Chapter Title: Gravitation

Sections: Newton's Law of Gravitation, Gravitation Near Earth's Surface, Gravitational Potential Energy, Kepler's laws of planetary motion

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Newton's Law of Gravitation

Before 1687, a large amount of data had been collected on the motions of the Moon and the planets, but a clear understanding of the forces related to these motions was not available.

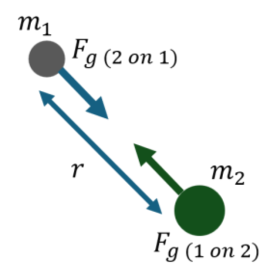
In that year, Isaac Newton provided the key that unlocked the secrets of the heavens.

He knew, from his first law, that a net force had to be acting on the Moon because without such a force the Moon would move in a straight-line path rather than in its almost circular orbit. Newton reasoned that this force was the gravitational attraction exerted by the Earth on the Moon.

Newton published the law of gravitation in 1687.

"Every particle of matter in the universe attracts every other particle with a force

that is directly proportional to the product of the masses of the particles and inversely proportional to the square of the distance between them."



$$F_g = \frac{Gm_1m_2}{r^2}$$

This is called law of gravitation.

Here, F_g is the magnitude of gravitational force on either particle, m_1 and m_2 are their masses, r is the distance between them, and G is a fundamental physical constant called the gravitational constant.

$$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$$

$$G = 6.67 \times 10^{-11} m^3/kgs^2$$

In vector form,

$$\vec{F}_g = \frac{Gm_1m_2}{r^2}\hat{r}$$

Gravitation Near Earth's Surface

If the particle is released, it will fall toward the center of Earth, as a result of the gravitational force $\vec{F}$, with an acceleration which is called the gravitational acceleration $\vec{a}_g$.

According to the Newton's second law,

$$F_{net} = F_g = ma_g$$

Applying Newton's gravitation law in the above equation,

$$a_g = \frac{GM}{r^2}$$

Using the equation, the magnitude of gravitational acceleration can be calculated

Gravitational Acceleration and Free fall Acceleration

Free fall acceleration:

The concept is developed based on several simplified models that results the same magnitude as $g = a_g = 9.8\, m/s^2$.

1) Earth is an inertial frame that neglects the earth's rotation.
2) Earth's mass is not uniform.
3) It has been considered that a_g and g have the same value.

$a_g = \frac{GM}{r^2}$ tells that the magnitude of a_g depends on three factors:

1) Earth’s mass is not distributed uniformly
2) Earth is not a perfect sphere
3) Earth rotates

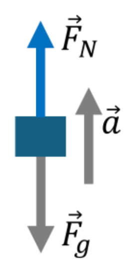
Influence of Earth's rotation on gravitational acceleration

Assuming a box of mass m is placed at the top on an earth. From a free-body diagram, the net force, $F_{net} = ma$ is,

$$F_N - F_g = ma$$

$$F_N = F_g + ma$$

Centripetal acceleration is $a = \omega^2 R$, ω is earth’s angular speed, and R is earth’s radius.



$$F_N = ma_g - m\omega^2 R$$

$$mg = ma_g - m\omega^2 R$$

$$g = a_g - \omega^2 R$$

Gravitational Potential Energy

It was assumed that the gravitational force on a body is constant in magnitude and direction. Therefore, potential energy is,

$$U = mgh$$

But the earth’s gravitational force on a body at any point outside the earth is given more generally by

$$F_g = \frac{Gm_{earth}m}{R^2}$$

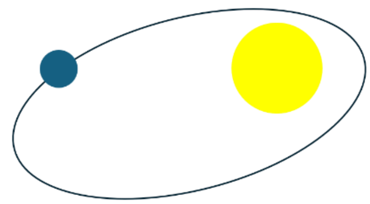
Here, the magnitude of R changes enough that the gravitational force can't be considered constant.

Therefore, gravitational potential energy,

$$U = -\frac{Gm_{earth}m}{R}$$

More generally, the above equation becomes,

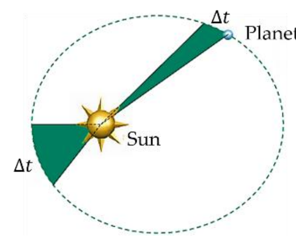
$$U = -\frac{GMm}{r}$$



Here, the sign is negative as the work has to be done to bring an object from infinity to distance r.

Kepler's laws of planetary motion

Law 1 - The law of orbits:

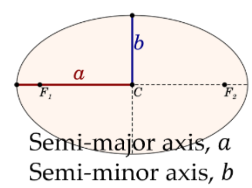


All planets move in elliptical orbits with the Sun at one focus.

Law 2 - The law of areas:

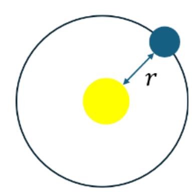
A line joining any planet to the Sun sweeps out equal areas in equal times.

Law 3 - The law of periods:



The square of the period of any planet is proportional to the cube of the semimajor axis of its orbit.

$$T^2 = Cr^3$$



Here, C is proportionality constant and $C = \frac{4\pi^2}{GM}$, M is the mass of the central body.

$$T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$