Chapter Title: Rotation

Sections: Kinetic Energy of Rotation, Parallel-Axis Theorem

Kinetic Energy of Rotation

A rigid body is composed of many particles and if the rigid body is rotating at an angular speed ω about an axis, the total kinetic energy K of the whole body is,

$$K = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2 + \cdots + \frac{1}{2}m_nv_n^2$$

$$K = \sum \frac{1}{2}m_iv_i^2$$

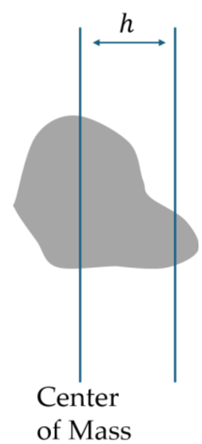
$$K = \sum \frac{1}{2}m_i(\omega r_i)^2 = \frac{1}{2}\left(\sum m_ir_i^2\right)\omega^2$$

Kinetic Energy of Rotation & Rotational Inertias

$$K = \frac{1}{2}\left(\sum m_ir_i^2\right)\omega^2 = \frac{1}{2}I\omega^2$$

$$I = \sum m_ir_i^2$$

The quantity, I is called rotational inertia (or moment of inertia) of the body with respect to the axis of rotation. It is a constant for a particular rigid body and a particular rotation axis.



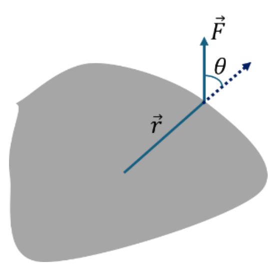
Parallel-Axis Theorem

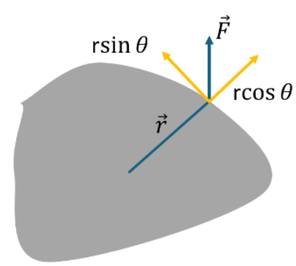
A rigid body may have many moment of inertia. There is a simple relationship between the moment of inertia I_{com} of a body of mass M about an axis through its center of mass and the moment of inertia I about any other axis parallel to the original one but displaced from it by a distance h. This relationship, called the parallel-axis theorem.

$$I = I_{com} + Mh^2$$

Torque

The tendency of a force to rotate an object about an axis is measured by a quantity called torque (or moment). It is presented by a Greek word τ. Torque is a vector quantity, and its SI unit is Newton-meter (Nm). Torque is calculated by,





$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau} = rF\sin\theta\,\hat{\eta}$$

The direction is measured by the right-hand rule.

Difference Between Torque & Work

Work: Work is a measure of energy transferred by a force acting through a displacement. It is calculated for any motion. It's expressed as $W = \vec{F}.\vec{s}$. It is a scalar quantity. Though its unit is Newton-meter (N.m) which is presented as Joule.

Torque: Torque measures the rotational effect of a force around a specific axis. It is applied in rotational motion, presented as $\vec{\tau} = \vec{r} \times \vec{F}$ and a vector quantity. Its unit is presented as Newton-meter (N.m).

Newton's Second Law for Rotation

Newton's second law for linear motion,

$$F_{net} = ma$$

Newton's second law for rotational motion,

$$\tau_{net} = I\alpha$$

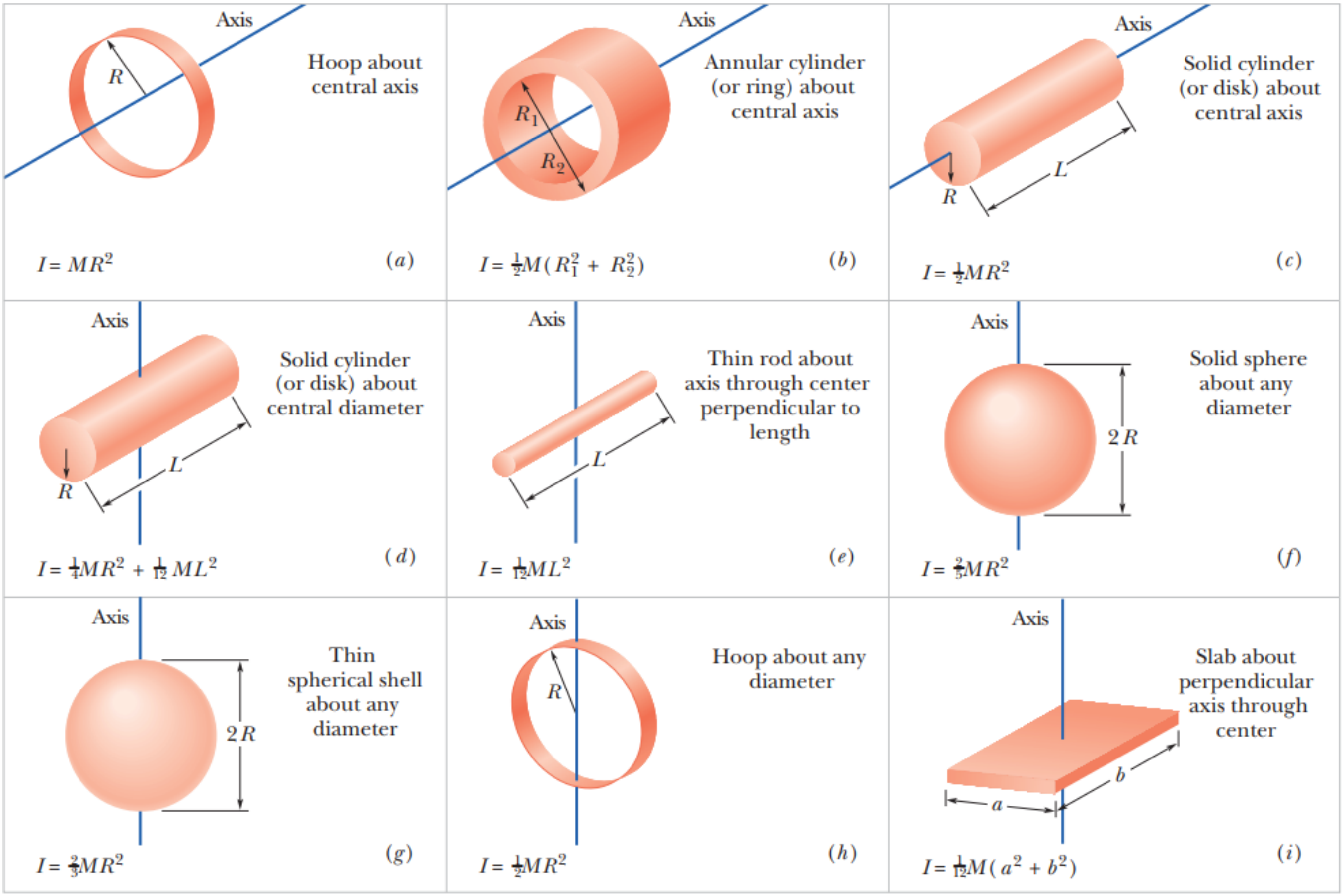
Here, I is moment of inertia and α is angular acceleration

Energy & Work for Rotational Motion

For rotational motion, Work-Kinetic Energy Theorem:

$$\Delta K = K_f - K_i = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = W$$

Here, I is the rotational inertia of the body about the fixed axis and ω_i and ω_f are the angular speeds of the body before and after the work is done.


Axis
Hoop about central axis
R
I = MR²
(a)
Axis
Annular cylinder (or ring) about central axis
R₁
R₂
I = ½M(R₁² + R₂²)
(b)
Axis
Solid cylinder (or disk) about central axis
R
L
I = ½MR²
(c)
Axis
Solid cylinder (or disk) about central diameter
R
L
I = ¼MR² + 1/12 ML²
(d)
Axis
Thin rod about axis through center perpendicular to length
L
I = 1/12 ML²
(e)
Axis
Solid sphere about any diameter
2R
I = ⅖MR²
(f)
Axis
Thin spherical shell about any diameter
2R
I = ⅔MR²
(g)
Axis
Hoop about any diameter
R
I = ½MR²
(h)
Axis
Slab about perpendicular axis through center
a
b
I = 1/12 M(a² + b²)
(i)