Introduction to Linear Algebra

Linear Systems

Linear Equations

A linear equation in n variables $x_1, x_2, x_3, ..., x_n$ is expressed in the form

$$a_1x_1 + a_2x_2 + ... + a_nx_n = b$$

where $a_1, a_2, ..., a_n$ and b are real constants

Solution of a linear equation
A solution of a linear equation is a sequence of numbers such that the equation is satisfied when we substitute the variables with the solution values.
A parameter is an arbitrary number which is assigned to a variable, the solution set is described in terms of this parameter.

Linear Systems

A finite set of linear equations in the variables $x_1, x_2, x_3, ..., x_n$ is called a linear system. Every system of linear equations has no solutions, or has exactly one solution, or has infinitely many solutions.

Consistent Equations

If there is at least on solution to a linear system it is said to be consistent.
System of equations with no solutions are called inconsistent.

Augmented Matrices

An augmented matrix is used to show a linear system in a matrix form where the elements of the matrix are all the coefficients of the linear system of equations in their respective order.

Elementary Row Operations

1. Multiplying an equation by a nonzero constant
2. Interchanging two equations
3. Add a multiple of one equation to another

Row - Echelon Form

1. If a row consists entirely of zeros, they are grouped together at the bottom of the matrix.
2. If a row does not consist entirely of zeros, the first non zero number should be a 1 (Leading 1).

3. If any two successive rows do not consist of entirely of zeros, the leading 1 in the lower row occurs farther right than the leading 1 in the higher row.

$$\begin{bmatrix} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 1 & * \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Reduced Row - Echelon Form

All qualities are same as a row echelon form except an extra point which states that;

1. Each column that contains a leading zero has zeros everywhere else in that column.

$$\begin{bmatrix} 1 & 0 & 0 & * \\ 0 & 1 & 0 & * \\ 0 & 0 & 1 & * \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Homogenous Equations

An equation is homogenous if all the constant terms are zero.
An homogenous equation always have a solution where all the values are 0. This is called the trivial solution, other solutions are called nontrivial solutions.
Possibilities of a homogenous system;

- The system has only the trivial solutions
- The system has infinitely many solutions in addition to the trivial solution

A homogenous system of linear equations with more unknowns than equations has infinitely many solutions.

Euclidean Vector Spaces