Chapter Title: Oscillations

Sections: Simple Harmonic Motion, The Force Law for Simple Harmonic Motion, Energy in Simple Harmonic Motion

Chapter Title: Waves - I

Sections: Types of Waves, Wavelength and Frequency, Amplitude and Phase, Wavelength and Angular Wave Number, Period, Angular Frequency, and Frequency, The Speed of a Traveling Wave, Wave Equation & Principle of Superposition for Waves, Interference of Waves

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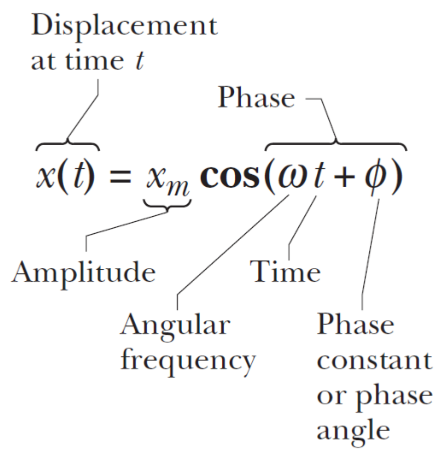
Oscillations: Simple Harmonic Oscillation

Displacement: In simple harmonic motion (SHM), the displacement of a particle from its equilibrium position is,

$$x(t) = x_m \cos(\omega t + \phi)$$



Velocity:

$$v(t) = \frac{dx(t)}{dt} = -\omega x_m \sin(\omega t + \phi)$$

Acceleration:

$$a(t) = \frac{dv(t)}{dt} = -\omega^2 x_m \cos(\omega t + \phi)$$

$$a(t) = -\omega^2 x(t)$$

In SHM, the acceleration a is proportional to the displacement x but opposite in sign, and the two quantities are related by the square of the angular frequency ω.

Here, the negative sign arises from the restoring force (F_{res}) that always acts to bring the object back to its equilibrium position. In other words, if an object is displaced to the right, the acceleration is directed to the left, and vice versa.

According to Newton's Second Law,

$$F = F_{res} = ma$$

$$a = \frac{F_{res}}{m} = \frac{-kx}{m} = -\left(\frac{k}{m}\right)x = -\omega^2 x$$

Units:

Angular frequency, ω is radian per second = rad/ sec	Phase constant, ϕ is radian = rad	Period, T is second = sec
Amplitude, x_m is meter = m Displacement, x is meter = m	Frequency, f is hertz = Hz = oscillation per second = /sec	Spring constant, k is newton per meter = N/m

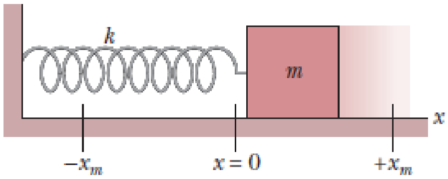
Force Law for Simple Harmonic Motion

Let us consider a mass, m is attached to a spring, and the other part of the spring is attached to a fixed wall. If the spring starts to oscillate, a restoring force works against the displacement.

The block–spring system is called a **linear simple harmonic oscillator** or simply **linear oscillator**.

According to Hook's law,

$$F = -kx$$



Here, k is a spring constant.

According to Newton's second law.

$$F_{net} = ma$$

$$F_{net} = F = -kx = ma$$

$$-kx = ma$$

$$-kx = m(-\omega^2 x)$$

$$k = m\omega^2$$

$$\omega = \sqrt{\frac{k}{m}}$$

Period:

The angular frequency, ω for the oscillation is,

$$\omega = \frac{2\pi}{T} = 2\pi f$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Energy in Simple Harmonic Motion

Potential energy,

$$U(t) = \frac{1}{2}kx^2 = \frac{1}{2}kx_m^2 cos^2(\omega t + \phi)$$

Kinetic energy,

$$K(t) = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 x_m^2 sin^2(\omega t + \phi)$$

$$K(t) = \frac{1}{2}kx_m^2 sin^2(\omega t + \phi)$$

Mechanical energy:

$$E = K + U$$

$$E = \frac{1}{2}kx_m^2$$

Chapter Title: Waves - I

Waves

Ripples on a pond, musical sounds, seismic tremors triggered by an earthquake - all these are wave phenomena.

Waves can occur whenever a system is disturbed from equilibrium.

When the disturbance can travel, or propagate, from one region of the system to another, a wave is created. As a wave propagates, it carries energy.

Wave: 3 Major Types

Mechanical waves: A mechanical wave is a disturbance that travels through some material or substance called the medium for the wave.

Electromagnetic waves: They are a form of radiation that are formed when an electric field couples with a magnetic field and propagates through a medium or vacuum.

Matter waves: Matter waves, also known as de Broglie waves, are a fundamental concept in quantum mechanics, describing the wavelike behavior of particles and illustrating the principle of wave-particle duality, where all matter exhibits both wavelike and particle-like properties. These waves are associated with electrons, protons, and other fundamental particles, and even atoms and molecules.

Travelling Wave: Types

Transverse Waves: A traveling wave or pulse that causes the elements of the disturbed medium to move perpendicular to the direction of propagation is called a transverse wave.

Example: Water wave, light wave or any other electromagnetic wave (like radio waves, microwaves, X-rays)

Longitudinal Waves: A traveling wave or pulse that causes the elements of the medium to move parallel to the direction of propagation is called a longitudinal wave

Example: Different types of sound waves and in various mediums.

Wavelength & Frequency

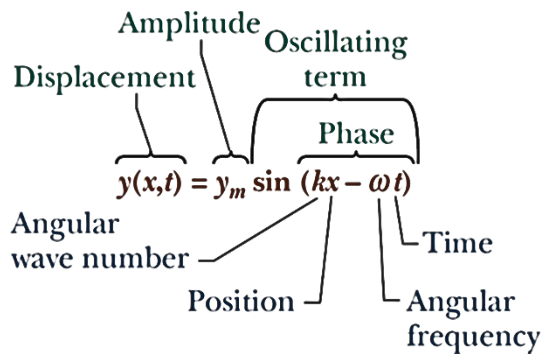
A wave on a string or simply a motion of any element along

Its length is presented through a function,

$$y = h(x, t)$$

Here, y is the transverse displacement of any string element as a function h of the time t and the position x of the element along the string.

In general, a sinusoidal shape like the wave can be described with h being either a sine or cosine function; both give the same general shape for the wave.



Let us consider a sinusoidal wave traveling in the positive direction of the x-axis. As the wave sweeps through succeeding elements of the string, the elements oscillate parallel to the y-axis. At time t, the displacement y of the element located at position x is given by,

$$y(x, t) = y_m \sin(kx - \omega t)$$

If there is a phase constant ϕ, then the wave function becomes,

$$y(x, t) = y_m \sin(kx - \omega t + \phi)$$

The wavelength λ of a wave is the distance between repetitions of the shape of the wave (or wave shape).

The angular wave number or simply wave number k of a wave is represented as,

$$k = \frac{2\pi}{\lambda}$$

Unit: The SI unit is the radian per meter, or the inverse meter. (Note: k is not the spring constant)

Wavelength & Frequency: Period, Angular Frequency, & Frequency

The angular frequency ω of a wave can be presented using the period of oscillation T of the wave.

$$\omega = \frac{2\pi}{T}$$

Unit of angular frequency: The SI unit is the radian per second.

The frequency of wave (number of oscillation per unit time) is related to the period,

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

Here, frequency represents an oscillation number that is made by a string element as the wave moves through it.

The Speed of a Traveling Wave

Let us consider a sinusoidal wave traveling in the positive direction of the x-axis.

$$y(x,t) = y_m \sin(kx - \omega t)$$

Speed of the travelling wave is,

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f$$

If the wave is traveling in the opposite direction, $y(x,t) = y_m \sin(kx + \omega t)$

$$v = -\frac{\omega}{k}$$

The minus sign indicates the wave is traveling in the negative x direction.

Wave Equation & Principle of Superposition for Waves

The wave equation is a differential equation that governs the travel of waves of all types.

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

Principle of Superposition for Waves: When two or more waves overlap, the resultant wave is the algebraic sum of the individual waves.

Let us consider two wave functions $y_1(x,t)$ and $y_2(x,t)$. According to the principle of superposition of waves is,

$$y(x,\ t) = y_1(x,t) + y_2(x,\ t)$$

Here, $y(x,\ t)$ is the resultant wave (or net wave).

Interference of Waves

If two sinusoidal waves of the same amplitude and wavelength travel in the same direction along a stretched string, they interfere to produce a resultant sinusoidal wave traveling in that direction.

Let us consider two wave functions $y_1(x,t)$ and $y_2(x,t)$.

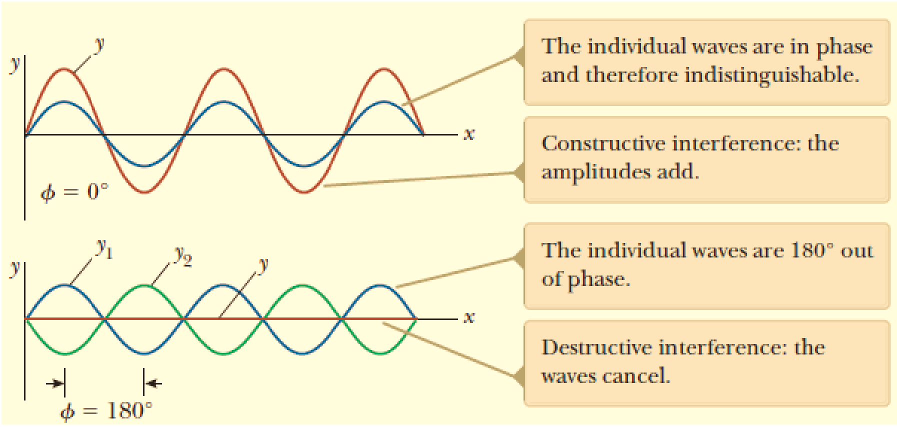
Here, $y_1(x,t) = A\sin(kx - \omega t)$ and $y_2(x,t) = A\sin(kx - \omega t + \phi)$

According to the principle of superposition, $y(x,t) = y_1(x,t) + y_2(x,t)$

After solving the equations,

$$y(x,t) = \left[2A\cos\frac{\phi}{2}\right]\sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

From the above equation, the resultant wave has an amplitude ($2A\cos\frac{\phi}{2}$) and a phase equal to half the difference between the phases of the original waves.



The phenomenon of two or more waves of the same, or almost the same, frequency superposing to produce an observable pattern in the intensity is called interference.

The combination of separate waves in the same region of space to produce a resultant wave is called interference.

If $\phi = 0\, rad$ (or 0°), the two interfering waves are exactly 'in phase'. The interference that produces the greatest possible amplitude is called fully constructive interference.

$$y(x,t) = 2A\sin(kx - \omega t)$$

If $\phi = \pi\, rad$ (or 180°), the two interfering waves are exactly 'out-of-phase'. The interference that produces the zero or null amplitude is called fully destructive interference.

$$y(x,t) = 0$$