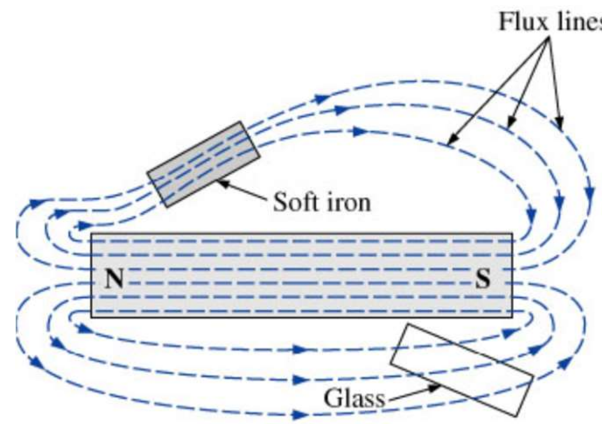
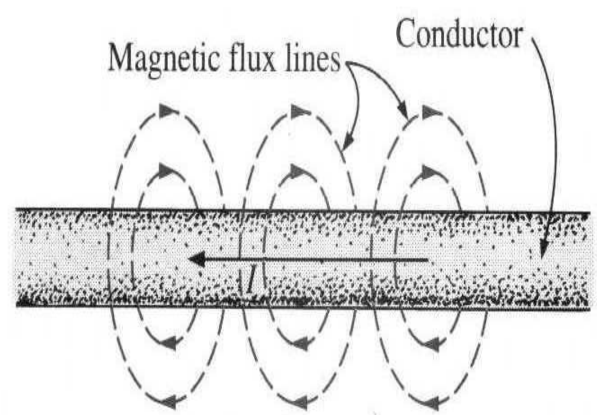


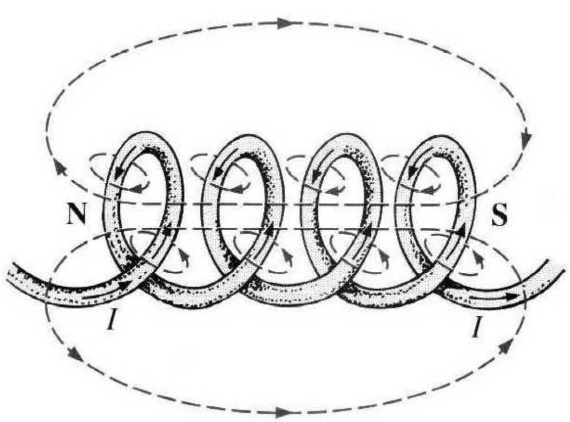
FIG. 11.3
Flux distribution for two adjacent, like poles.

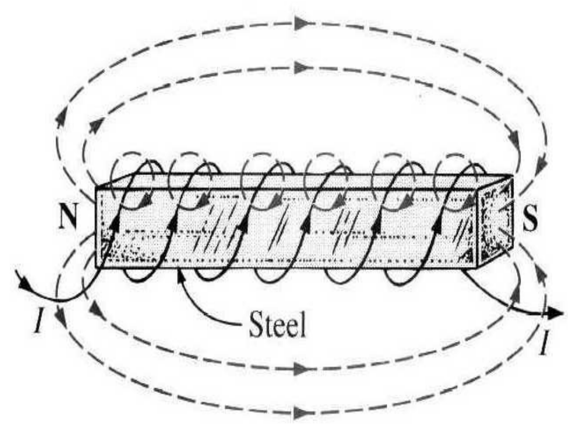


MAGNETIC EFFECTS OF ELECTRIC CURRENT

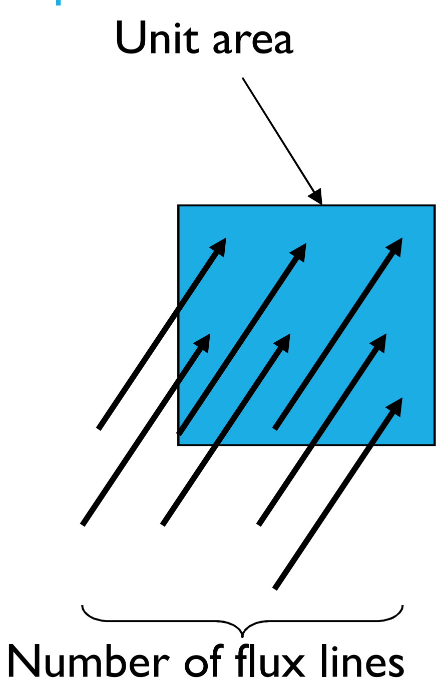
- A conductor carrying an electric current will be surrounded by a magnetic field
- A conductor may be wound into a **coil** to increase the strength of the magnetic field
- An **iron core** placed inside the coil will greatly increase the strength magnetic field







FLUX DENSITY



The strength of a magnetic field is given by **flux density B**

Flux density **B** is the number of (imaginary) flux lines Φ passing through a unit area **A**

$$\mathbf{B = \Phi / A}$$

B = Tesla (T)

Φ = Weber (Wb)

A = square metres (m^2)

PERMEABILITY

- To permeate means to penetrate or pass through. The permeability of a magnetic material is a measure of how easy it is for a magnetic field to pass through it.
- Iron has a very high permeability (It is easy for magnetic flux to pass through it)
- Air has a low permeability (It is hard for magnetic flux to pass through it)
- Permeability of free space:

$$\mu_0 = 4\Pi \text{ x } 10^{-7} \text{ Wb/(At.m)}$$

PERMEABILITY

Materials that have permeability slightly less than that of free space are said to be **diamagnetic** and those with permeability slightly greater than that of free space are said to be **paramagnetic**

Magnetic materials, such as iron, nickel, steel and alloys of these materials, have permeability hundreds and even thousands of times that of free space and are referred to as **ferromagnetic**

Relative permeability is the permeability of the material compared to the permeability of free space:

$$\mu_r = \frac{\mu}{\mu_0}$$

RELUCTANCE

The resistance of a material to the flow of charge (current) is determined for electric circuits by the equation

$$R = \rho \frac{l}{A}$$

The **reluctance** of a material to the setting up of magnetic flux lines in a material is determined by the following equation

$$\boxed{\mathcal{R} = \frac{l}{\mu A}} \qquad \text{(rels, or At/Wb)}$$

OHM'S LAW FOR MAGNETIC CIRCUITS

Ohm's law

$$\text{Effect} = \frac{\text{cause}}{\text{opposition}}$$

For magnetic circuits, the effect is the flux Φ

The cause is the **magnetomotive force (mmf)** $\mathcal{F}$**,** which is the external force (or "pressure") required to set up the magnetic flux lines within the magnetic material

The opposition to the setting up of the flux Φ is the reluctance $\mathcal{R}$

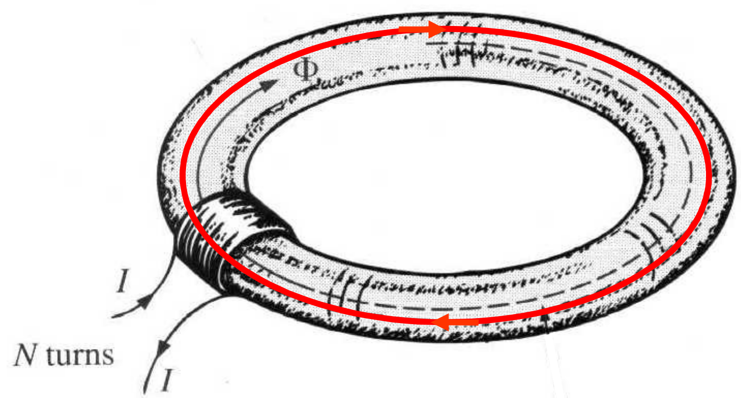
OHM'S LAW FOR MAGNETIC CIRCUITS

Substituting

$$\boxed{\Phi = \frac{\mathscr{F}}{\mathscr{R}}}$$

The magnetomotive force F is proportional to the product of the number of turns around the core (in which the flux is to be established) and the current through the turns of wire

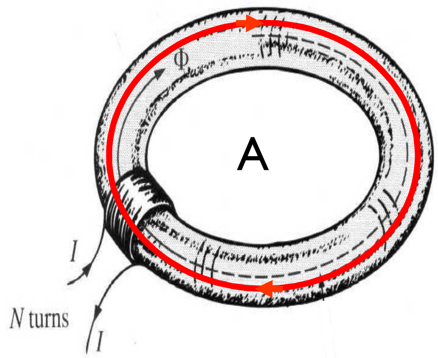
$$\boxed{\mathscr{F} = NI}$$ (ampere-turns, At)

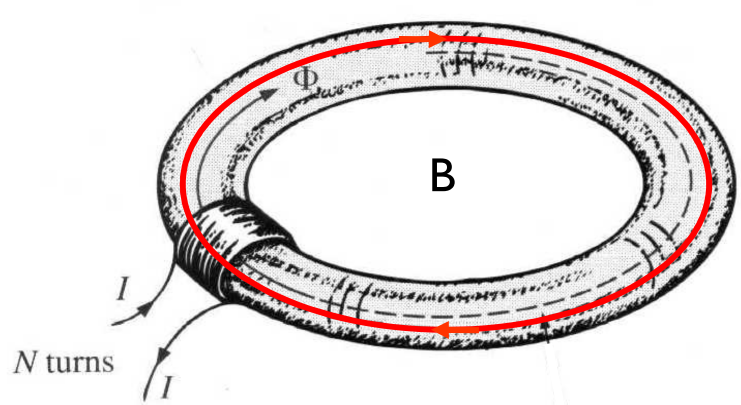


MAGNETISING FORCE H

Imagine two magnetic circuits each with the same MMF (say 500At)

- The loop length of circuit A is shorter than the loop length of circuit B - **flux travels a shorter distance**
- The MMF **per unit length** over the loop will be greater for A than B
- The **field strenath** will be **greater** in A than B





$$500\text{At}\begin{cases} N = 100 \\ I = 5\text{A} \end{cases}$$

MAGNETISING FORCE H

Magnetising force takes into account the **loop length** ℓ as well as the MMF

Magnetising Force is given the symbol H

H = MMF/length

$$\mathbf{H} = \frac{\mathbf{NI}}{\ell}$$

I = current (A)
N = turns (t)
ℓ = length of magnetic circuit (m)

Magnetizing force is independent of the type of core material

Magnetizing force is determined solely by the number of turns, the current and the length of the core

B VERSUS H FOR NON MAGNETIC MATERIALS

Magnetising force H propels the magnetic flux around the circuit

The larger the H the stronger (denser) the flux

For a **vacuum** and (approx) for **non magnetic** materials:

$B = \mu_0 H$, $\mu_0 = 4\Pi \times 10^{-7}$ Wb/(At.m)

- Relation between magnetic field intensity H and magnetic field density B (measured in Tesla):

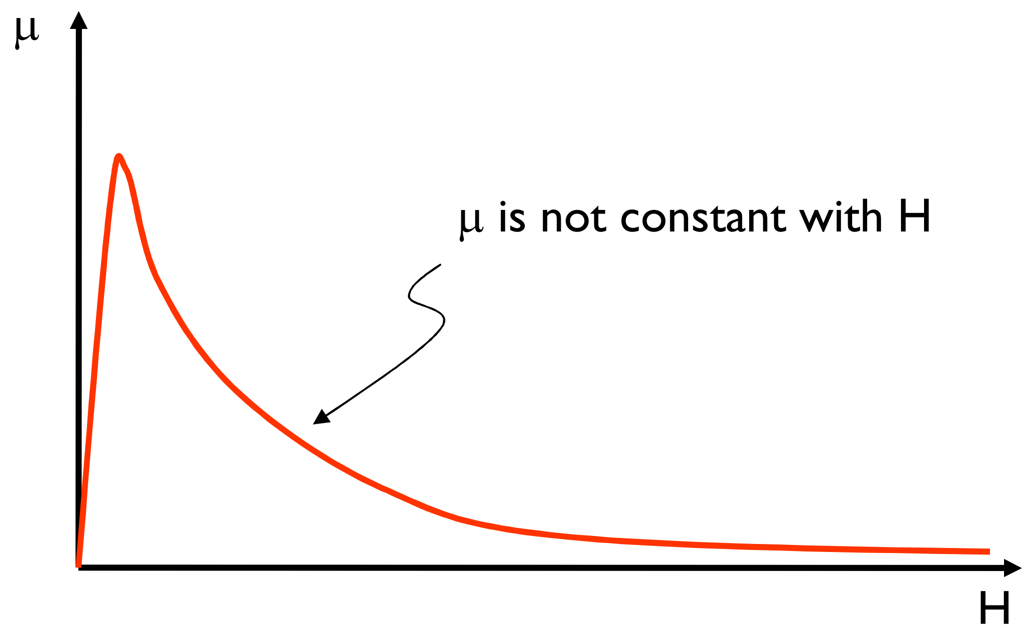
$$B = \mu H = (\mu_r \mu_0) H$$

where is μ_r is the relative permeability of the medium (unit-less), is μ_o is the permeability of free space ($4\pi x10^{-7}$ H/m).

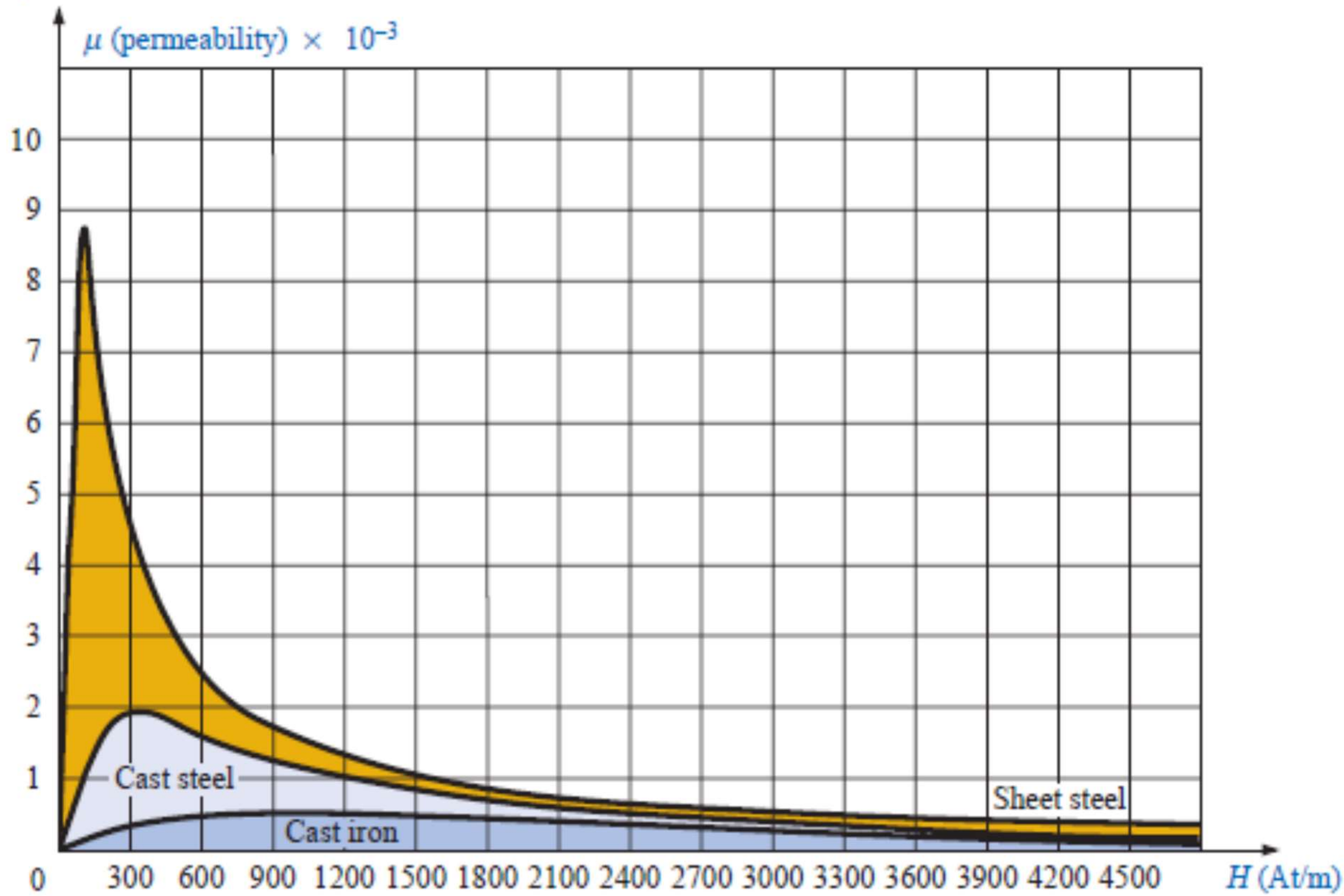
B VERSUS H

$B = \mu H$, Important: μ is not constant for magnetic materials

- Magnetic materials exhibit a phenomenon known as **saturation**



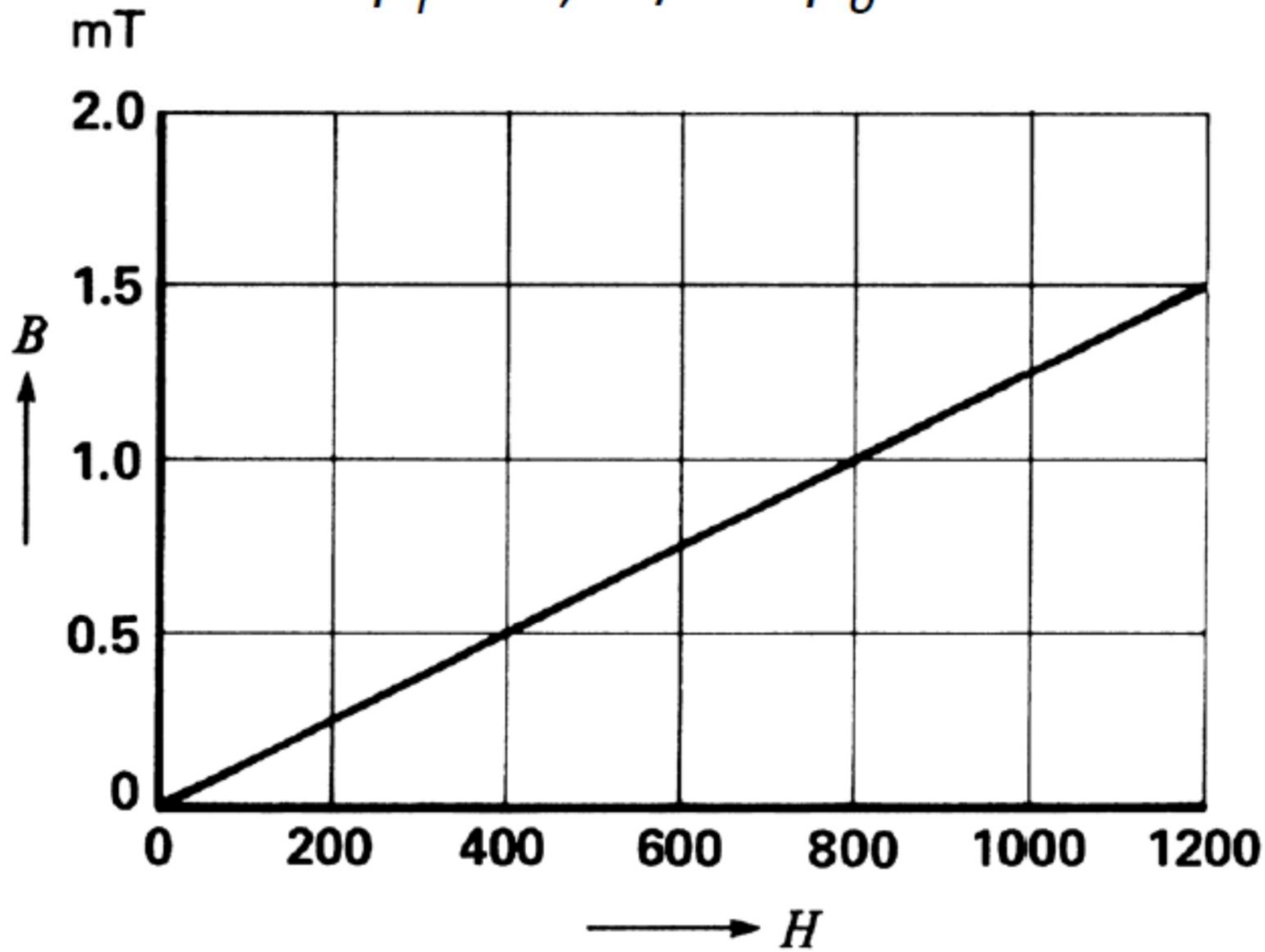
μ VERSUS H



μ (permeability) × 10⁻³
10
9
8
7
6
5
4
3
2
1
0
300
600
900
1200
1500
1800
2100
2400
2700
3000
3300
3600
3900
4200
4500
H (At/m)
Cast steel
Cast iron
Sheet steel

B-H CURVE IN AIR AND NON-FERROMAGNETIC MATERIAL

$$\mu_r = 1,\ B/H = \mu_o = 4\pi x 10^{-7}$$



DOMAIN THEORY OF MAGNETISATION

An electron spinning around a nucleus produces a magnetic field

In most elements the electron spins are randomly aligned resulting in a zero net magnetic field

In ferro magnetic materials the magnetic fields of atoms are aligned in groups

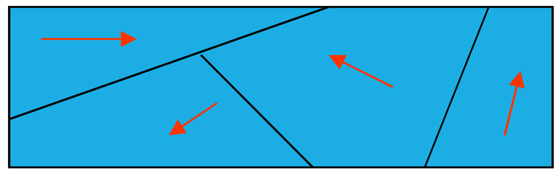
- These groups are called **domains**

The domains themselves are randomly aligned resulting in a zero net magnetic field

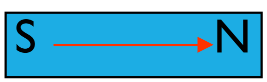
An external MMF can be used to align the domains

- Aligned domains produce a strong magnetic field

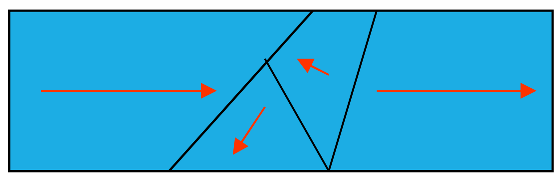
DOMAIN THEORY OF MAGNETISATION



Non magnetised material



External magnet



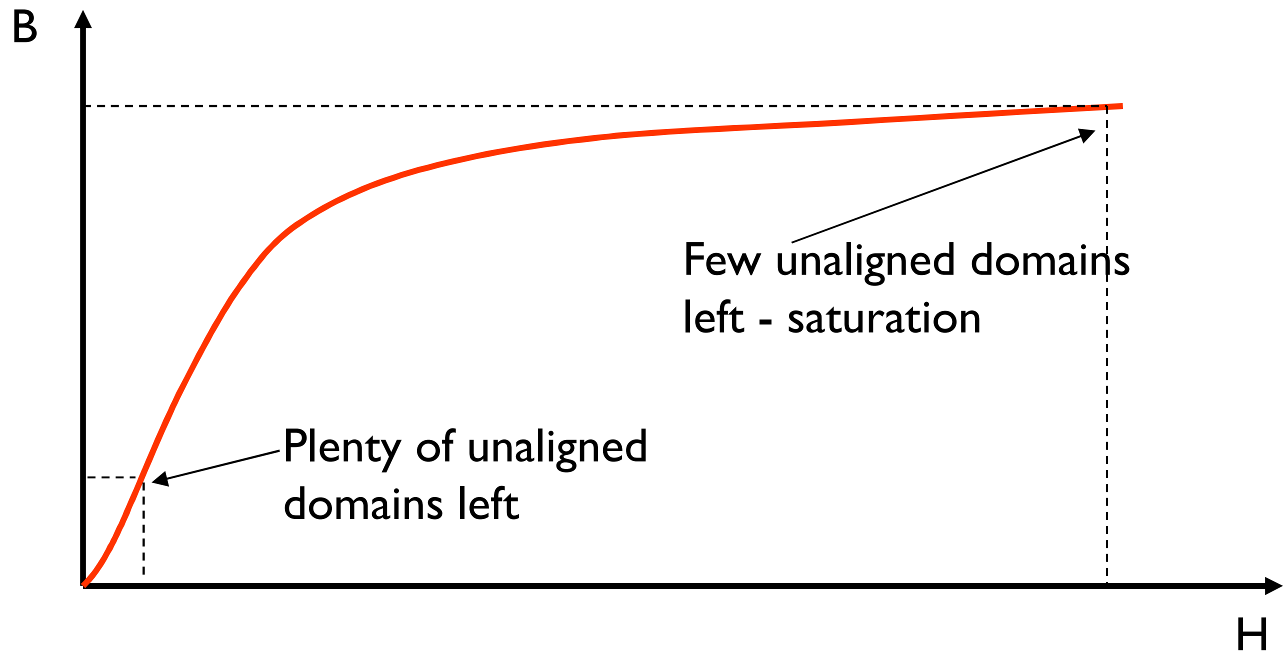
Material magnetised by external magnetic field

DOMAIN THEORY OF MAGNETISATION

Magnetic field strength B increases as more domains become aligned

Once **all** the domains are aligned the can be no more increase in B no matter how much the magnetising force H increases

The phenomenon is known as **saturation**



ELECTROMAGNETS

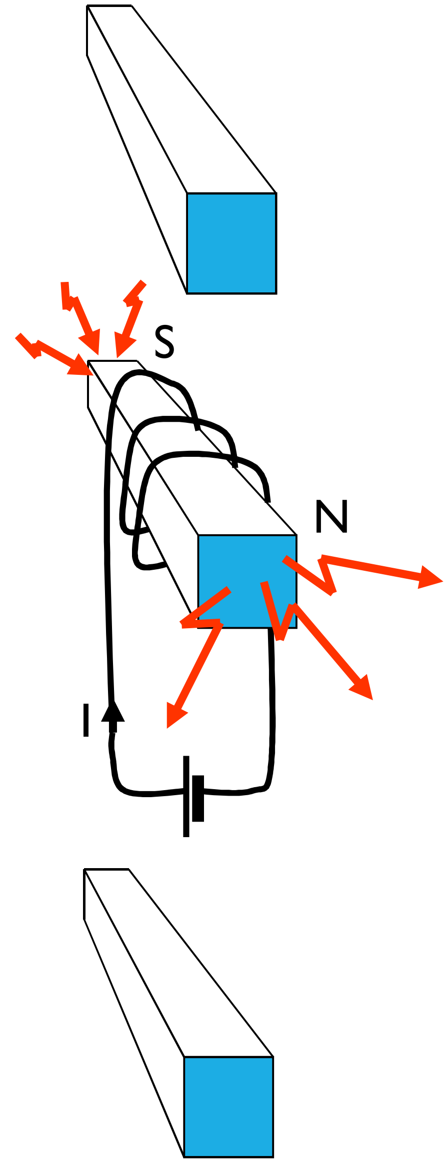
Coil of wire wound onto a core of **non magnetised** ferro magnetic material

Current is passed through the coil

Ferro magnetic core becomes magnetised

When current is removed ferro magnetic core is no longer magnetised

- **Magnetism is not retained**



PERMANENT MAGNETS

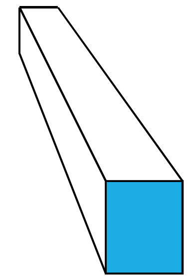
Coil of wire wound onto a core of **non magnetised** ferro magnetic material

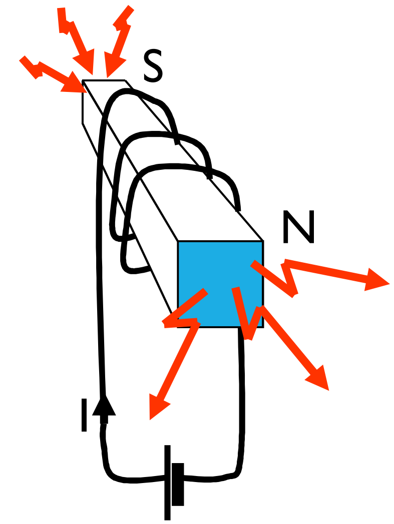
Current is passed through the coil

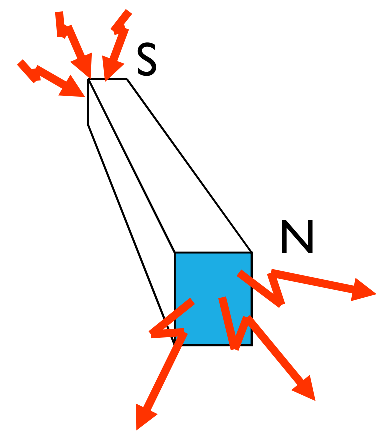
Ferro magnetic core becomes magnetised

When current is removed ferro magnetic core remains magnetised

- **A permanent magnet has been created**





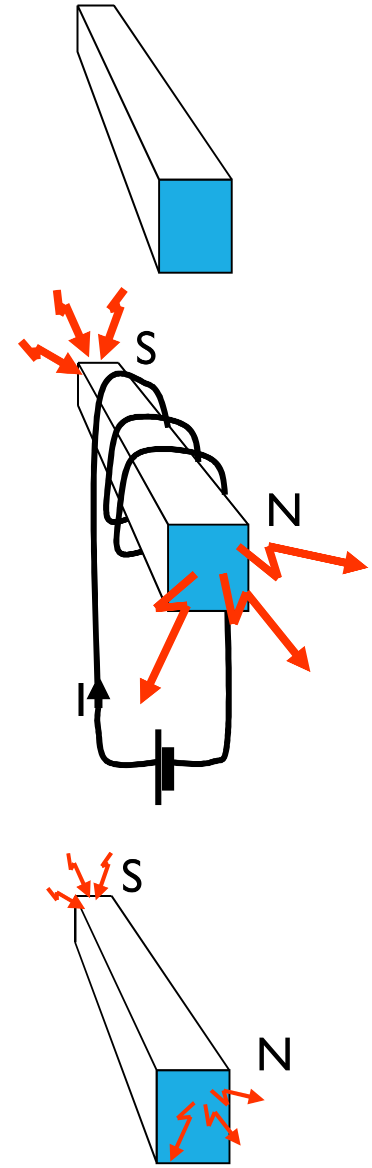


HYSTERESIS

When an non-magnetised magnetic material is placed in a magnetic field it becomes magnetised

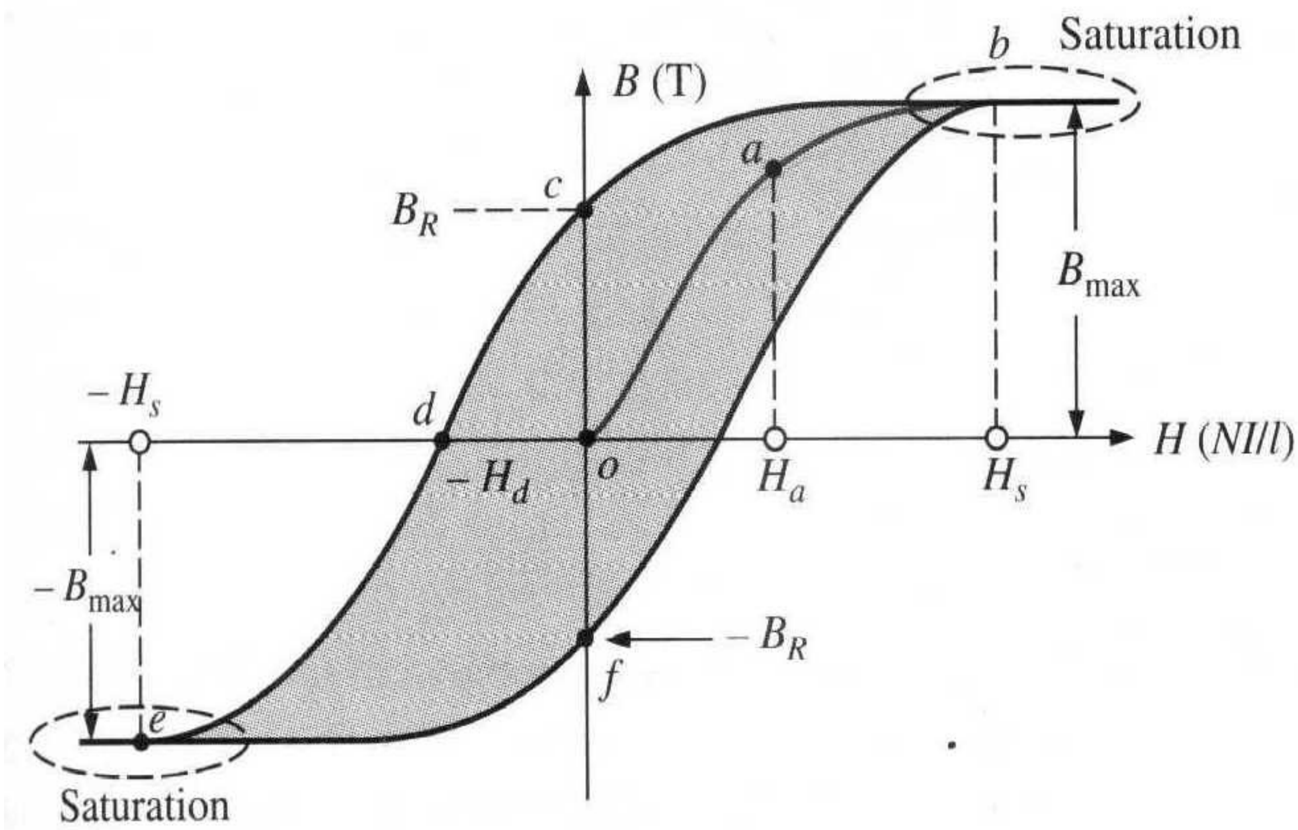
When the external field is removed the originally non-magnetised material retains some permanent magnetism

An external magnetic field has to be applied in the opposite direction to remove the residual magnetism

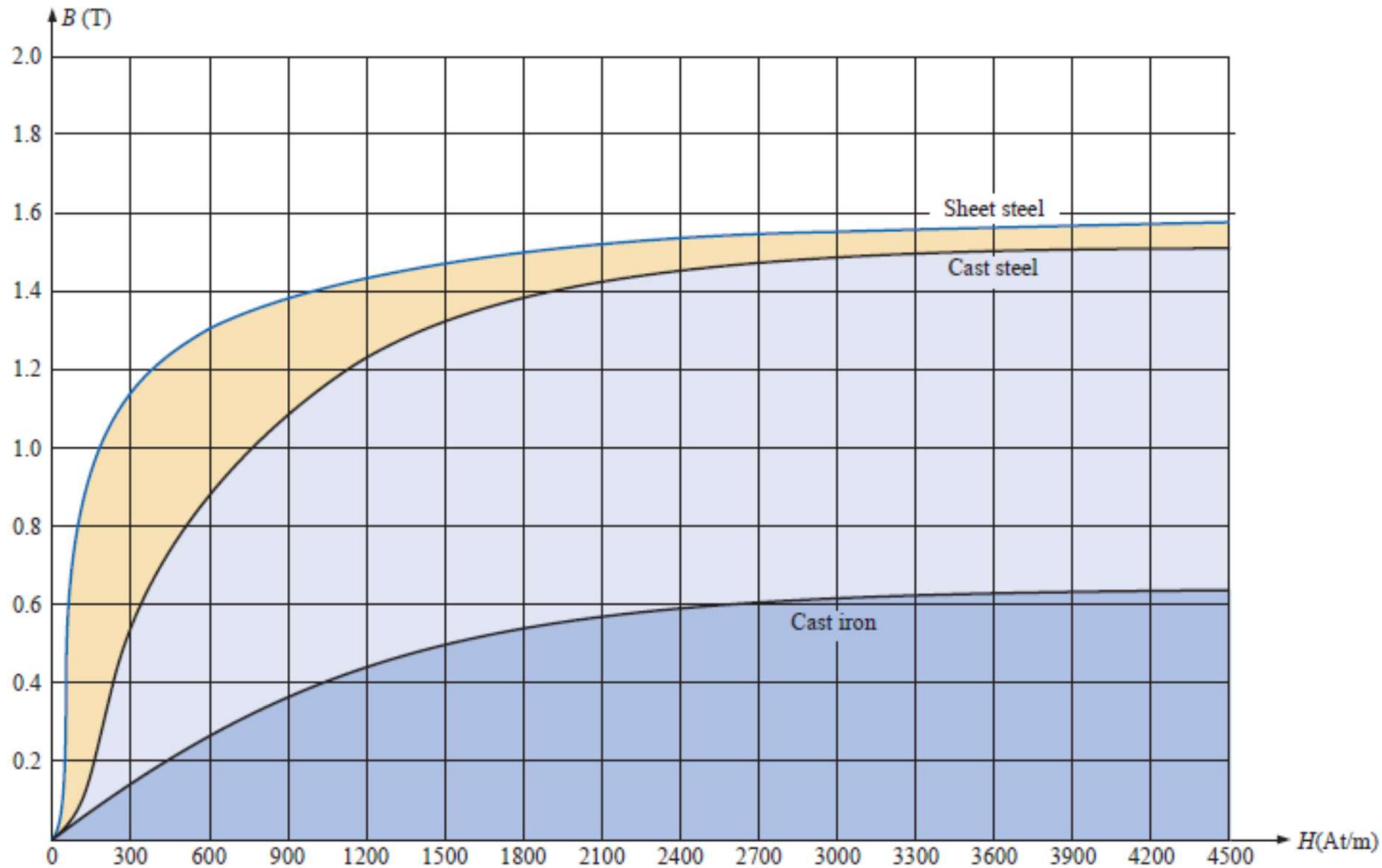


HYSTERESIS

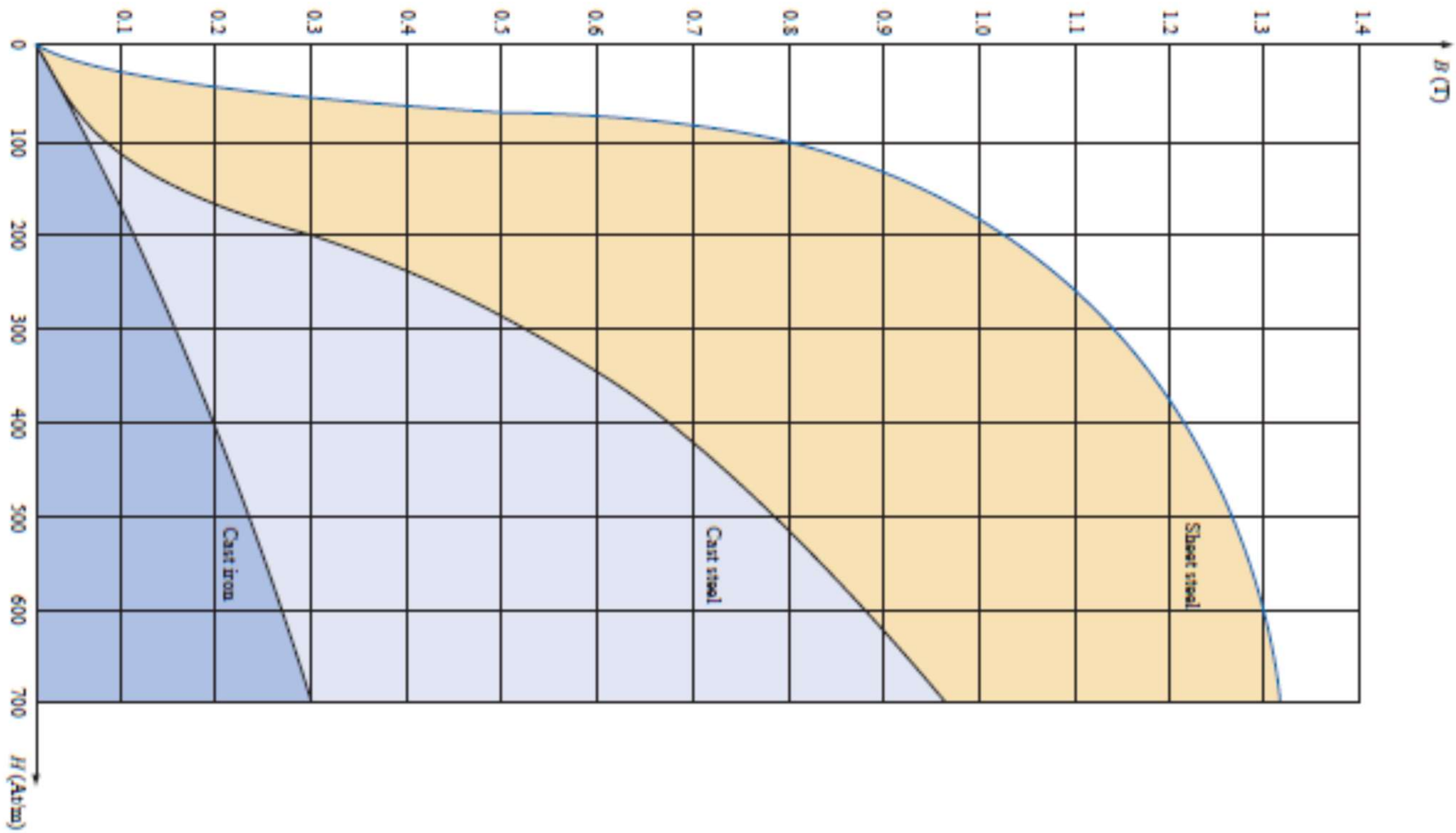
The entire curve represented by bcdefb is called the hysteresis curve named from the Greek *hysterein*, meaning “to lag behind”



B-H CURVE OF 3 FERROMAGNETIC MATERIALS



B-H CURVE OF 3 FERROMAGNETIC MATERIALS



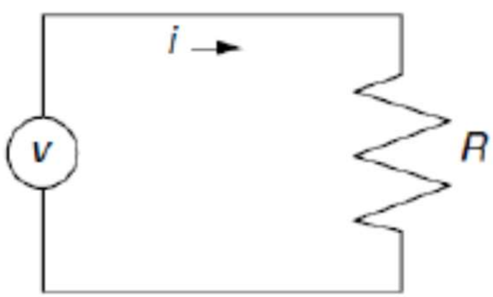
ANALOGY BETWEEN ELECTRIC AND MAGNETIC CIRCUITS

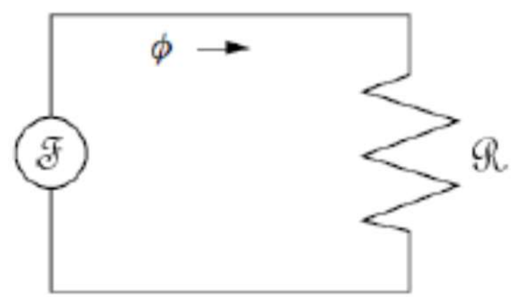
Electrical	Magnetic	Magnetic Units
Voltage v	Magnetomotive force $\mathfrak{F} = Ni$	Amp-turns
Current i	Magnetic flux ϕ	Webers Wb
Resistance R	Reluctance $\mathfrak{R}$	Amp-turns/Wb
Conductivity $1/\rho$	Permeability μ	Wb/A-t-m
Current density J	Magnetic flux density B	$\mathrm{Wb/m^2}$ = teslas T
Electric field E	Magnetic field intensity H	Amp-turn/m

Electrical

Magnetic

EQUIVALENT CIRCUITS





AMPÈRE'S CIRCUITAL LAW

If we apply the "cause" analogy to Kirchhoff's voltage law ($\Sigma_{\circlearrowright} V = 0$), we obtain the following:

$$\boxed{\Sigma_{\circlearrowright} \mathcal{F} = 0} \qquad \text{(for magnetic circuits)} \qquad \textbf{(11.9)}$$

which, in words, states that the algebraic sum of the rises and drops of the mmf around a closed loop of a magnetic circuit is equal to zero; that is, the sum of the rises in mmf equals the sum of the drops in mmf around a closed loop.

Equation (11.9) is referred to as **Ampère's circuital law.** When it is applied to magnetic circuits, sources of mmf are expressed by the equation

$$\boxed{\mathcal{F} = NI} \qquad \text{(At)} \qquad \textbf{(11.10)}$$

The equation for the mmf drop across a portion of a magnetic circuit can be found by applying the relationships listed in Table 11.1; that is, for electric circuits,

$$V = IR$$

resulting in the following for magnetic circuits:

$$\boxed{\mathcal{F} = \Phi\mathcal{R}} \qquad \text{(At)} \qquad \textbf{(11.11)}$$

AMPÈRE'S CIRCUITAL LAW

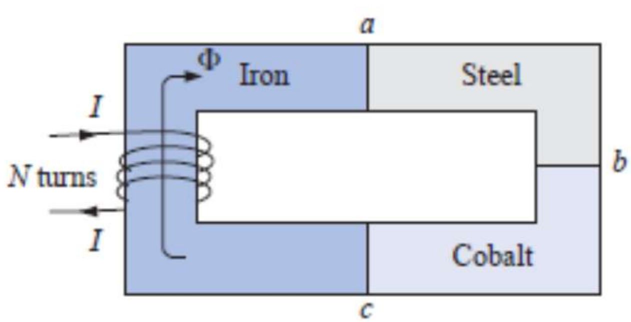


FIG. 11.27
Series magnetic circuit of three different materials.

$$\boxed{\mathscr{F} = Hl} \qquad \text{(At)} \qquad \textbf{(11.12)}$$

as derived from Eq. (11.6), where H is the magnetizing force on a section of a magnetic circuit and l is the length of the section.

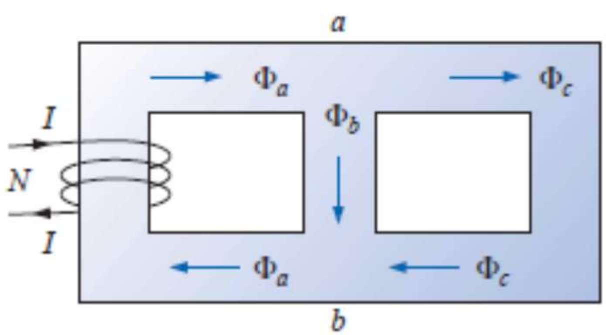
As an example of Eq. (11.9), consider the magnetic circuit appearing in Fig. 11.27 constructed of three different ferromagnetic materials. Applying Ampère's circuital law, we have

$$\Sigma_{\circlearrowright}\, \mathscr{F} = 0$$

$$\underbrace{+NI}_{\text{Rise}} - \underbrace{H_{ab}l_{ab}}_{\text{Drop}} - \underbrace{H_{bc}l_{bc}}_{\text{Drop}} - \underbrace{H_{ca}l_{ca}}_{\text{Drop}} = 0$$

$$\underbrace{NI}_{\substack{\text{Impressed}\\ \text{mmf}}} = \underbrace{H_{ab}l_{ab} + H_{bc}l_{bc} + H_{ca}l_{ca}}_{\text{mmf drops}}$$

THE FLUX Φ



If we continue to apply the relationships described in the previous section to Kirchhoff's current law, we will find that the sum of the fluxes entering a junction is equal to the sum of the fluxes leaving a junction; that is, for the circuit of Fig. 11.28,

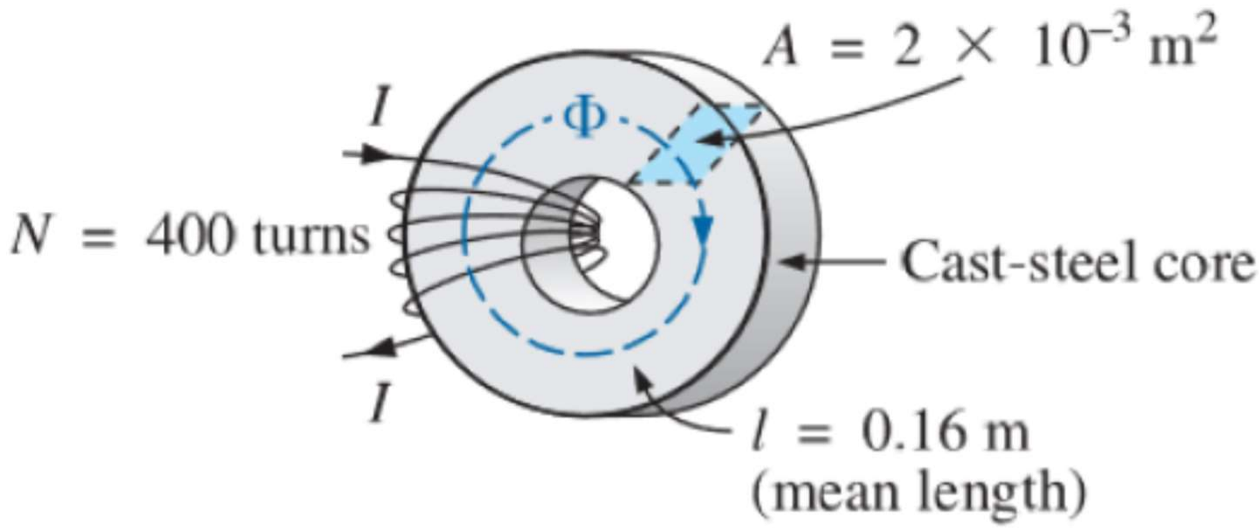
$$\Phi_a = \Phi_b + \Phi_c \qquad \text{(at junction } a\text{)}$$

or

$$\Phi_b + \Phi_c = \Phi_a \qquad \text{(at junction } b\text{)}$$

both of which are equivalent.

EXAMPLE



1. Find the value of I that will develop a magnetic flux of 0.4 mWb.
2. Determine μ_r of the material under the above conditions.

Answer:

1. B =0.2 T, H = 170 At/m, I = 68 mA
2. $\mu = 1.176 \times 10^{-3}$, $\mu_r = 935.8$

EXAMPLE

The electromagnet to the right has picked up a piece of cast iron (bottom section). Calculate the current required to establish the indicated flux in the core.

Answer:

(convert lengths to m and area to m^2)

B = 0.542 T

H(steel) = 70 At/m

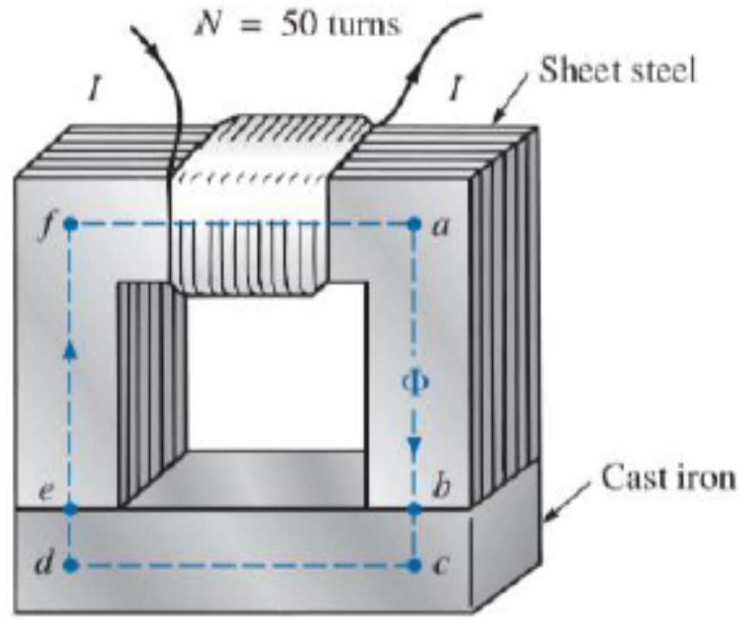
H(cast iron) = 1600 At/m

I = 4.49 A

$$12\,\cancel{\text{in.}}\left(\frac{1\text{ m}}{39.37\,\cancel{\text{in.}}}\right) = 304.8 \times 10^{-3}\text{ m}$$

$$5\,\cancel{\text{in.}}\left(\frac{1\text{ m}}{39.37\,\cancel{\text{in.}}}\right) = 127 \times 10^{-3}\text{ m}$$

$$1\,\cancel{\text{in.}}^2\left(\frac{1\text{ m}}{39.37\,\cancel{\text{in.}}}\right)\left(\frac{1\text{ m}}{39.37\,\cancel{\text{in.}}}\right) = 6.452 \times 10^{-4}\text{ m}^2$$



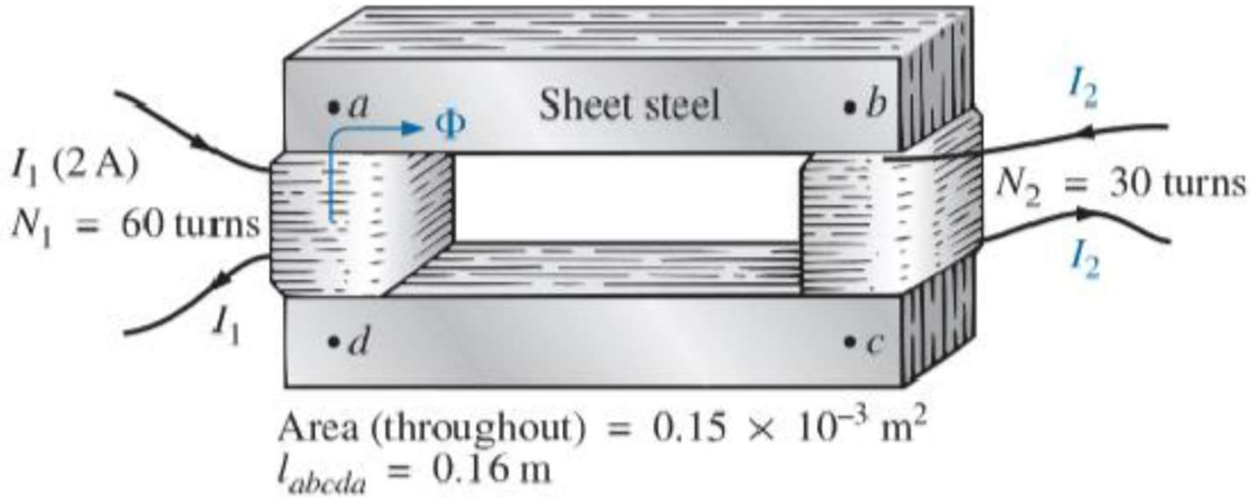
$l_{ab} = l_{cd} = l_{ef} = l_{fa} = 4$ in.

$l_{bc} = l_{de} = 0.5$ in.

Area (throughout) = 1 in.^{2}

$\Phi = 3.5 \times 10^{-4}$ Wb

EXAMPLE



Determine the current I_2 of the resultant clockwise flux is 15 μWb.

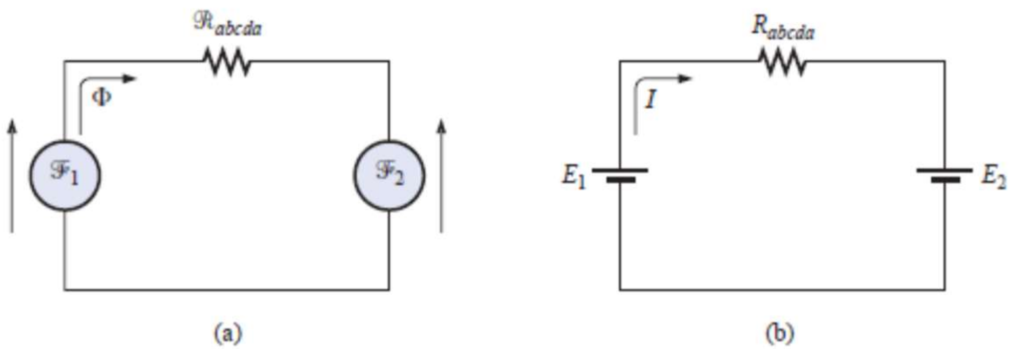
Assume both current flow in a counterclockwise direction.

Answer:

B = 0.1 T

H(steel) = 20 At/m

I = 3.89 A



EFFECTS OF AIR GAPS ON A MAGNETIC CIRCUIT

- The flux density of the air gap is given by

$$\boxed{B_g = \frac{\Phi_g}{A_g}}$$

- Where:
  - $\Phi_g = \Phi_{core}$
  - $A_g = A_{core}$
- The permeability of air is taken to be equal to that of free space.
- The magnetizing force of the air gap is then determined by

$$\boxed{H_g = \frac{B_g}{\mu_o}}$$

- And the mmf drop across the air gap is equal to $H_g\, l_g$

$$\boxed{H_g = (7.96 \times 10^5) B_g} \qquad \text{(At/m)}$$

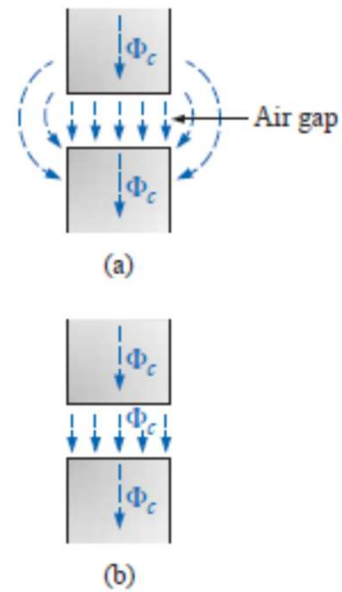


FIG. 11.35
Air gaps: (a) with fringing; (b) ideal.

EXAMPLE

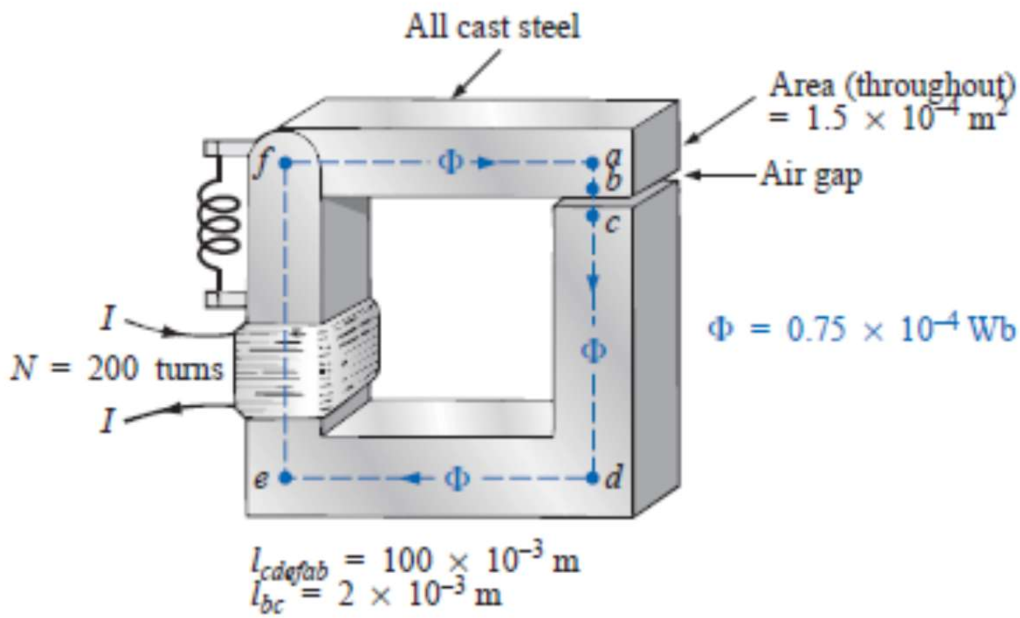
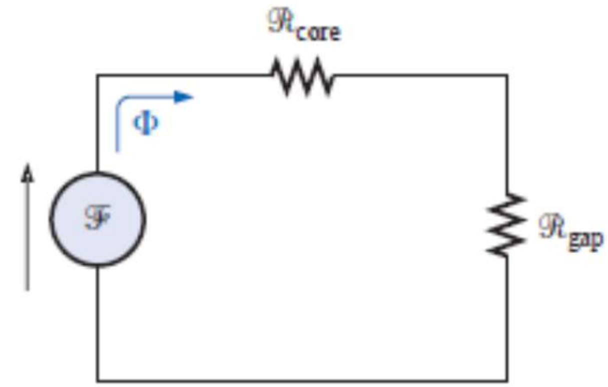
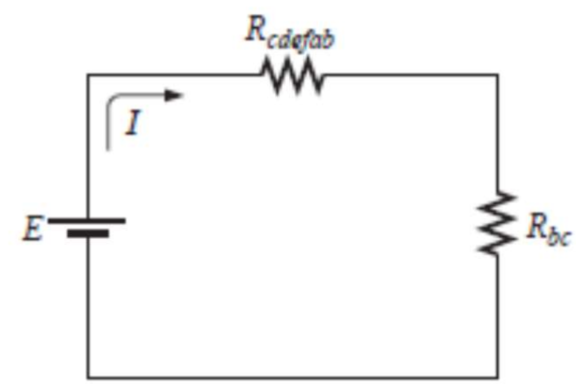


FIG. 11.36
Relay for Example 11.6.



(a)



EXAMPLE

$$H\text{ (cast steel)} \cong 280\text{ At/m}$$

Applying Eq. (11.15),

$$H_g = (7.96 \times 10^5)B_g = (7.96 \times 10^5)(0.5\text{ T}) = 3.98 \times 10^5\text{ At/m}$$

The mmf drops are

$$H_{core}l_{core} = (280\text{ At/m})(100 \times 10^{-3}\text{ m}) = 28\text{ At}$$
$$H_g l_g = (3.98 \times 10^5\text{ At/m})(2 \times 10^{-3}\text{ m}) = 796\text{ At}$$

Applying Ampère's circuital law,

$$NI = H_{core}l_{core} + H_g l_g$$
$$= 28\text{ At} + 796\text{ At}$$

$$(200\text{ t})I = 824\text{ At}$$
$$I = \mathbf{4.12\text{ A}}$$

SERIES-PARALLEL MAGNETIC CIRCUITS

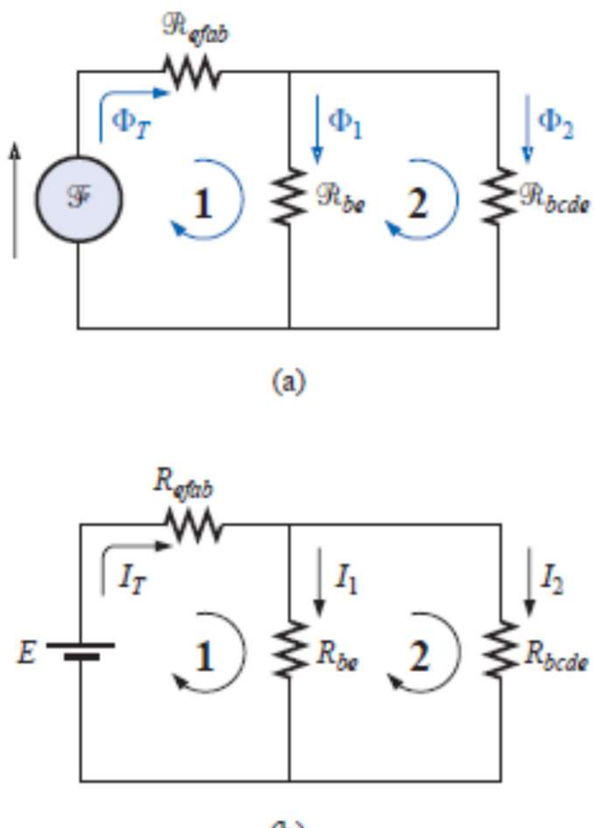


FIG. 11.39

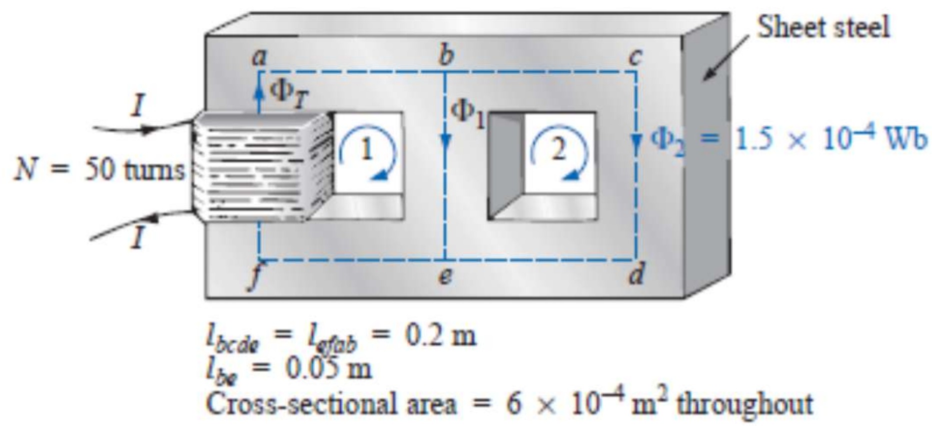


FIG. 11.38
Example 11.7.

Solution: The equivalent magnetic circuit and the electric circuit analogy appear in Fig. 11.39. We have

$$B_2 = \frac{\Phi_2}{A} = \frac{1.5 \times 10^{-4}\,\text{Wb}}{6 \times 10^{-4}\,\text{m}^2} = 0.25\ \text{T}$$

From Fig. 11.24,

$$H_{bcde} \cong 40\ \text{At/m}$$

EXAMPLE

$$\Sigma_{\circlearrowright} \mathcal{F} = 0$$

$$H_{be}l_{be} - H_{bcde}l_{bcde} = 0$$

$$H_{be}(0.05 \text{ m}) - (40 \text{ At/m})(0.2 \text{ m}) = 0$$

$$H_{be} = \frac{8 \text{ At}}{0.05 \text{ m}} = 160 \text{ At/m}$$

From Fig. 11.24,

$$B_1 \cong 0.97 \text{ T}$$

and

$$\Phi_1 = B_1 A = (0.97 \text{ T})(6 \times 10^{-4} \text{ m}^2) = 5.82 \times 10^{-4} \text{ Wb}$$

EXAMPLE

$$\Phi_T = \Phi_1 + \Phi_2 = 5.82 \times 10^{-4}\text{ Wb} + 1.5 \times 10^{-4}\text{ Wb}$$
$$= 7.32 \times 10^{-4}\text{ Wb}$$

$$B = \frac{\Phi_T}{A} = \frac{7.32 \times 10^{-4}\text{ Wb}}{6 \times 10^{-4}\text{ m}^2}$$
$$= 1.22\text{ T}$$

From Fig. 11.23,

$$H_{efab} \cong 400\text{ At}$$

Applying Ampère's circuital law,

$$+NI - H_{efab}l_{efab} - H_{be}l_{be} = 0$$
$$NI = (400\text{ At/m})(0.2\text{ m}) + (160\text{ At/m})(0.05\text{ m})$$
$$(50\text{ t})I = 80\text{ At} + 8\text{ At}$$
$$I = \frac{88\text{ At}}{50\text{ t}} = \mathbf{1.76\text{ A}}$$

DETERMINING Φ

When determining magnetic circuits with more than one section, there is no set order of steps that will lead to an exact solution for every problem on the first attempt

Find the impressed mmf for a calculated guess of the flux Φ and then compare this with the specified value of mmf

- Make adjustments to the guess to bring it closer to the actual value
- For most applications, a value within $\pm 5\%$ of the actual Φ or specified NI is acceptable.

EXAMPLE

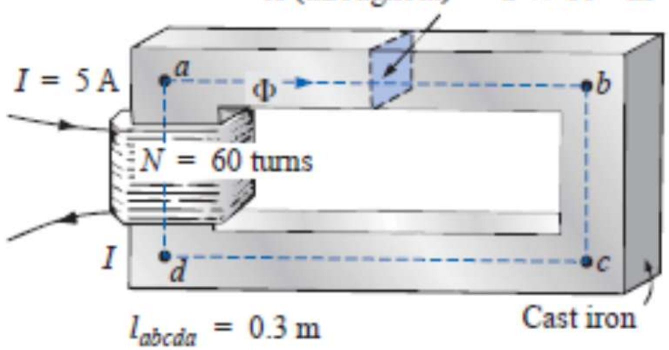


FIG. 11.40
Example 11.8.

EXAMPLE 11.8 Calculate the magnetic flux Φ for the magnetic circuit of Fig. 11.40.

Solution: By Ampère's circuital law,

$$NI = H_{abcda} l_{abcda}$$

or

$$H_{abcda} = \frac{NI}{l_{abcda}} = \frac{(60 \text{ t})(5 \text{ A})}{0.3 \text{ m}} = \frac{300 \text{ At}}{0.3 \text{ m}} = 1000 \text{ At/m}$$

and

$$B_{abcda} \text{ (from Fig. 11.23)} \cong 0.39 \text{ T}$$

Since $B = \Phi/A$, we have

$$\Phi = BA = (0.39 \text{ T})(2 \times 10^{-4} \text{ m}^2) = \mathbf{0.78 \times 10^{-4} \text{ Wb}}$$

EXAMPLE

EXAMPLE 11.9 Find the magnetic flux Φ for the series magnetic circuit of Fig. 11.41 for the specified impressed mmf.

Solution: Assuming that the total impressed mmf NI is across the air gap,

$$NI = H_g l_g$$

or

$$H_g = \frac{NI}{l_g} = \frac{400\text{ At}}{0.001\text{ m}} = 4 \times 10^5\text{ At/m}$$

and

$$B_g = \mu_o H_g = (4\pi \times 10^{-7})(4 \times 10^5\text{ At/m}) = 0.503\text{ T}$$

The flux

$$\Phi_g = \Phi_{core} = B_g A = (0.503\text{ T})(0.003\text{ m}^2)$$
$$\Phi_{core} = 1.51 \times 10^{-3}\text{ Wb}$$

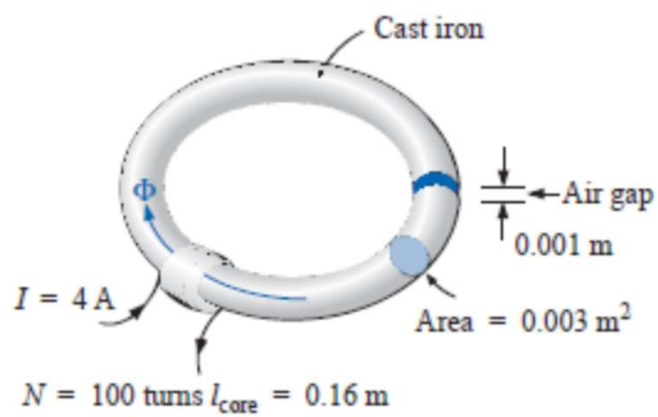


FIG. 11.41
Example 11.9.

EXAMPLE

$$H_{core}l_{core} = (1500 \text{ At/m})(0.16 \text{ m}) = 240 \text{ At}$$

Applying Ampère's circuital law results in

$$\begin{aligned} NI &= H_{core}l_{core} + H_g l_g \\ &= 240 \text{ At} + 400 \text{ At} \\ &= 640 \text{ At} > 400 \text{ At} \end{aligned}$$

Since we neglected the reluctance of all the magnetic paths but the air gap, the calculated value is greater than the specified value. We must therefore reduce this value by including the effect of these reluctances. Since approximately (640 At − 400 At)/640 At = 240 At/640 At ≅ 37.5% of our calculated value is above the desired value, let us reduce Φ by 30% and see how close we come to the impressed mmf of 400 At:

$$\begin{aligned} \Phi &= (1 - 0.3)(1.51 \times 10^{-3} \text{ Wb}) \\ &= 1.057 \times 10^{-3} \text{ Wb} \end{aligned}$$

EXAMPLE

$$B = \frac{\Phi}{A} = \frac{1.057 \times 10^{-3}\ \text{Wb}}{0.003\ \text{m}^2} \cong 0.352\ \text{T}$$

$$\begin{aligned} H_g l_g &= (7.96 \times 10^5) B_g l_g \\ &= (7.96 \times 10^5)(0.352\ \text{T})(0.001\ \text{m}) \\ &\cong 280.19\ \text{At} \end{aligned}$$

From the *B-H* curves,

$$H_{\text{core}} \cong 850\ \text{At/m}$$

$$H_{\text{core}} l_{\text{core}} = (850\ \text{At/m})(0.16\ \text{m}) = 136\ \text{At}$$

Applying Ampère's circuital law yields

$$\begin{aligned} NI &= H_{\text{core}} l_{\text{core}} + H_g l_g \\ &= 136\ \text{At} + 280.19\ \text{At} \\ &= \mathbf{416.19\ At} > 400\ \text{At} \quad \text{(but within } \pm 5\% \\ &\qquad\qquad\qquad\qquad \text{and therefore acceptable)} \end{aligned}$$

The solution is, therefore,

$$\Phi \cong \mathbf{1.057 \times 10^{-3}\ Wb}$$