EEE 141
ELECTRICAL CIRCUITS

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CIRCUIT THEOREMS

- Linearity Property
- Superposition
- Thevenin's Theorem
- Norton's Theorem
- Maximum Power Transfer

LINEARITY PROPERTY

It is the property of an element describing a linear relationship between cause and effect.

A linear circuit is one whose output is linearly related (or directly proportional) to its input.

Homogeneity (scaling) property

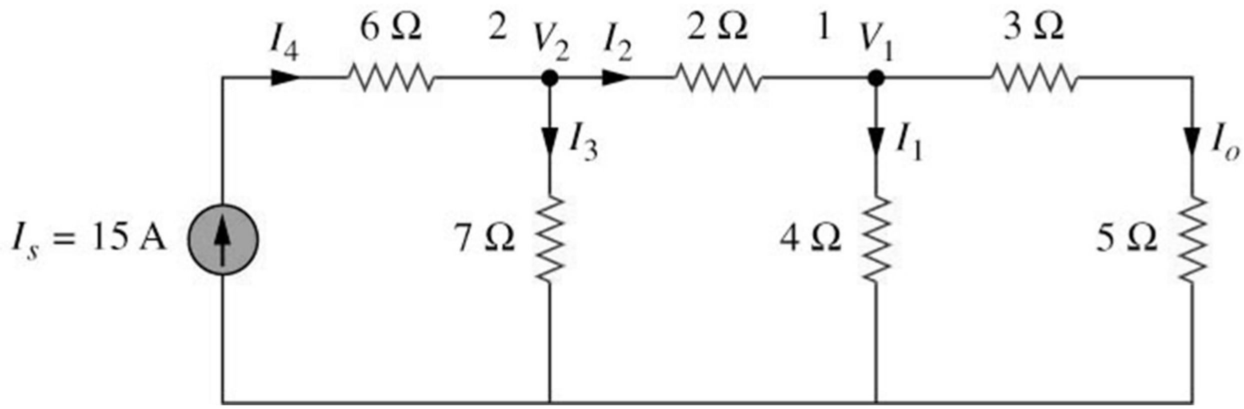
$$v = i\,R \quad \rightarrow \quad k\,v = k\,i\,R$$

Additive property

$$v_1 = i_1\,R \text{ and } v_2 = i_2\,R$$
$$\rightarrow \quad v = (i_1 + i_2)\,R = v_1 + v_2$$

EXAMPLE

By assume I_o = 1 A, use linearity to find the actual value of Io in the circuit shown below.



If $I_o = 1$ A, then $V_1 = (3+5)I_o = 8$ V and $I_1 = V_1/4 = 2$ A. Applying KCL at node 1 gives

$$I_2 = I_1 + I_o = 3 \text{ A}$$

$$V_2 = V_1 + 2I_2 = 8 + 6 = 14 \text{ V}, \qquad I_3 = \frac{V_2}{7} = 2 \text{ A}$$

Applying KCL at node 2 gives

$$I_4 = I_3 + I_2 = 5 \text{ A}$$

Therefore, $I_s = 5$ A. This shows that assuming $I_o = 1$ gives $I_s = 5$ A, the actual source current of 15 A will give $I_o = 3$ A as the actual value.

*Refer to in-class illustration, text book, answer I_o = 3A

SUPERPOSITION THEOREM

It states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltage across (or currents through) that element due to EACH independent source acting alone.

The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately.

SUPERPOSITION THEOREM

Steps to apply superposition principle

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.

2. Repeat step 1 for each of the other independent sources.

3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

SUPERPOSITION THEOREM

Two things have to be keep in mind:

1. When we say turn off all other independent sources:
   - Independent voltage sources are replaced by 0 V (short circuit) and
   - Independent current sources are replaced by 0 A (open circuit).
2. Dependent sources are left intact because they are controlled by circuit variables.

SUPERPOSITION THEOREM

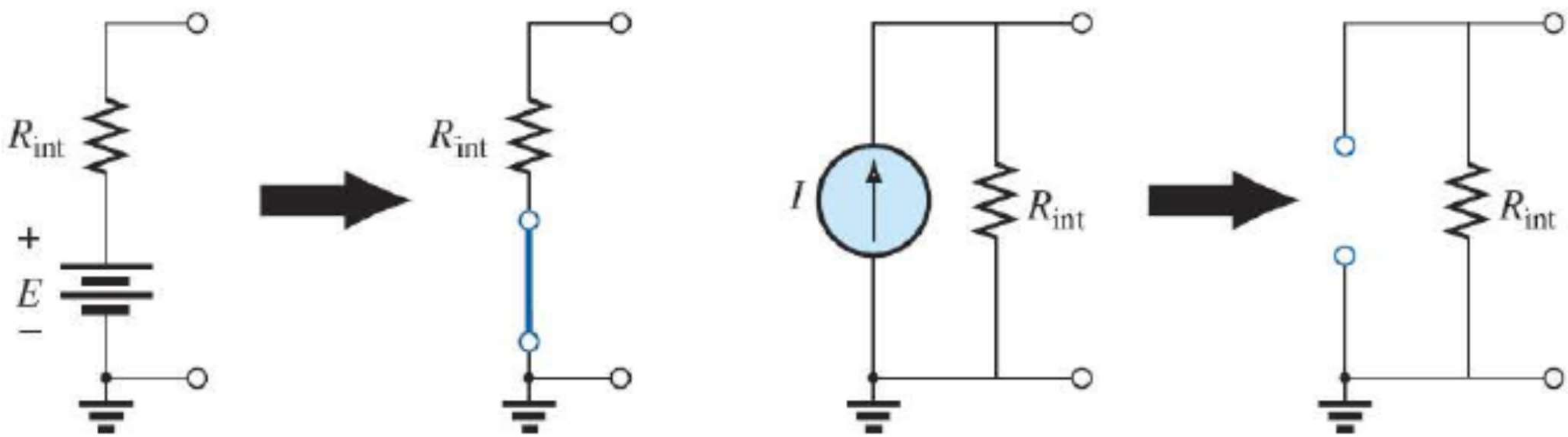
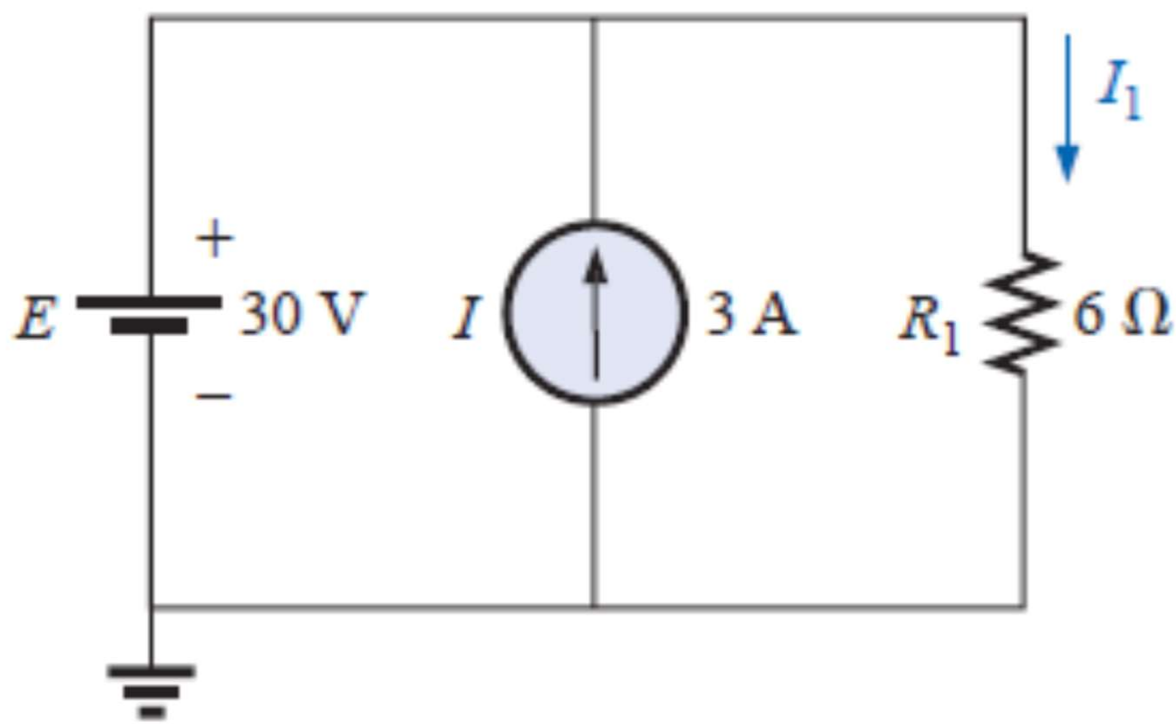
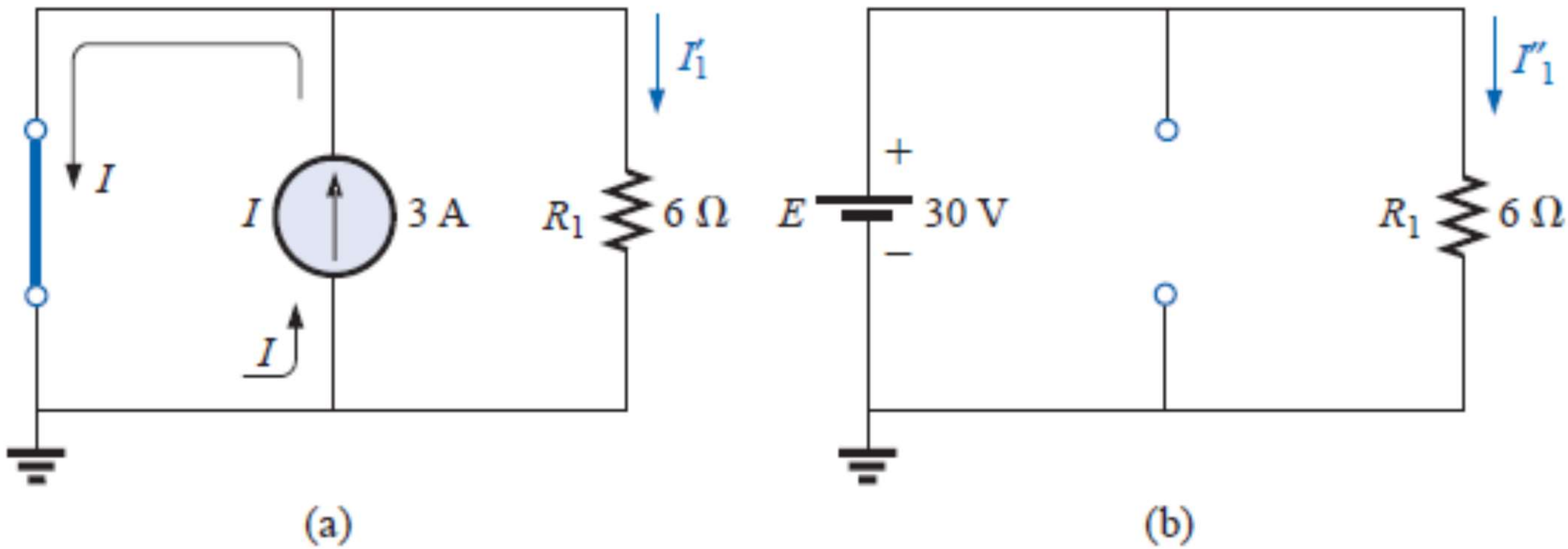


FIG. 9.1 *Removing a voltage source and a current source to permit the application of the superposition theorem.*

EXAMPLE

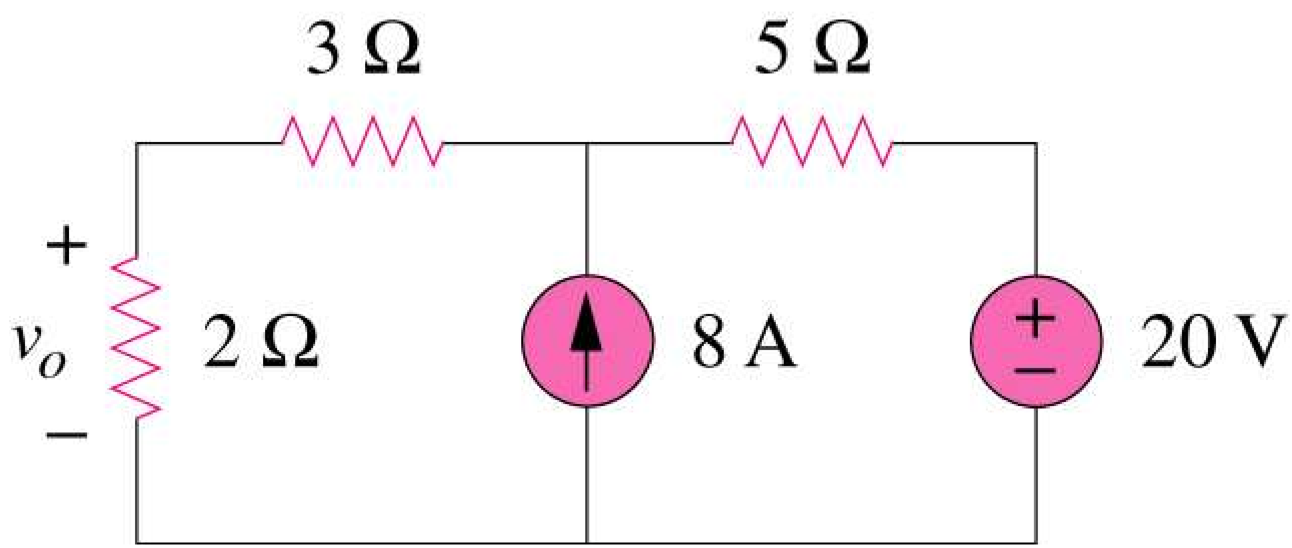


SOLUTION



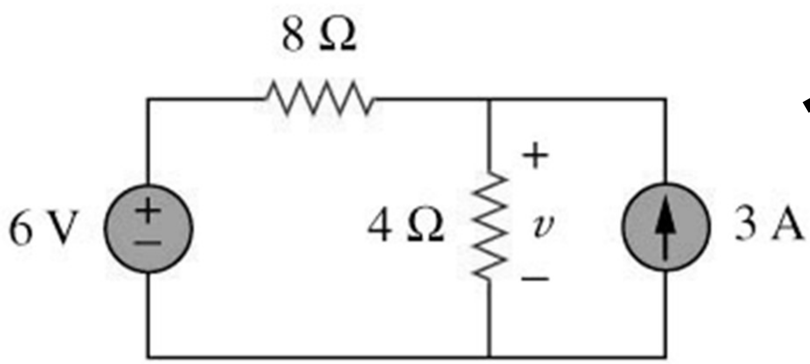
EXAMPLE

We consider the effects of 8A and 20V one by one, then add the two effects together for final v_o.

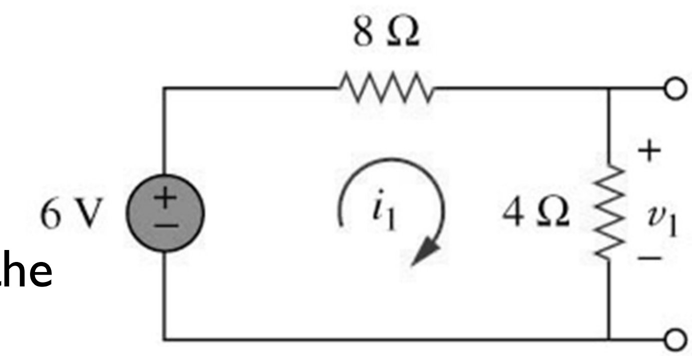


EXAMPLE

Use the superposition theorem to find v in the circuit shown below.

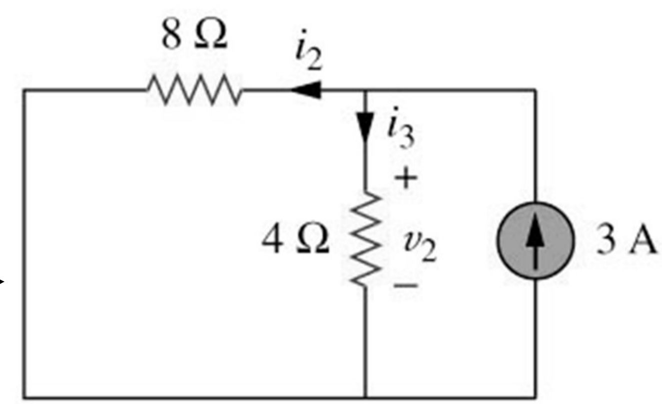


3A is discarded by open-circuit



(a)

6V is discarded by short-circuit



(b)

*Refer to in-class illustration, text book, answer v = 10V

EXAMPLE

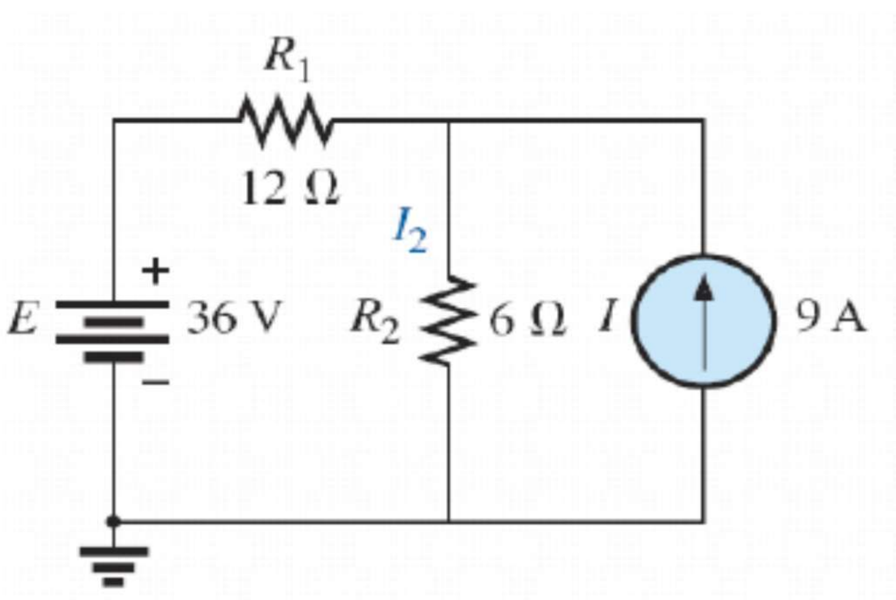


FIG. 9.2 *Network to be analyzed in Example 9.1 using the superposition theorem.*

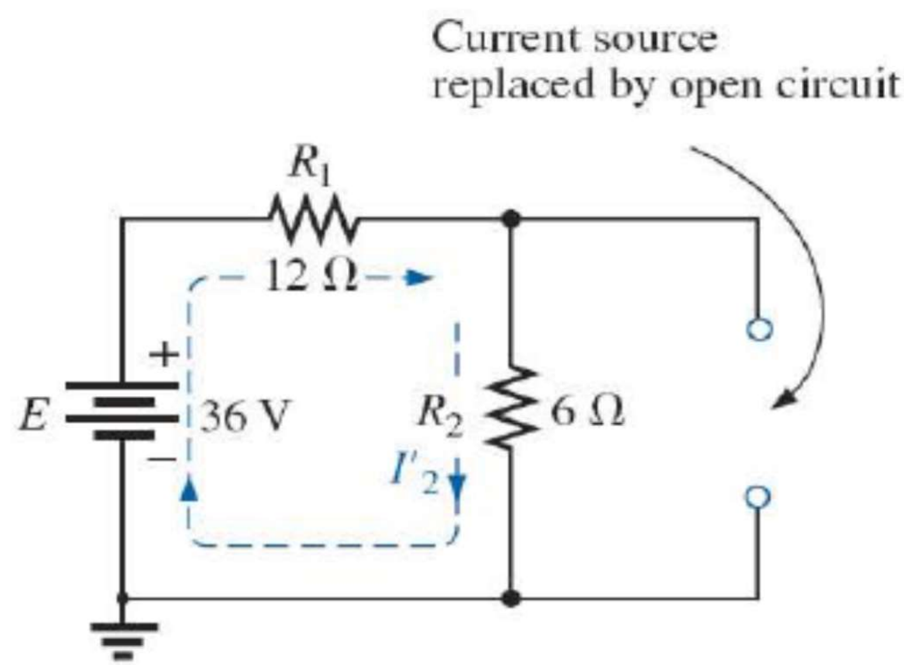


FIG. 9.3 *Replacing the 9 A current source in Fig. 9.2 by an open circuit to determine the effect of the 36 V voltage source on current I_2.*

SOLUTION

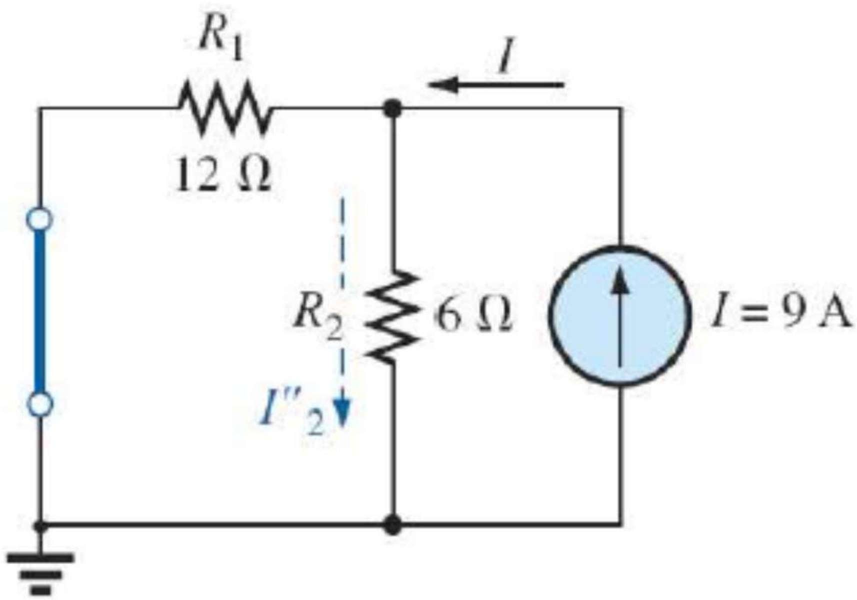


FIG. 9.4 *Replacing the 36 V voltage source by a short-circuit equivalent to determine the effect of the 9 A current source on current I_2.*

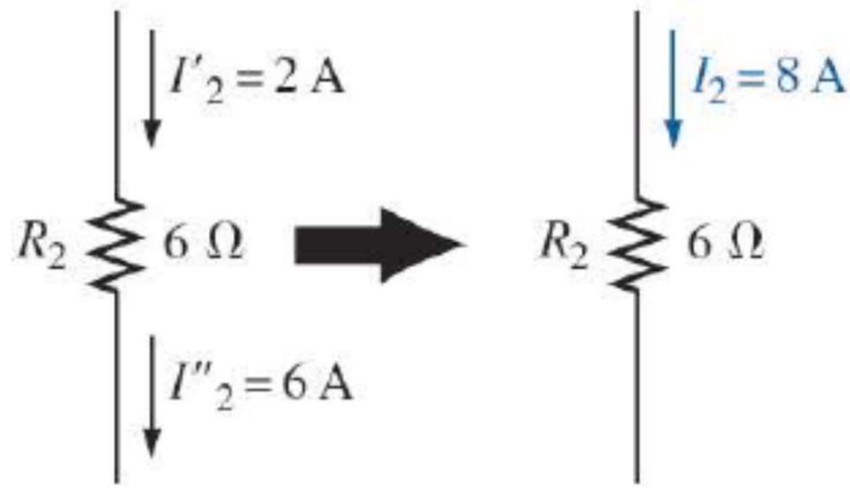


FIG. 9.5 *Using the results of Figs. 9.3 and 9.4 to determine current I_2 for the network in Fig. 9.2.*

POWER VERSUS CURRENT

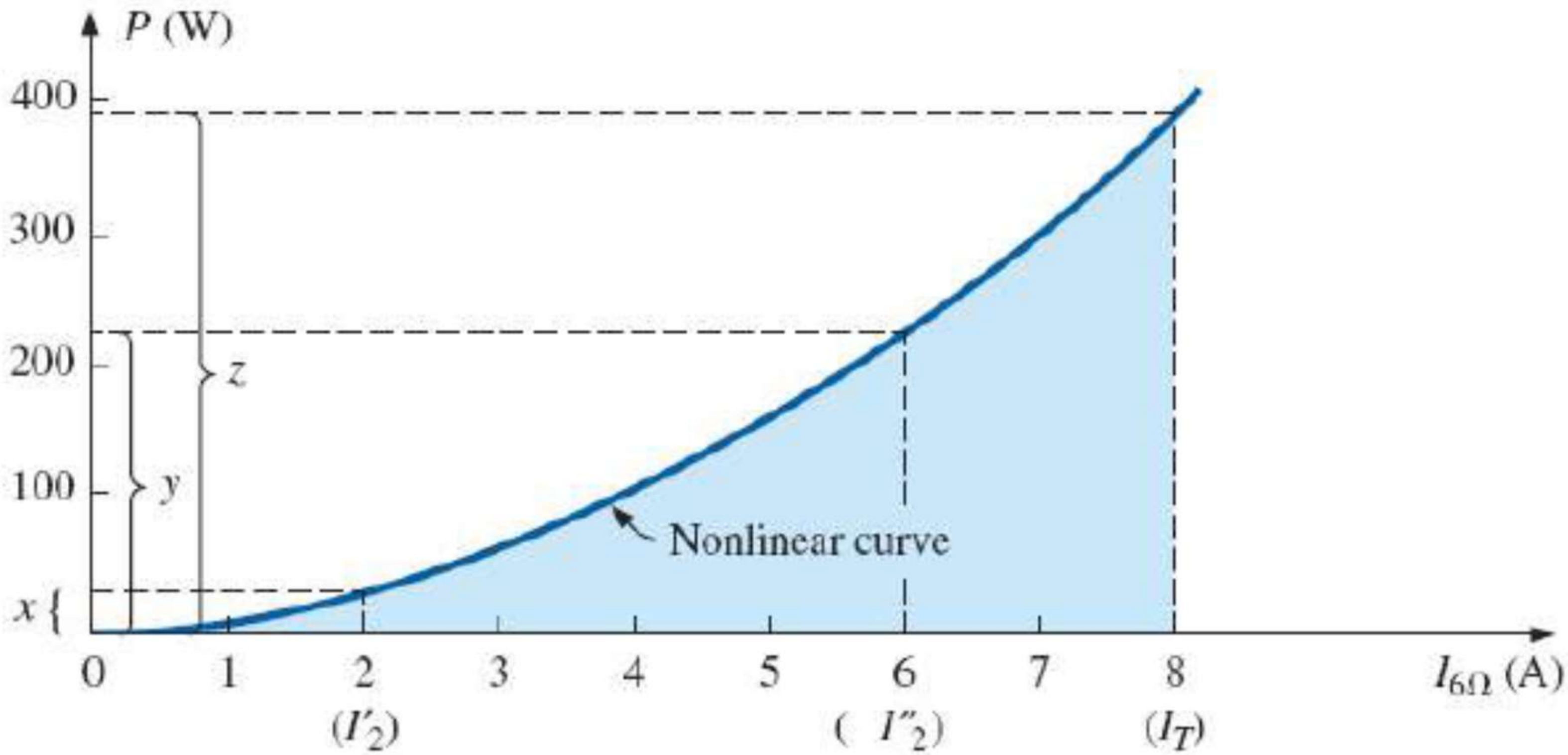


FIG. 9.6 *Plotting power delivered to the 6Ω resistor versus current through the resistor.*

I VERSUS V

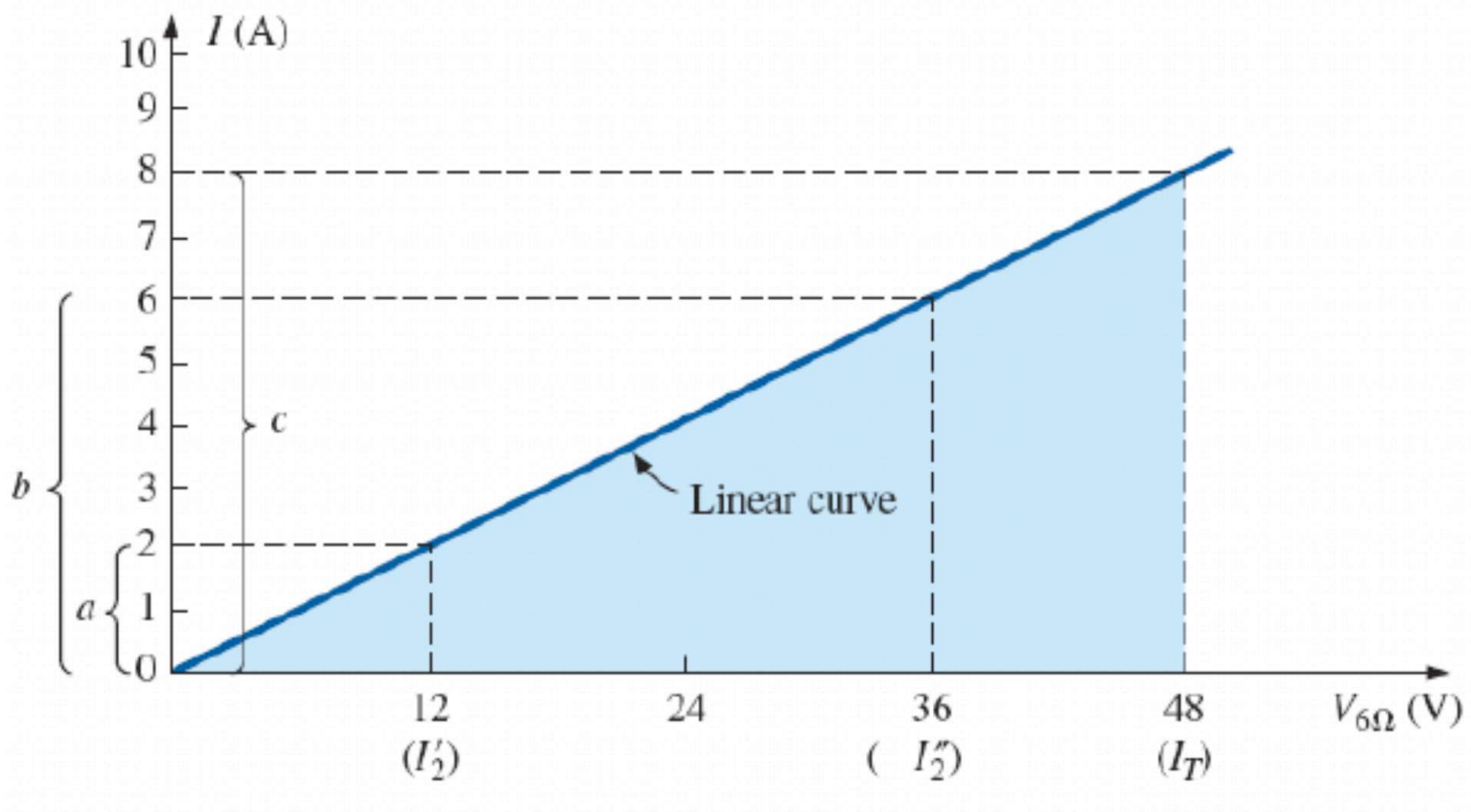


FIG. 9.7 *Plotting I versus V for the 6Ω resistor.*

EXAMPLE

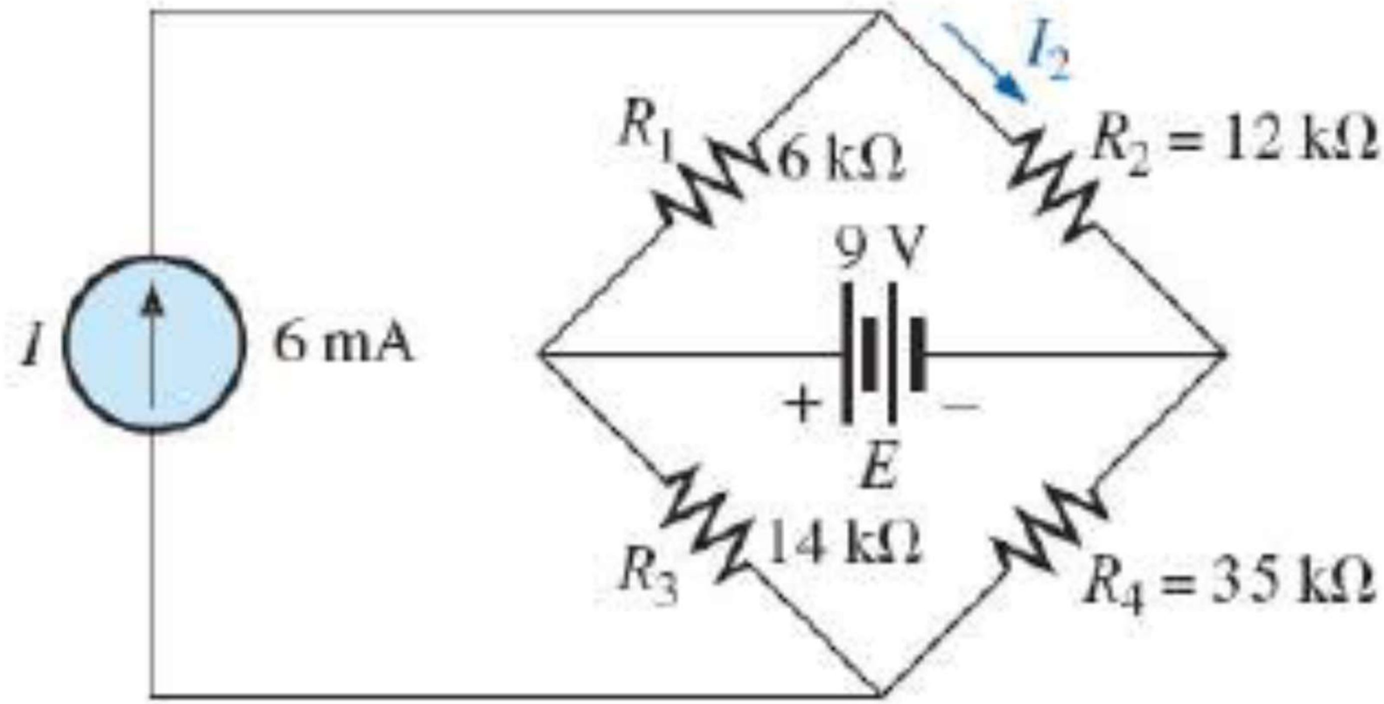


FIG. 9.15 *Example 9.4.*

SOLUTION

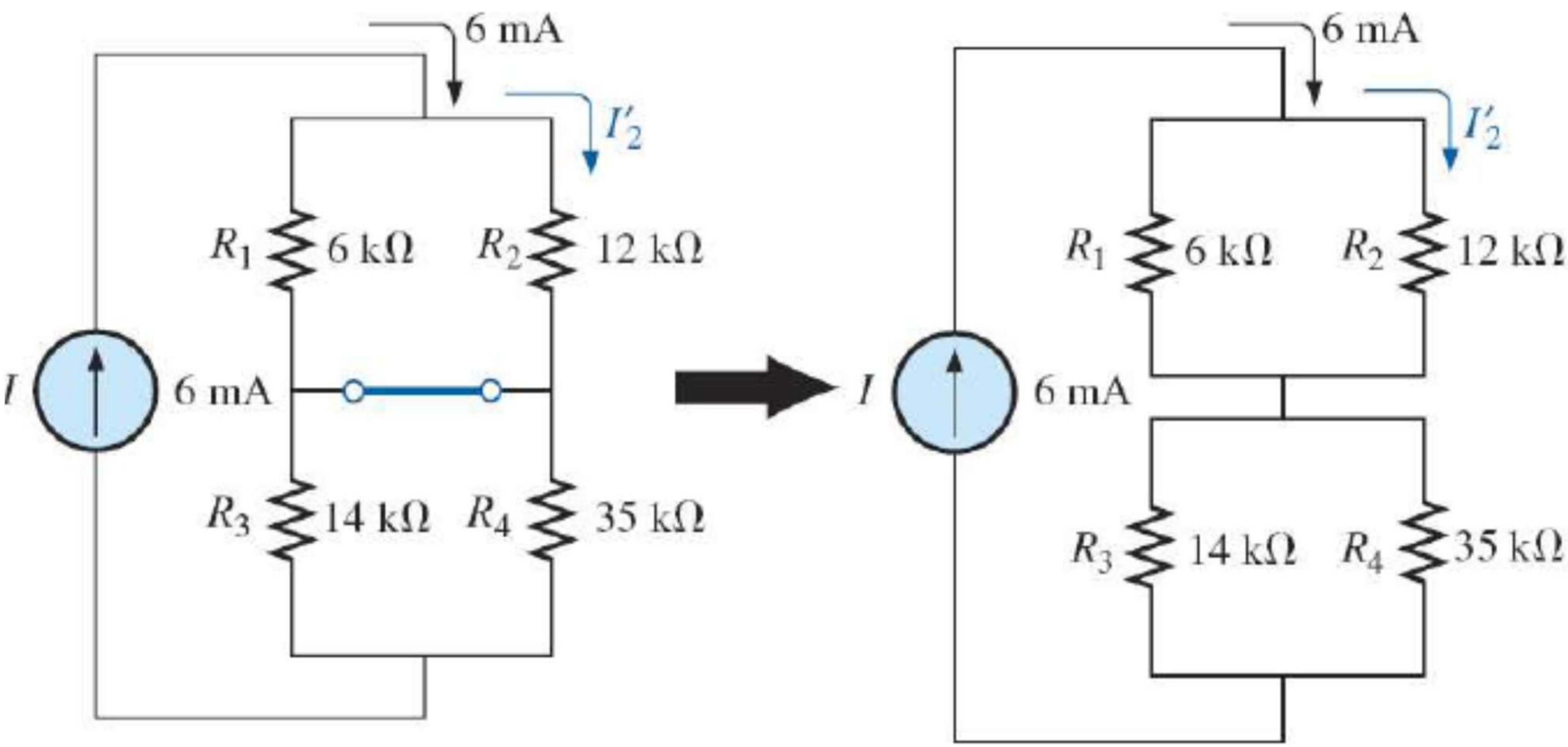


FIG. 9.16 *The effect of the current source I on the current I_2.*

SOLUTION

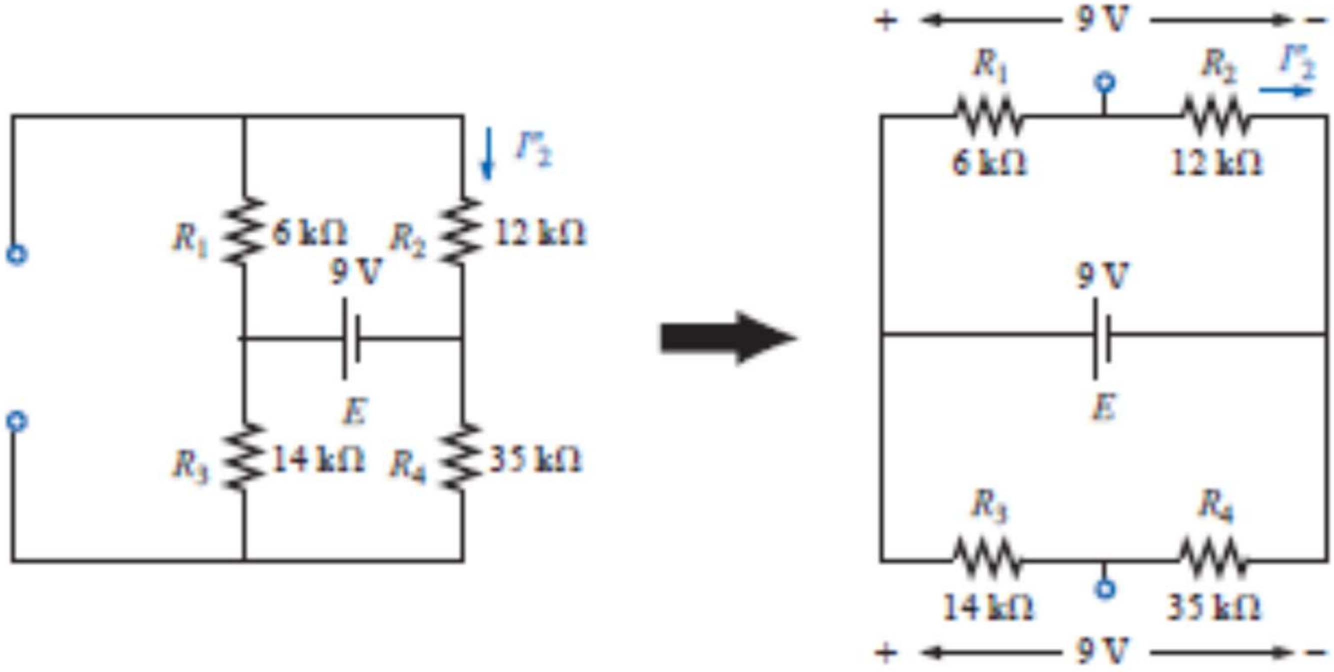


FIG. 9.18

The effect of the voltage source E on the current I_2.

Since I'_2 and I''_2 have the same direction through R_2, the desired current is the sum of the two:

$$
\begin{aligned}
I_2 &= I'_2 + I''_2 \\
&= 2\text{ mA} + 0.5\text{ mA} \\
&= \mathbf{2.5\text{ mA}}
\end{aligned}
$$

EXAMPLE

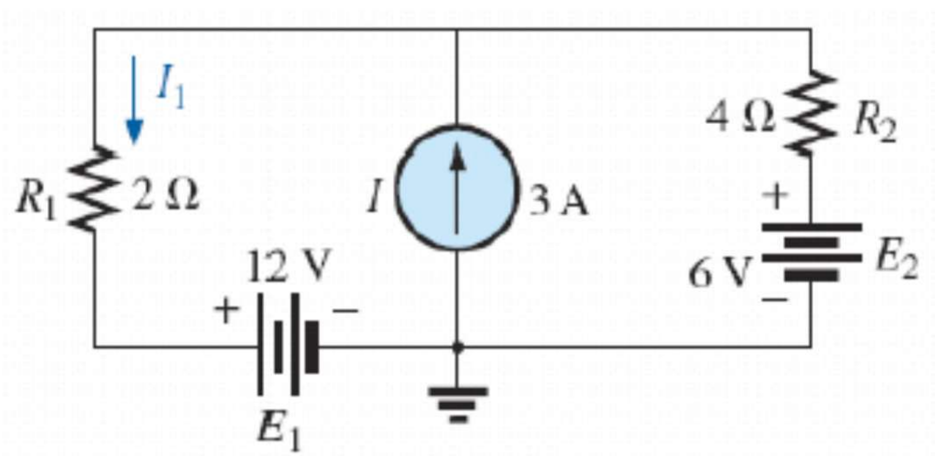


FIG. 9.18 *Example 9.5.*

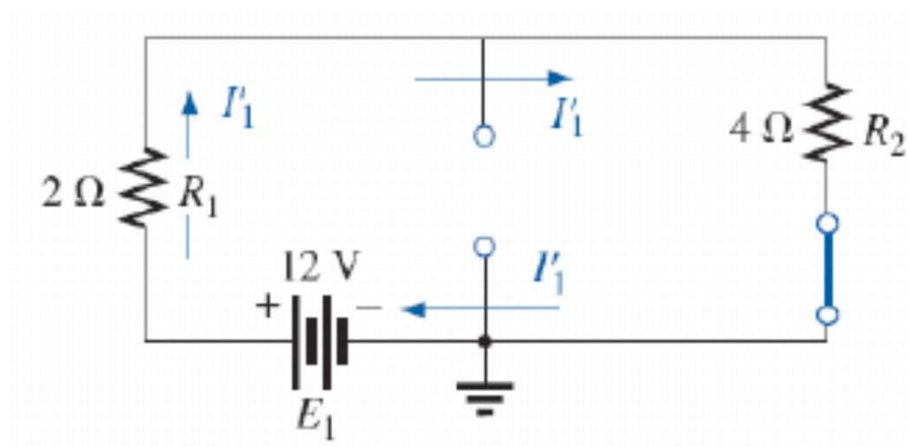


FIG. 9.19 *The effect of E_1 on the current I.*

SOLUTION

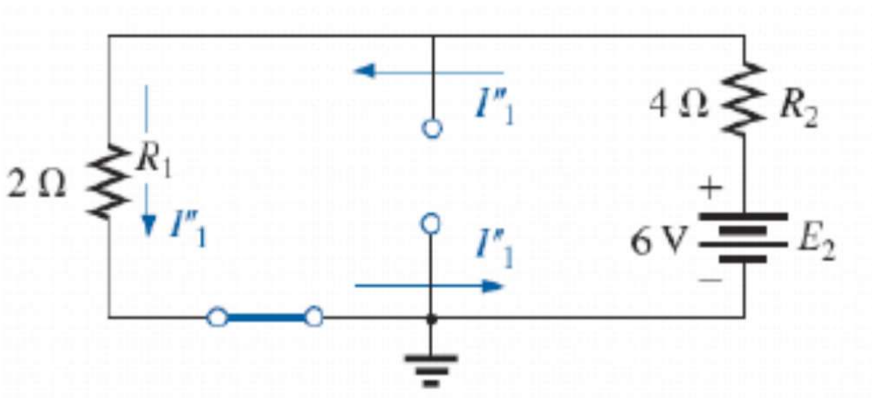


FIG. 9.20 *The effect of E_2 on the current I_1.*

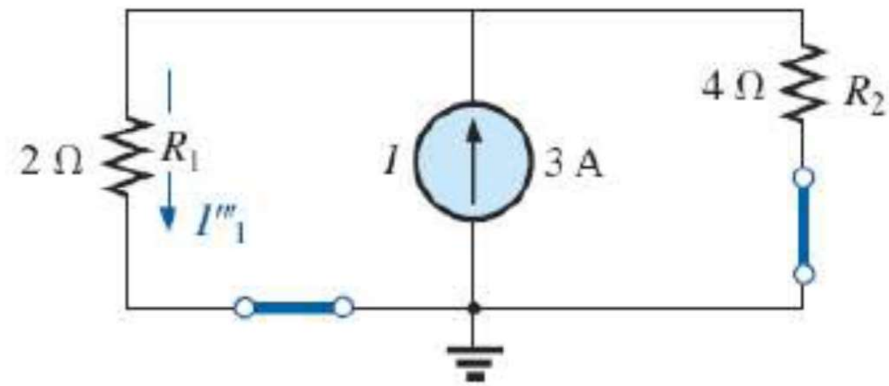


FIG. 9.21 *The effect of I on the current I_1*

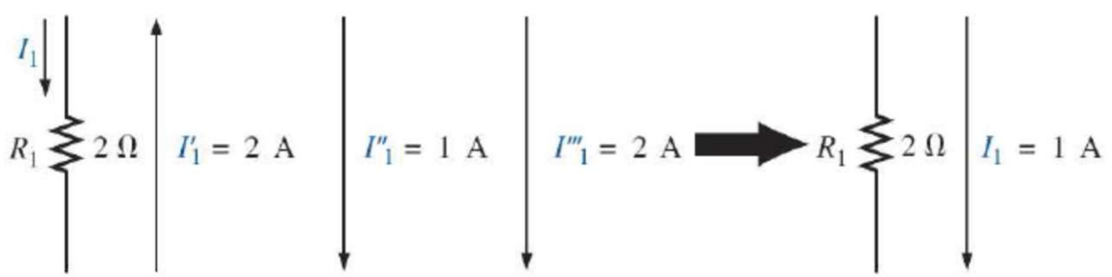
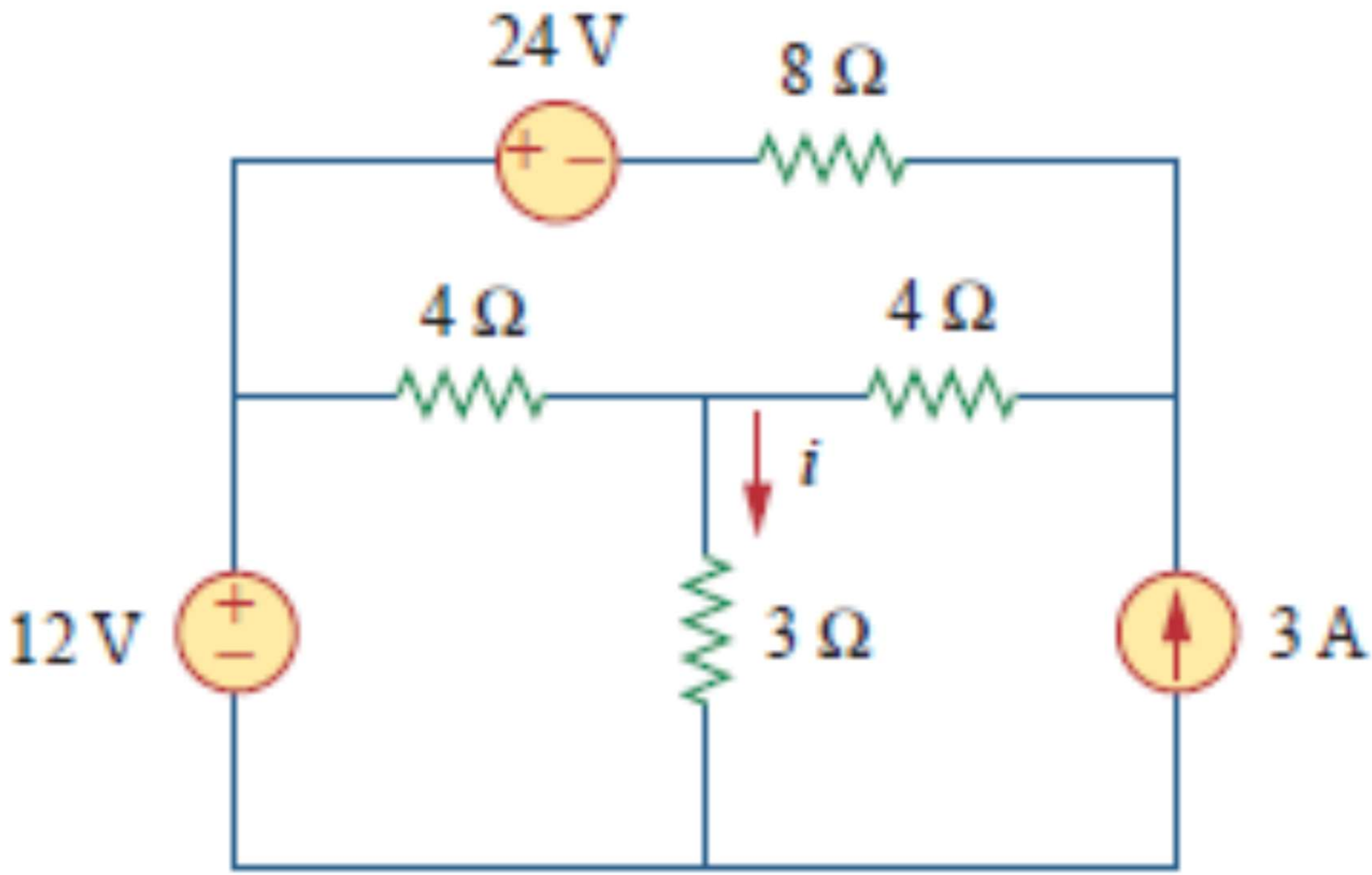
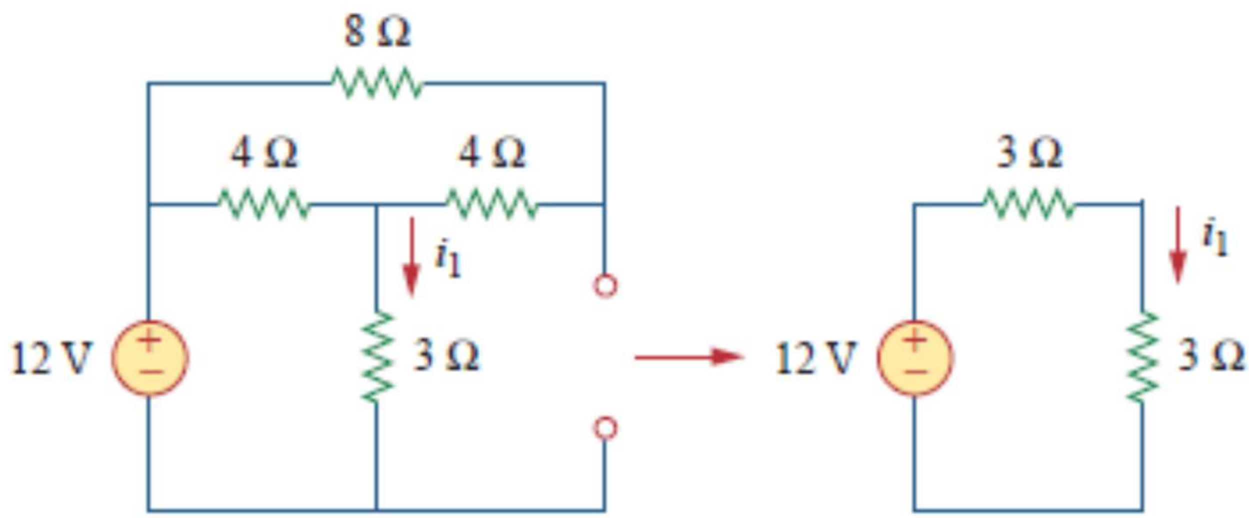


FIG. 9.22 *The resultant current I_1.*

EXAMPLE



SOLUTION



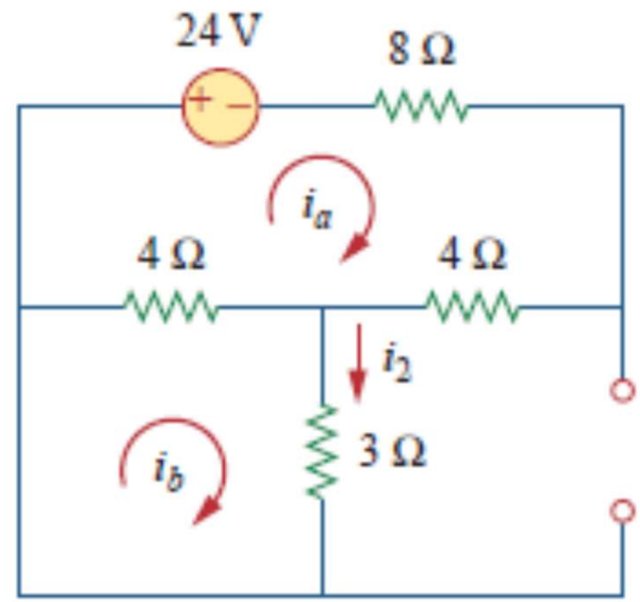
(a)

$$i = i_1 + i_2 + i_3$$

where i_1, i_2, and i_3 are due to the 12-V, 24-V, and 3-A sources respectively. To get i_1, consider the circuit in Fig. 4.13(a). Combining 4 Ω (on the right-hand side) in series with 8 Ω gives 12 Ω. The 12 Ω in parallel with 4 Ω gives $12 \times 4/16 = 3\ \Omega$. Thus,

$$i_1 = \frac{12}{6} = 2\ \text{A}$$

SOLUTION



To get i_2, consider the circuit in Fig. 4.13(b). App
gives

(b)

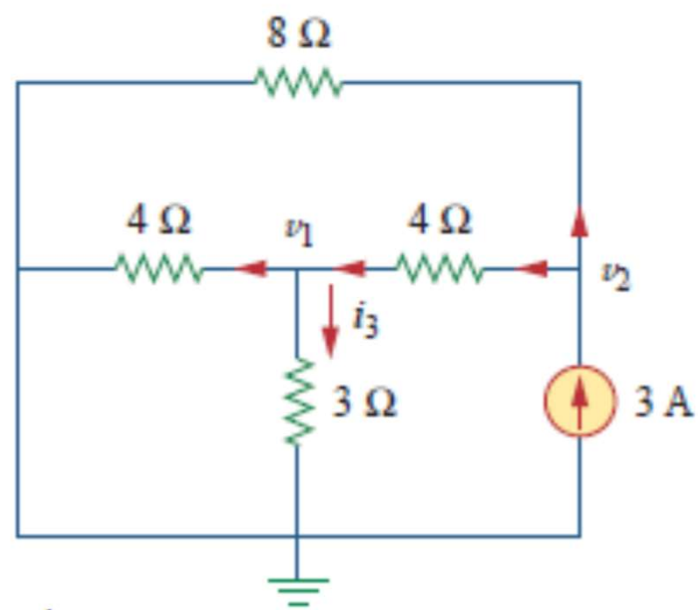
$$16i_a - 4i_b + 24 = 0 \quad \Rightarrow \quad 4i_a - i_b = -6 \qquad \textbf{(4.5.1)}$$

$$7i_b - 4i_a = 0 \quad \Rightarrow \quad i_a = \frac{7}{4}i_b \qquad \textbf{(4.5.2)}$$

Substituting Eq. (4.5.2) into Eq. (4.5.1) gives

$$i_2 = i_b = -1$$

SOLUTION



(c)

To get i_3, consider the circuit in Fig. 4.13(c). Using nodal analysis gives

$$3 = \frac{v_2}{8} + \frac{v_2 - v_1}{4} \quad \Rightarrow \quad 24 = 3v_2 - 2v_1 \qquad \textbf{(4.5.3)}$$

$$\frac{v_2 - v_1}{4} = \frac{v_1}{4} + \frac{v_1}{3} \quad \Rightarrow \quad v_2 = \frac{10}{3}v_1 \qquad \textbf{(4.5.4)}$$

Substituting Eq. (4.5.4) into Eq. (4.5.3) leads to $v_1 = 3$ and

$$i_3 = \frac{v_1}{3} = 1 \text{ A}$$

Thus,

$$i = i_1 + i_2 + i_3 = 2 - 1 + 1 = 2 \text{ A}$$