Chapter Title: Vectors

Sections: Vectors, Adding Vectors Geometrically, Components of Vectors

Vectors

We have seen in past lectures; physical quantities are determined by measuring physical phenomena. Examples, Temperature, pressure, energy, displacement, etc.

Some physical quantities are sufficient if only measurement magnitude is provided. Examples, Temperature, energy

However, for some other physical quantities, magnitude not enough rather needs direction. Examples, displacement, velocity.

Physical quantities can be divided into two categories:

(a) Quantities need direction and magnitude

(b) Quantities need only magnitude.

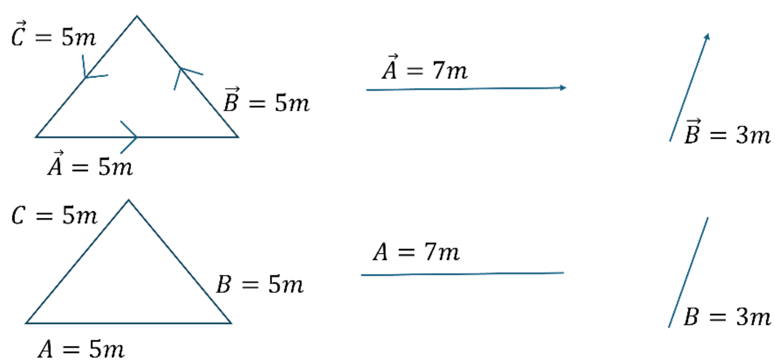
The Quantities need direction, and the magnitude of a physical phenomenon is called vector quantities. Vectors and scalars are similar to collective names representing groups of quantities: one requires both magnitude and direction, while the other needs only magnitude.

How to present a vector quantity?

$\vec{A}$, $\vec{B}$, and $\vec{C}$ are vectors of a triangle.

How to present a scalar quantity?

A, B and C are scalar quantities of a triangle.

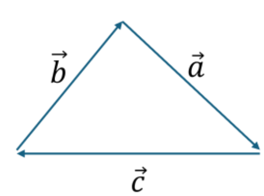


Vector: Addition & Subtraction

If $\vec{a}$ and $\vec{b}$ are two vectors, then vector sum or resultant can be written as,

$\vec{s} = \vec{a} + \vec{b}$ $\quad$ (***Equation* 1**)

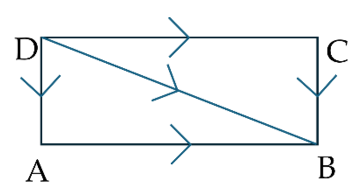
If a triangle is considered that has arms $\vec{a}$, $\vec{b}$, and $\vec{c}$, the directions are,



Similarly, if a rectangle is drawn, let's consider ABCD is a rectangle, the diagonal arm represents the sum or resultant of the corresponding vectors.

$$\overrightarrow{DB} = \overrightarrow{DA} + \overrightarrow{AB}$$

$$\overrightarrow{DB} = \overrightarrow{DC} + \overrightarrow{CB}$$



$$\overrightarrow{DA} + \overrightarrow{AB} = \overrightarrow{DC} + \overrightarrow{CB} \quad (Equation\ 2)$$

If we assume that $\overrightarrow{DA} = \overrightarrow{CB} = \vec{b}$ and $\overrightarrow{AB} = \overrightarrow{DC} = \vec{a}$, then

$\vec{b} + \vec{a} = \vec{a} + \vec{b}$ Commutative Law of Addition

$\vec{A} + \vec{B} = \vec{B} + \vec{A}$

If another vector $\vec{C}$ is added, the equation becomes

$\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$ Associative Law of Addition

Negative Vector

If we assume a vector $\overrightarrow{AB} = \vec{a}$ and presented by the following diagrams,

$$A \xrightarrow{\vec{a}} B$$

$$A \xleftarrow{-\vec{a}} B$$

The vector $\overrightarrow{BA} = -\vec{a}$ that becomes a negative when the direction is changed.

$\overrightarrow{AB} = -\overrightarrow{BA}$

If $\vec{A} + (-\vec{A}) = 0$ is called a null vector.

In equation (1), the direction of $\vec{b}$ is changed, that means the vector becomes -$\vec{b}$, then the resultant or sum of the vectors $\vec{a}$ and $\vec{b}$ becomes,

$$\vec{s} = \vec{a} - \vec{b}$$

Multiplying a Vector by a Scalar

If m is a scalar, a vector can be multiplied by a scalar quantity.

Example, $m\vec{A}$, $-m\vec{b}$, if $m = 12$, then $12\vec{a}$

Components of Vectors

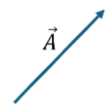
The graphical method of adding vectors is not recommended whenever high accuracy is required or in three-dimensional problems.

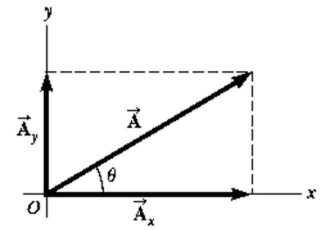
Therefore, the method of adding vectors uses the projection of vectors along coordinate axes.

These projections are called the *components of the vector* or *its rectangular components*.

$$A_x = A\cos\theta$$

$$A_y = A\sin\theta$$





Magnitude, $A = \sqrt{A_x^2 + A_y^2}$ Angle, $\theta = \tan^{-1}\frac{A_y}{A_x}$