# EEE 141
# ELECTRICAL CIRCUITS

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# BASIC LAWS

- Wye-Delta Transformations.
- Series-Parallel Networks.
- Ladder Network

# WYE-DELTA TRANSFORMATIONS

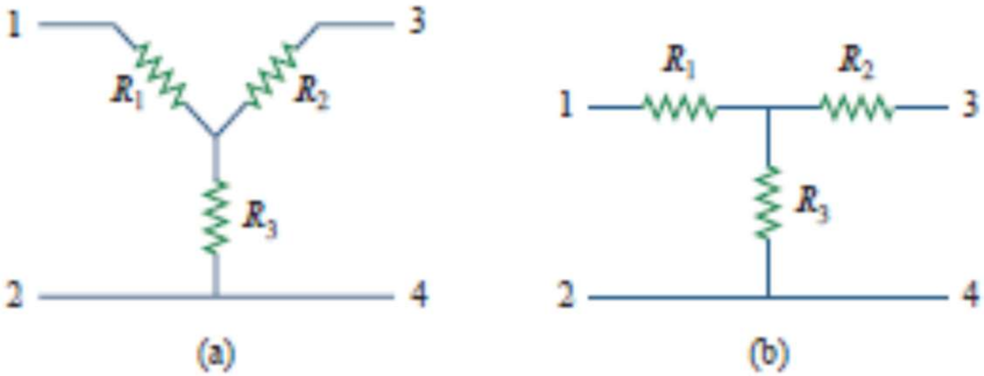



**Figure 2.47**
Two forms of the same network: (a) Y, (b) T.

$$R_{12}(Y) = R_1 + R_3$$
$$R_{12}(\Delta) = R_b \parallel (R_a + R_c)$$

Setting $R_{12}(Y) = R_{12}(\Delta)$ gives

$$R_{12} = R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c}$$

Similarly,

$$R_{13} = R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c}$$
$$R_{34} = R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c}$$

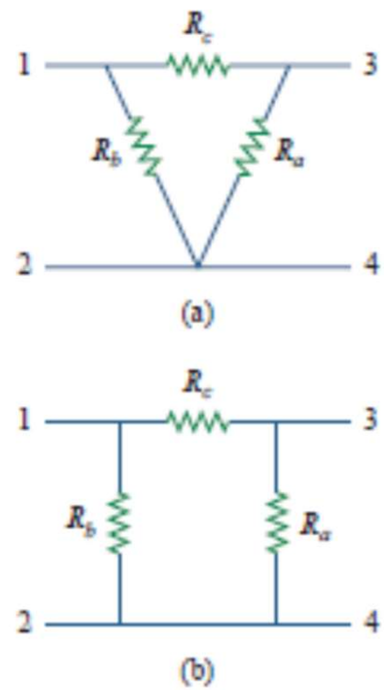
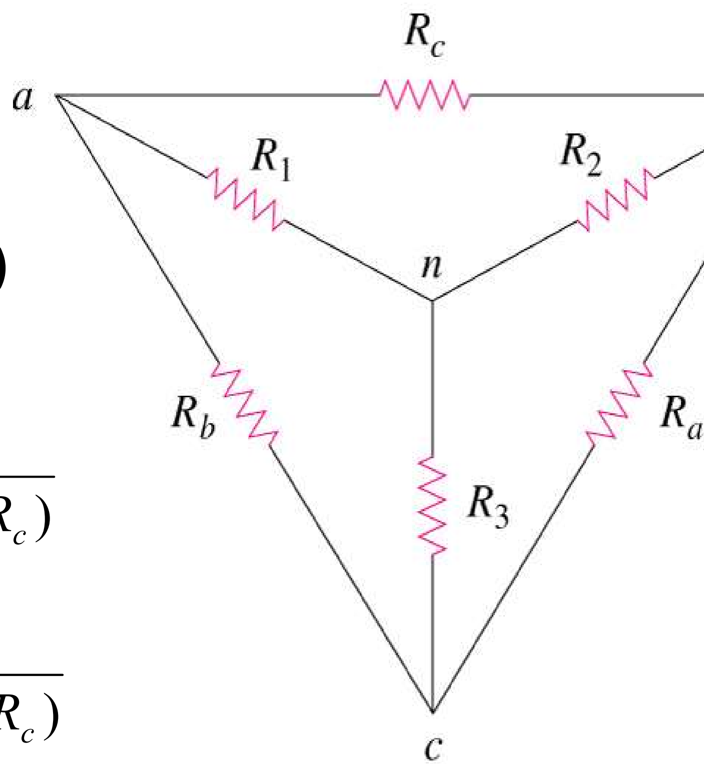



**Figure 2.48**
Two forms of the same network: (a) $\Delta$, (b) $\Pi$.

# WYE-DELTA TRANSFORMATIONS




**Delta -> Star (Wye)**

$$R_1 = \frac{R_b R_c}{(R_a + R_b + R_c)}$$

$$R_2 = \frac{R_c R_a}{(R_a + R_b + R_c)}$$

$$R_3 = \frac{R_a R_b}{(R_a + R_b + R_c)}$$

**Star (Wye) -> Delta**

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

# EXAMPLE

**EXAMPLE 8.29** Find the total resistance of the network of Fig. 8.80, where $R_A = 3\ \Omega$, $R_B = 3\ \Omega$, and $R_C = 6\ \Omega$.

***Solution:***

Two resistors of the Δ were equal; therefore, two resistors of the Y will be equal.

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{12\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(3\ \Omega)(3\ \Omega)}{12\ \Omega} = \frac{9\ \Omega}{12} = \mathbf{0.75\ \Omega}$$

Replacing the Δ by the Y, as shown in Fig. 8.81, yields

$$R_T = 0.75\ \Omega + \frac{(4\ \Omega + 1.5\ \Omega)(2\ \Omega + 1.5\ \Omega)}{(4\ \Omega + 1.5\ \Omega) + (2\ \Omega + 1.5\ \Omega)}$$

$$= 0.75\ \Omega + \frac{(5.5\ \Omega)(3.5\ \Omega)}{5.5\ \Omega + 3.5\ \Omega}$$

$$= 0.75\ \Omega + 2.139\ \Omega$$

$$R_T = \mathbf{2.889\ \Omega}$$

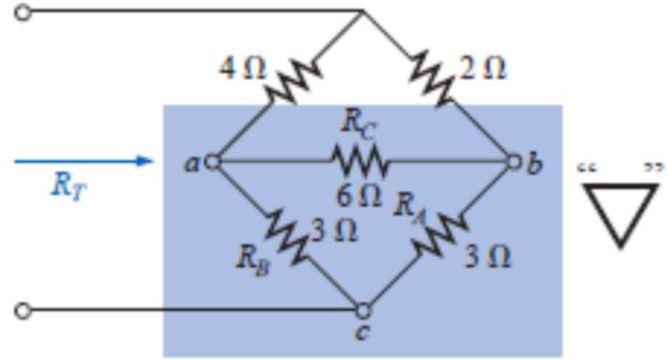



**FIG. 8.80**
*Example 8.29.*

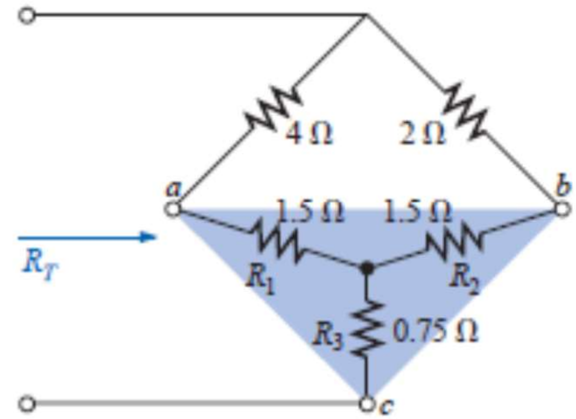
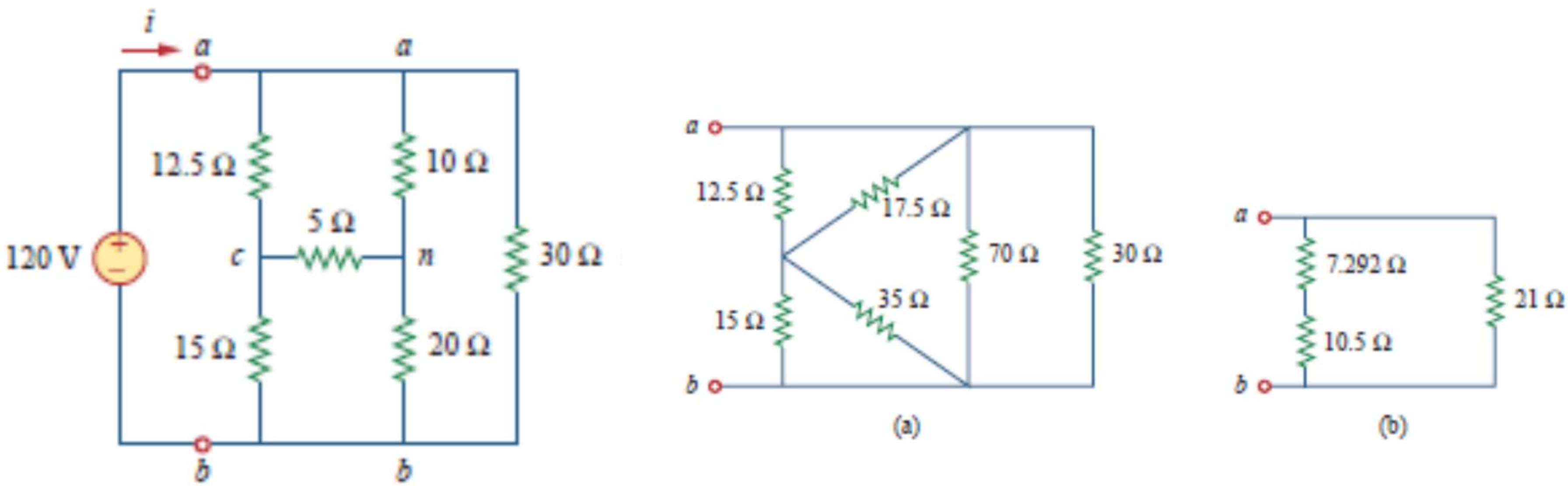



**FIG. 8.81**
*Substituting the Y equivalent for the bottom Δ of Fig. 8.80.*

# EXAMPLE

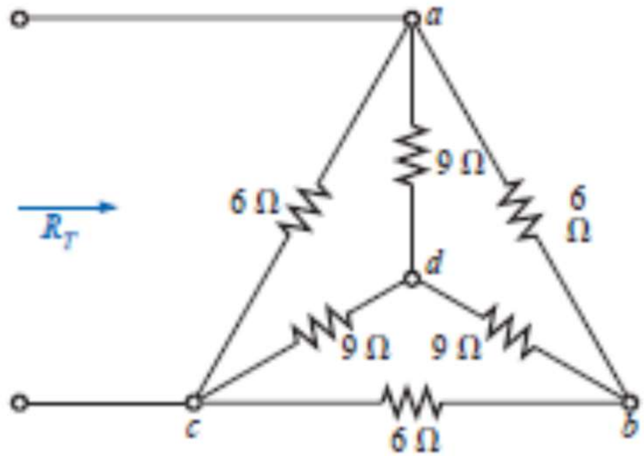



$$R_{ab} = (7.292 + 10.5) \| 21 = \frac{17.792 \times 21}{17.792 + 21} = \mathbf{9.632\ \Omega}$$

Then

$$i = \frac{v_s}{R_{ab}} = \frac{120}{9.632} = \mathbf{12.458\ A}$$

# EXAMPLE




$$R_Y = \frac{R_\Delta}{3} = \frac{6\ \Omega}{3} = 2\ \Omega \qquad \text{(Fig. 8.83)}$$

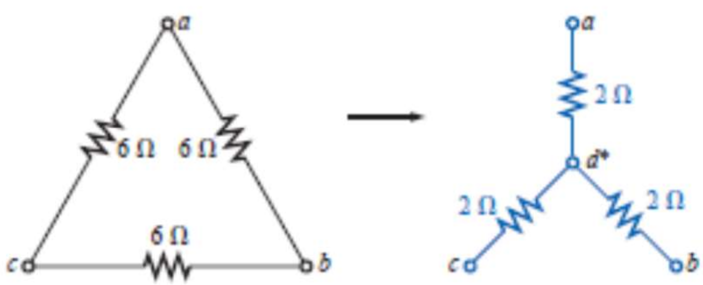
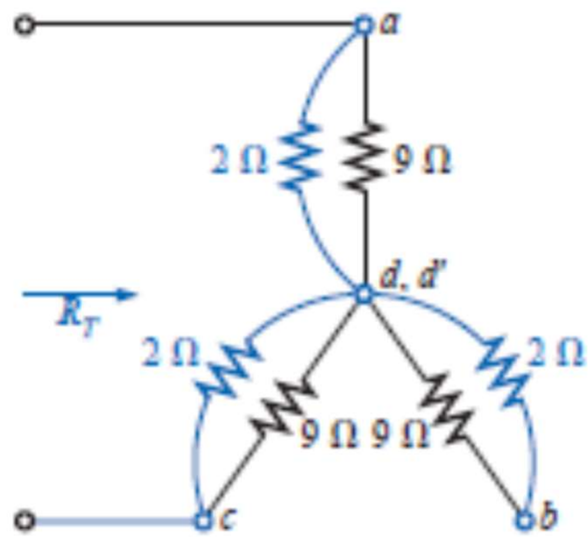



**FIG. 8.83**
*Converting the Δ configuration of Fig. 8.82 to a Y configuration.*

The network then appears as shown in Fig. 8.84.

$$R_T = 2\left[\frac{(2\ \Omega)(9\ \Omega)}{2\ \Omega + 9\ \Omega}\right] = \mathbf{3.2727\ \Omega}$$




b. *Converting the Y to a Δ:*

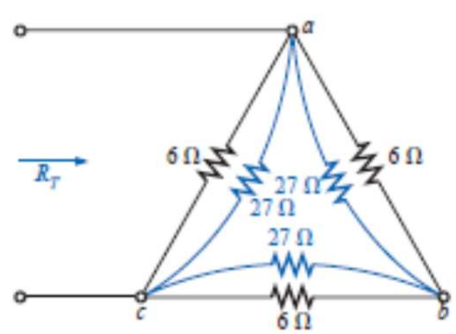
$$R_\Delta = 3R_Y = (3)(9\ \Omega) = 27\ \Omega \qquad \text{(Fig. 8.85)}$$

$$R'_T = \frac{(6\ \Omega)(27\ \Omega)}{6\ \Omega + 27\ \Omega} = \frac{162\ \Omega}{33} = 4.9091\ \Omega$$

$$R_T = \frac{R'_T(R'_T + R'_T)}{R'_T + (R'_T + R'_T)} = \frac{R'_T 2R'_T}{3R'_T} = \frac{2R'_T}{3}$$

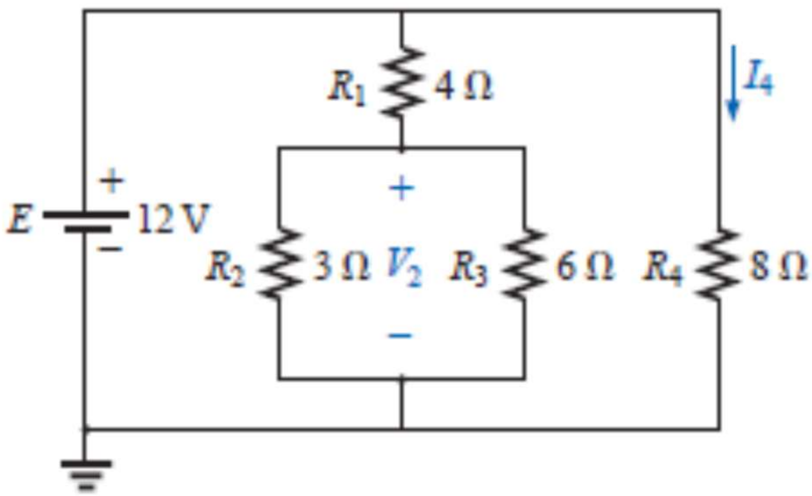
$$= \frac{2(4.9091\ \Omega)}{3} = \mathbf{3.2727\ \Omega}$$

which checks with the previous solution.



# SERIES-PARALLEL NETWORKS

Example



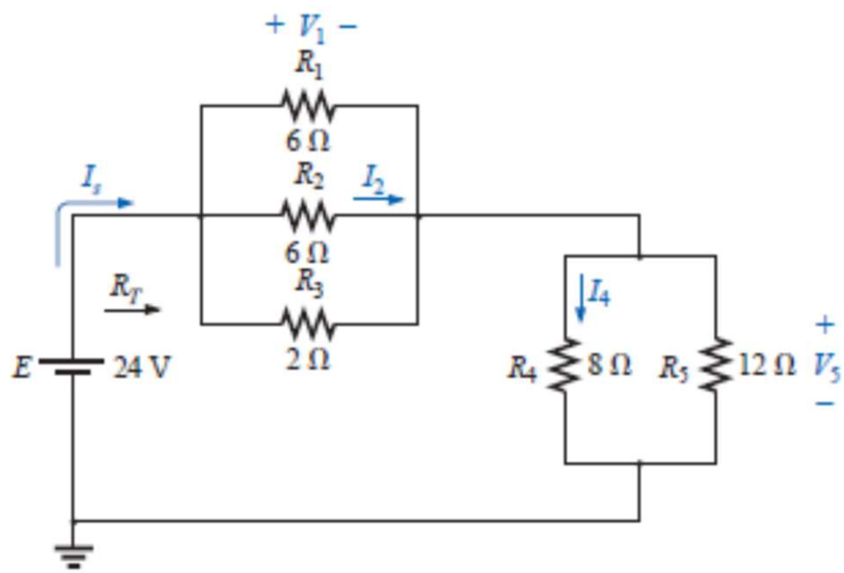

$$I_4 = \frac{E}{R_B} = \frac{E}{R_4} = \frac{12V}{8\Omega} = 1.5\ A$$

$$R_D = R_2 // R_3 = 3\Omega // 6\Omega = 2\Omega$$

$$V_2 = \frac{R_D\,E}{R_D + R_C}$$

$$= \frac{(2\Omega)(12V)}{2\Omega + 4\Omega} = 4\ V$$

# EXAMPLE




$$R_{1\|2} = \frac{R}{N} = \frac{6\ \Omega}{2} = 3\ \Omega$$

$$R_A = R_{1\|2\|3} = \frac{(3\ \Omega)(2\ \Omega)}{3\ \Omega + 2\ \Omega} = \frac{6\ \Omega}{5} = 1.2\ \Omega$$

$$R_B = R_{4\|5} = \frac{(8\ \Omega)(12\ \Omega)}{8\ \Omega + 12\ \Omega} = \frac{96\ \Omega}{20} = 4.8\ \Omega$$

The reduced form of Fig. 7.13 will then appear as shown in Fig. 7.15, and

$$R_T = R_{1\|2\|3} + R_{4\|5} = 1.2\ \Omega + 4.8\ \Omega = \mathbf{6\ \Omega}$$

$$I_s = \frac{E}{R_T} = \frac{24\ \text{V}}{6\ \Omega} = \mathbf{4\ A}$$

with

$$V_1 = I_s R_{1\|2\|3} = (4\ \text{A})(1.2\ \Omega) = \mathbf{4.8\ V}$$

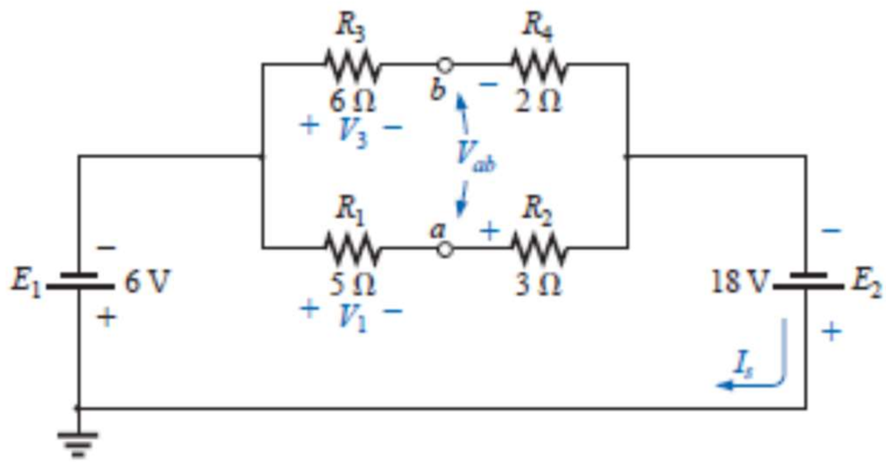
$$V_5 = I_s R_{4\|5} = (4\ \text{A})(4.8\ \Omega) = \mathbf{19.2\ V}$$

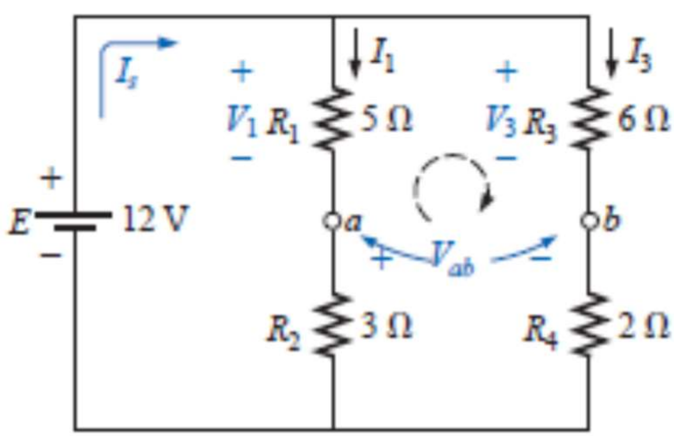
Applying Ohm's law,

$$I_4 = \frac{V_5}{R_4} = \frac{19.2\ \text{V}}{8\ \Omega} = \mathbf{2.4\ A}$$

$$I_2 = \frac{V_2}{R_2} = \frac{V_1}{R_2} = \frac{4.8\ \text{V}}{6\ \Omega} = \mathbf{0.8\ A}$$

# EXAMPLE




$$V_1 = \frac{R_1 E}{R_1 + R_2} = \frac{(5\ \Omega)(12\ \text{V})}{5\ \Omega + 3\ \Omega} = \frac{60\ \text{V}}{8} = \mathbf{7.5\ V}$$

$$V_3 = \frac{R_3 E}{R_3 + R_4} = \frac{(6\ \Omega)(12\ \text{V})}{6\ \Omega + 2\ \Omega} = \frac{72\ \text{V}}{8} = \mathbf{9\ V}$$

The open-circuit voltage $V_{ab}$ is determined by applying Kirchhoff's voltage law around the indicated loop of Fig. 7.17 in the clockwise direction starting at terminal $a$.

$$+V_1 - V_3 + V_{ab} = 0$$

and $$V_{ab} = V_3 - V_1 = 9\ \text{V} - 7.5\ \text{V} = \mathbf{1.5\ V}$$

By Ohm's law,

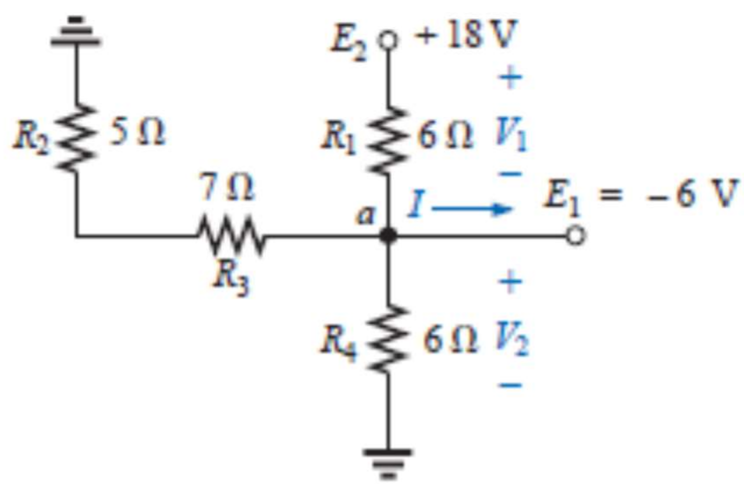
$$I_1 = \frac{V_1}{R_1} = \frac{7.5\ \text{V}}{5\ \Omega} = 1.5\ \text{A}$$

$$I_3 = \frac{V_3}{R_3} = \frac{9\ \text{V}}{6\ \Omega} = 1.5\ \text{A}$$

Applying Kirchhoff's current law,

$$I_s = I_1 + I_3 = 1.5\ \text{A} + 1.5\ \text{A} = \mathbf{3\ A}$$

# EXAMPLE



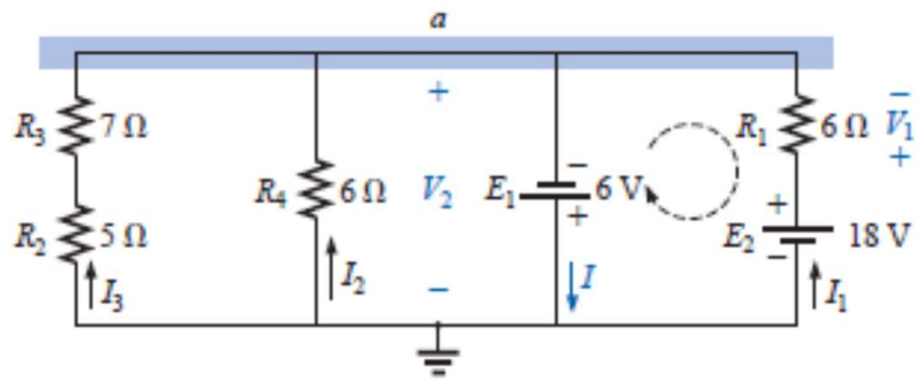




**FIG. 7.19**
*Network of Fig. 7.18 redrawn.*

and $$V_1 = E_2 + E_1 = 18\text{ V} + 6\text{ V} = \mathbf{24\text{ V}}$$

Applying Kirchhoff's current law to node $a$ yields

$$I = I_1 + I_2 + I_3$$
$$= \frac{V_1}{R_1} + \frac{E_1}{R_4} + \frac{E_1}{R_2 + R_3}$$
$$= \frac{24\text{ V}}{6\ \Omega} + \frac{6\text{ V}}{6\ \Omega} + \frac{6\text{ V}}{12\ \Omega}$$
$$= 4\text{ A} + 1\text{ A} + 0.5\text{ A}$$
$$I = \mathbf{5.5\text{ A}}$$

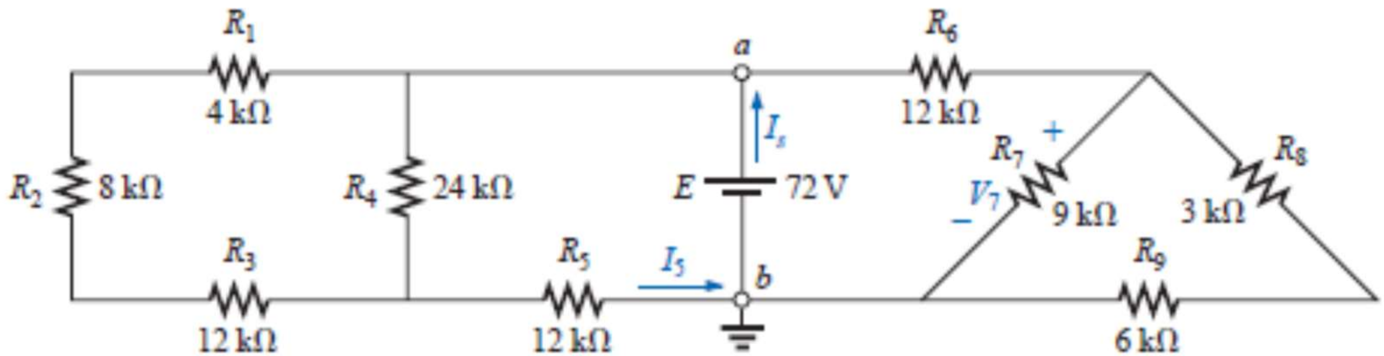
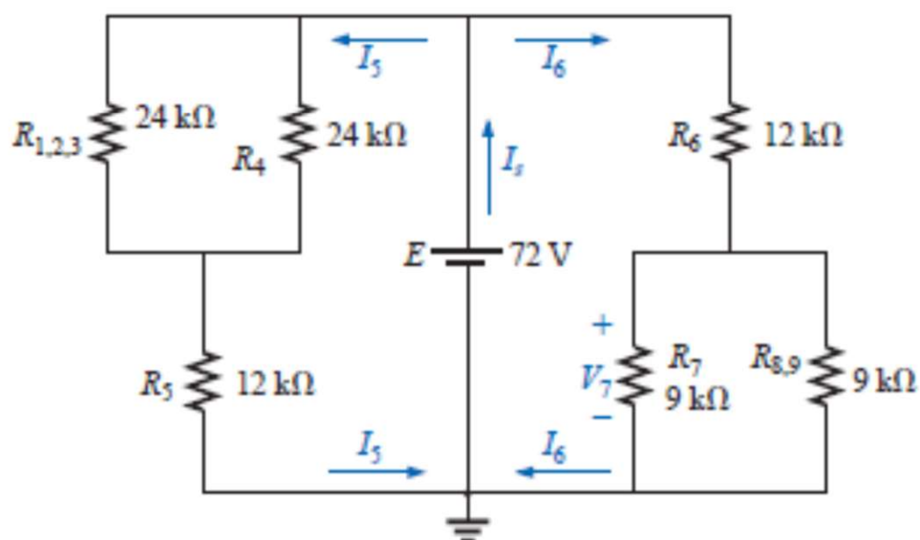



**FIG. 7.22**
*Example 7.9.*

**Solution:** Redrawing the network after combining series elements yields Fig. 7.23, and



$$I_5 = \frac{E}{R_{(1,2,3)\|4} + R_5} = \frac{72\text{ V}}{12\text{ k}\Omega + 12\text{ k}\Omega} = \frac{72\text{ V}}{24\text{ k}\Omega} = \mathbf{3\text{ mA}}$$

with

$$V_7 = \frac{R_{7\|(8,9)}E}{R_{7\|(8,9)} + R_6} = \frac{(4.5\text{ k}\Omega)(72\text{ V})}{4.5\text{ k}\Omega + 12\text{ k}\Omega} = \frac{324\text{ V}}{16.5} = \mathbf{19.6\text{ V}}$$

$$I_6 = \frac{V_7}{R_{7\|(8,9)}} = \frac{19.6\text{ V}}{4.5\text{ k}\Omega} = \mathbf{4.35\text{ mA}}$$

and $\qquad I_s = I_5 + I_6 = 3\text{ mA} + 4.35\text{ mA} = \mathbf{7.35\text{ mA}}$

Since the potential difference between points *a* and *b* of Fig. 7.22 is fixed at *E* volts, the circuit to the right or left is unaffected if the network is reconstructed as shown in Fig. 7.24.

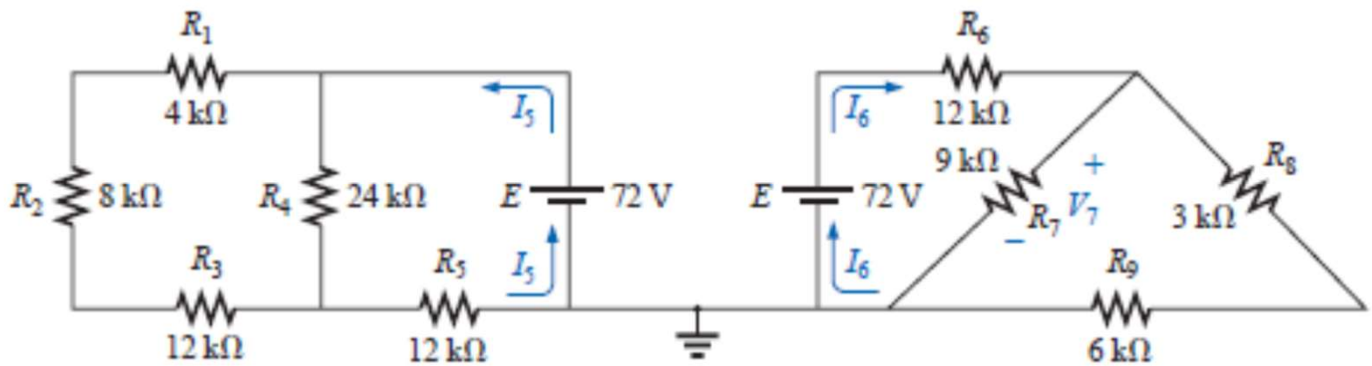



**FIG. 7.24**
*An alternative approach to Example 7.9.*

We can find each quantity required, except $I_s$, by analyzing each circuit independently. To find $I_s$, we must find the source current for each

# EXAMPLE

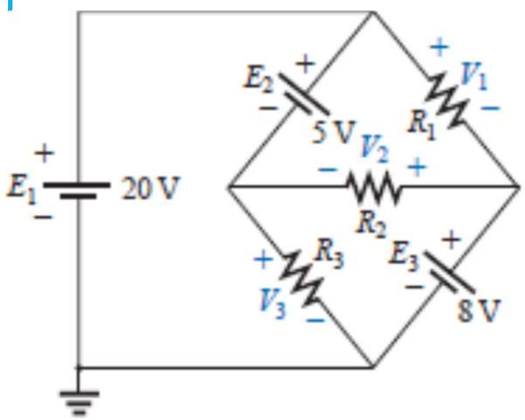
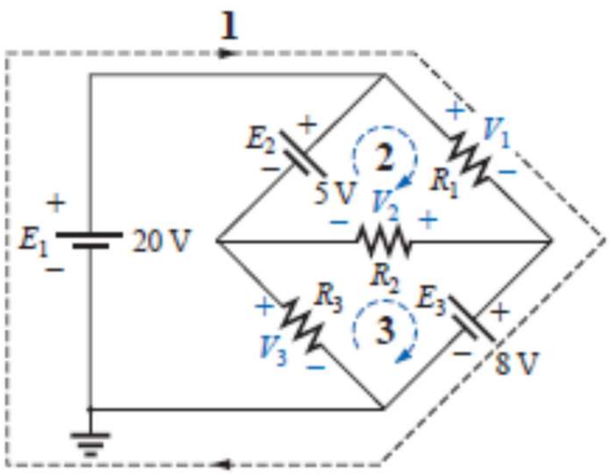



**FIG. 7.25**
*Example 7.10.*




**EXAMPLE 7.10** This example demonstrates the power of Kirchhoff's voltage law by determining the voltages $V_1$, $V_2$, and $V_3$ for the network of Fig. 7.25. For path 1 of Fig. 7.26,

$$E_1 - V_1 - E_3 = 0$$

and $$V_1 = E_1 - E_3 = 20\text{ V} - 8\text{ V} = \mathbf{12\text{ V}}$$

For path 2,

$$E_2 - V_1 - V_2 = 0$$

and $$V_2 = E_2 - V_1 = 5\text{ V} - 12\text{ V} = \mathbf{-7\text{ V}}$$

indicating that $V_2$ has a magnitude of 7 V but a polarity opposite to that appearing in Fig. 7.25. For path 3,

$$V_3 + V_2 - E_3 = 0$$

and $$V_3 = E_3 - V_2 = 8\text{ V} - (-7\text{ V}) = 8\text{ V} + 7\text{ V} = \mathbf{15\text{ V}}$$

Note that the polarity of $V_2$ was maintained as originally assumed, requiring that −7 V be substituted for $V_2$.

# EXAMPLE (LADDER NETWORK)

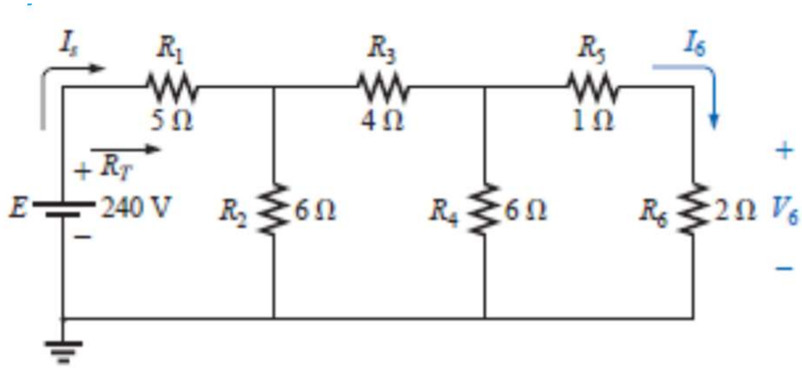
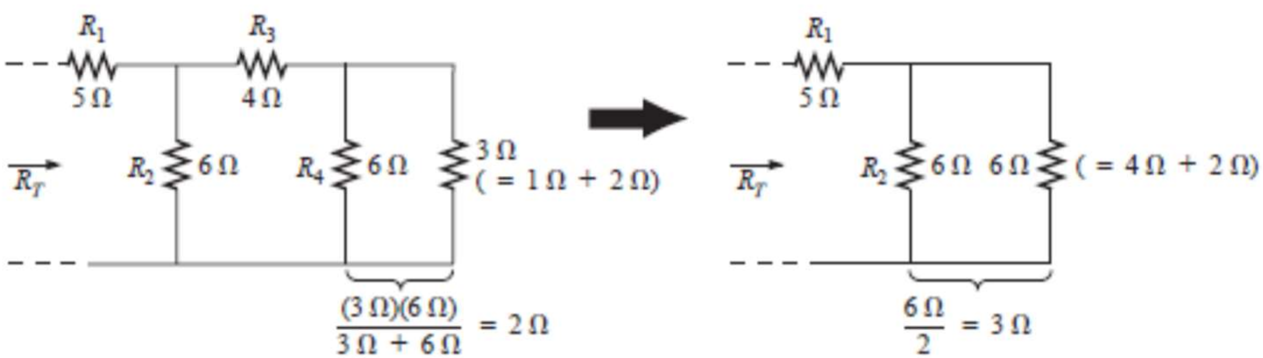



FIG. 7.27
*Ladder network.*



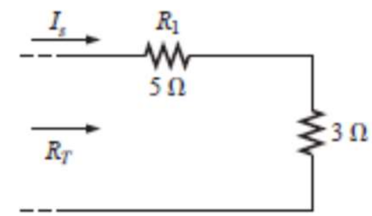




FIG. 7.29
*Calculating $R_T$ and $I_s$.*

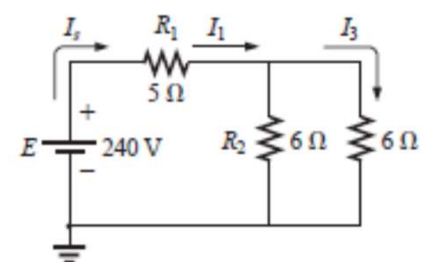



FIG. 7.30
*Working back toward $I_6$.*

# EXAMPLE

$$R_T = 5\ \Omega + 3\ \Omega = 8\ \Omega$$

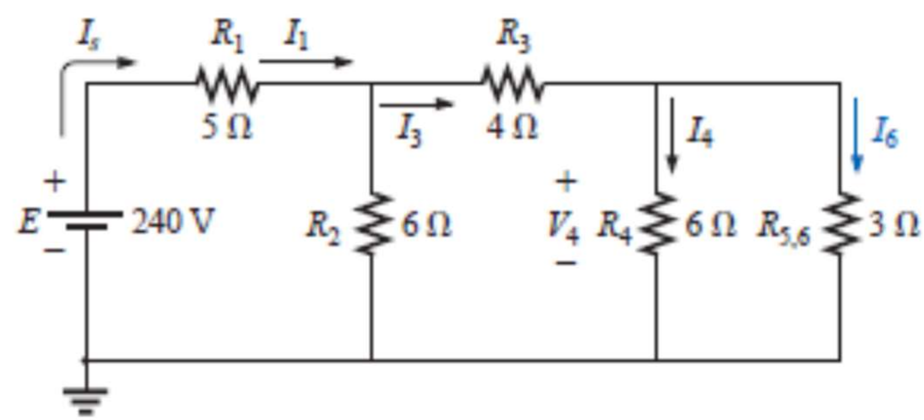
$$I_s = \frac{E}{R_T} = \frac{240\ \text{V}}{8\ \Omega} = 30\ \text{A}$$

Working our way back to $I_6$ (Fig. 7.30), we find that

$$I_1 = I_s$$

and
$$I_3 = \frac{I_s}{2} = \frac{30\ \text{A}}{2} = 15\ \text{A}$$

and, finally (Fig. 7.31),




$$I_6 = \frac{(6\ \Omega)I_3}{6\ \Omega + 3\ \Omega} = \frac{6}{9}(15\ \text{A}) = 10\ \text{A}$$

$$V_6 = I_6 R_6 = (10\ \text{A})(2\ \Omega) = 20\ \text{V}$$

# EXAMPLE

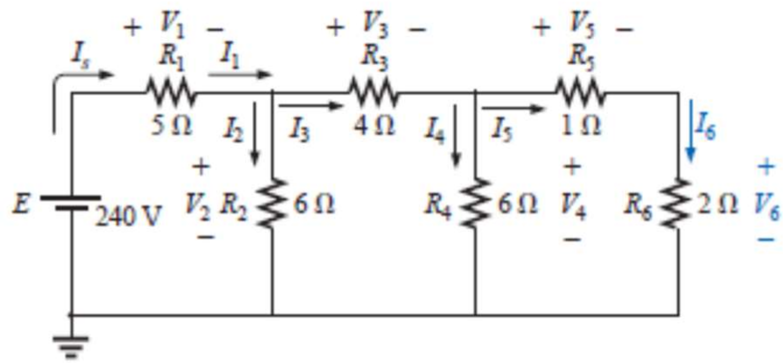



**FIG. 7.32**
*An alternative approach for ladder networks.*

The assigned notation for the current through the final branch is $I_6$:

$$I_6 = \frac{V_4}{R_5 + R_6} = \frac{V_4}{1\,\Omega + 2\,\Omega} = \frac{V_4}{3\,\Omega}$$

or
$$V_4 = (3\,\Omega)I_6$$

so that
$$I_4 = \frac{V_4}{R_4} = \frac{(3\,\Omega)I_6}{6\,\Omega} = 0.5I_6$$

and
$$I_3 = I_4 + I_6 = 0.5I_6 + I_6 = 1.5I_6$$
$$V_3 = I_3R_3 = (1.5I_6)(4\,\Omega) = (6\,\Omega)I_6$$

Also,
$$V_2 = V_3 + V_4 = (6\,\Omega)I_6 + (3\,\Omega)I_6 = (9\,\Omega)I_6$$

so that
$$I_2 = \frac{V_2}{R_2} = \frac{(9\,\Omega)I_6}{6\,\Omega} = 1.5I_6$$

and
$$I_s = I_2 + I_3 = 1.5I_6 + 1.5I_6 = 3I_6$$

with
$$V_1 = I_1R_1 = I_sR_1 = (5\,\Omega)I_s$$

so that
$$E = V_1 + V_2 = (5\,\Omega)I_s + (9\,\Omega)I_6$$
$$= (5\,\Omega)(3I_6) + (9\,\Omega)I_6 = (24\,\Omega)I_6$$

and
$$I_6 = \frac{E}{24\,\Omega} = \frac{240\text{ V}}{24\,\Omega} = 10\text{ A}$$

with
$$V_6 = I_6R_6 = (10\text{ A})(2\,\Omega) = \mathbf{20\text{ V}}$$