Chapter Title: Gravitation

Sections: Escape Speed, Satellites: Orbits and Energy

Chapter Title: Oscillations

Sections: Simple Harmonic Motion

Satellites: Orbits and Energy

Satellite:

As a satellite orbits Earth in an elliptical path, both its speed, which fixes its kinetic energy K, and its distance from the center of Earth, which fixes its gravitational potential energy U, fluctuates with fixed periods.

However, the mechanical energy E of the satellite remains constant.

- Depending on energy, the orbits can be classified into two systems.
  1. If total energy $E < 0$, the system is bound
  2. If total energy $E \geq 0$, the system is unbound

When E is negative, its absolute value $|E|$ is called the binding energy.

The binding energy is the energy that must be added to the system to bring the total energy up to zero.

$$E = E_{circular} = -\frac{GMm}{2r}$$

$$E = E_{elliptical} = -\frac{GMm}{2a}$$

Here, if a satellite is in a circular orbit, r is the radius of the satellite's orbit, M and m are the masses of Earth and the satellite, respectively, and a is the semimajor axis if a satellite is in an elliptical orbit.

If a satellite's orbital period matches Earth's rotation period (24 hours) and circles in a circular orbit is called a geostationary orbit.

- Based on system category, Orbits can be classified into closed and open orbit systems.

Closed: When the path of the projectile returns to its starting point.

Open: When the path of the projectile never returns to its starting point.

Escape Speed

An object will escape the gravitational pull of an astronomical body if the object's speed near the body's surface is at least equal to the escape speed. It is represented by v_e or v_{esc}.

From the principle of conservation of energy, its total energy at the planet's surface must also have been zero.

$$K + U = 0$$

$$\frac{1}{2}mv_{esc}^2 + \left(-\frac{GMm}{R}\right) = 0$$

$$\frac{1}{2}mv_{esc}^2 - \frac{GMm}{R} = 0$$

$$\frac{1}{2}mv_{esc}^2 = \frac{GMm}{R}$$

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

If we substitute the values of G, M, and R for the earth, the escape velocity of the earth can be determined. $V_{earth} = 11.2\ km/s \approx 7mi/s \approx 25{,}000mi/h$.

Difference Between Maximum Height of Projectile Motion & Escape Speed

Maximum Height	**Escape Speed**
At maximum height, the velocity is zero.	It is coming to the rest at infinity.
For projectile upward flight, it is the final speed of that motion.	It is the minimum initial speed at which a projectile may initiate its motion.
For projectile downward flight, it is the initial speed of that motion.	It never returns.

Body	Mass (kg)	Radius (m)	Escape Speed (km/s)
Ceres[a]	1.17×10^{21}	3.8×10^{5}	0.64
Earth's moon[a]	7.36×10^{22}	1.74×10^{6}	2.38
Earth	5.98×10^{24}	6.37×10^{6}	11.2
Jupiter	1.90×10^{27}	7.15×10^{7}	59.5
Sun	1.99×10^{30}	6.96×10^{8}	618
Sirius B[b]	2×10^{30}	1×10^{7}	5200
Neutron star[c]	2×10^{30}	1×10^{4}	2×10^{5}

[a] The most massive of the asteroids.
[b] A *white dwarf* (a star in a final stage of evolution) that is a companion of the bright star Sirius.
[c] The collapsed core of a star that remains after that star has exploded in a *supernova* event.

Chapter Title: Oscillations

Sections: Simple Harmonic Motion

Oscillations

We studied two different types of motion before:

1. Translational Motion
2. Rotational Motion

What about the motion if it repeats itself in equal intervals of time?

The motion is called period motion.

Periodic motion is motion of an object that regularly returns to a given position after a fixed time interval.

Examples, a rocking chair, a water waves, Earth's motion

A special type of periodic motion is called **Oscillation**.

A body that undergoes periodic motion always has a **stable equilibrium position**.

Do Oscillation and Vibration describe the same meaning?

Although they are often interchangeably used in different literature or cultures. Oscillation" is used in a broader aspect that refers to any repetitive motion around a central point called an equilibrium position.



Vibration is an oscillation that can be characterized based on the system, such as a rapid, small movement in a mechanical system.

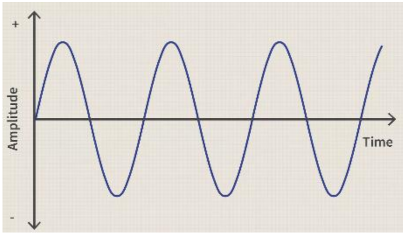
When it is moved away from this position and released, a force or torque comes into play to pull it back toward equilibrium.

Oscillations: Simple Harmonic Oscillation

One of the simplest types of oscillation is called Simple Harmonic Oscillation.

Simple Harmonic Motion (SHM) describes objects oscillating back and forth around a central point or equilibrium position

For examples, pendulums and guitar strings.

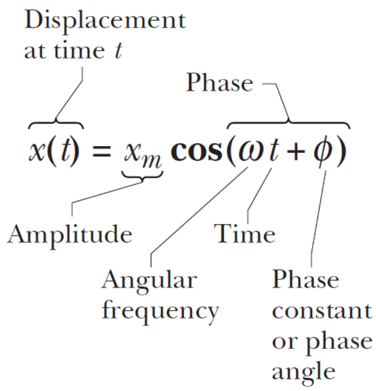


How does it come back and forth around a central point of this oscillation?

A restoring force acts on the oscillatory system that is proportional to displacement, leading to oscillations.

How do you demonstrate oscillation?

What are the variables/ parameters needed to describe an oscillation?



Frequency: The frequency f of the oscillation is the number of times per second that it completes a full oscillation (a cycle) and has the unit of hertz (abbreviated Hz).

$$1\ hertz = 1\ Hz = 1\ oscillation\ per\ second = 1\ s^{-1}$$

Period: The time for one full cycle is the period T of the oscillation.

$$T = \frac{1}{f}$$

Displacement: If an arbitrary cosine function is chosen, the displacement is,

$$x(t) = x_m \cos(\omega t + \phi)$$