Chapter Title: Vectors

Sections: Unit Vectors, Multiplying Vectors

Unit Vectors

It is a vector with a magnitude of 1, no units, and a direction in a particular orientation.

Purpose: To point a direction in space.

It has been distinguished from a vector by inserting a caret or hat ($\widehat{\ }$) sign.

Example, $\hat{\eta}$ is a unit vector.

In a 3D co-ordinate system, unit vectors are presented as, $\hat{\imath}$, j, and $\hat{k}$ along the x, y, and z axis.

Adding Vectors by Components

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

$$\vec{R} = \vec{A} + \vec{B}$$

$$\vec{R} = \left(A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}\right) + (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$$

$$\vec{R} = (A_x + B_x)\hat{\imath} + \left(A_y + B_y\right)j + (A_z + B_z)\hat{k}$$

$$\vec{R} = R_x\hat{\imath} + R_yj + R_z\hat{k}$$

Multiplying Vectors

A vector can be multiplied in 3 ways.

1. Multiplying a Vector by a Scalar
2. Multiplying a Vector by a Vector: That has two types:
    a) Scalar Product
    b) Vector Product

Multiplying Vectors: Multiplying a Vector by a Scalar

$$m\vec{A} = \vec{R}$$

Where m is a scalar and $\vec{R}$ is a new vector with the same direction as $\vec{A}$.

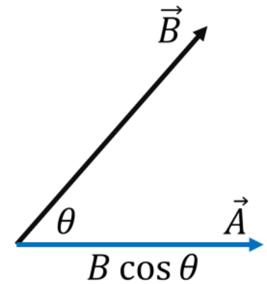
Multiplying Vectors: Multiplying a Vector by a Vector

Scalar Product

$$\vec{A}.\vec{B} = AB\cos\theta$$

It is also known as the dot product.

Here, θ is the angle between $\vec{A}$ and $\vec{B}$. $\vec{A}.\vec{B} = A(B\cos\theta) = A \times (Horizontal\ Projection)$. The dot product uses the horizontal or parallel projection of a vector, as it measures how much one vector aligns with or contributes to the direction of the other. If $\theta = 0°$, $B\cos\theta = B\cos 0° = B$, which gives a perfect projection value. Then, it reduces to $\vec{A}.\vec{B} = AB$, which is like a regular multiplication of two scalar numbers.



$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

$$\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

Scalar Product of Unit Vectors

$$\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$$

$$\hat{\imath}.\hat{\jmath} = \hat{\imath}.\hat{k} = \hat{\jmath}.\hat{k} = 0$$

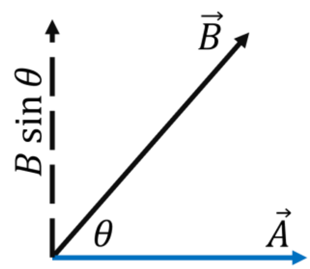
$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

$$\vec{A}.\vec{B} = \left(A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}\right).\left(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}\right)$$

$$\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

Multiplying Vectors: Multiplying a Vector by a Vector



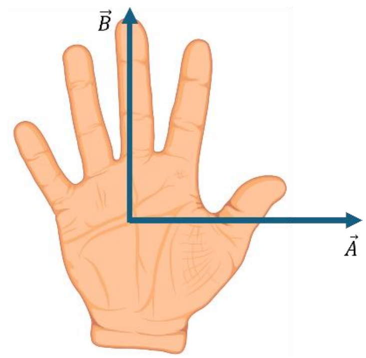
Vector Product

$$\vec{A} \times \vec{B} = AB\sin\theta\,\hat{\eta}$$

It is also known as the cross product.

Here, θ is the angle between $\vec{A}$ and $\vec{B}$. $\vec{A} \times \vec{B} = A(B\sin\theta) = A \times (Vertical\ Projection)$. The cross product uses the vertical or perpendicular projection of a vector, as it measures the area of the parallelogram formed by two vectors (*i.e.*, area of a parallelogram = Bh = base × height). Here, $B\sin\theta$ works as the height of a parallelogram, and A is the base of that parallelogram. If $\theta = 90°$, $B\sin\theta = B\sin 90° = B$, which gives a perfect projection value. Then, it reduces to

$\vec{A} \times \vec{B} = AB$, but the area is a vector that works perpendicular to the surface. Therefore, a unit vector is placed to provide the direction of the vector.



If $\vec{C}$ is a third vector such that,

$$\vec{C} = \vec{A} \times \vec{B}$$

The direction of $\vec{C}$ = ?

Use Right-Hand Rule

The direction of $\vec{B} \times \vec{A}$

$$\vec{C} = -\vec{B} \times \vec{A}$$

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

$$\vec{A} \times \vec{B} = ?$$

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

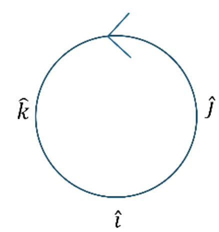
Vector Product of Unit Vectors

$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$$

$$\hat{\imath} \times \hat{\jmath} = -\hat{\jmath} \times \hat{\imath} = \hat{k}$$

$$\hat{\jmath} \times \hat{k} = -\hat{k} \times \hat{\jmath} = \hat{\imath}$$

$$\hat{k} \times \hat{\imath} = -\hat{\imath} \times \hat{k} = \hat{\jmath}$$



Anti-Clockwise

$$\vec{A} \times \vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) \times (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$$

$$\vec{A} \times \vec{B} = \left(A_yB_z - B_yA_z\right)\hat{\imath} + (A_zB_x - B_zA_x)\hat{\jmath} + (A_xB_y - B_xA_y)\hat{k}$$