EEE 141
ELECTRICAL CIRCUITS

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METHODS OF ANALYSIS

Source Conversion.

Branch Current Analysis.

Mesh Analysis.

Nodal Analysis.

INTRODUCTION

The steps shown in the previous chapters cannot be applied if the sources are not in series or parallel.

Other methods have been developed to solve network with any arrangement of sources.

These methods apply also for single source network,

These methods are:

1- Branch-current analysis

2- Mesh analysis

3- Nodal analysis

Any of these method can be applied, the best method cannot be defined.

All the methods can be applied to **linear bilateral** networks.

Linear ≡ elements of the network are independent of the voltage applied across or the current through them,

Bilateral ≡ no change in the behavior if current or voltage is reversed.

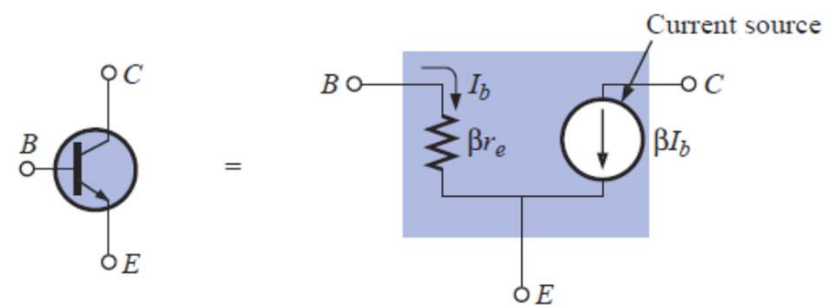
CURRENT SOURCES

Current source is the dual of the voltage source:

Current source supplies fixed current to the branch in which it is located; the voltage across its terminal can vary.

The interest in current source is due to semiconductor devices (transistors),

Current-controlled devices



(a) Transistor symbol (b) Transistor equivalent circuit

FIG. 8.1
Current source within the transistor equivalent circuit.

Transistor behaves like a current source.

Symbol of current source ≡ circle with an arrow.

Arrow indicates the direction of current supplied.

IDEAL VOLTAGE AND CURRENT SOURCE

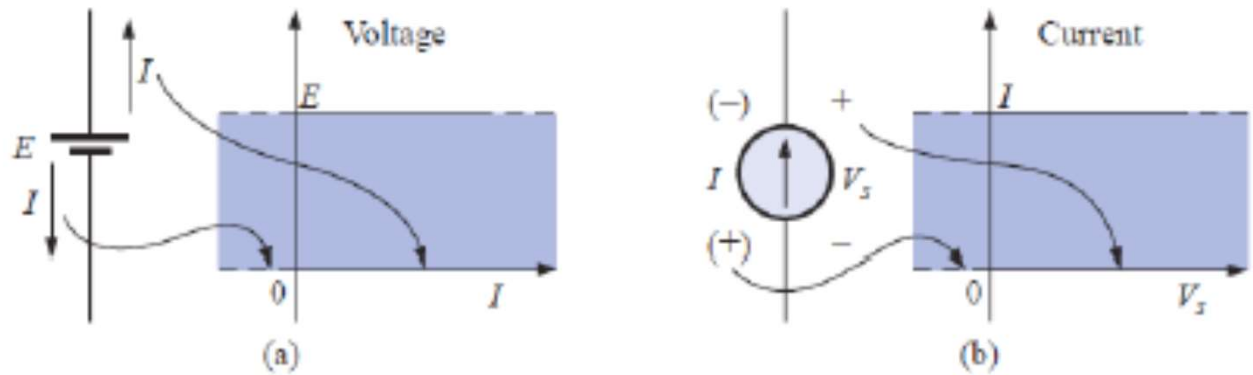
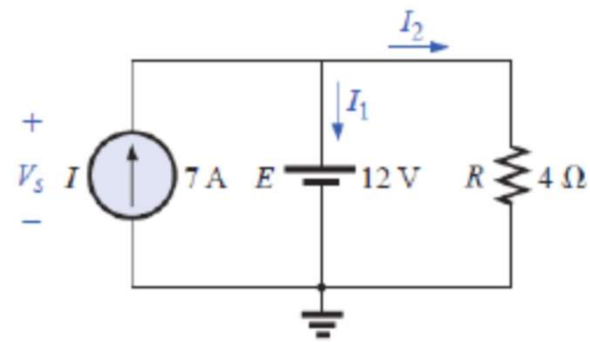


FIG. 8.2
Comparing the characteristics of an ideal voltage and current source.

- ***A current source determines the current in the branch in which it is located***
- ***The magnitude and polarity of the voltage across a current source are a function of the network to which it is applied.***

EXAMPLE

EXAMPLE 8.2 Find the voltage V_s and the currents I_1 and I_2 for the network of Fig. 8.4.



Solution:

$$V_s = E = \mathbf{12\ V}$$

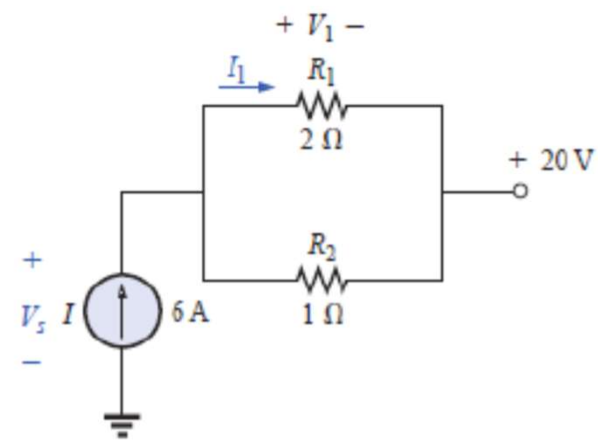
$$I_2 = \frac{V_R}{R} = \frac{E}{R} = \frac{12\ \text{V}}{4\ \Omega} = \mathbf{3\ A}$$

Applying Kirchhoff's current law:

$$I = I_1 + I_2$$

and $$I_1 = I - I_2 = 7\ \text{A} - 3\ \text{A} = \mathbf{4\ A}$$

EXAMPLE 8.3 Determine the current I_1 and the voltage V_s for the network of Fig. 8.5.



Solution: Using the current divider rule:

$$I_1 = \frac{R_2 I}{R_2 + R_1} = \frac{(1\ \Omega)(6\ \text{A})}{1\ \Omega + 2\ \Omega} = \mathbf{2\ A}$$

The voltage V_1 is

$$V_1 = I_1 R_1 = (2\ \text{A})(2\ \Omega) = 4\ \text{V}$$

and, applying Kirchhoff's voltage law,

$$+V_s - V_1 - 20\ \text{V} = 0$$

and $$V_s = V_1 + 20\ \text{V} = 4\ \text{V} + 20\ \text{V} = \mathbf{24\ V}$$

Note the polarity of V_s as determined by the multisource network.

SOURCE CONVERSIONS

The current (voltage) source described are ***ideal sources*** $\Rightarrow$ No internal resistance
Real sources (voltage or current) have internal resistance

Real Voltage source:

- $E \equiv$ voltage source rating
- $R_s \equiv$ internal resistance (series)

$R_s = 0$ (much smaller than any series resistor)

$\Rightarrow$ **Ideal voltage source**

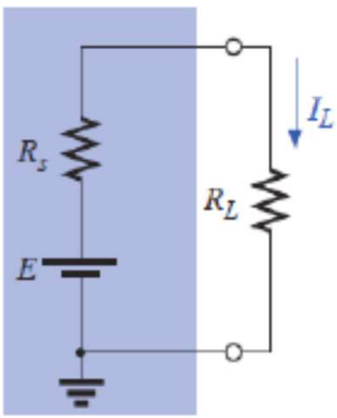


FIG. 8.6
Practical voltage source.

$$I_L = \frac{E}{R_s + R_L}$$

Real Current source:

- $I \equiv$ current source rating
- $R_s \equiv$ internal resistance (parallel)

$R_s = \infty$ (much larger than any parallel resistor)

$\Rightarrow$ **Ideal Current source**

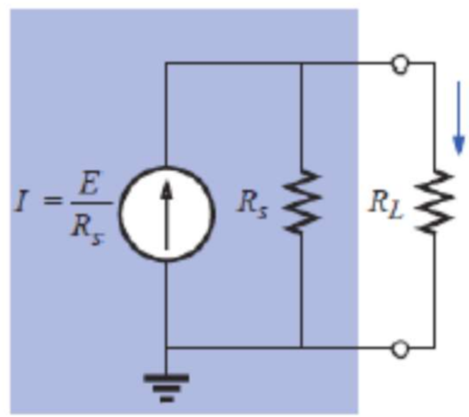


FIG. 8.7
Practical current source.

$$I_L = \frac{R_s \cdot I}{R_s + R_L}$$

SOURCE CONVERSIONS

Sources with their internal resistance included can be converted from one type to the other type.

Source conversions are equivalent only at their external terminals.

- R_s is the same (position changed)
- $E = I \cdot R_s$

As far as terminal point (a) and (b) and the circuit in between there is no difference:

- Same current
- Same voltage
- Same power

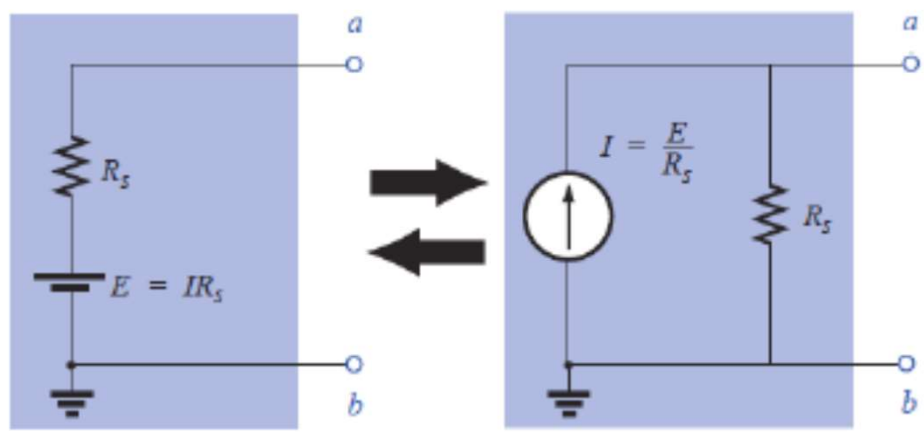


FIG. 8.8
Source conversion.

EXAMPLE

EXAMPLE 8.4

a. Convert the voltage source of Fig. 8.9(a) to a current source, and calculate the current through the 4-Ω load for each source.
b. Replace the 4-Ω load with a 1-kΩ load, and calculate the current I_L for the voltage source.
c. Repeat the calculation of part (b) assuming that the voltage source is ideal ($R_s = 0\ \Omega$) because R_L is so much larger than R_s. Is this one of those situations where assuming that the source is ideal is an appropriate approximation?

Solutions:

a. See Fig. 8.9.

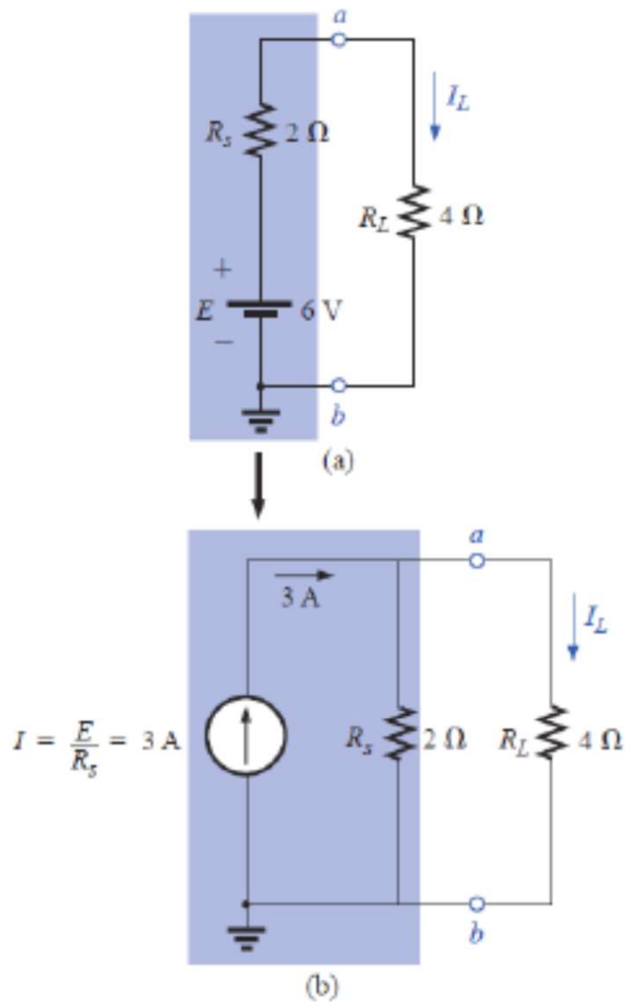
$$\text{Fig. 8.9(a):}\quad I_L = \frac{E}{R_s + R_L} = \frac{6\text{ V}}{2\ \Omega + 4\ \Omega} = \mathbf{1\ A}$$

$$\text{Fig. 8.9(b):}\quad I_L = \frac{R_s I}{R_s + R_L} = \frac{(2\ \Omega)(3\text{ A})}{2\ \Omega + 4\ \Omega} = \mathbf{1\ A}$$

b. $$I_L = \frac{E}{R_s + R_L} = \frac{6\text{ V}}{2\ \Omega + 1\text{ k}\Omega} \cong \mathbf{5.99\ mA}$$

c. $$I_L = \frac{E}{R_L} = \frac{6\text{ V}}{1\text{ k}\Omega} = \mathbf{6\ mA} \cong 5.99\text{ mA}$$

Yes, $R_L \gg R_s$ (voltage source).



EXAMPLE

EXAMPLE 8.5

a. Convert the current source of Fig. 8.10(a) to a voltage source, and find the load current for each source.

b. Replace the 6-kΩ load with a 10-Ω load, and calculate the current I_L for the current source.

c. Repeat the calculation of part (b) assuming that the current source is ideal ($R_s = \infty\ \Omega$) because R_L is so much smaller than R_s. Is this one of those situations where assuming that the source is ideal is an appropriate approximation?

Solutions:

a. See Fig. 8.10.

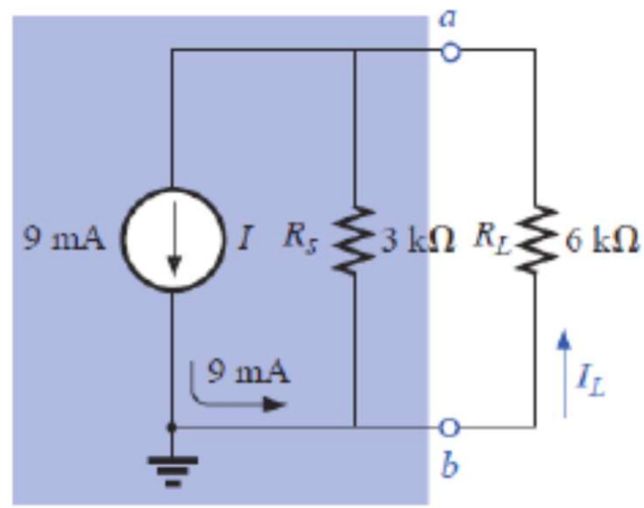
$$\text{Fig. 8.10(a):} \quad I_L = \frac{R_s I}{R_s + R_L} = \frac{(3\ \text{k}\Omega)(9\ \text{mA})}{3\ \text{k}\Omega + 6\ \text{k}\Omega} = \mathbf{3\ mA}$$

$$\text{Fig. 8.10(b):} \quad I_L = \frac{E}{R_s + R_L} = \frac{27\ \text{V}}{3\ \text{k}\Omega + 6\ \text{k}\Omega} = \frac{27\ \text{V}}{9\ \text{k}\Omega} = \mathbf{3\ mA}$$

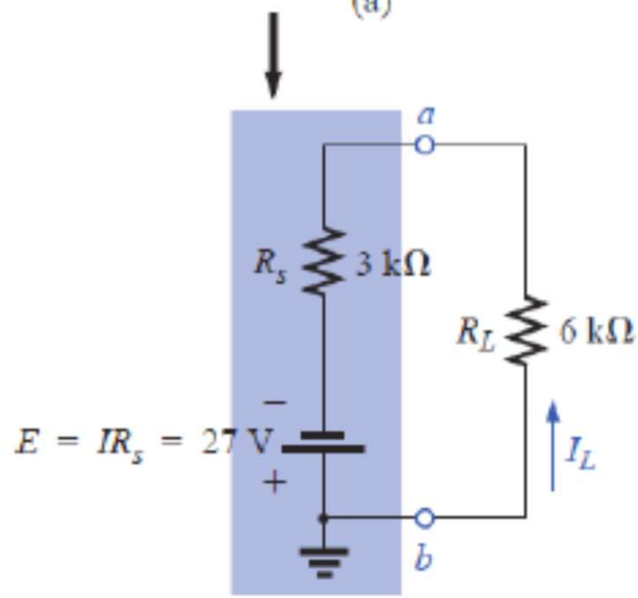
b. $$I_L = \frac{R_s I}{R_s + R_L} = \frac{(3\ \text{k}\Omega)(9\ \text{mA})}{3\ \text{k}\Omega + 10\ \Omega} = \mathbf{8.97\ mA}$$

c. $I_L = I = \mathbf{9\ mA} \cong 8.97\ \text{mA}$

Yes, $R_s \gg R_L$ (current source).



(a)



(b)

CURRENT SOURCES IN PARALLEL

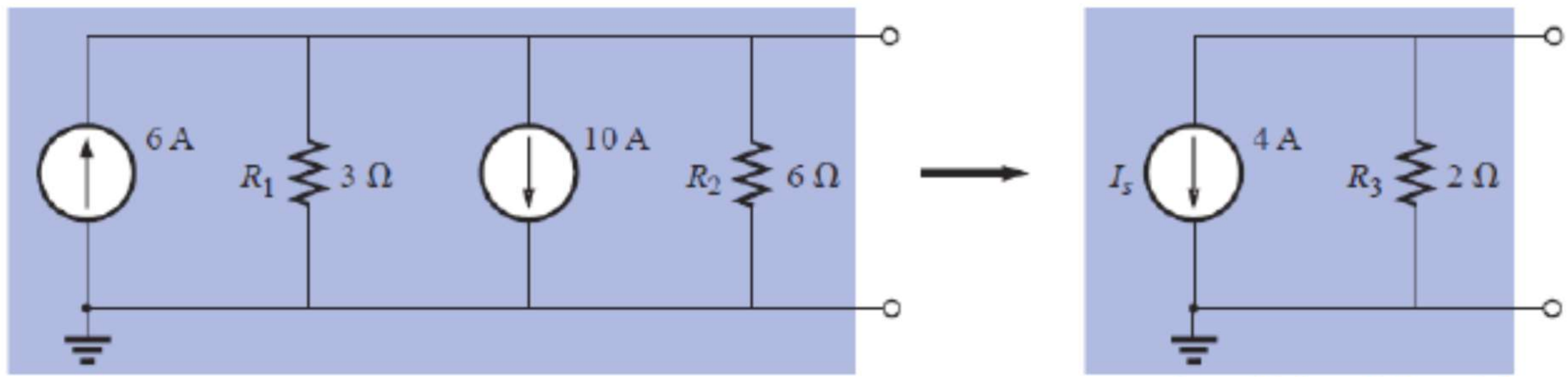
Current sources in parallel: they may all be replaced by one current source having the magnitude and direction of the resultant:

Summing the currents in one direction

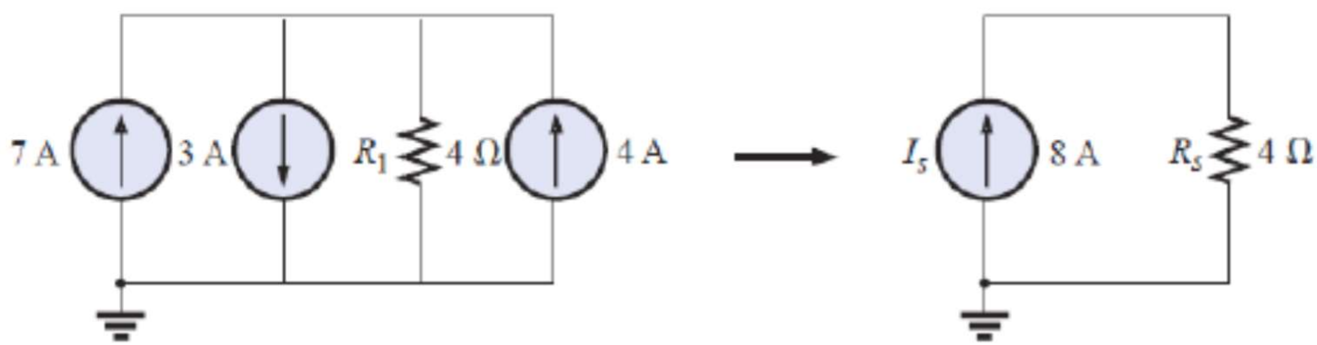
Subtracting the sum of the currents in the opposite direction

New parallel resistance is found like any parallel resistors in parallel

EXAMPLE



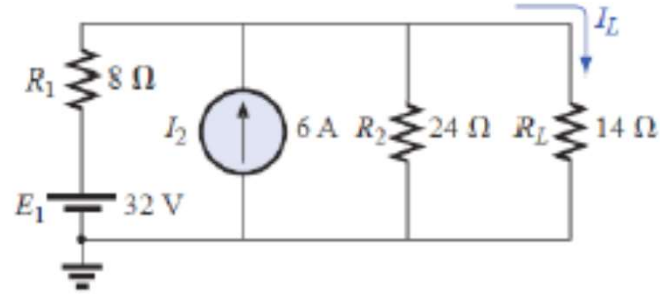
$I_s = 10\text{ A} - 6\text{ A} = \mathbf{4\ A}$
$R_s = 3\ \Omega \parallel 6\ \Omega = \mathbf{2\ \Omega}$



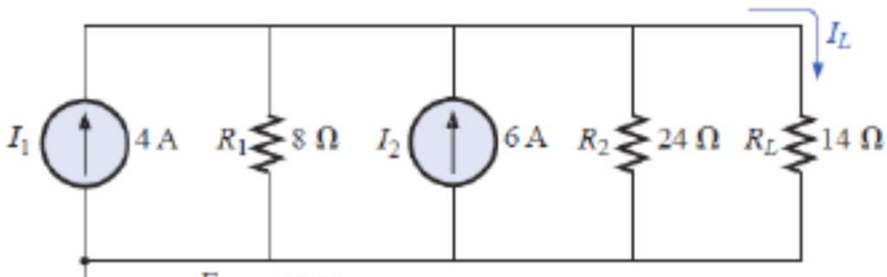
$I_s = 7\text{ A} + 4\text{ A} - 3\text{ A} = \mathbf{8\ A}$
$R_s = R_1 = \mathbf{4}\ \Omega$

EXAMPLE

EXAMPLE 8.7 Reduce the network of Fig. 8.13 to a single current source, and calculate the current through R_L.



Solution: In this example, the voltage source will first be converted to a current source as shown in Fig. 8.14. Combining current sources,



$$I_1 = \frac{E_1}{R_1} = \frac{32\text{ V}}{8\ \Omega} = 4\text{ A}$$

$$I_s = I_1 + I_2 = 4\text{ A} + 6\text{ A} = \mathbf{10\text{ A}}$$

and

$$R_s = R_1 \parallel R_2 = 8\ \Omega \parallel 24\ \Omega = \mathbf{6\ \Omega}$$

Applying the current divider rule to the resulting network of Fig. 8.15,

$$I_L = \frac{R_s I_s}{R_s + R_L} = \frac{(6\ \Omega)(10\text{ A})}{6\ \Omega + 14\ \Omega} = \frac{60\text{ A}}{20} = \mathbf{3\text{ A}}$$

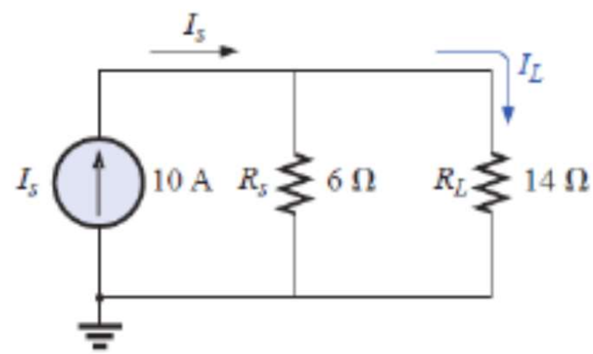


FIG. 8.15
Network of Fig. 8.14 reduced to its simplest form.

CURRENT SOURCES IN SERIES

current sources of different current ratings are not connected in series,

Voltage sources of different voltage ratings are not connected in parallel.

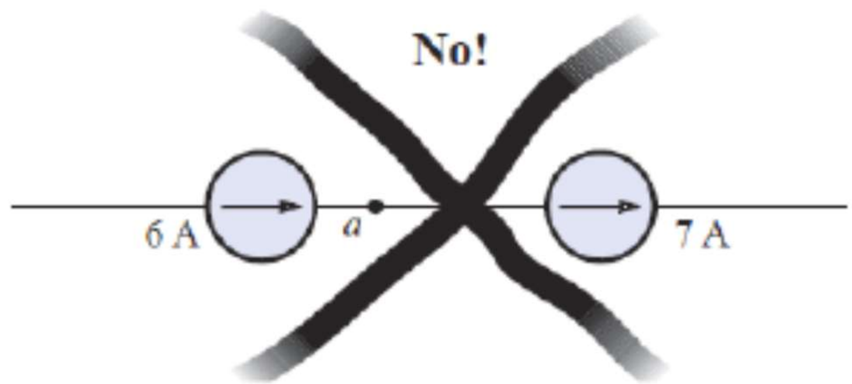


FIG. 8.18
Invalid situation.

MESH ANALYSIS

Mesh ≡ closed loop (like a fence)

1. *Assign a current in clockwise direction to each independent, closed loop of the network.*

    - *Not necessarily the clockwise direction.*
    - *Any direction can be chosen ---*
    - *Remaining steps follow the choice properly.*

However, by choosing the clockwise direction as a standard, we can develop a shorthand method for writing the required equations that will save time and possibly prevent some common errors.

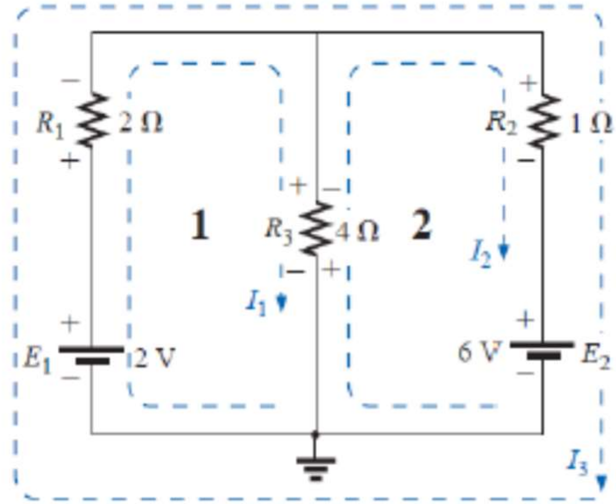


FIG. 8.26
Defining the mesh currents for a "two-window" network.

Independent closed loop

MESH ANALYSIS

2. *Indicate the polarities within each loop for each resistor as determined by the assumed direction of loop current for that loop.*

Note the requirement that the polarities be placed within each loop. This requires, as shown in Fig. 8.26, that the 4Ω resistor have two sets of polarities across it.

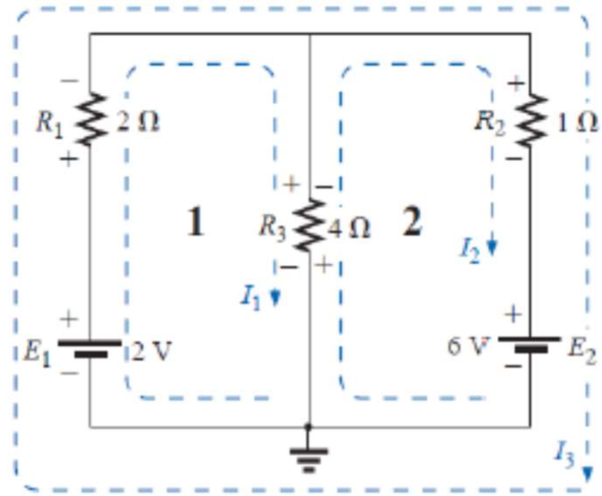
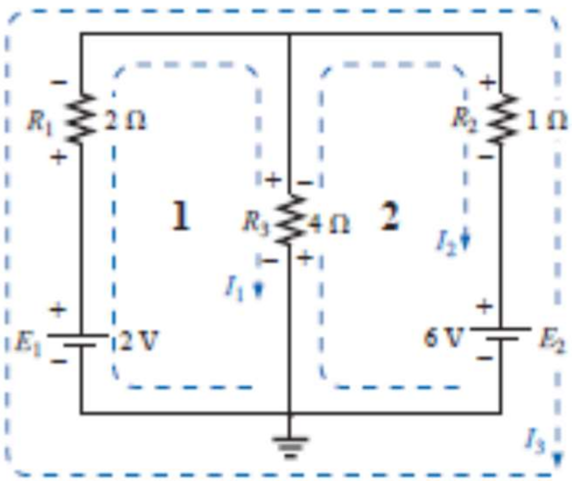


FIG. 8.26
Defining the mesh currents for a "two-window" network.

3. *Apply Kirchhoff's voltage law around each closed loop.*
   Again, the clockwise direction is chosen to establish uniformity.
   a. *If a resistor has two or more assumed currents through it,* $I_T = \sum I$
   b. *The polarity of a voltage source is unaffected by the direction of the assigned loop currents.*
4. *Solve the resulting simultaneous linear equations for the assumed loop currents.*

EXAMPLE



loop 1: $+E_1 - V_1 - V_3 = 0$ (clockwise starting at point a)

$$+2\text{ V} - (2\ \Omega)\, I_1 - \overbrace{(4\ \Omega)\underbrace{(I_1 - I_2)}_{\substack{\text{Total current}\\\text{through}\\\text{4-}\Omega\text{ resistor}}}}^{\substack{\text{Voltage drop across}\\\text{4-}\Omega\text{ resistor}}} = 0$$

Subtracted since I_2 is opposite in direction to I_1.

loop 2: $-V_3 - V_2 - E_2 = 0$ (clockwise starting at point b)

$$-(4\ \Omega)(I_2 - I_1) - (1\ \Omega)I_2 - 6\text{ V} = 0$$

Step 4: The equations are then rewritten as follows (without units for clarity):

loop 1: $+2 - 2I_1 - 4I_1 + 4I_2 = 0$
loop 2: $-4I_2 + 4I_1 - 1I_2 - 6 = 0$

and

loop 1: $+2 - 6I_1 + 4I_2 = 0$
loop 2: $-5I_2 + 4I_1 - 6 = 0$

or

loop 1: $-6I_1 + 4I_2 = -2$
loop 2: $+4I_1 - 5I_2 = +6$

Applying determinants will result in

$$I_1 = \mathbf{-1\ A} \quad \text{and} \quad I_2 = \mathbf{-2\ A}$$

EXAMPLE

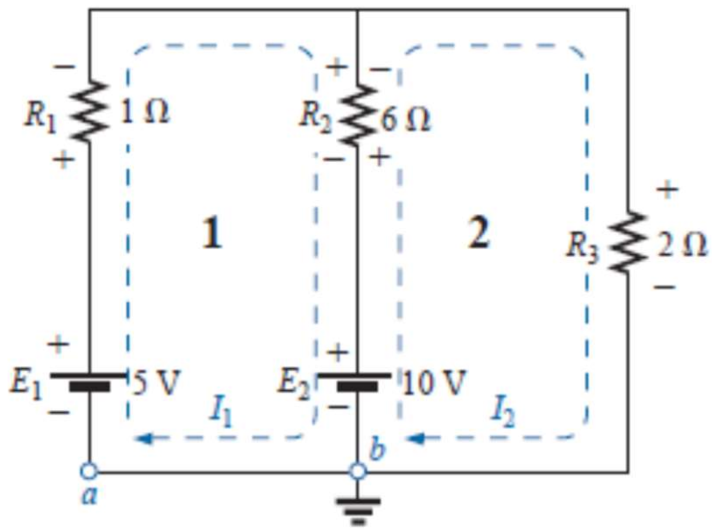


FIG. 8.27
Example 8.12.

loop 1: $+E_1 - V_1 - V_2 - E_2 = 0$ (clockwise starting at point a)

$$+5\text{ V} - (1\ \Omega)I_1 - (6\ \Omega)(I_1 - I_2) - 10\text{ V} = 0$$

I_2 flows through the 6-Ω resistor in the direction opposite to I_1.

loop 2: $E_2 - V_2 - V_3 = 0$ (clockwise starting at point b)

$$+10\text{ V} - (6\ \Omega)(I_2 - I_1) - (2\ \Omega)I_2 = 0$$

The equations are rewritten as

$$\left.\begin{aligned} 5 - I_1 - 6I_1 + 6I_2 - 10 &= 0 \\ 10 - 6I_2 + 6I_1 - 2I_2 &= 0 \end{aligned}\right\} \begin{aligned} -7I_1 + 6I_2 &= 5 \\ +6I_1 - 8I_2 &= -10 \end{aligned}$$

Step 4:

$$I_1 = \frac{\begin{vmatrix} 5 & 6 \\ -10 & -8 \end{vmatrix}}{\begin{vmatrix} -7 & 6 \\ 6 & -8 \end{vmatrix}} = \frac{-40 + 60}{56 - 36} = \frac{20}{20} = \mathbf{1\ A}$$

$$I_2 = \frac{\begin{vmatrix} -7 & 5 \\ 6 & -10 \end{vmatrix}}{20} = \frac{70 - 30}{20} = \frac{40}{20} = \mathbf{2\ A}$$