EEE 141
ELECTRICAL CIRCUITS

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CAPACITORS

Transient

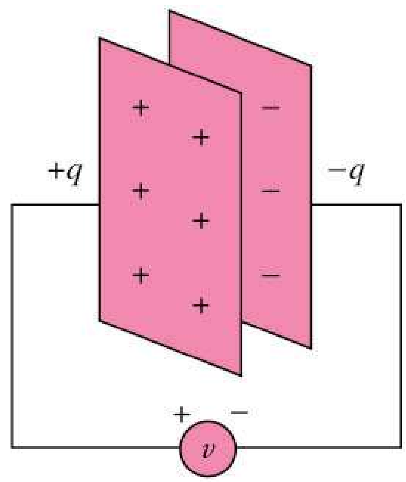
Charging

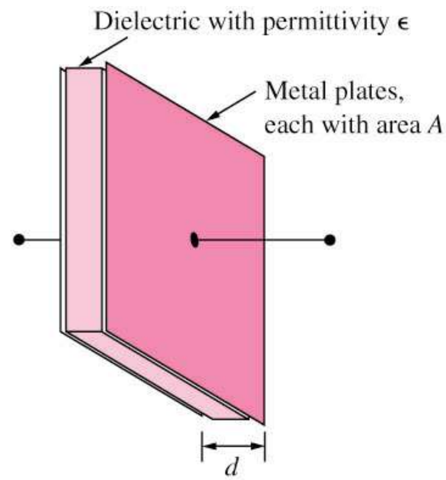
Discharging

Series and Parallel Capacitors

CAPACITORS

A capacitor is a passive element designed to **store energy** in its **electric field.**

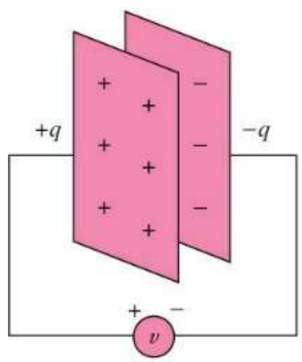




- A **capacitor** consists of two conducting plates separated by an insulator (or dielectric).

CAPACITORS

Capacitance C is the ratio of the charge *q* on one plate of a capacitor to the voltage difference *v* between the two plates, measured in farads (F).



$$q = C\,v \qquad \text{and} \qquad C = \frac{\varepsilon\,A}{d}$$

- Where ε is the permittivity of the dielectric material between the plates, A is the surface area of each plate, d is the distance between the plates.
- Unit: F, pF (10^{-12}), nF (10^{-9}), and **μF (10^{-6})**

TRANSIENTS IN CAPACITIVE NETWORKS: CHARGING PHASE

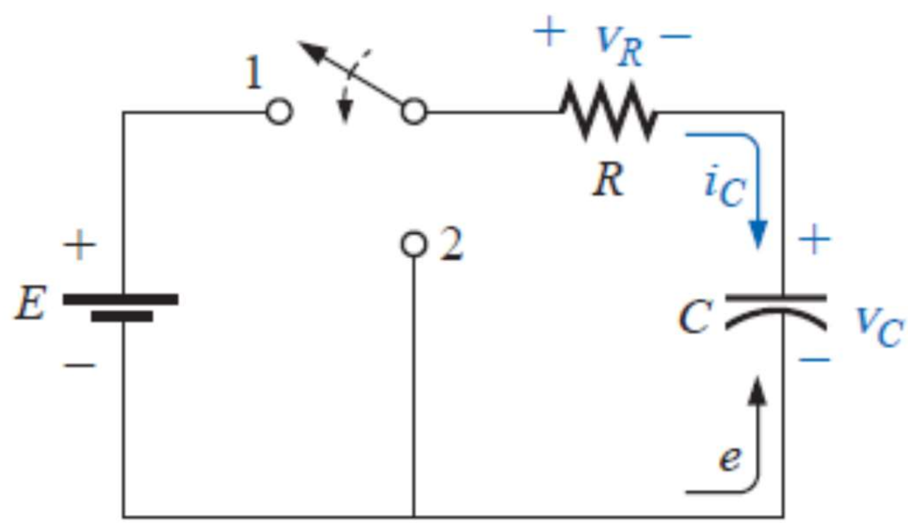


FIG. 10.24
Basic charging network.

CHARGING PHASE

- At the instant the switch is closed, electrons are drawn from the top plate and deposited on the bottom plate by the battery, resulting in a net positive charge on the top plate and a negative charge on the bottom plate.
- The transfer of electrons is very rapid at first, slowing down as the potential across the capacitor approaches the applied voltage of the battery.
- When the voltage across the capacitor equals the battery voltage, the transfer of electrons will cease and the plates will have a net charge determined by $Q = CV_C = CE$.

CHARGING PHASE

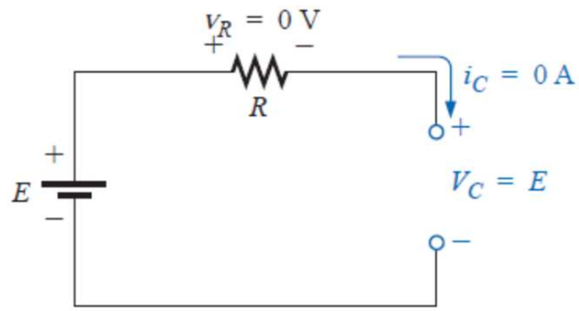


FIG. 10.27
Open-circuit equivalent for a capacitor following the charging phase.

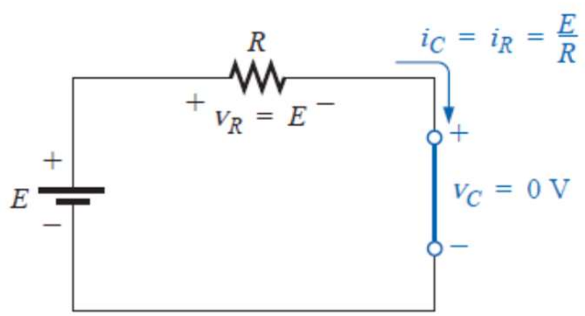


FIG. 10.28
Short-circuit equivalent for a capacitor (switch closed, $t = 0$).

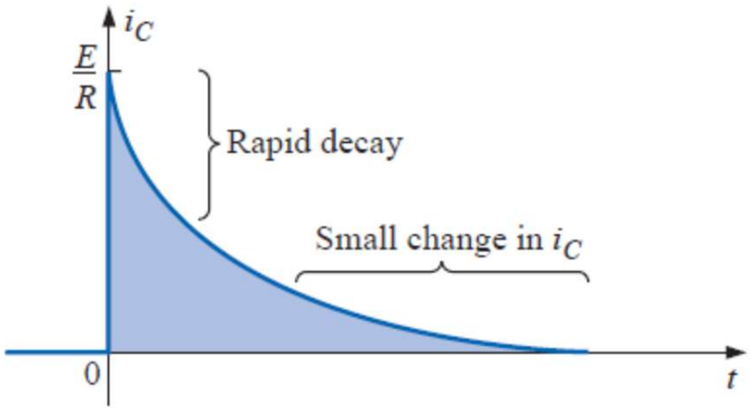


FIG. 10.25
i_C during the charging phase.

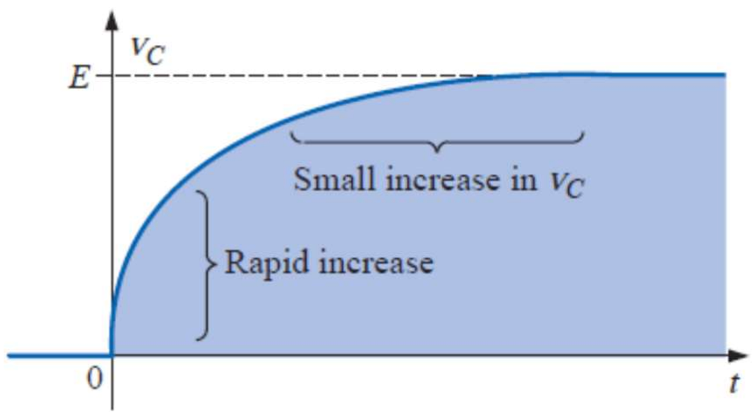


FIG. 10.26
v_C during the charging phase.

CHARGING PHASE

A capacitor can be replaced by an open-circuit equivalent once the charging phase in a dc network has passed.

$$\boxed{i_C = \frac{E}{R} e^{-t/RC}}$$

The factor RC is called the time constant.

TIME CONSTANT

$$RC = \left(\frac{V}{I}\right)\left(\frac{Q}{V}\right) = \left(\frac{\cancel{V}}{\cancel{Q}/t}\right)\left(\frac{\cancel{Q}}{\cancel{V}}\right) = t$$

Its symbol is the Greek letter τ (tau), and its unit of measure is the second. Thus,

$$\boxed{\tau = RC} \qquad \text{(seconds, s)} \qquad \textbf{(10.14)}$$

If we substitute $\tau = RC$ into the exponential function $e^{-t/RC}$, we obtain $e^{-t/\tau}$. In one time constant, $e^{-t/\tau} = e^{-\tau/\tau} = e^{-1} = 0.3679$, or the function equals 36.79% of its maximum value of 1. At $t = 2\tau$, $e^{-t/\tau} = e^{-2\tau/\tau} = e^{-2} = 0.1353$, and the function has decayed to only 13.53% of

CAPACITORS

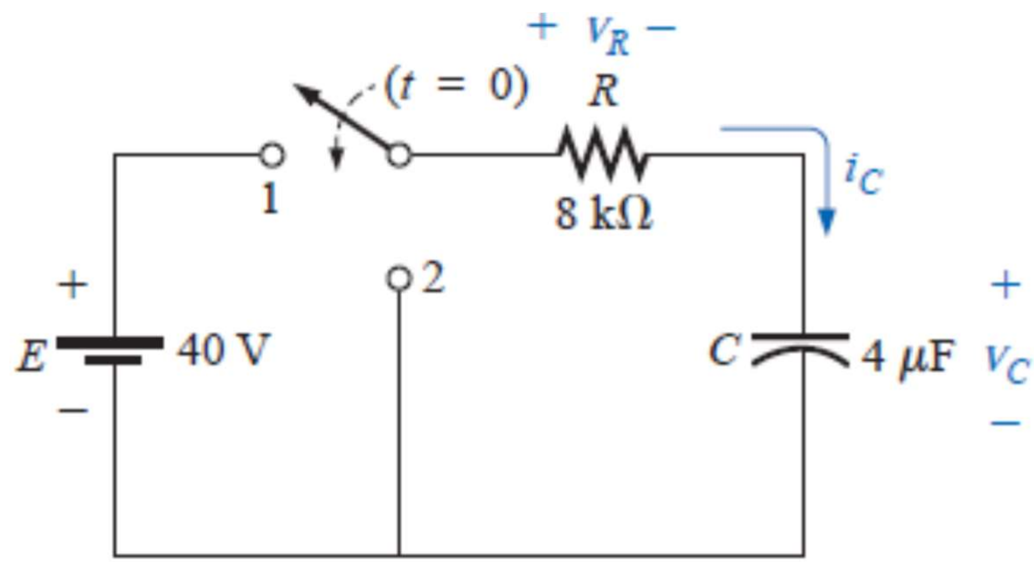
The current i_C of a capacitive network is essentially zero after five time constants of the charging phase have passed in a dc network.

Since C is usually found in microfarads or picofarads, the time constant $\tau = RC$ will never be greater than a few seconds unless R is very large.

Let us now turn our attention to the charging voltage across the capacitor. Through further mathematical analysis, the following equation for the voltage across the capacitor can be determined:

$$\boxed{v_C = E(1 - e^{-t/RC})} \qquad \textbf{(10.15)}$$

EXAMPLE



a. Find the mathematical expressions for the transient behavior of v_C, i_C, and v_R for the circuit of Fig. 10.35 when the switch is moved to position 1. Plot the curves of v_C, i_C, and v_R.

b. How much time must pass before it can be assumed, for all practical purposes, that $i_C \cong 0$ A and $v_C \cong E$ volts?

Solutions:

a. $\tau = RC = (8 \times 10^3\ \Omega)(4 \times 10^{-6}\ \text{F}) = 32 \times 10^{-3}\ \text{s} = \mathbf{32\ ms}$

By Eq. (10.15),

$$v_C = E(1 - e^{-t/\tau}) = \mathbf{40(1 - e^{-t/(32\times10^{-3})})}$$

EXAMPLE

$$i_C = \frac{E}{R}e^{-t/\tau} = \frac{40\text{ V}}{8\text{ k}\Omega}e^{-t/(32\times10^{-3})}$$

$$= \mathbf{(5 \times 10^{-3})}\boldsymbol{e^{-t/(32\times10^{-3})}}$$

By Eq. (10.17),

$$v_R = Ee^{-t/\tau} = \mathbf{40}\boldsymbol{e^{-t/(32\times10^{-3})}}$$

The curves appear in Fig. 10.36.

b. $5\tau = 5(32\text{ ms}) = \mathbf{160\text{ ms}}$

DISCHARGING

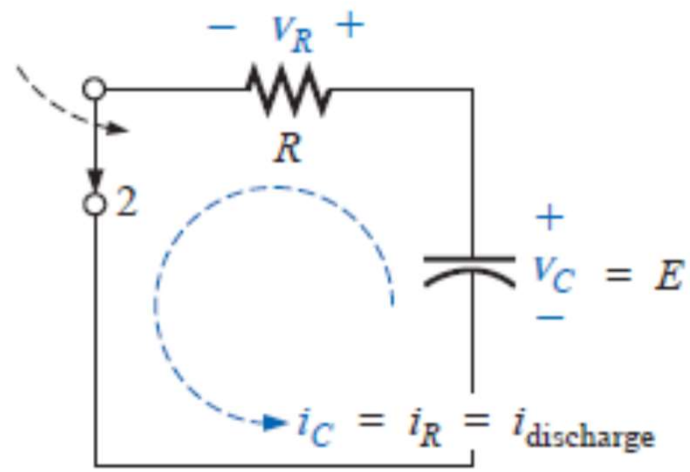


FIG. 10.38
Demonstrating the discharge behavior of a capacitive network.

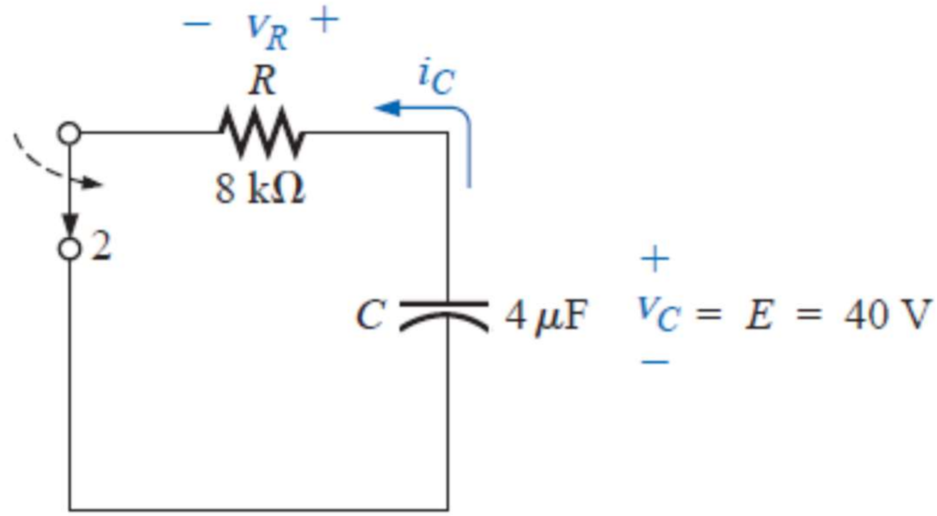
$$\boxed{v_C = Ee^{-t/RC}}$$ *discharging*

$$\boxed{i_C = \frac{E}{R}e^{-t/RC}}$$ *discharging*

The voltage $v_R = v_C$, and

$$\boxed{v_R = Ee^{-t/RC}}$$ *discharging*

EXAMPLE



EXAMPLE 10.6 After v_C in Example 10.5 has reached its final value of 40 V, the switch is thrown into position 2, as shown in Fig. 10.40. Find the mathematical expressions for the transient behavior of v_C, i_C,

and v_R after the closing of the switch. Plot the curves for v_C, i_C, and v_R using the defined directions and polarities of Fig. 10.35. Assume that $t = 0$ when the switch is moved to position 2.

SOLUTION

$$\tau = 32 \text{ ms}$$

,

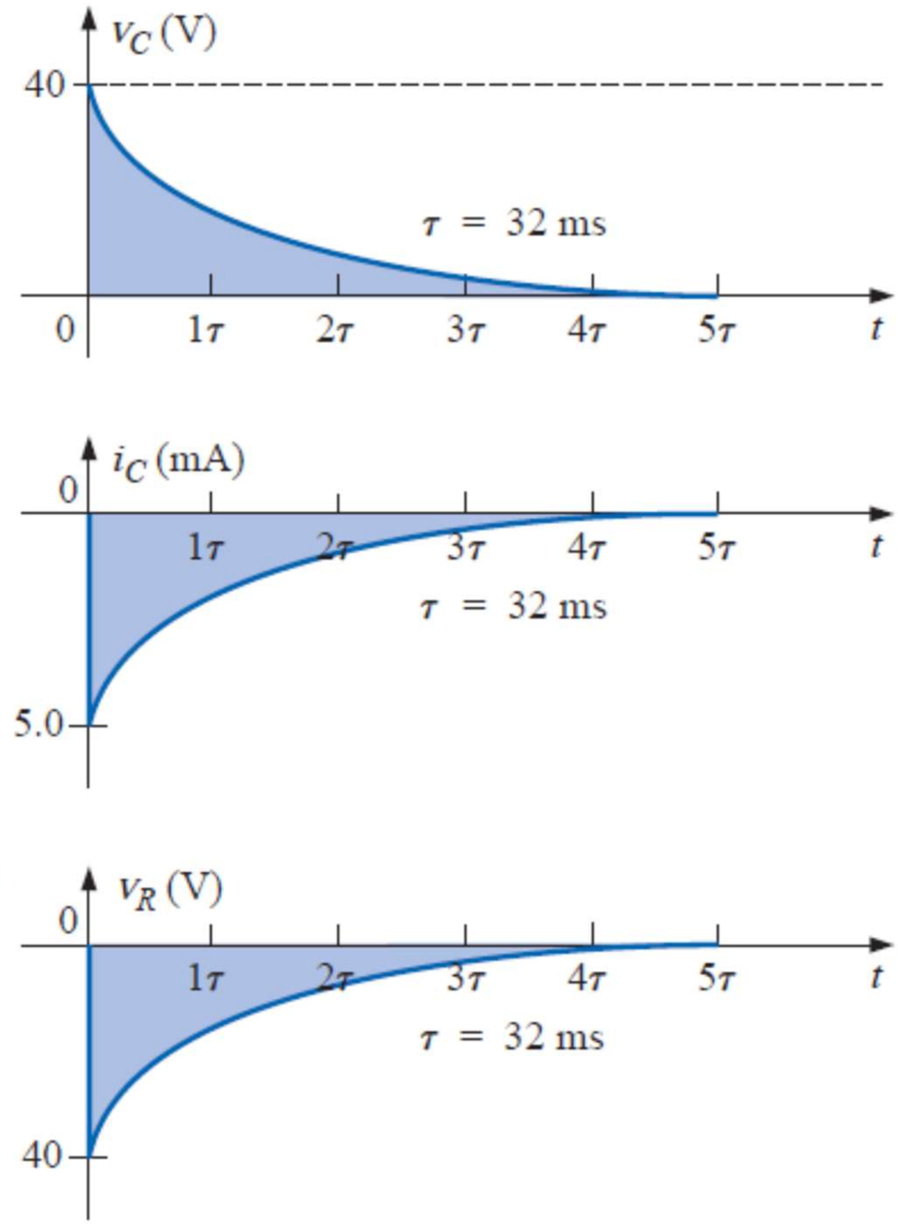
$$v_C = Ee^{-t/\tau} = \mathbf{40e^{-t/(32\times10^{-3})}}$$

,

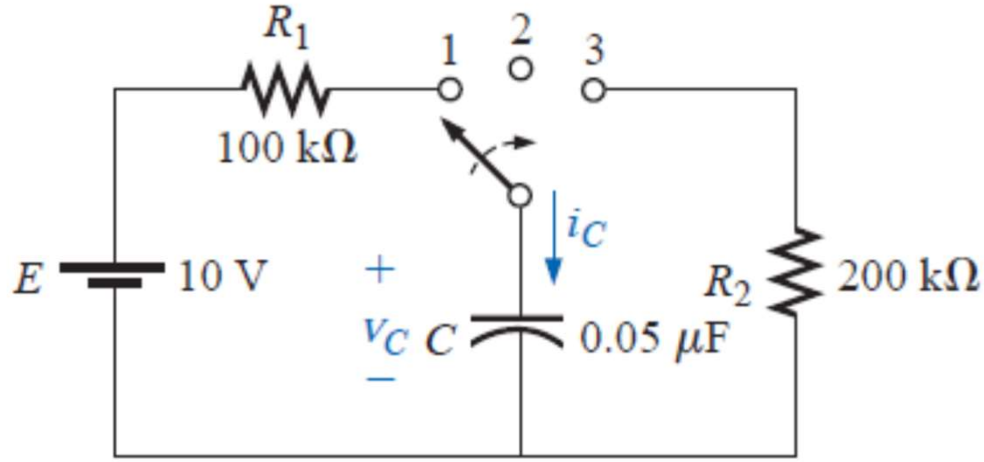
$$i_C = -\frac{E}{R}e^{-t/\tau} = \mathbf{-(5\times10^{-3})e^{-t/(32\times10^{-3})}}$$

,

$$v_R = -Ee^{-t/\tau} = \mathbf{-40e^{-t/(32\times10^{-3})}}$$



EXAMPLE



a. Find the mathematical expression for the transient behavior of the voltage across the capacitor of Fig. 10.42 if the switch is thrown into position 1 at $t = 0$ s.
b. Repeat part (a) for i_C.
c. Find the mathematical expressions for the response of v_C and i_C if the switch is thrown into position 2 at 30 ms (assuming that the leakage resistance of the capacitor is infinite ohms).
d. Find the mathematical expressions for the voltage v_C and current i_C if the switch is thrown into position 3 at $t = 48$ ms.
e. Plot the waveforms obtained in parts (a) through (d) on the same time axis for the voltage v_C and the current i_C using the defined polarity and current direction of Fig. 10.42.

SOLUTION

a. *Charging phase:*

$$v_C = E(1 - e^{-t/\tau})$$

$$\tau = R_1C = (100 \times 10^3\ \Omega)(0.05 \times 10^{-6}\ \text{F}) = 5 \times 10^{-3}\ \text{s}$$
$$= 5\ \text{ms}$$

$$v_C = \mathbf{10(1 - e^{-t/(5\times10^{-3})})}$$

b. $$i_C = \frac{E}{R_1}e^{-t/\tau}$$

$$= \frac{10\ \text{V}}{100 \times 10^3\ \Omega}e^{-t/(5\times10^{-3})}$$

$$i_C = \mathbf{(0.1 \times 10^{-3})e^{-t/(5\times10^{-3})}}$$

c. *Storage phase:*

$$v_C = E = \mathbf{10\ V}$$

$$i_C = \mathbf{0\ A}$$

SOLUTION

d. *Discharge phase* (starting at 48 ms with $t = 0$ s for the following equations):

$$v_C = Ee^{-t/\tau'}$$

$$\tau' = R_2C = (200 \times 10^3\ \Omega)(0.05 \times 10^{-6}\ \text{F}) = 10 \times 10^{-3}\ \text{s}$$
$$= 10\ \text{ms}$$

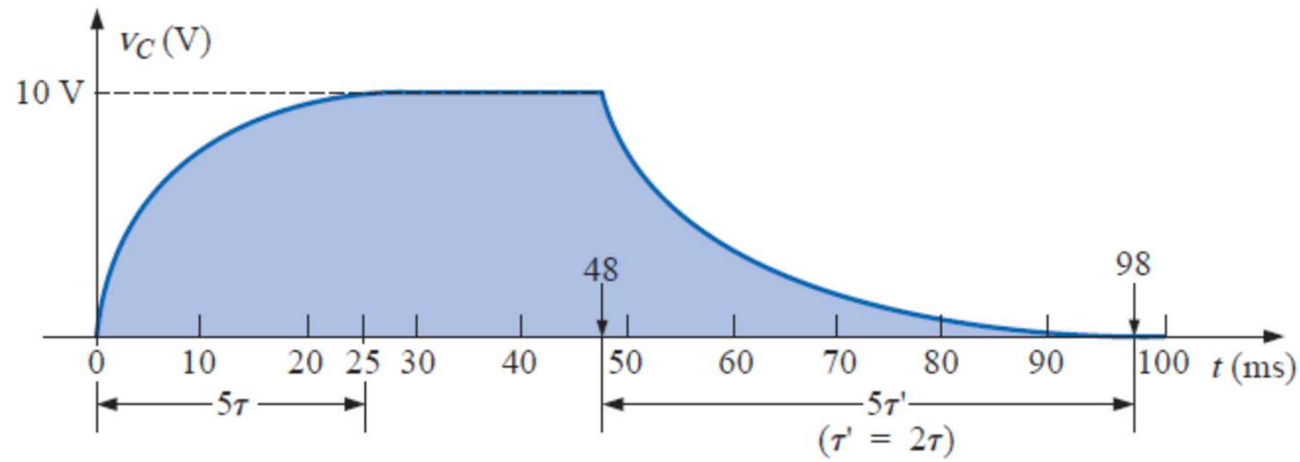
$$v_C = \mathbf{10}\boldsymbol{e}^{\boldsymbol{-t/(10\times10^{-3})}}$$

$$i_C = -\frac{E}{R_2}e^{-t/\tau'}$$

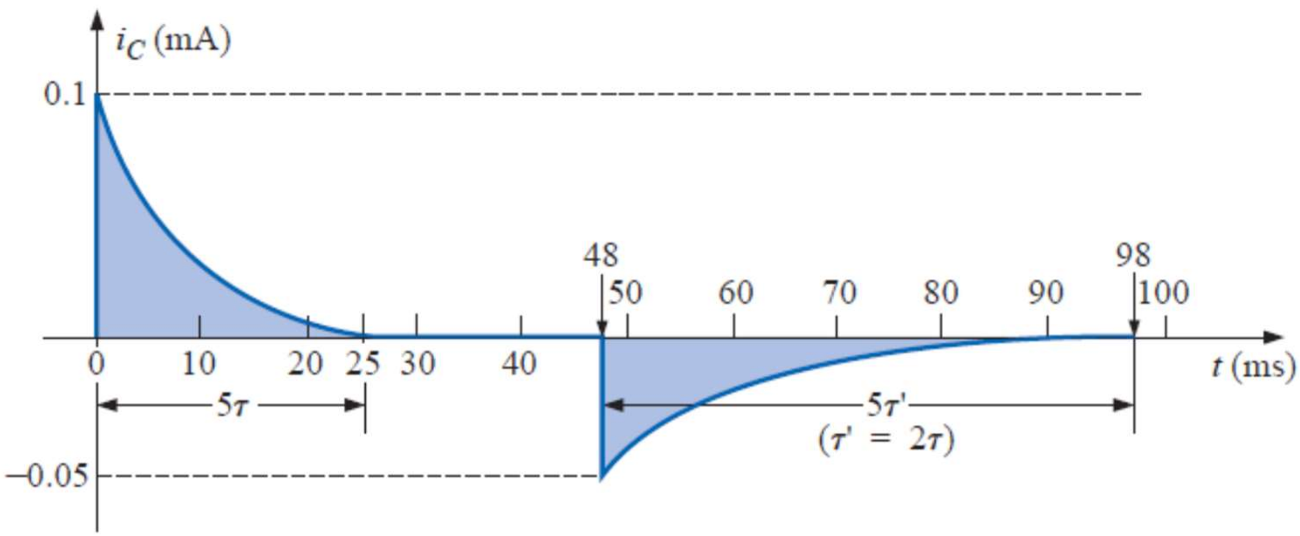
$$= -\frac{10\ \text{V}}{200 \times 10^3\ \Omega}e^{-t/(10\times10^{-3})}$$

$$i_C = \mathbf{-(0.05 \times 10^{-3})}\boldsymbol{e}^{\boldsymbol{-t/(10\times10^{-3})}}$$

SOLUTION



v_C (V)
10 V
48
98
0
10
20
25
30
40
50
60
70
80
90
100
t (ms)
5τ
5τ'
(τ' = 2τ)

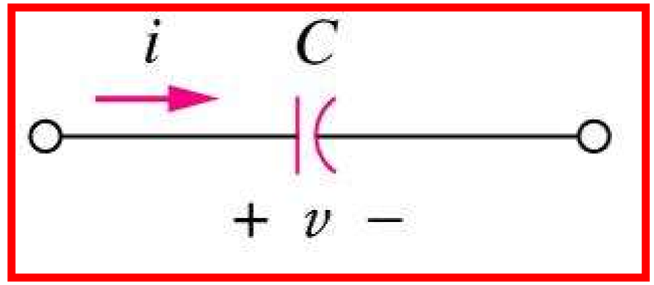


i_C (mA)
0.1
48
98
50
60
70
80
90
100
0
10
20
25
30
40
t (ms)
5τ
5τ'
(τ' = 2τ)
–0.05

CAPACITORS

If *i* is flowing into the +ve terminal of C

- Charging => *i* is +ve
- Discharging => *i* is –ve



- The current-voltage relationship of capacitor according to above convention is

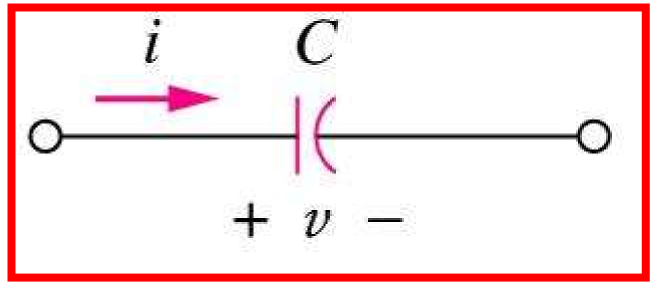
$$\boxed{i = C\frac{dv}{dt}}$$

and

$$\boxed{v = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0)}$$

CAPACITORS

The energy, **w,** stored in the capacitor is

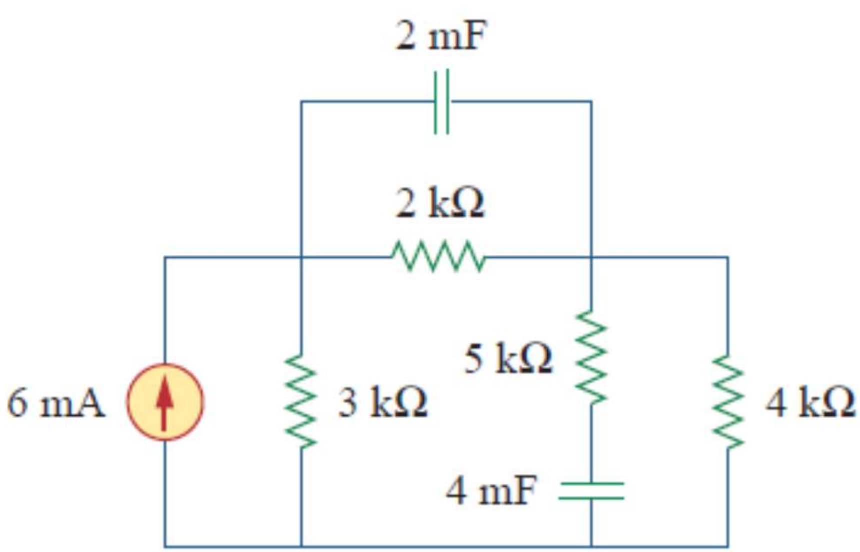


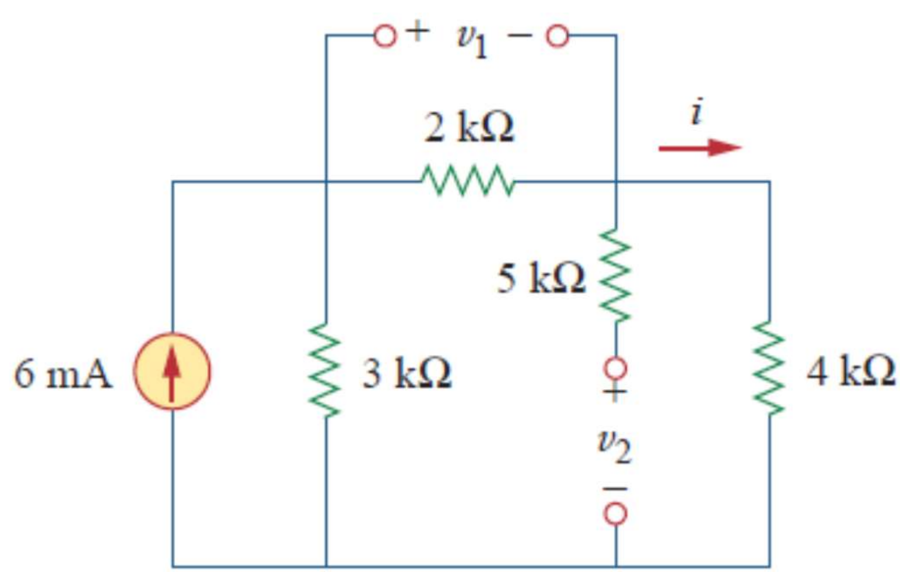
$$\boxed{w = \frac{1}{2} C v^2}$$

- A capacitor is
  - an **open circuit** to dc (dv/dt = 0).
  - its voltage **cannot change abruptly.**

EXAMPLE

Obtain the energy stored in each capacitor under dc conditions.





SOLUTION

$$i = \frac{3}{3 + 2 + 4}(6 \text{ mA}) = 2 \text{ mA}$$

Hence, the voltages v_1 and v_2 across the capacitors are

$$v_1 = 2000i = 4 \text{ V} \qquad v_2 = 4000i = 8 \text{ V}$$

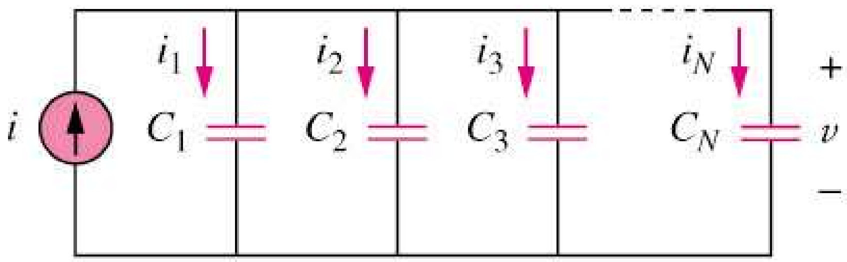
and the energies stored in them are

$$w_1 = \frac{1}{2}C_1 v_1^2 = \frac{1}{2}(2 \times 10^{-3})(4)^2 = 16 \text{ mJ}$$

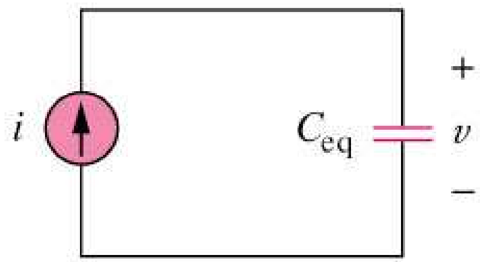
$$w_2 = \frac{1}{2}C_2 v_2^2 = \frac{1}{2}(4 \times 10^{-3})(8)^2 = 128 \text{ mJ}$$

SERIES AND PARALLEL CAPACITORS

The equivalent capacitance of N **parallel-connected** capacitors is the sum of the individual capacitances.



(a)

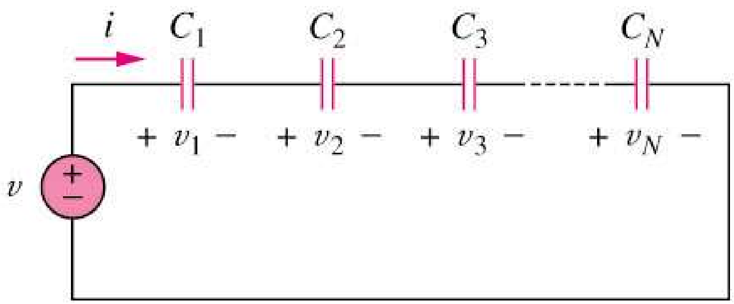


(b)

$$C_{eq} = C_1 + C_2 + \ldots + C_N$$

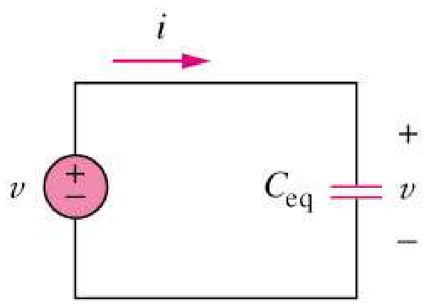
SERIES AND PARALLEL CAPACITORS

The equivalent capacitance of *N* **series-connected** capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.



(a)

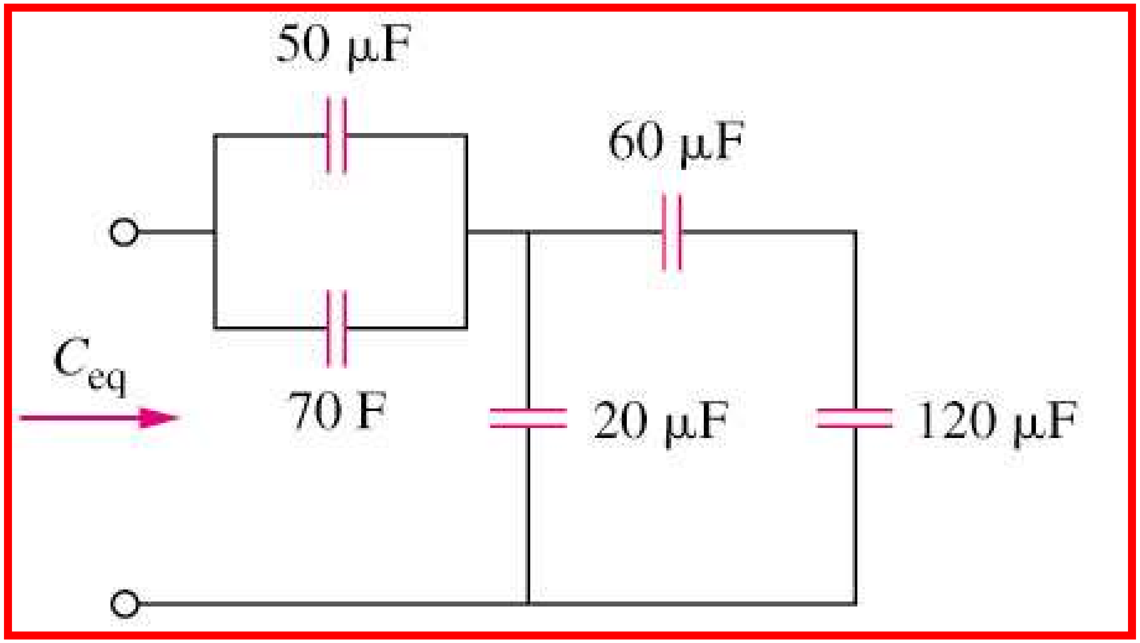
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_N}$$



(b)

SERIES AND PARALLEL CAPACITORS (EXAMPLE)

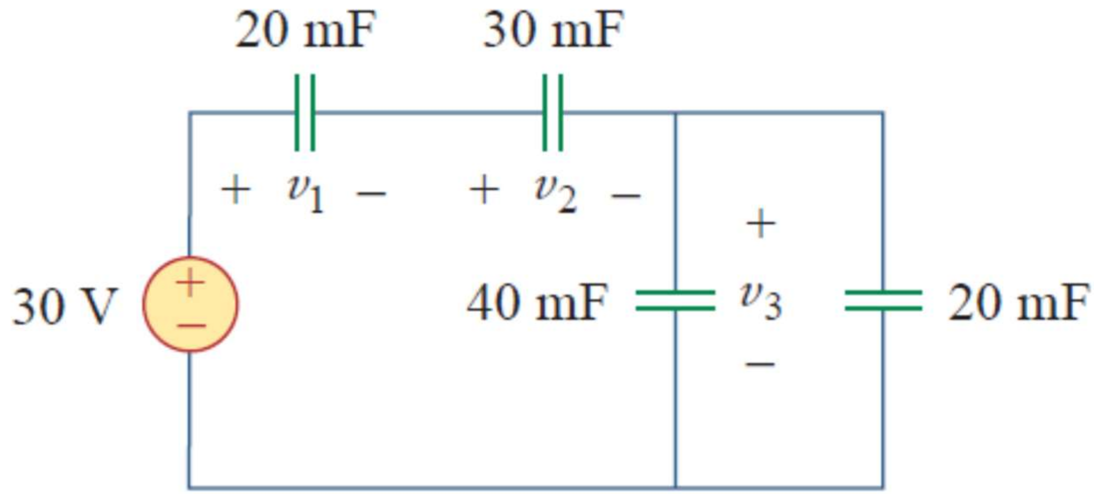
Find the equivalent capacitance seen at the terminals of the circuit in the circuit shown below:



Answer:

C_{eq} = 40μF

EXAMPLE



$$C_{eq} = \frac{1}{\frac{1}{60} + \frac{1}{30} + \frac{1}{20}} \text{ mF} = 10 \text{ mF}$$

The total charge is

$$q = C_{eq}v = 10 \times 10^{-3} \times 30 = 0.3 \text{ C}$$

EXAMPLE

This is the charge on the 20-mF and 30-mF capacitors, because they are in series with the 30-V source. (A crude way to see this is to imagine that charge acts like current, since $i = dq/dt$.) Therefore,

$$v_1 = \frac{q}{C_1} = \frac{0.3}{20 \times 10^{-3}} = 15 \text{ V} \qquad v_2 = \frac{q}{C_2} = \frac{0.3}{30 \times 10^{-3}} = 10 \text{ V}$$

Having determined v_1 and v_2, we now use KVL to determine v_3 by

$$v_3 = 30 - v_1 - v_2 = 5 \text{ V}$$

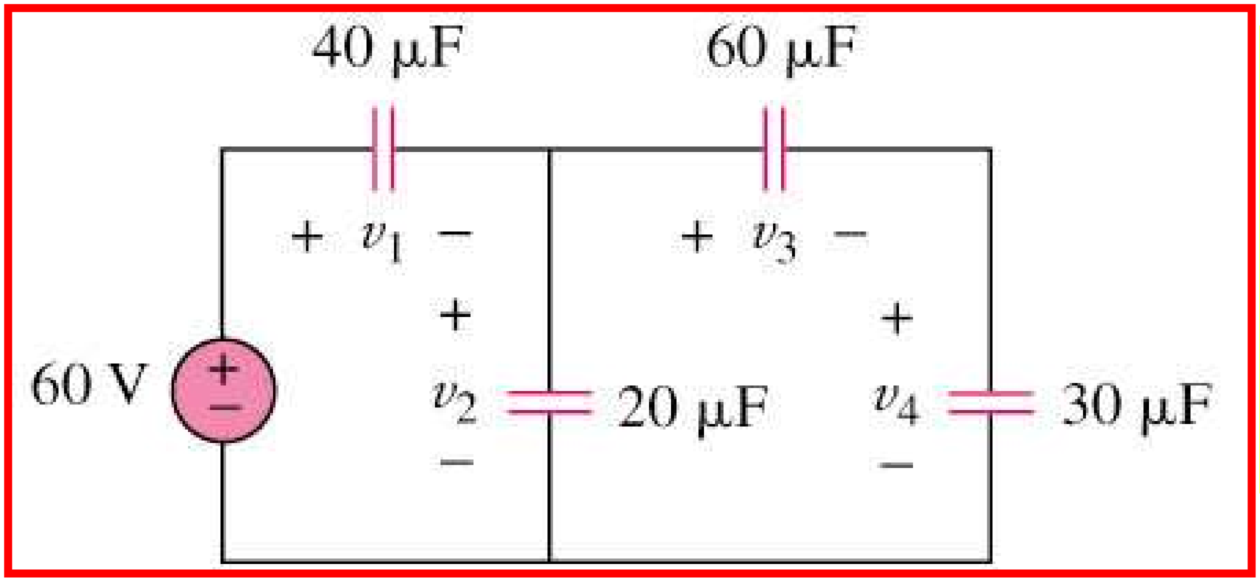
EXAMPLE

Alternatively, since the 40-mF and 20-mF capacitors are in parallel, they have the same voltage v_3 and their combined capacitance is $40 + 20 = 60$ mF. This combined capacitance is in series with the 20-mF and 30-mF capacitors and consequently has the same charge on it. Hence,

$$v_3 = \frac{q}{60 \text{ mF}} = \frac{0.3}{60 \times 10^{-3}} = 5 \text{ V}$$

SERIES AND PARALLEL CAPACITORS (EXAMPLE)

Find the voltage across each of the capacitors in the circuit shown below:



Answer:

v_1 = 30V

v_2 = 30V

v_3 = 10V

v_4 = 20V