

# **BJT AC Analysis**

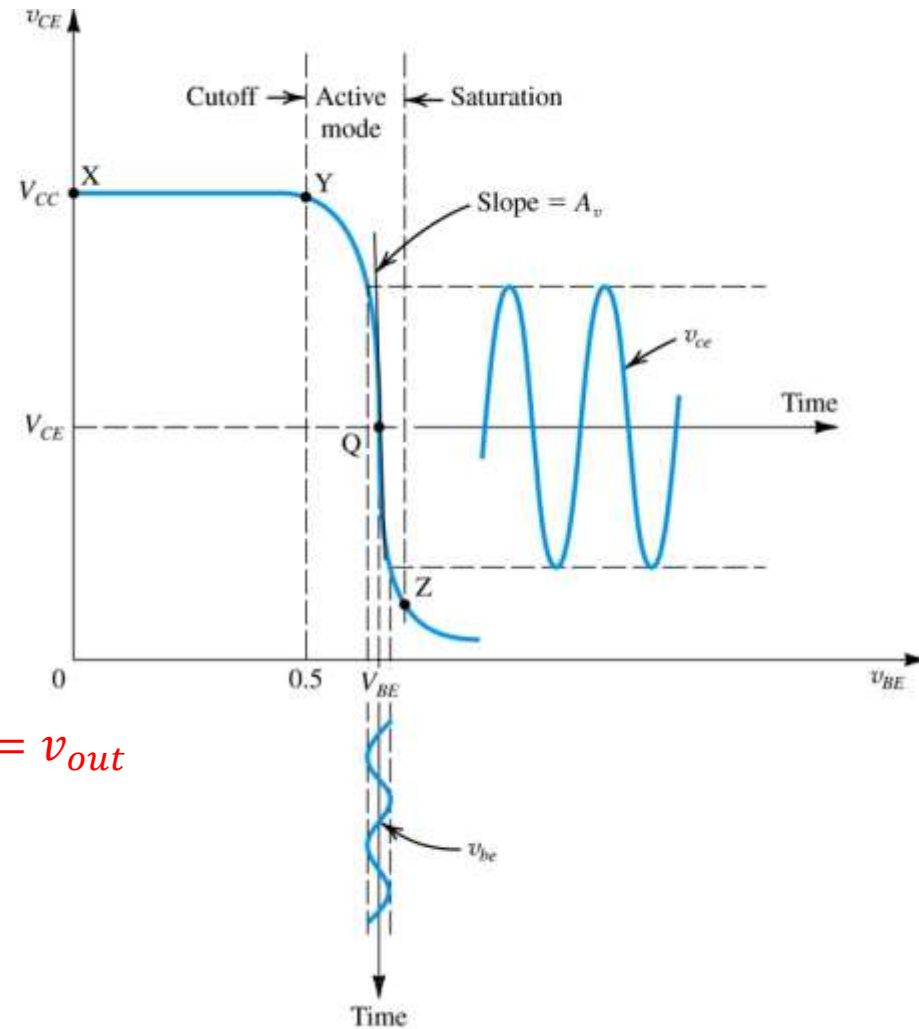
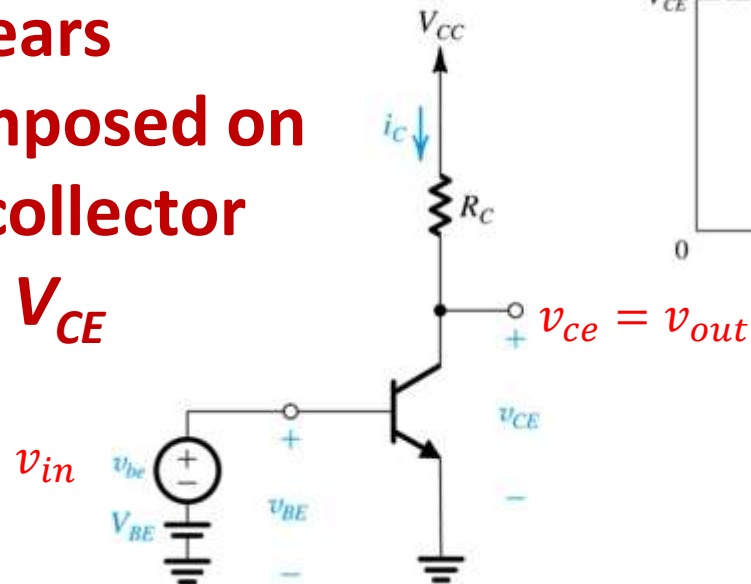
Topic 5 (Chapter 5)

## 5.2 AC Amplification

- After a transistor has been biased with the Q point near the middle of the load line, we can couple a small ac voltage into the base (input).
  - This will produce an ac collector voltage (output).
  - The ac collector voltage looks like the ac base voltage, except that it's a lot bigger.
    - In other words, the ac collector voltage is an *amplified* version of the ac base voltage.
- The invention of amplifying devices, first vacuum tubes and later transistors, was crucial to the evolution of electronics.
  - Without amplification, there would be no radio, no television, and no computers.

# BJT amplifier biased at a point Q

- A small signal voltage  $v_{be}$  is applied
- The output signal  $v_{ce}$  appears superimposed on the dc collector voltage  $V_{CE}$



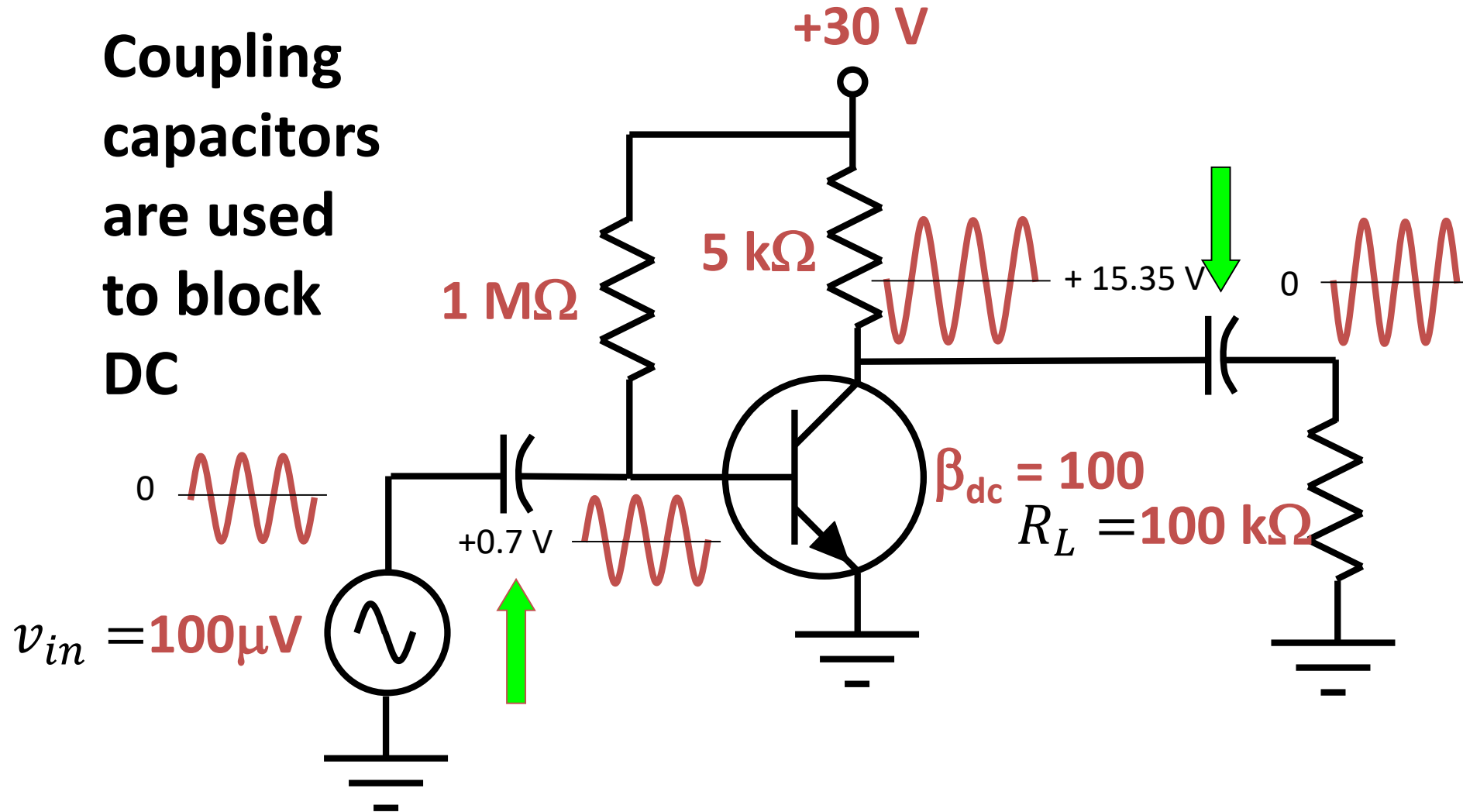
- The amplitude of  $v_{ce}$  is larger than that of  $v_{be}$  by the voltage gain  $A_v$

# Base-biased amplifier

- **AC** input is applied into base
- $v_{be} = v_{in}$
- Coupling capacitors are used to block DC
  - The **reactance** of a coupling capacitor is small for AC signal
- Amplified and inverted **output** at the collector.  $v_{ce} = v_{out}$
- **AC** output coupled to the load ( $R_L$ )
- Voltage Gain:  $A_v = -\frac{v_{out}}{v_{in}} = -\frac{v_{ce}}{v_{be}}$

# A base-biased amplifier with capacitive coupling

Coupling capacitors are used to block DC



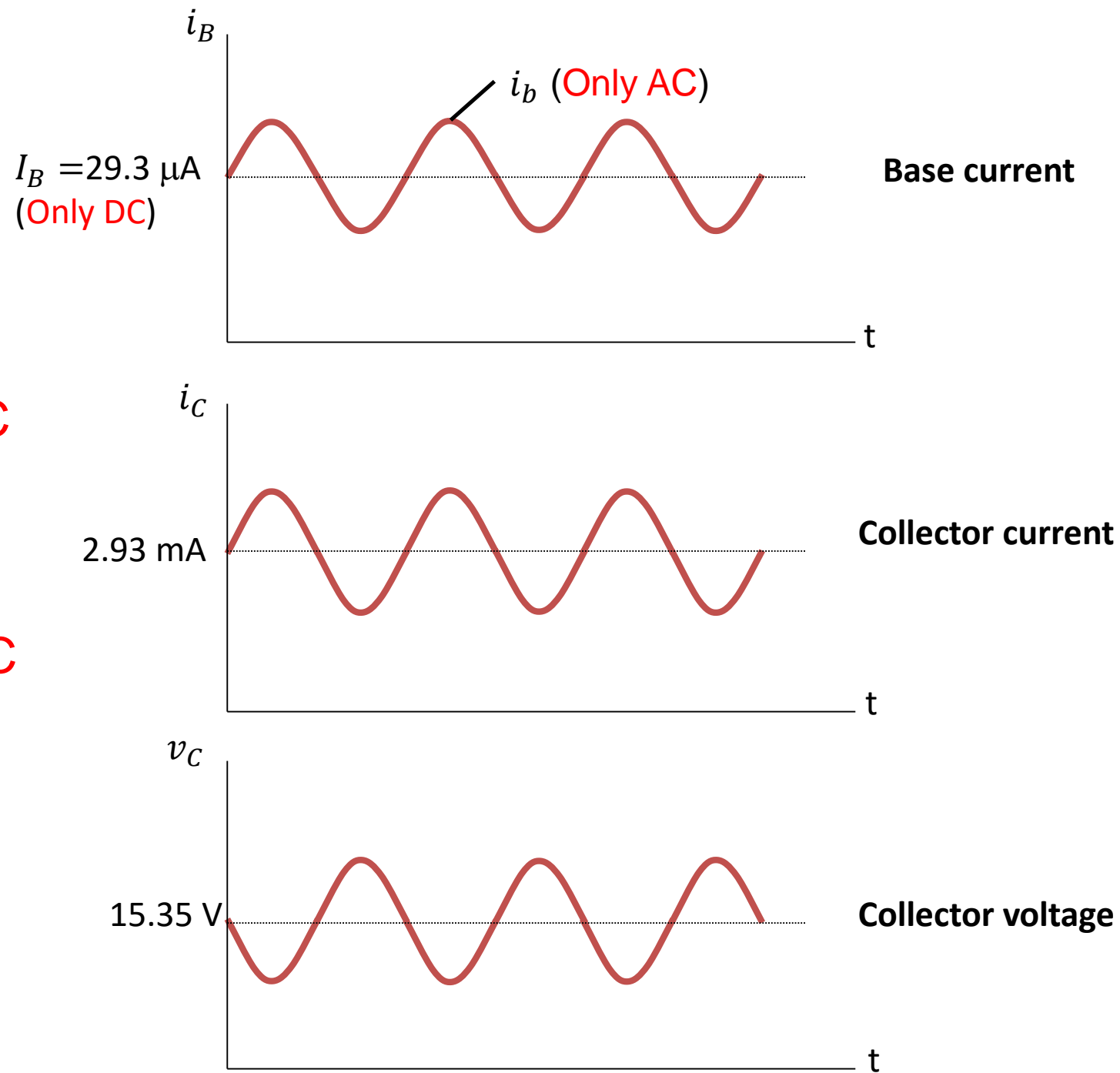
DC analysis gives:  $I_B = 29.3 \mu\text{A}$ ,  $I_C = 2.93 \text{ mA}$  and  $V_C = 15.35 \text{ V}$

## Waveforms

$I_B$ : Only DC

$i_b$ : Only AC

$i_B$ : DC + AC



# The coupling capacitor

Capacitor current,

$$i_c = C \frac{dv}{dt}.$$

DC Analysis:  $v$  is  
fixed.  $\frac{dv}{dt} = 0$

$i_c = 0$ . **Open**

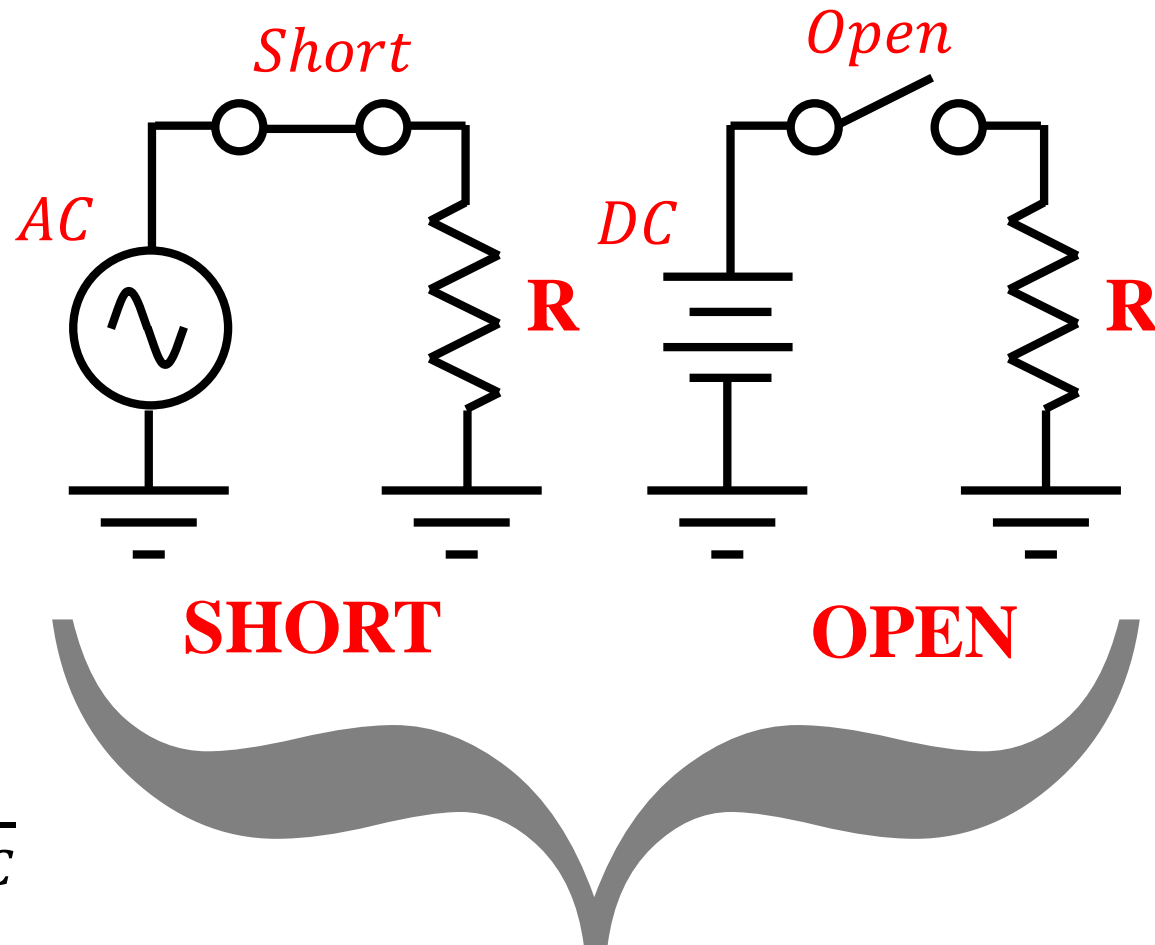
AC Analysis:

Reactance  $X = \frac{1}{2\pi fC}$

$C = \text{Large}$

$X = 0$ . **Short**

$$i_c = C \frac{dv}{dt} = \text{Large}$$



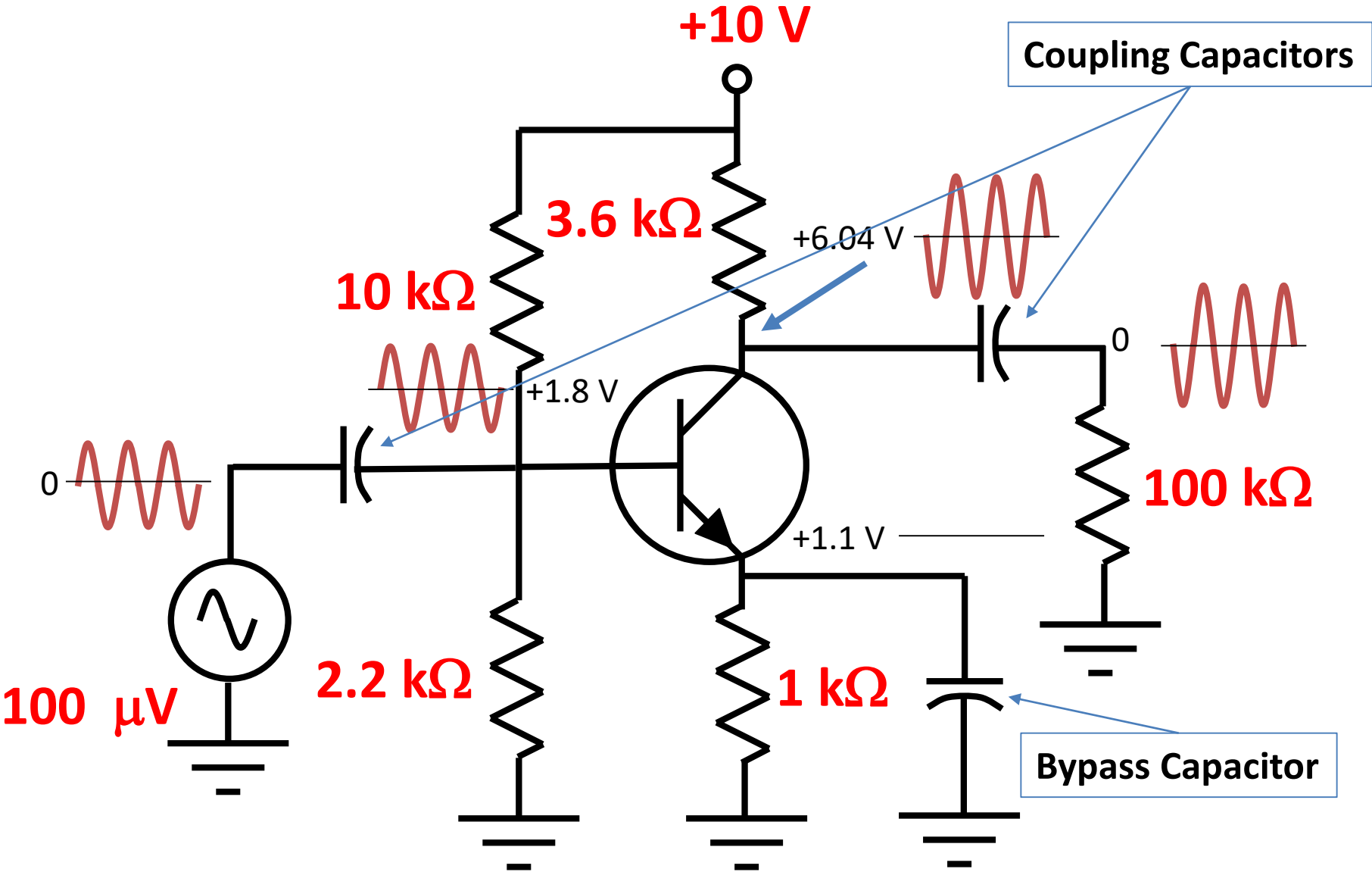
1. For **ac** analysis, the capacitor is a **short**.
2. For **dc** analysis, the capacitor is **open**.

# BJT Amplifier Analysis

- **Superposition Theorem (DC + AC analysis)**
- **DC** voltages and currents are calculated mentally by opening capacitors: DC Analysis (Chapter 4)
- The **AC** signal is coupled via a coupling capacitor
- Coupling capacitor: couples between BJT & input AND BJT & output
- The **bypass** capacitor causes an AC signal to appear across the base-emitter junction and provides higher gain
- **Bypass** capacitor: Need in DC. Not need in AC.



# VDB Amplifier



The dc current gain is given as:

$$\beta_{DC} = \frac{I_C}{I_B}$$

The ac current gain is given as:

$$\beta_{ac} = \frac{i_c}{i_b}$$

Use **CAPITAL** letters for dc quantities  
and **lowercase** letters for ac.

# Formula for ac emitter resistance

Derived by using solid-state physics and calculus:

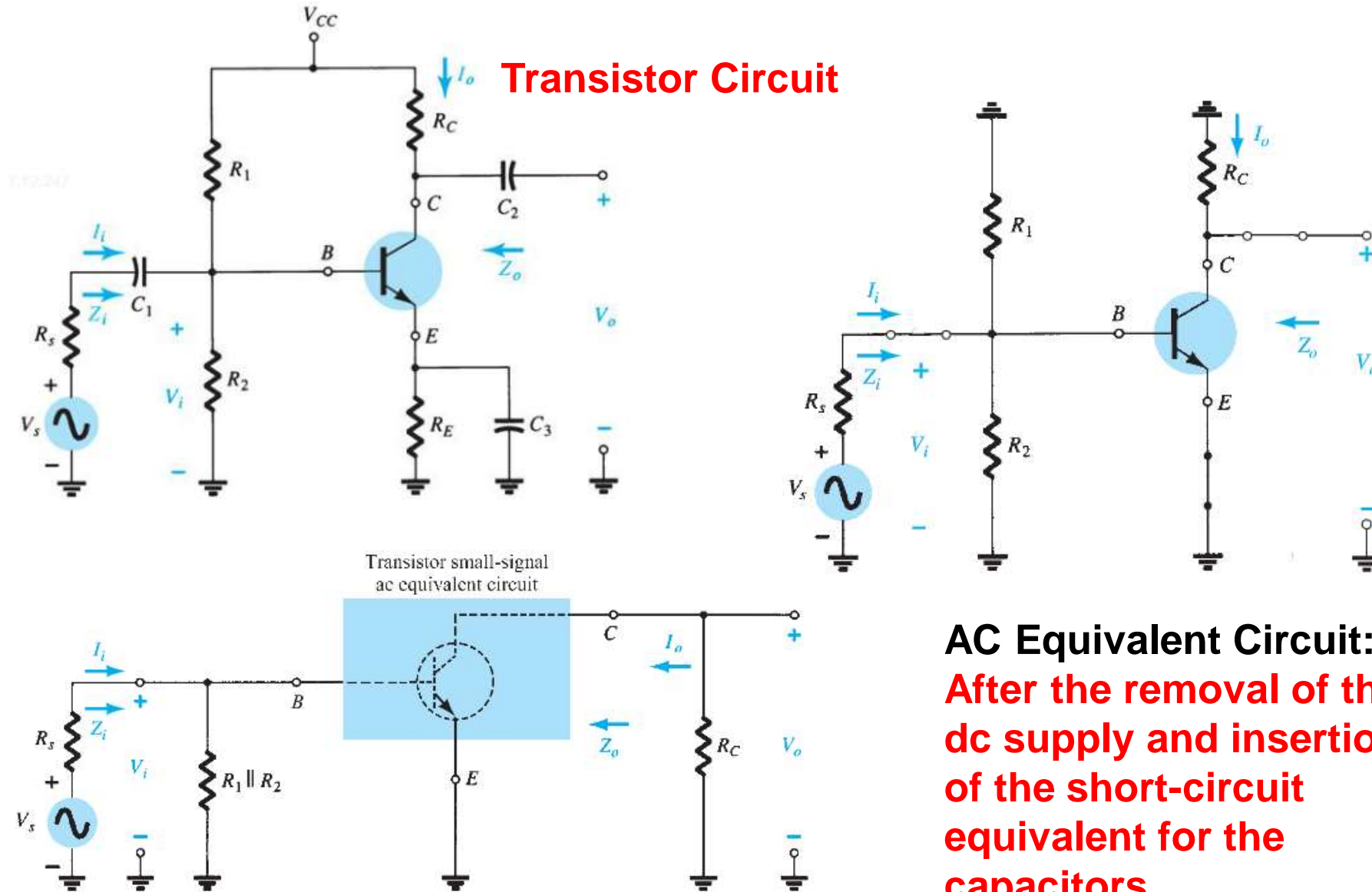
Diode AC Resistance,  $r_{ac} = \frac{nV_T}{I_D}$  Chapter 1

$$r_e' = \frac{nV_T = 26 \text{ mV}}{I_E}$$

$r_e$  is AC emitter resistance

$I_E$  is DC emitter current

## 5.3 BJT TRANSISTOR MODELING



Redrawn for small-signal ac analysis

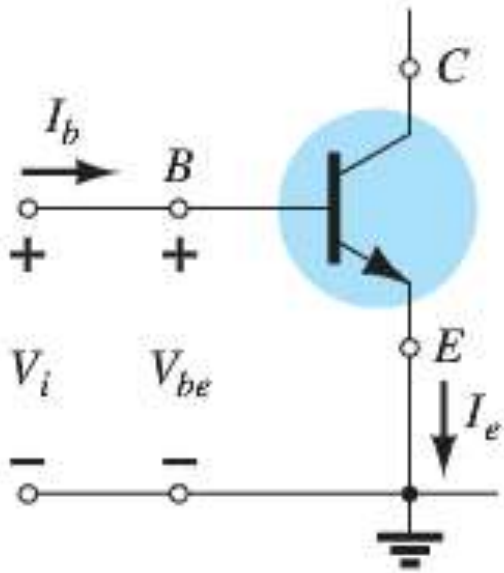
**AC Equivalent Circuit:**  
After the removal of the dc supply and insertion of the short-circuit equivalent for the capacitors.

# Transistor AC models

- AC equivalent circuit for a transistor
- Simulates how a transistor behaves when an ac signal is present
- There are two models commonly used in small signal AC analysis of a transistor:
  - $r_e$  model
    - T model and  $\pi$  type models are widely used
  - Hybrid equivalent model ( $h$  parameter model)

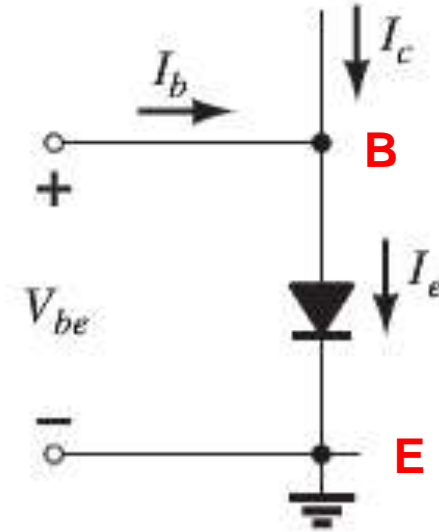
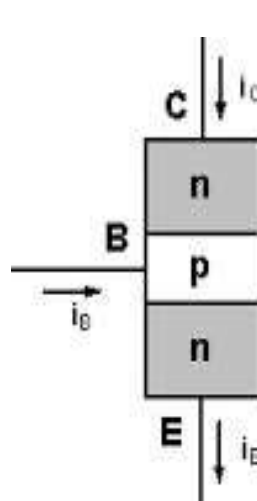
## 5.4 The $r_e$ Transistor Model

(The Input Equivalent Circuit)



**FIG. 5.8**

*Finding the input equivalent circuit for a BJT transistor.*



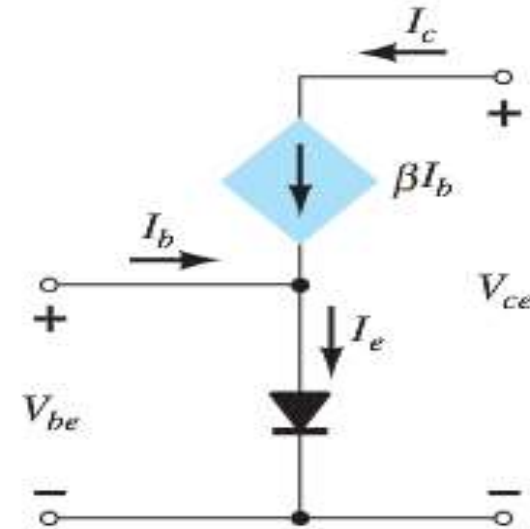
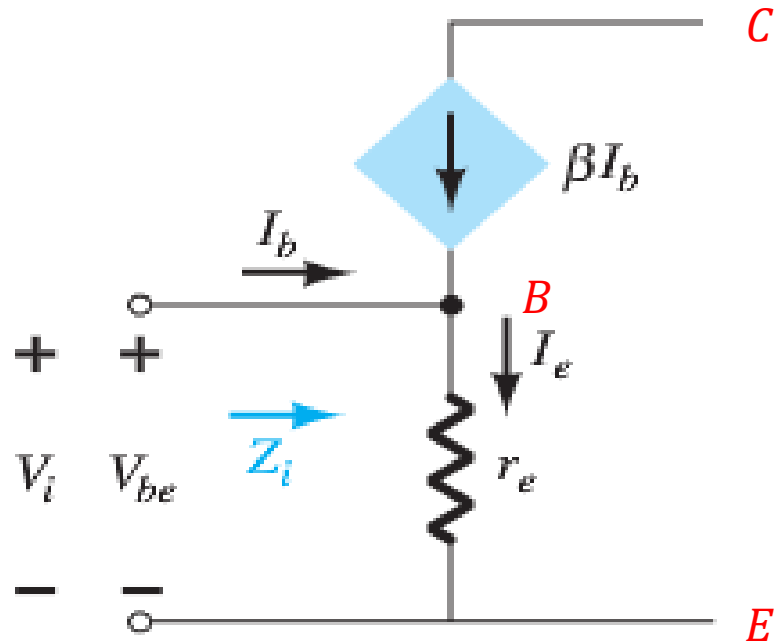
**FIG. 5.10**

*Equivalent circuit for the input side of a BJT transistor.*

BJT similar to two diodes, emitter-base (FB) and collector-base (RB)  
 $r_e$  Transistor Model: Diode between Emitter and Base

# The $r_e$ Transistor Model

(The BJT Equivalent Circuit – T Model)



**FIG. 5.12**

*BJT equivalent circuit.*

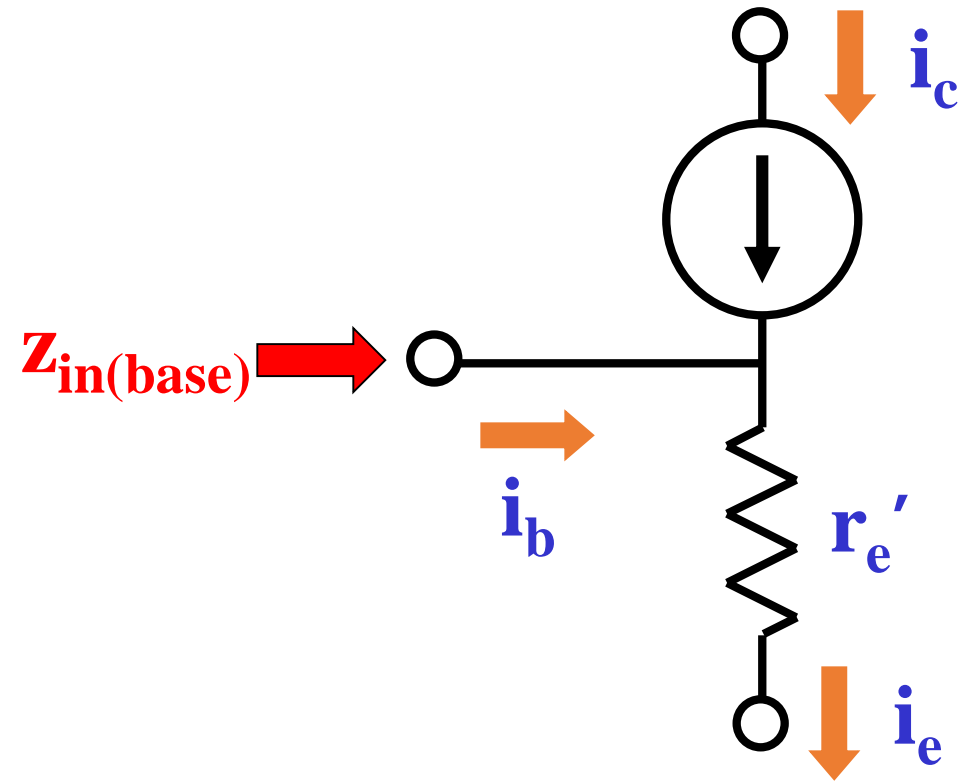
- BJT similar to two diodes, emitter-base (FB) and collector-base (RB)
- $r_e$  Transistor Model: AC resistance ( $r_e$ ) between Emitter and Base (input/ base side)
- Output/ collector side:  $i_c = \beta i_b$ ; output current =  $\beta \times$  input current
- Output side: Current dependent current source ( diamond shape)

# Deriving $\pi$ model from T model

$$v_{be} = i_e \times r_e$$

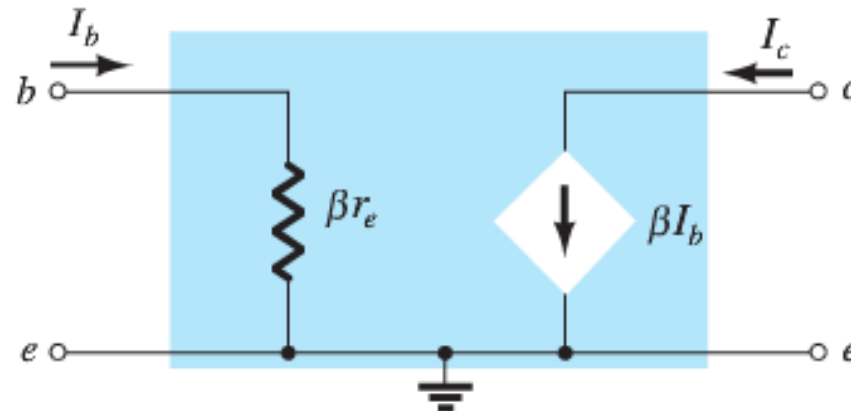
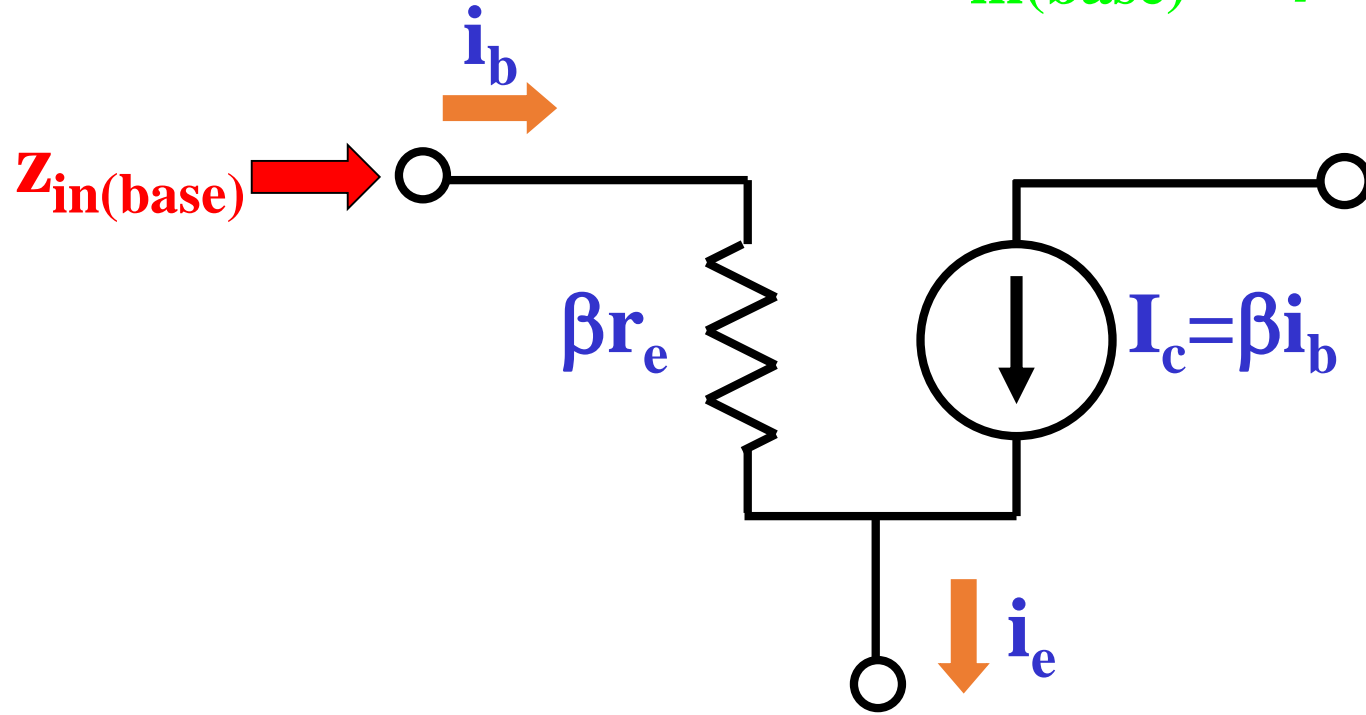
$$= (\beta + 1) i_b r_e$$

$$\cong i_b \times \beta r_e$$



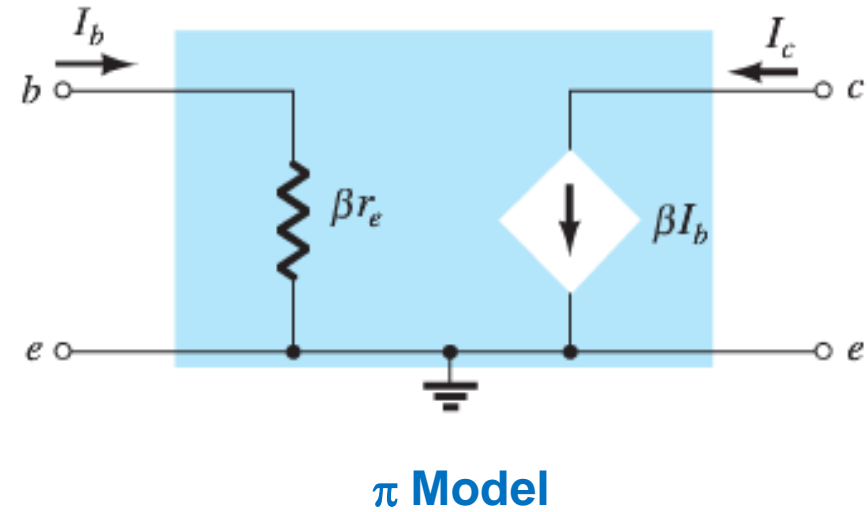
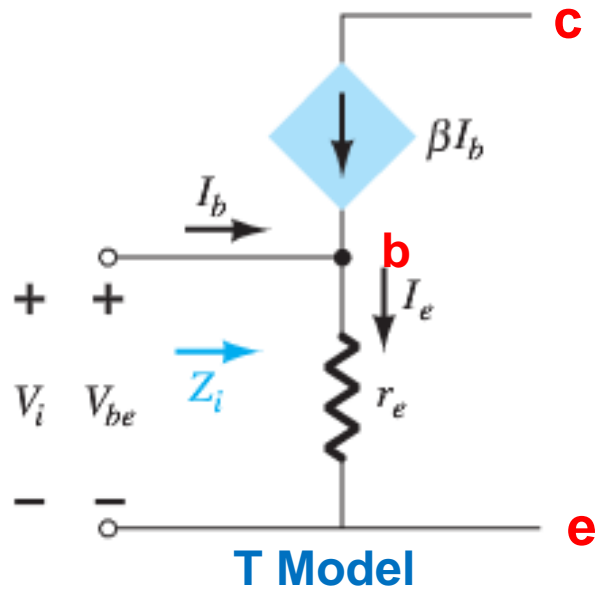


The  $\pi$  model of a transistor  
is based on  $z_{in(base)} = \beta r_e$ :



# Overview of $r_e$ Transistor Models

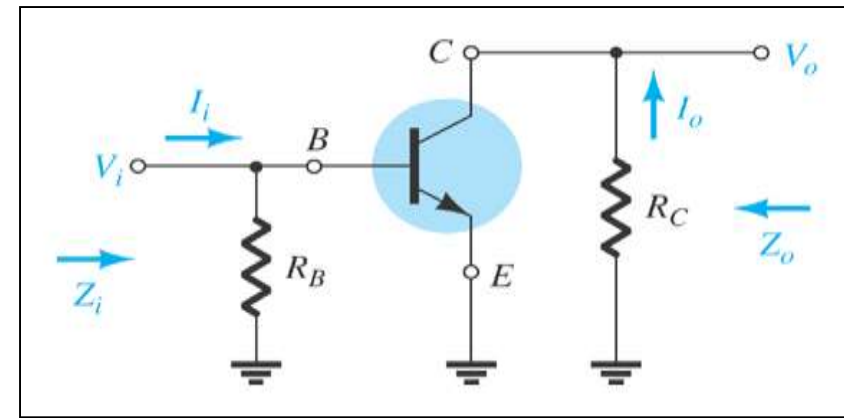
(T and  $\pi$  Model)



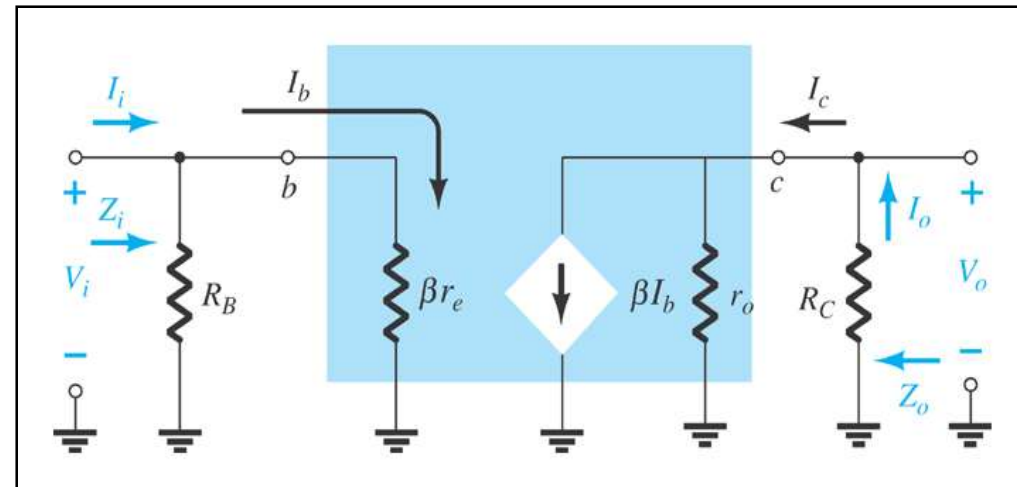
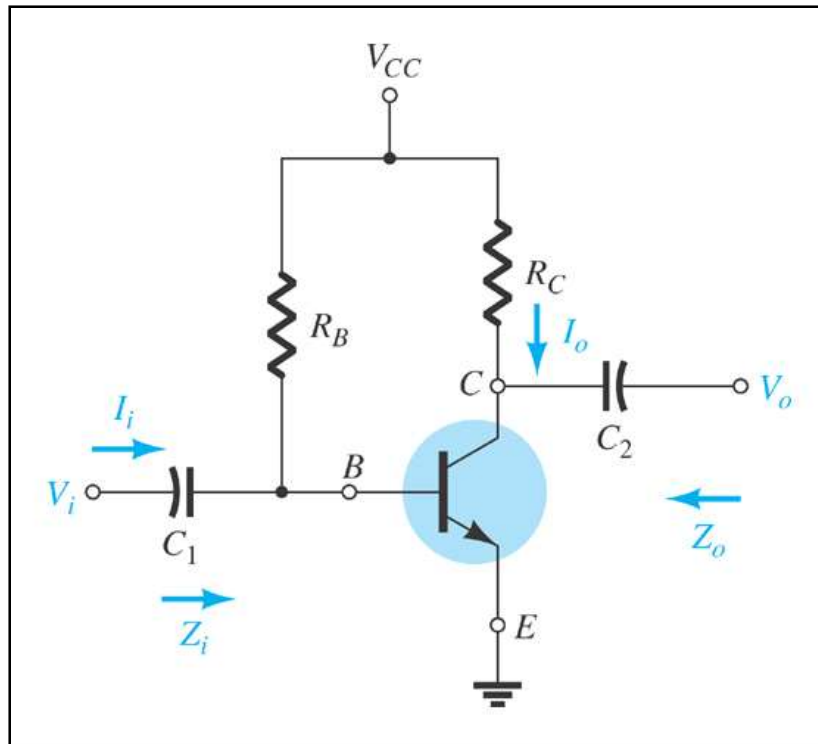
# BJT Amplifier AC analysis

- Superposition theorem (DC + AC analysis)
- Perform a complete DC analysis (Chapter 4)
- Short all coupling and bypass capacitors for ac signals
- Visualize all DC supply voltages as grounds
- Replace the transistor by its  $\pi$  or  $\text{T}$  model
- Draw the AC equivalent circuit

# 5.5 Common-Emitter Fixed-Bias Configuration

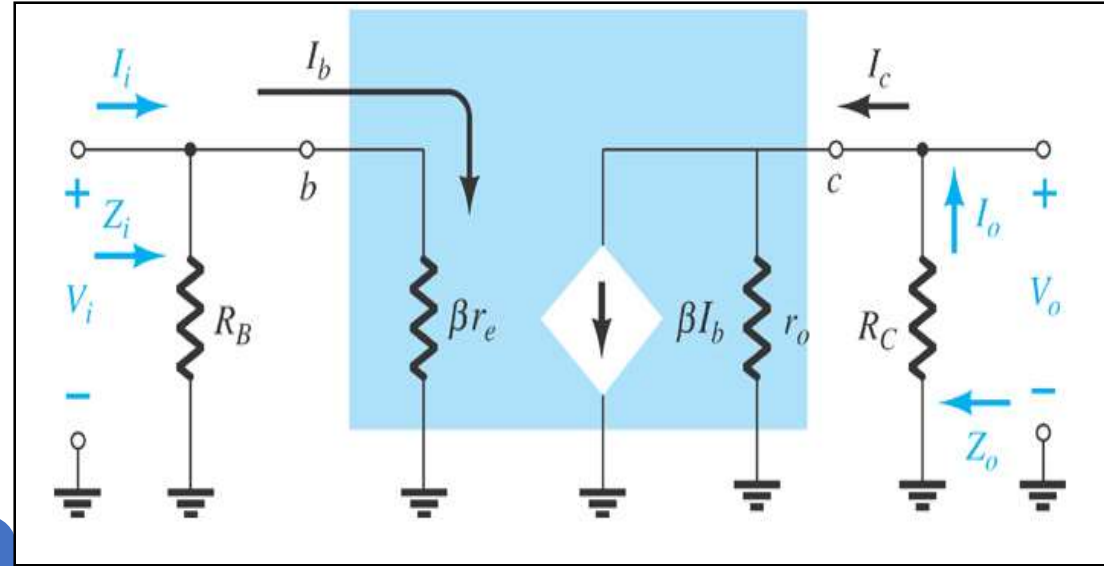


AC equivalent



$r_e$  model

# Common-Emitter Fixed-Bias Calculations



Input  
impedance:

$$Z_i = R_B \parallel \beta r_e$$

$$Z_i \cong \beta r_e \Big|_{R_E \geq 10 \beta r_e}$$

Output  
impedance:

$$Z_o = R_C \parallel r_o$$

$$Z_o \cong R_C \Big|_{r_o \geq 10 R_C}$$

Voltage gain:

$$A_v = \frac{V_o}{V_i} = - \frac{(R_C \parallel r_o)}{r_e}$$

$$A_v = - \frac{R_C}{r_e} \Big|_{r_o \geq 10 R_C}$$

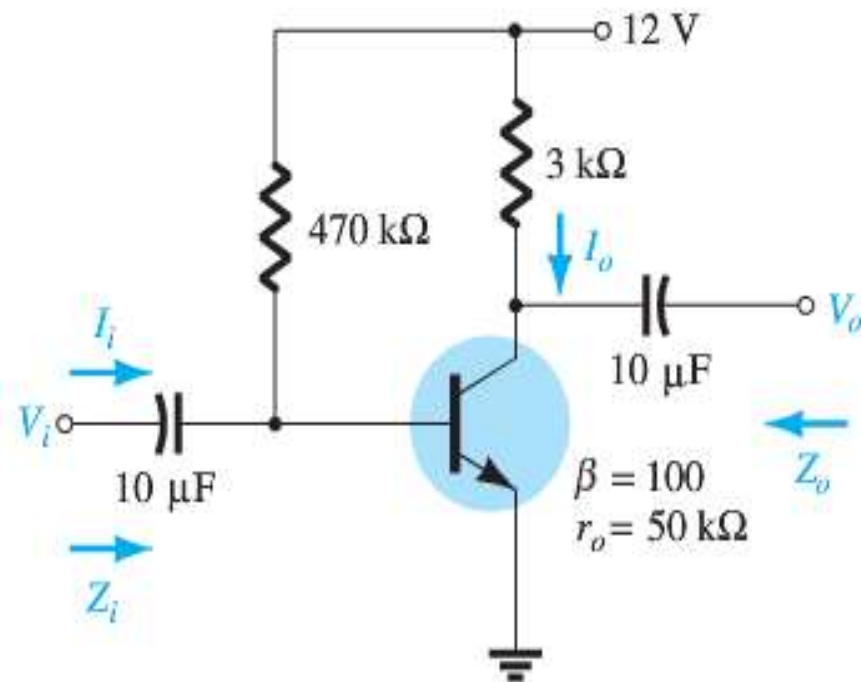
Current gain:

$$A_i = \frac{I_o}{I_i} = \frac{\beta R_B r_o}{(r_o + R_C)(R_B + \beta r_e)}$$

$$A_i \cong \beta \Big|_{r_o \geq 10 R_C, R_B \geq 10 \beta r_e}$$

**EXAMPLE 5.1** For the network of Fig. 5.25:

- Determine  $r_e$ .
- Find  $Z_i$  (with  $r_o = \infty \Omega$ ).
- Calculate  $Z_o$  (with  $r_o = \infty \Omega$ ).
- Determine  $A_v$  (with  $r_o = \infty \Omega$ ).



1. Short all coupling and bypass capacitors for ac signals

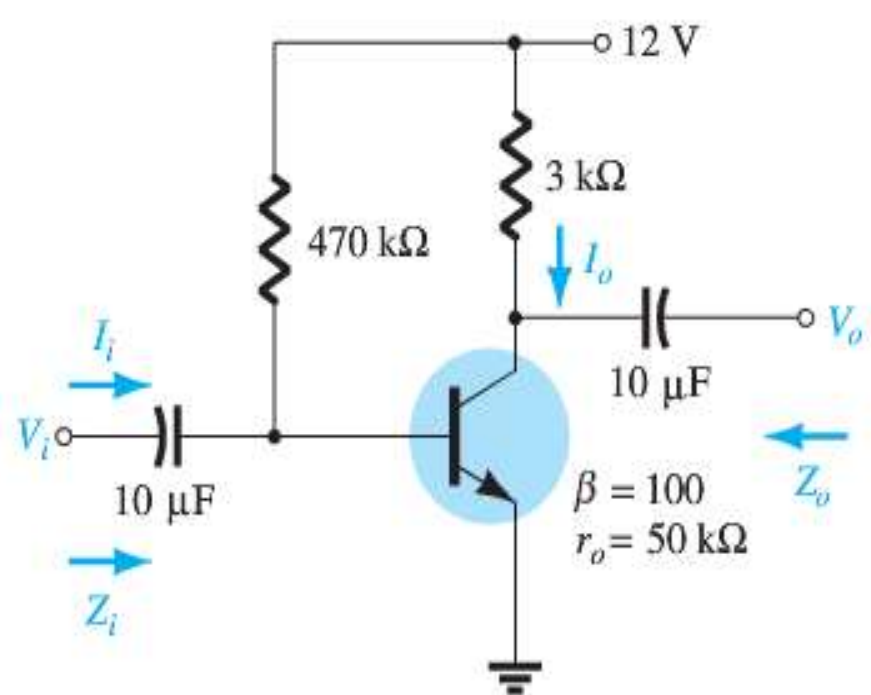
2. DC supply voltages as grounds

3. Replace the transistor by its  $\pi$  or  $\underline{\text{T}}$  model

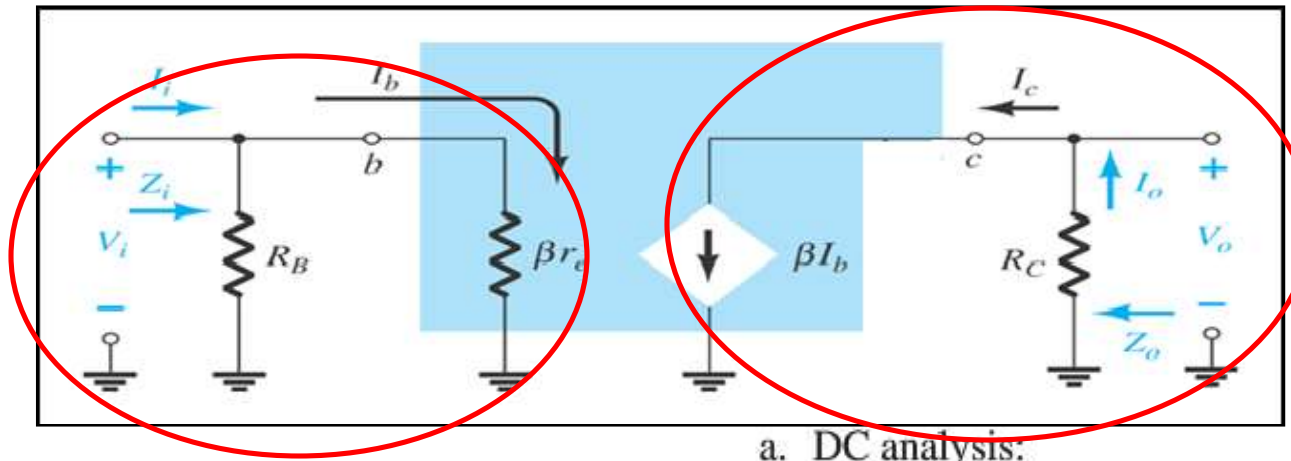
$\pi$  Model: CE

$\underline{\text{T}}$  model: CB

4. Draw the AC equivalent circuit



- Determine  $r_e$ .
- Find  $Z_i$  (with  $r_o = \infty \Omega$ ).
- Calculate  $Z_o$  (with  $r_o = \infty \Omega$ ).
- Determine  $A_v$  (with  $r_o = \infty \Omega$ ).



**INPUT**

**OUTPUT**

a. DC analysis:

$$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{12 \text{ V} - 0.7 \text{ V}}{470 \text{ k}\Omega} = 24.04 \mu\text{A}$$

$$I_E = (\beta + 1)I_B = (101)(24.04 \mu\text{A}) = 2.428 \text{ mA}$$

$$r_e = \frac{26 \text{ mV}}{I_E} = \frac{26 \text{ mV}}{2.428 \text{ mA}} = \mathbf{10.71 \Omega}$$

$$\text{b. } \beta r_e = (100)(10.71 \Omega) = 1.071 \text{ k}\Omega$$

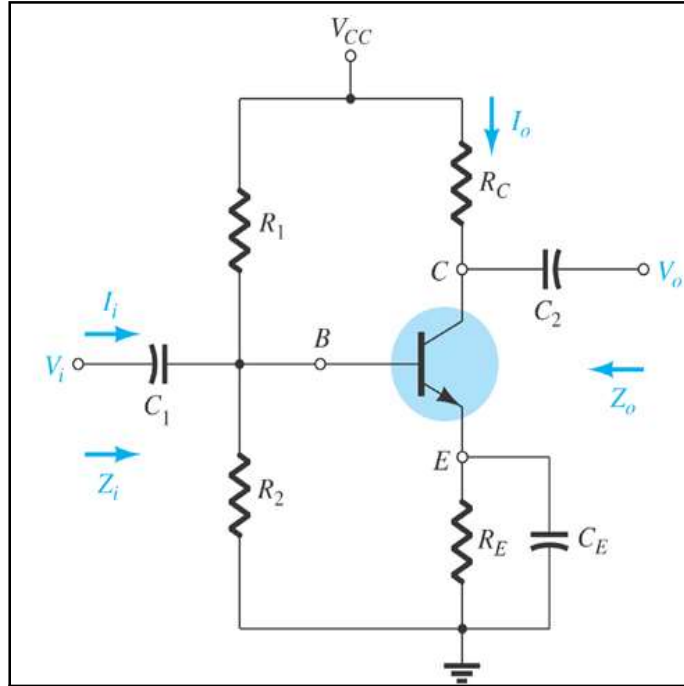
$$Z_i = R_B \parallel \beta r_e = 470 \text{ k}\Omega \parallel 1.071 \text{ k}\Omega = \mathbf{1.07 \text{ k}\Omega}$$

$$\text{c. } Z_o = R_C = \mathbf{3 \text{ k}\Omega}$$

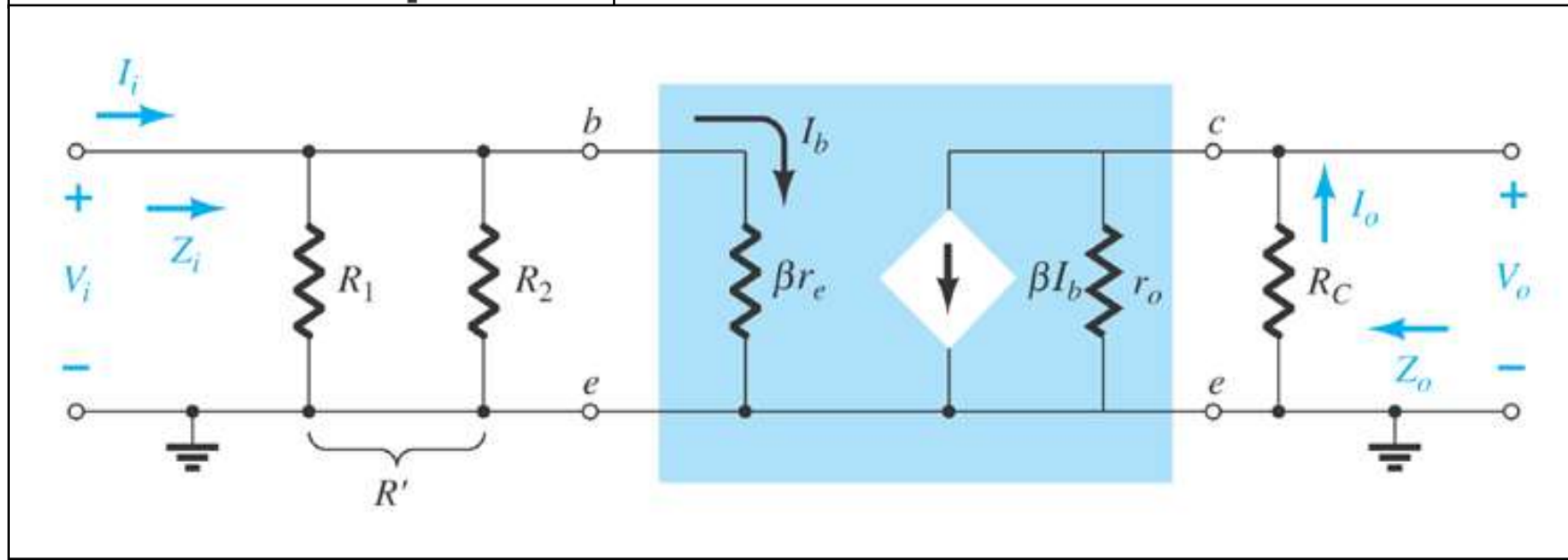
$$\text{d. } A_v = -\frac{R_C}{r_e} = -\frac{3 \text{ k}\Omega}{10.71 \Omega} = \mathbf{-280.11}$$



## 5.6 Common-Emitter Voltage-Divider Bias



$r_e$  model requires you to determine  $\beta$ ,  $r_e$ , and  $r_o$ .



# 5.6 Common-Emitter Voltage-Divider Bias

Input impedance

$$R' = R_1 \parallel R_2$$

$$Z_i = R' \parallel \beta r_e$$

Output impedance

$$Z_o = R_C \parallel r_o$$

$$Z_o \cong R_C \big|_{r_o \geq 10R_C}$$

Voltage gain

$$A_v = \frac{V_o}{V_i} = \frac{-R_C \parallel r_o}{r_e}$$

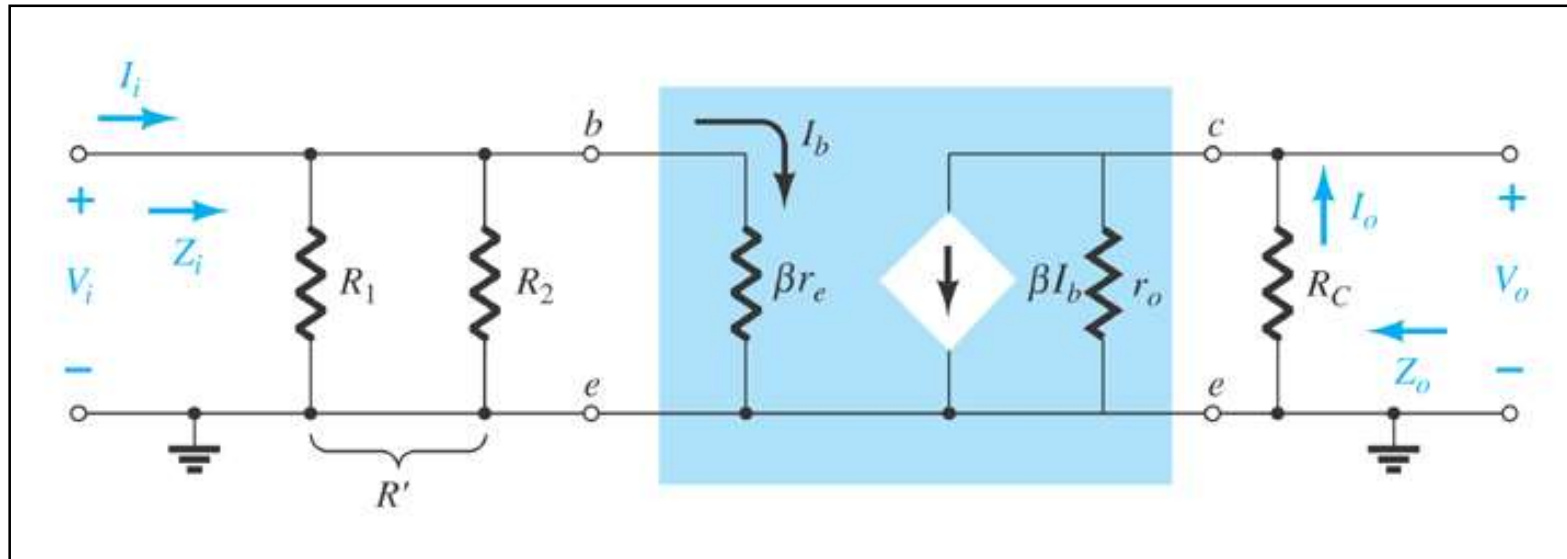
$$A_v = \frac{V_o}{V_i} \cong -\frac{R_C}{r_e} \big|_{r_o \geq 10R_C}$$

Current gain

$$A_i = \frac{I_o}{I_i} = \frac{\beta R' r_o}{(r_o + R_C)(R' + \beta r_e)}$$

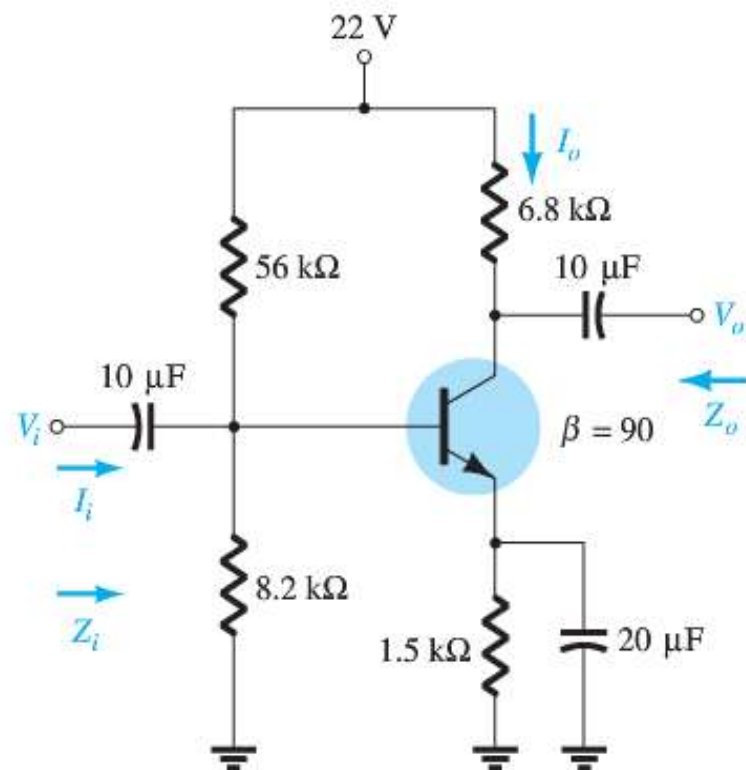
$$A_i = \frac{I_o}{I_i} \cong \frac{\beta R'}{R' + \beta r_e} \big|_{r_o \geq 10R_C}$$

$$A_i = \frac{I_o}{I_i} \cong \beta \big|_{r_o \geq 10R_C, R' \geq 10\beta r_e}$$



**EXAMPLE 5.2** For the network of Fig. 5.28, determine:

- $r_e$ .
- $Z_i$ .
- $Z_o$  ( $r_o = \infty \Omega$ ).
- $A_v$  ( $r_o = \infty \Omega$ ).



**FIG. 5.28**  
Example 5.2.

$$V_B = \frac{R_2}{R_1 + R_2} V_{CC} = \frac{(8.2 \text{ k}\Omega)(22 \text{ V})}{56 \text{ k}\Omega + 8.2 \text{ k}\Omega} = 2.81 \text{ V}$$

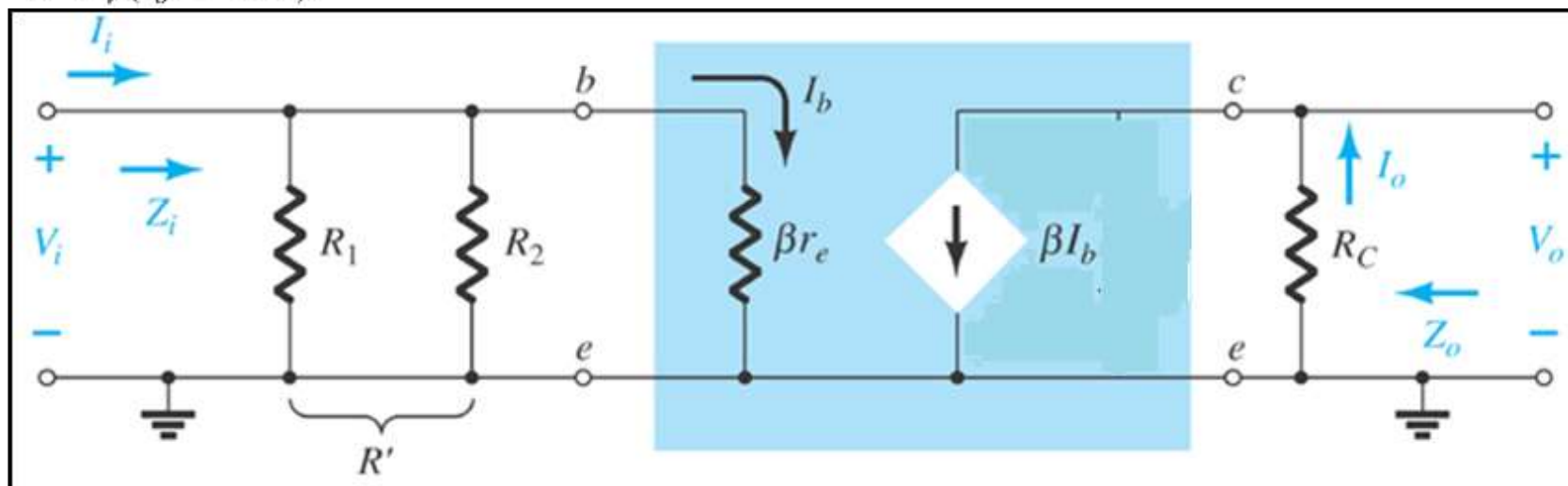
$$V_E = V_B - V_{BE} = 2.81 \text{ V} - 0.7 \text{ V} = 2.11 \text{ V}$$

$$I_E = \frac{V_E}{R_E} = \frac{2.11 \text{ V}}{1.5 \text{ k}\Omega} = 1.41 \text{ mA}$$

$$r_e = \frac{26 \text{ mV}}{I_E} = \frac{26 \text{ mV}}{1.41 \text{ mA}} = \mathbf{18.44 \Omega}$$

**EXAMPLE 5.2** For the network of Fig. 5.28, determine:

- $r_e$ .
- $Z_i$ .
- $Z_o$  ( $r_o = \infty \Omega$ ).
- $A_v$  ( $r_o = \infty \Omega$ ).



$$\text{b. } R' = R_1 \parallel R_2 = (56 \text{ k}\Omega) \parallel (8.2 \text{ k}\Omega) = 7.15 \text{ k}\Omega$$

$$Z_i = R' \parallel \beta r_e = 7.15 \text{ k}\Omega \parallel (90)(18.44 \Omega) = 7.15 \text{ k}\Omega \parallel 1.66 \text{ k}\Omega$$

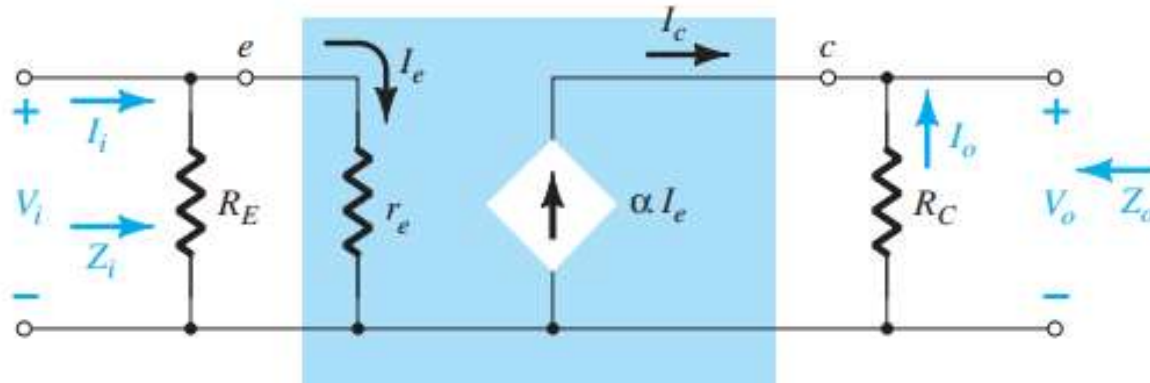
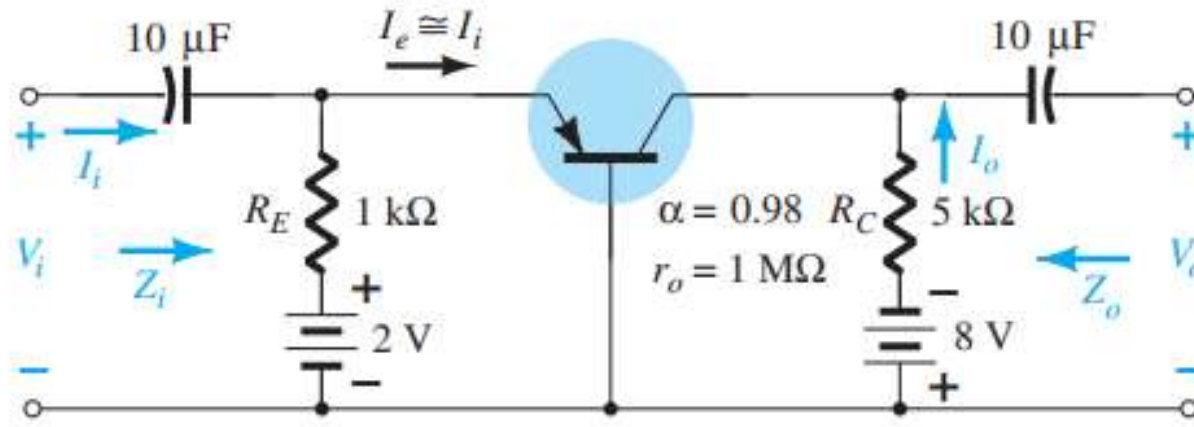
$$= 1.35 \text{ k}\Omega$$

$$\text{c. } Z_o = R_C = 6.8 \text{ k}\Omega$$

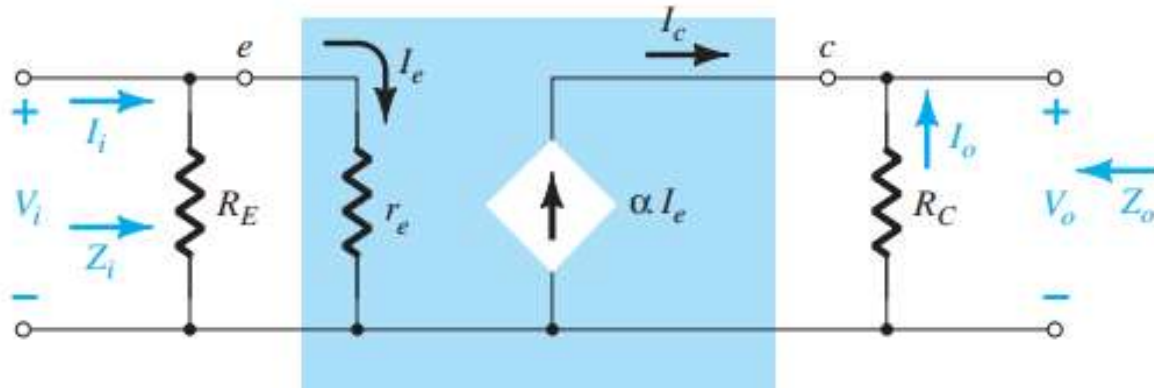
$$\text{d. } A_v = -\frac{R_C}{r_e} = -\frac{6.8 \text{ k}\Omega}{18.44 \Omega} = -368.76$$

# Example 5.8: Common Base BJT Amplifier

- Common base PNP:  $r_e$  T model
- Input: emitter; output: collector
- Output Current:  $I_C = \beta I_B = \alpha I_E = \alpha \times \text{input current}$



## Example 5.8: Common Base BJT Amplifier



$$I_E = \frac{V_{EE} - V_{BE}}{R_E} = \frac{2 \text{ V} - 0.7 \text{ V}}{1 \text{ k}\Omega} = \frac{1.3 \text{ V}}{1 \text{ k}\Omega} = 1.3 \text{ mA}$$

$$r_e = \frac{26 \text{ mV}}{I_E} = \frac{26 \text{ mV}}{1.3 \text{ mA}} = \mathbf{20 \text{ }\Omega}$$

$$Z_i = R_E \parallel r_e = 1 \text{ k}\Omega \parallel 20 \text{ }\Omega \\ = \mathbf{19.61 \text{ }\Omega} \cong r_e$$

$$Z_o = R_C = \mathbf{5 \text{ k}\Omega}$$

$$A_v \cong \frac{R_C}{r_e} = \frac{5 \text{ k}\Omega}{20 \text{ }\Omega} = \mathbf{250}$$