Chapter Title: Rotation

Sections: Kinetic Energy of Rotation, Torque, Angular Momentum

Work for Rotational Motion

One-dimensional Work Done:

$$W = \int_{x_i}^{x_f} F dx$$

Work Done about Rotational Axis:

$$W = \int_{\theta_i}^{\theta_f} \tau d\theta$$

Here, τ is the torque doing the work W, and θ_i and θ_f are the body's angular positions before and after the work is done, respectively.

Power for Rotational Motion

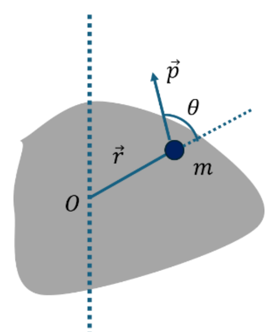
One-dimensional Power:

$$P = \frac{dW}{dt} = Fv$$

Work Done about Rotational Axis:

$$P = \frac{dW}{dt} = \tau\omega$$

Angular Momentum



Linear momentum is expressed as a particle of mass m is moving at a velocity $\vec{v}$ is,

$$\vec{p} = m\vec{v}$$

Angular momentum of a particle about its rotational axis is,

$$\vec{l} = \vec{r} \times \vec{p}$$

$$\vec{l} = m(\vec{r} \times \vec{v})$$

Its unit is kilogram meter-squared per second (kgm^2/s), equivalent to the joule-second (Js).

The magnitude of angular momentum is,

$$l = mvr \sin\theta$$

Newton's Second Law in Angular Form:

$$\vec{\tau}_{net} = \frac{d\vec{l}}{dt}$$

That means, the (vector) sum of all the torques acting on a particle is equal to the time rate of change of the angular momentum of that particle.

Angular Momentum of a Rigid Body

The total angular momentum $\vec{L}$ of the system is the (vector) sum of the angular momenta $\vec{l}$ of the individual particles (here with label i):

$$\vec{L} = \vec{l}_1 + \vec{l}_2 + \vec{l}_3 \ldots + \vec{l}_n = \sum_{i=1}^{n} \vec{l}_i$$

With time, the angular momenta of individual particles may change because of interactions between the particles or with the outside, so, the resulting change in $\vec{L}$ is,

$$\frac{d\vec{L}}{dt} = \sum_{i=1}^{n} \frac{d\vec{l}_i}{dt} = \sum_{i=1}^{n} \vec{\tau}_{net,i}$$

That is, the rate of change of the system's angular momentum is equal to the vector sum of the torques on its individual particles. Those torques include internal torques (due to forces between the particles) and external torques (due to forces on the particles from bodies external to the system). Net External Torque:

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$$

The net external torque $\vec{\tau}_{net}$ acting on a system of particles is equal to the time rate of change of the system's total angular momentum $\vec{L}$.

Angular Momentum of a Rigid Body Rotating About a Fixed Axis

If a rigid body rotates about a fixed axis with an angular speed ω. the angular momentum is L,

$$L = I\omega$$

[Here, $I = mr^2$ and $v = \omega r$]

From the conservation of angular momentum,

$$I_i\omega_i = I_f\omega_f$$

Conservation of Angular Momentum

When the net external torque acting on a system is zero, the total angular momentum of the system is constant (conserved) (*i.e.*; $\vec{L} =$ Constant, isolated system).

If the total angular momentum of a system is $\vec{L}$, then the net torque is,

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt} = 0$$

That means, $L_i = L_f$. The torques of the internal forces can transfer angular momentum from one body to the other, but they can't change the total angular momentum of the system.