Chapter Title: Equilibrium and Elasticity

Sections: Equilibrium, The Requirements of Equilibrium, The Center of Gravity, Elasticity, Tension and Compression, Shearing, Hydraulic Stress

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Equilibrium

An object is considered in equilibrium conditions if the following two conditions are satisfied.

1. The linear momentum $\vec{P}$ of its center of mass is constant. ($\vec{P} = \text{Constant}$)
2. Its angular momentum $\vec{L}$ about its center of mass, or about any other point, is also constant ($\vec{L} = \text{Constant}$)

If Newton's 2nd Law of Motions are applied, those conditions are reduced to,

$$\vec{F}_{net} = \frac{d\vec{P}}{dt} = 0$$

Here, that means, all the forces are balanced. Similarly,

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt} = 0$$

All the torques are balanced. There is no translational and rotational motion. Such conditions are called in static equilibrium.

Conditions for static equilibrium:

1. The vector sum of all the external forces that act on the body must be zero.
2. The vector sum of all external torques that act on the body, measured about any possible point, must also be zero.

If a body returns to a state of static equilibrium after having been displaced from that state by a force, the body is said to be in *stable static equilibrium*.

If a small force can displace the body and end the equilibrium, the body is in *unstable static equilibrium*.

Center of Gravity

The gravitational force on a body is the vector sum of all the gravitational forces acting on the individual elements (the atoms) of the body.

Instead of considering all those individual elements, the quantity 'center of gravity' is used.

The gravitational force $\vec{F}_g$ on a body effectively acts at a single point, called the center of gravity (cog) of the body. Here, the word "effectively" means that if the *gravitational forces* on the individual elements were somehow turned off and the *gravitational force* $\vec{F}_g$ at the center of gravity were turned on, therefore, the net force and the net torque (about any point) acting on the body would not change.

Location: If $\vec{g}$ is the same for all elements of a body, then the body's center of gravity (cog) is coincident with the body's center of mass (com). That means,

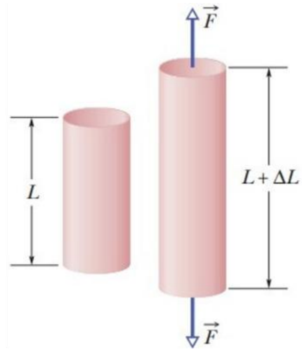
$$x_{cog} = x_{com}$$

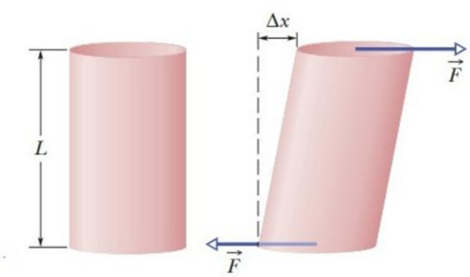
Elasticity

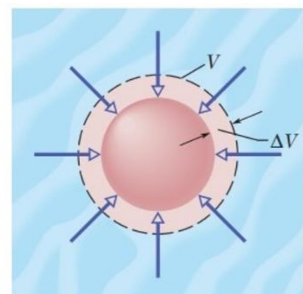
All real "rigid" bodies are to some extent elastic, which means that we can change their dimensions slightly by pulling, pushing, twisting, or compressing them. The rigid body is a useful idealized model, but the stretching, squeezing, and twisting of real bodies when forces are applied, is significantly important to consider.

Rigid body deformation

A rigid body can be deformed in the following three ways.







Tension & Compression Stretching or Shearing Hydraulic Stress

Elastic: If the object returns to its original shape when the forces are removed, it is said to be elastic.

Elastic Limit: Most objects are elastic for forces up to a certain maximum, called the elastic limit.

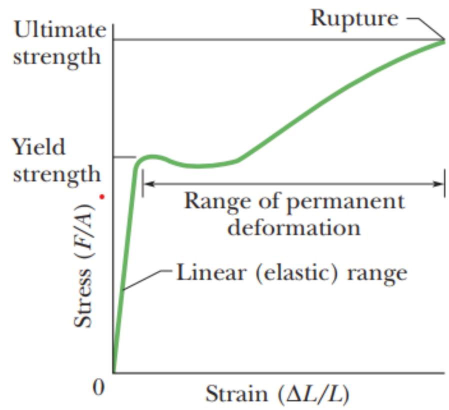
Stress: If the forces exceed the elastic limit, the object does not return to its original shape but is permanently deformed. For each kind of deformation, a quantity called *stress* is introduced that characterizes the strength of the forces causing the deformation, on a "force per unit area" basis.

Strain is another quantity used for characterizing deformation, describes the resulting deformation.

When the stress and strain are small enough, the two are directly proportional, and the proportionality constant is called an *elastic modulus* or *modulus of elasticity*.

That means, the harder you pull on something, the more it stretches; the more you squeeze it, the more it compresses.

$$\frac{Stress}{Strain} = Elastic\ Modulus$$



The proportionality of stress and strain (under certain conditions) is called *Hooke's law*, named after Robert Hooke (1635–1703).

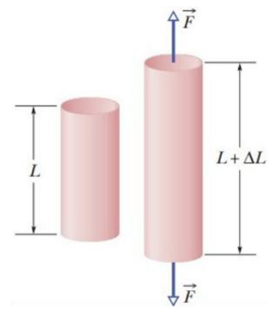
Yield Strength: If the stress is increased beyond the yield strength of the specimen, the specimen becomes permanently deformed.

Ultimate Strength: If the stress continues to increase, the specimen eventually ruptures, at a stress called the ultimate strength

Tension and Compression

The modulus for tensile and compressive stresses is called the *Young's modulus* and is represented in engineering practice by the symbol E or Y.

$$\frac{F}{A} = E\frac{\Delta L}{L}$$



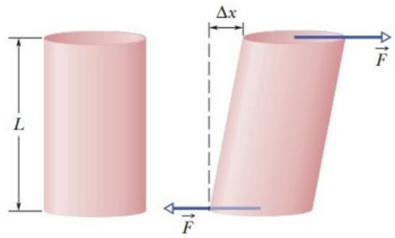
Tension & Compression

Shearing

In the case of shearing, the stress is also a force per unit area, but the force vector lies in the plane of the area rather than perpendicular to it.

The corresponding modulus, which is given the symbol G in engineering practice, is called the *shear modulus*.

$$\frac{F}{A} = G\frac{\Delta x}{L}$$



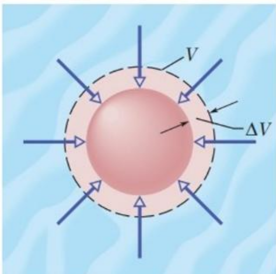
Shearing

Hydraulic Stress

The stress is a uniform pressure on all sides, and the resulting deformation is a volume change and the terms are used as bulk stress (or volume stress) and bulk strain (or volume strain).

When Hooke's law is obeyed, an increase in pressure (bulk stress) produces a proportional bulk strain (fractional change in volume). The corresponding elastic modulus (ratio of stress to strain) is called the *bulk modulus*, denoted by B. The object is said to be under *hydraulic compression*, and the pressure can be called the *hydraulic stress*.

$$p = B\frac{\Delta V}{V}$$



Hydraulic Stress