Expt-3: Demonstration of Hooke's Law using spiral spring

Objectives:

1. To measure the spring constant of a spiral spring and corresponding elastic potential energy using the Hooke's Law.
2. To determine the effective mass of the spring.

Apparatus:

Stand with a clamp, a set of slotted masses, spiral spring, meter scale, weighing scale and stop watch.

Theory:

Hooke's law of elasticity states that, for relatively small deformations of an object, the displacement or size of the deformation is directly proportional to the deforming force or load.

Consider a spring in its **relaxed state** that is, neither compressed nor extended. One end is fixed, and a particle-like object a block, say is attached to the other, free end. If we stretch the spring by pulling the block, the spring pulls on the block in the opposite direction. Similarly, if we compress the spring by pushing the block, the spring now pushes on the block in the opposite direction. This is because a spring force acts to restore the relaxed state, called the *restoring force*.

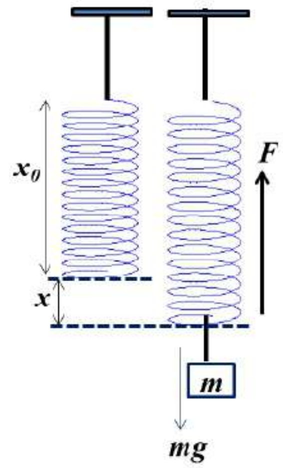

Figure 1: Spring mass system setup for Hooke's Law demonstration

To a good approximation for many springs, the force from a spring is proportional to the displacement of the free end from its position when the spring is in the relaxed state. The *spring force* is given by

$$F = -kx \tag{1}$$

which is known as **Hooke's law** for a spring mass system. The minus sign indicates that the direction of the spring force is always opposite the direction of the displacement of the spring's free end.

The constant k is called the **spring constant** (or **force constant**) and is a measure of the stiffness of the spring. The larger k is, the stiffer the spring; that is, the larger k is, the stronger the spring's pull or push for a given displacement. The SI unit for k is the Newton per meter.

In figure 1, x_o is the length of the spring with the mass holder hanging at rest at the equilibrium point. The displacement, x, is measured relative to the equilibrium point. Hooke's Law is valid within the elastic limit of the spring.

The work done by the spring force: $W = -\int_{x_i}^{x_f} F dx$ (since force and displacement oppositely directed)

$$= \int_{x_i}^{x_f} kx\, dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 \qquad (2)$$

If $x_i = 0$, $W = -\frac{1}{2}kx^2$ (work done by a spring force)

We know that the work done by a variable force could also be represented by the area under the *force-displacement* curve.

Also the corresponding elastic potential energy can be expressed as,

$$U = -W = \frac{1}{2}kx^2 \qquad (3)$$

The theoretical period of a system composed of a mass M oscillating at the end of a mass less spring of force constant k is given by,

$$T = 2\pi\sqrt{\frac{M}{k}} \qquad (4)$$

In a real spring–mass system, since no spring is mass less, the equation should modify by considering the effective mass m_e of the spring, which is defined as the mass that needs to be considered to correctly predict the behavior of the system.

$$T = 2\pi\sqrt{\frac{m_0+m_e}{k}}, \qquad (5)$$

where m_0 is the applied load.

The effective mass of the spring, in a spring-mass system when using an ideal spring of uniform linear density is 1/3 of the mass of the spring, i.e. $m_e = \frac{1}{3}m_s$ and is independent of the direction of the spring-mass system. Note that this effective spring mass is responsible for elongation when the spring is vertical.

Procedure:

1. Measure the mass of the spring M by using the digital balance.
2. Hang the spring vertically from the clamp and measure the length with meter ruler.
3. Add masses, one at a time, beginning with 150 grams. Increment the mass by 50 grams and record the length of the spring, *X*.
4. Compute the length of the spring for each added mass. Record the data in Table 1.
5. Determine the Force = mg (N) applied for each mass and record it in Table-1.
6. Slightly change the position of the mass holder and then release it - do not apply any external forces. The mass should be oscillating straight up and down. If not, stop it and try again until you get vertical oscillation.
7. With the digital stop watch count the time for 10 cycles for each mass added twice and then find the average time period. Note: All of your time values should have to the same first two digits - otherwise your experimental setup might have some flaws. Record your results in Table 1.
8. To gather the best results, one has to be very careful and consistent. The base of the apparatus should be firmly held and not allowed to move. Any motion of the apparatus outside the spring and the holder will increase the error in your results.
9. Plot the Force Applied (*F*) vs. Total Elongation (*L*), and determine the slope of the line. See Figure 2.
10. Using the slope of the graph determine the value of the spring constant, *k*.

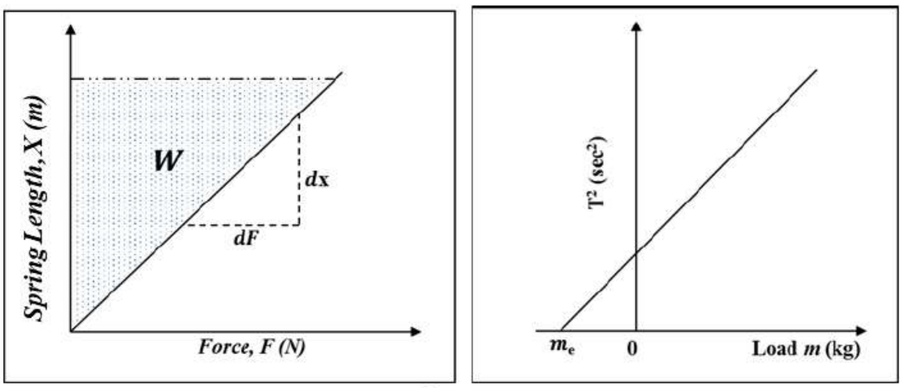

Figure 2: Extension vs Force and T^2 vs load graph for a spring mass system

11. For a specific load and extension find the area from F-L graph, for a triangle: area = ½ × base× height, this is the work done, W.
12. For the above load and extension find the corresponding elastic potential energy using the relation, $U = \frac{1}{2}kx^2$
13. Plot another graph with m (abscissa) against T^2 (ordinate) as shown in Figure 2 (right). Find out the effective mass (m_e) by taking the point of intercept of the resulting lines on *horizontal* axis.