Calculus and Analytic Geometry

$$\int du = u + C$$

$$\int a\ du = a \int du = au + C$$

$$\int u^r\ du = \frac{u^{r+1}}{r+1} + C, r \neq -1$$

$$\int \frac{du}{u} = \ln|u| + C$$

$$\int e^u\ du = e^u + C$$

$$\int b^u\ du = \frac{b^u}{\ln b} + C, b > 0, b \neq -1$$

$$\int \sin u\ du = -\cos u + C$$

$$\int \cos u\ du = \sin u + C$$

$$\int \sec^2 u\ du = \tan u + C$$

$$\int \csc^2 u\ du = -\cot u + C$$

$$\int \sec u \tan u\ du = \sec u + C$$

$$\int \csc u \cot u\ du = -\csc u + C$$

$$\int \tan u\ du = -\ln|\cos u| + C$$

$$\int \cot u\ du = \ln|\sin u| + C$$

$$\int \sinh u\ du = \cosh u + C$$

$$\int \cosh u \, du = \sinh u + C$$

$$\int \operatorname{sech}^2 u \, du = \tanh u + C$$

$$\int \operatorname{csch}^2 u \, du = -\coth u + C$$

$$\int \operatorname{sech} u \tanh u \, du = -\operatorname{sech} u + C$$

$$\int \operatorname{csch} u \coth u \, du = -\operatorname{csch} u + C$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \left| \frac{u}{a} \right| + C$$

$$\int \frac{du}{\sqrt{a^2 + u^2}} = \ln(u + \sqrt{u^2 + a^2}) + C$$

$$\int \frac{du}{\sqrt{u^2 - a^2}} = \ln \left| u + \sqrt{u^2 - a^2} \right| + C$$

$$\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{a + u}{a - u} \right| + C$$

$$\int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

$$\int \frac{du}{u\sqrt{a^2 + u^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 + u^2}}{u} \right| + C$$

$$\int Uv = UV - \int uV \; ; U = \int u \; ; V = \int v$$

$$\int \sin^n x \, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x \, dx$$

$$\int \cos^n x \, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx$$

For $\int \sin^m x \cos^n x \, dx$;

n is odd, $u = \sin x$, $\cos^2 x = 1 - \sin^2 x$

m is odd, $u = \cos x$, $\sin^2 x = 1 - \cos^2 x$
m and n is even, $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$, $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$$\int \tan^n \ dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \ dx$$

$$\int \sec^n x \ dx = \frac{\sec^{n-2} x \ tanx}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x \ dx$$

$$\int \tan x \ dx = \ln|\sec x| + C$$

$$\int \sec x \ dx = \ln|\sec x + \tan x| + C$$

For $\int \tan^m x \sec^n x \ dx$;
n is even, $u = \tan x$, $\sec^2 x = \tan^2 x + 1$
m is odd, $u = \sec x$, $\tan^2 x = \sec^2 x - 1$
m is even and n is odd, $\tan^2 x = \sec^2 x - 1$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha-\beta) + \sin(\alpha+\beta)]$$

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$

Trigonometric substitutions;
$\sqrt{a^2 - x^2} \to x = a\sin\theta \to a^2 - a^2\sin^2\theta \to a^2(1-\sin^2\theta) \to a^2\cos^2\theta$
$\sqrt{a^2 + x^2} \to x = a\tan\theta \to a^2 + a^2\tan^2\theta \to a^2(1+\tan^2\theta) \to a^2\sec^2\theta$
$\sqrt{x^2 - a^2} \to x = a\sec\theta \to a^2\sec^2\theta - a^2 \to a^2(\sec^2\theta - 1) \to a^2\tan^2\theta$

Area between two curves
For two continuous functions f and g , if $f(x) \geq g(x)$ for $a \leq x \leq b$, the area between these functions is defined as;

$$A = \int_a^b [f(x) - g(x)] \ dx$$

Volume of a solid with a cross sectional area of $A(x)$

$$V = \int_a^b A(x)\ dx$$

Volume by disks perpendicular to the x-axis

$$V = \int_a^b A(x)\ dx = \int_a^b \pi[f(x)]^2\ dx$$

Volume by washers perpendicular to the x-axis

$$V = \int_a^b A(x)\ dx = \pi \int_a^b [f(x)]^2 - [g(x)]^2\ dx$$

Volume of solids of revolution

$$V = \int_a^b 2\pi x f(x)\ dx$$

Length of arc

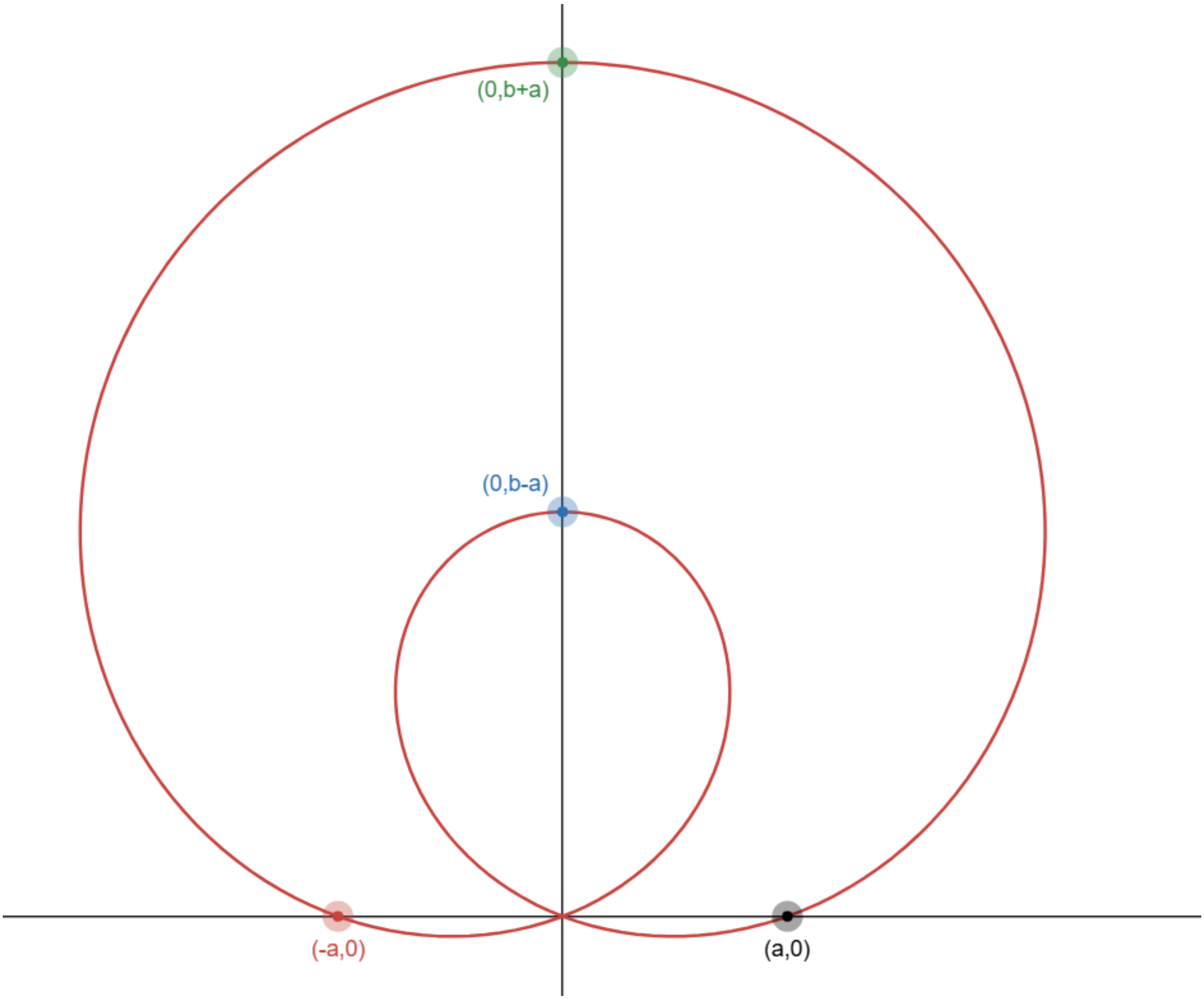
$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\ dx$$

Surface area of a surface of revolution

$$S = \int_a^b 2\pi f(x)\sqrt{1 + [f'(x)]^2}\ dx$$

Properties of Cardioid equations in the form ;

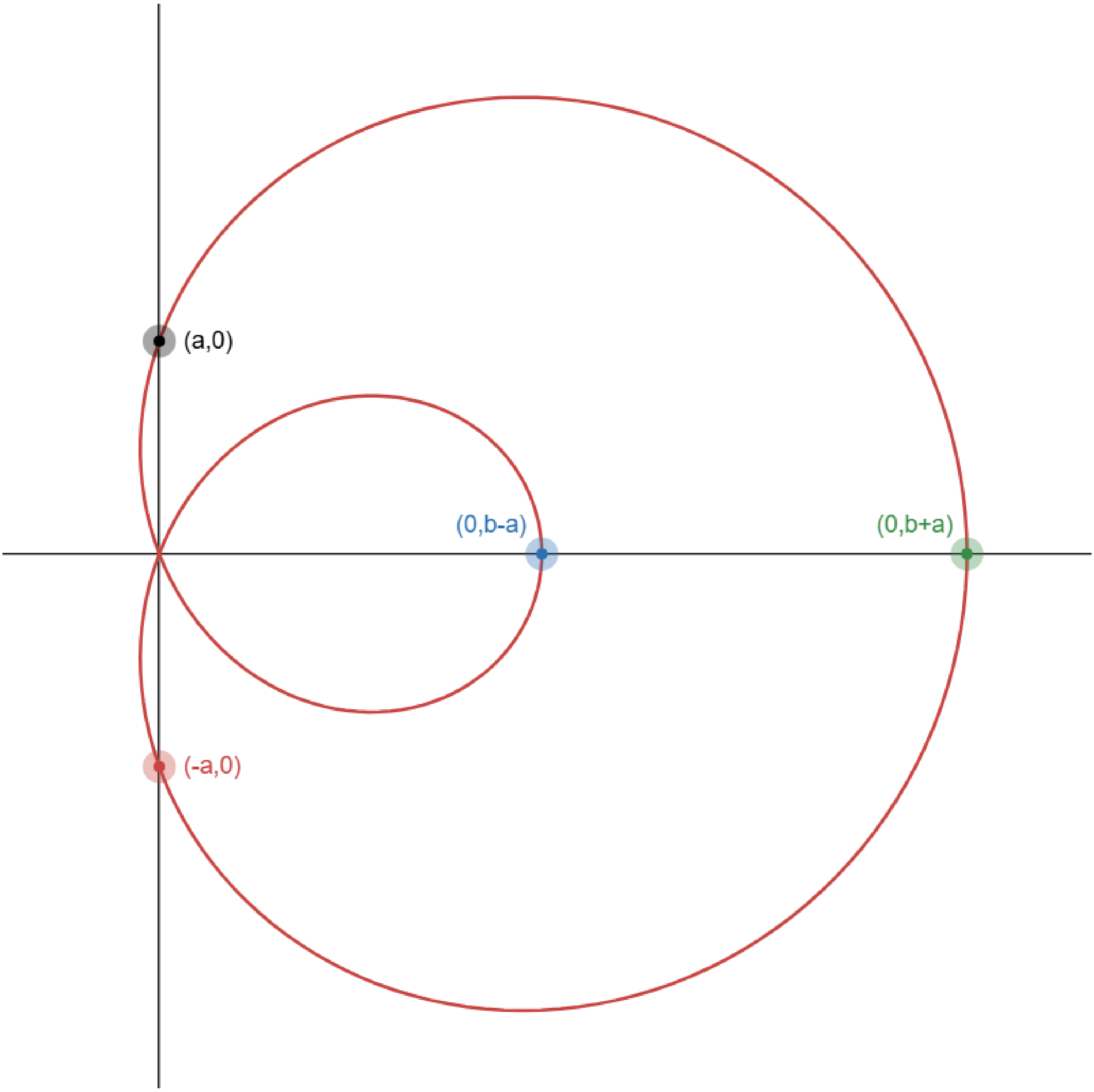
$$r = a + b\sin(n\theta)$$



$$A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} r^2 \, d\theta$$

Properties of Cardioid equations in the form;

$$r = a + b\cos(n\theta)$$



$$A = \int_0^{\pi} r^2 \, d\theta$$

Area of rose equations;
where a = 0;

	$\sin$	$\cos$
n is even	$n \int_0^{\frac{\pi}{n}} r^2 \, d\theta$	$2n \int_0^{\frac{\pi}{2n}} r^2 \, d\theta$

	$\sin$	$\cos$
n is odd	$\frac{n}{2}\int_0^{\frac{\pi}{n}} r^2 \, d\theta$	$n\int_0^{\frac{\pi}{2n}} r^2 \, d\theta$