HW: 0

1) Identify the propositions, and show their relation using appropriate logical operators

a) If you are happy and watch movies, then your parents ask you to study

b) You are a Bangladeshi or if you are not a Bangladeshi, then your friend is European.

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a) p: You are happy

q: You watch movies

r: Your parents ask you to study

$$(p \wedge q) \longrightarrow r$$

b) p: You are Bangladeshi

q: Your friend is European

$$p \vee (\neg p \longrightarrow q)$$

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2) Use a series of logical equivalence to show that $(p \wedge (p \to q)) \to q$ is a Tautology / Contradiction / Contingency

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$(p \wedge (p \to q)) \to q \equiv (p \wedge (\neg p \vee q)) \to q$ $[A \to B \equiv \neg A \vee B]$

$\equiv [(p \wedge \neg p) \vee (p \wedge q)] \to q$ $[A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)]$

$\equiv [F \vee (p \wedge q)] \to q$ $[A \wedge \neg A \equiv F]$

$[A \vee F \equiv A]$

$\equiv \neg (p \wedge q) \vee q$

$\equiv \neg p \vee \neg q \vee q$ $[A \vee T \equiv T]$

$\equiv \neg p \vee (\neg q \vee q)$

$\equiv \neg p \vee T \equiv T$

3) Construct the Truth Table for the compound proposition defined as $(\neg q) \rightarrow (p \vee \neg r)$

p	q	r	$\neg q$	$\neg r$	$(p \vee \neg r)$	$(\neg q) \rightarrow (p \vee \neg r)$
T	T	T	F	F	T	T
T	T	F	F	T	T	T
T	F	T	T	F	T	T
T	F	F	T	T	T	T
F	T	T	F	F	F	T
F	T	F	F	T	T	T
F	F	T	T	F	F	F
F	F	F	T	T	T	T

4) Identify the propositions and show their relations
"For you to get a good joob in Pathao, it is sufficient for you to learn CSE 173".

p: You get a good job in Pathao
q: You learn CSE 173

$(q \rightarrow p)$

HW : 1

1) Determine if the below conditional statements are True or False.

a) If $2+7=6$, then crocodiles can fly

b) If $5+5=10$, then dogs can talk like humans

c) If -3 is a negative number, then birds can fly.

d) If $1+1=2$, then $5+7=12$

a) $p: 2+7=6$, $p=F$

q: crocodiles can fly, $q=F$

$\therefore p \rightarrow q \equiv F \rightarrow F \equiv$ True

b) $p: 5+5=10$, $p=T$

q: dogs can talk like humans, $q=F$

$\therefore p \rightarrow q \equiv T \rightarrow F \equiv$ False

c) p: -3 is a negative number, $p=T$

q: birds can fly, $q=T$

$\therefore p \rightarrow q \equiv T \rightarrow T \equiv$ True

d) $p: 1+1=2$, $p=T$

$q: 5+7=12$, $q=T$

$\therefore p \rightarrow q \equiv T \rightarrow T \equiv$ True

2) Let us assume that p and q are two propositions. Using p and q you are asked to form a number of compound proposition as shown in Table 2. Fill up the truth table for all the propositions

p	q	$\neg(\neg p \wedge q)$	$(p \vee q) \leftrightarrow \neg(\neg p \wedge q)$	$\neg(p \wedge \neg q)$	$(p \rightarrow q) \leftrightarrow \neg(p \wedge \neg q)$
T	T	T	T	T	T
T	F	T	T	F	T
F	T	F	F	T	T
F	F	T	F	T	T

3) Translate the below English sentences into propositional logic, making the propositional variables as clear as possible

a) Neither the thunderstorm nor the heavy rain did any damage to the house.

b) If global warming is not controlled, low lying land will go under water within the next few decades

c) Uber/Pathao should not drive more than 60 miles per hour nor violate traffic signals, or they will be penalised.

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a) p: Thunderstorm did damage to the house
q: Heavy rain did damage to the house

$$\neg p \wedge \neg q$$

b) p: Global warming is controlled
q: Low lying land will go underwater within next few decades

$$\neg p \rightarrow q$$

c) p: uber/Pathao driver drives more than 60 mph
q: Uber/Pathao driver violates traffic signals
r: Uber/Pathao driver is penalised

$$(\neg p \wedge \neg q) \longrightarrow \neg r$$

4) Show if ~~the~~ the compound proposition $(p \lor q) \land (\neg p \lor r) \rightarrow (q \lor r)$ is a Tautology or a contradiction

~~p q r ¬p p∨q ¬p∨r (p∨q)∧(¬p∨r) (p∨q)∧(¬p∨r) →~~

p	q	r	$\neg p$	$p \lor q$	$\neg p \lor r$	$q \lor r$	$(p \lor q) \land (\neg p \lor r)$	$(p \lor q) \land (\neg p \lor r) \rightarrow (q \lor r)$
T	T	T	F	T	T	T	T	T
T	T	F	F	T	F	T	F	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	F	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	T	F	T	T	F	T
F	F	F	T	F	T	F	F	T

All truth values are true,

$\therefore (p \lor q) \land (\neg p \lor r) \rightarrow (q \lor r)$ is a Tautology

5) Suppose that you are doing a part time job at the library at NSU. students visited the library often leave books on tables. Librarians asked you to check the condition of all the books, and the below statements are made available for you. Use predicate, Quantifiers and Logical Operators the statements made by the librarians:

a) Some books are not ~~at~~ at the right place

b) All books are at the right place and are in excellent condition

c) Every book is in the right place and is in excellent condition.

d) Nothing in the library is at the right place and is in excellent condition

e) One of the books is not in the right place, but it is in ~~ex~~ excellent condition

a) $P(x)$: x is in the right place

$Q(x)$: x is a book

$\forall x\,(Q(x) \rightarrow P(x))$

b) $P(x)$: x is in the right place

$Q(x)$: x is in excellent condition

$R(x)$: x is a book

$\forall x\,(R(x) \rightarrow (P(x) \wedge Q(x)))$

c) $P(x)$: x is in the right place

$Q(x)$: x is in excellent condition

$R(x)$: x is a book

$\forall x\,(R(x) \rightarrow (P(x) \wedge Q(x)))$

d) $P(x)$: x is in the right place

$Q(x)$: x is in excellent condition

$\neg \exists x\,(P(x) \wedge Q(x))$

$\equiv \forall x\,(\neg P(x) \vee \neg Q(x))$

e) $P(x)$: x is in the right place

$Q(x)$: x is in excellent condition

$R(x)$: x is a book

$\exists x\,[R(x) \wedge (\neg P(x) \wedge Q(x))]$

6) Write a compound proposition involving the propositional variables p, q and r that is true when p or q are true and r is false, the proposition is false other wise.

$(p \vee q) \wedge \neg r$

HW: 2

1.) Use quantifiers to express the below statements, and then derive the negation of the statement. Make sure that no negation lies in the left of the quantifier. Finally, express the obtained negation in English text.

a) All cats have parasites.

b) There is a cow that can add two numbers

c) Every monkey you encounter can climb

d) There is a fish that can speak Bengali

e) There exists a horse that can fly and catch bird as needed.

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a) $C(x)$: x is a cat

$P(x)$: x has parasites

$\neg \forall x (C(x) \to P(x))$

$\equiv \neg \forall x (\neg C(x) \vee P(x))$

$\equiv \exists x \neg (\neg C(x) \vee P(x))$

$\equiv \exists x (C(x) \wedge \neg P(x))$

There exists a cat which does not have parasites

b) $C(x)$: x is a cow

$A(x, n)$: x can add n numbers

$\neg \exists x (C(x) \wedge A(x, 2))$

$\equiv \forall x (\neg C(x) \vee \neg A(x,2))$

$\equiv \forall x (C(x) \to (\neg A(x,2)))$

All cows cannot add two numbers

c) $M(x)$: x is a monkey

$E(x)$: You encounter x

$C(x)$: x can climb

$$\neg \cancel{\exists x} \forall x [[M(x) \land E(x)] \rightarrow C(x)]$$

$$\equiv \exists x \neg (\neg (M(x) \land E(x)) \lor C(x))$$

$$\equiv \exists x [\neg (\neg M(x) \lor \neg E(x)) \land \neg (C(x))$$

$$\equiv \exists x ((M(x) \land E(x)) \land \neg C(x))$$

You will encounter atleast one monkey ~~with~~ which cannot climb

d) $F(x)$: x is a fish

$B(x)$: x can speak Bengali

$$\neg \exists x [F(x) \land B(x)]$$

$$\equiv \forall x \neg [F(x) \land B(x)]$$

$$\equiv \forall x [\neg F(x) \lor \neg B(x)]$$

$$\equiv \forall x (F(x) \rightarrow \neg B(x))$$

Not a single fish can speak Bengali

e) $H(x)$: x is a horse

$F(x)$: x can fly

$B(x)$: x can catch birds

$$\neg \exists x [H(x) \land F(x) \land B(x)]$$

$$\equiv \forall x \neg (H(x) \land F(x) \land B(x)]$$

$$\equiv \forall x [\neg H(x) \lor \neg F(x) \lor \neg B(x)]$$

Not a single horse can fly or catch birds

2) Assume $Q(x,y)$ as the statement saying student x in CSE173 class is a contestant on TV reality show y. Express the below sentences using $Q(x,y)$, and other logical connectives. Consider all students in CSE173 class as the domain for x and all TV reality shows as the domain for y.

a) There is a student at CSE173 who is a contestant on a TV reality show

b) No student at CSE173 has ever been a contestant on a TV reality show

c) There is a student at CSE173 who is a contestant on Close Up and Bangladeshi Idol

d) Every TV reality show aired so far had a student from CSE173 as a contestant.

e) Atleast two students from CSE173 are the contestants on Bangladeshi Idol.

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a) $\exists x \exists y\, Q(x,y)$

b) $\forall x \forall y\, \neg Q(x,y)$

c) $\exists x [Q(x, \text{Close Up}) \land Q(x, \text{Bangladesh Idol})]$

d) $\forall y \exists x\, Q(x,y)$

e) $\exists x_1 \exists x_2 [(x_1 \neq x_2) \land Q(x_1, \text{Bangladesh Idol}) \land Q(x_2, \text{Bangladesh Idol})]$

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3) Derive the negation of the below logical expressions; use logical equivalences and move the negation operator onto the smallest element possible. For instance, negation of $\forall x [P(x) \rightarrow Q(x)]$ is obtained as per the criteria stated as follows: $\neg \forall x [P(x) \rightarrow Q(x)]$, convert this to $\exists x [\neg(P(x) \rightarrow Q(x))]$, and finally to $\exists x [P(x) \land \neg Q(x)]$

a) $\forall x [P(x) \lor Q(x)]$

b) $\exists y [P(y) \lor (Q(y) \lor R(y))]$

c) $\exists x [(P(x) \land Q(x)) \lor (Q(x) \land \neg P(x))]$

a) $\neg \forall x \, (P(x) \vee Q(x))$

$\equiv \exists x \neg (P(x) \vee Q(x))$

$\equiv \exists x (\neg P(x) \wedge \neg Q(x))$

b) $\neg \exists y \, [P(y) \vee (Q(y) \vee R(y))]$

$\equiv \forall y \neg [P(y) \vee (Q(y) \vee R(y))]$

$\equiv \forall y (\neg P(y) \wedge \neg Q(y) \wedge \neg R(y))$

c) $\neg \exists x \, [(P(x) \wedge Q(x)) \vee (Q(x) \wedge \neg P(x))]$

$\equiv \forall x \neg [(P(x) \wedge Q(x)) \vee (Q(x) \wedge \neg P(x))]$

$\equiv \forall x \, [\neg (P(x) \wedge Q(x)) \wedge \neg (Q(x) \wedge \neg P(x))]$

$\equiv \forall x \, [(\neg P(x) \vee \neg Q(x)) \wedge (\neg Q(x) \vee P(x))]$

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4) Use predicates, quantifiers, logical connectives and mathematical operators to express the below mathematical statements. Consider all integers as the domain

a) If m and n are both negative, their product is always positive

b) Assume m and n are positive, then average of m and n is positive.

c) If m and n are negative, m-n is not necessarily negative

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a) $\cancel{[(m<0) \wedge (n<0)] \rightarrow (mn>0)}$, $\cancel{n, m \in \mathbb{Z}}$

a) $\cancel{\forall m \forall n \, P(m,n)}$; $\cancel{P(m,n) : ((m<0) \wedge (n<0)) \rightarrow (mn>0)}$, $n \in \mathbb{Z}$, $m \in \mathbb{Z}$

b) $\cancel{\forall m \forall n \, P(m,n)}$; $\cancel{P(m,n) : ((m>0) \wedge (n>0)) \rightarrow \left(\frac{m+n}{2} > 0\right)}$, $n \in \mathbb{Z}$, $m \in \mathbb{Z}$

a) 2

a) $P(m): m<0$

$Q(n): n<0$

$R(m,n): mn>0$

$\forall m \forall n \left[[P(m) \land Q(n)] \rightarrow R(m,n) \right]$

b) $P(m): m>0$

$Q(n): n>0$

$R(m,n): \frac{m+n}{2} > 0$

$\forall m \forall n \left[(P(m) \land Q(n)) \rightarrow R(m,n) \right]$

c) $P(m): m<0$

$Q(n): n<0$

$R(\cancel{x}m,n): m-n<0$

$\exists \cancel{\forall} m \exists n \left[(P(m) \land Q(n)) \rightarrow R(m,n) \right]$