EEE 141
ELECTRICAL CIRCUITS

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BASIC LAWS

Series Resistors and Voltage Division.

Parallel Resistors and Current Division.

Wye-Delta Transformations.

PARALLEL RESISTORS AND CURRENT DIVISION

Parallel: Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them.

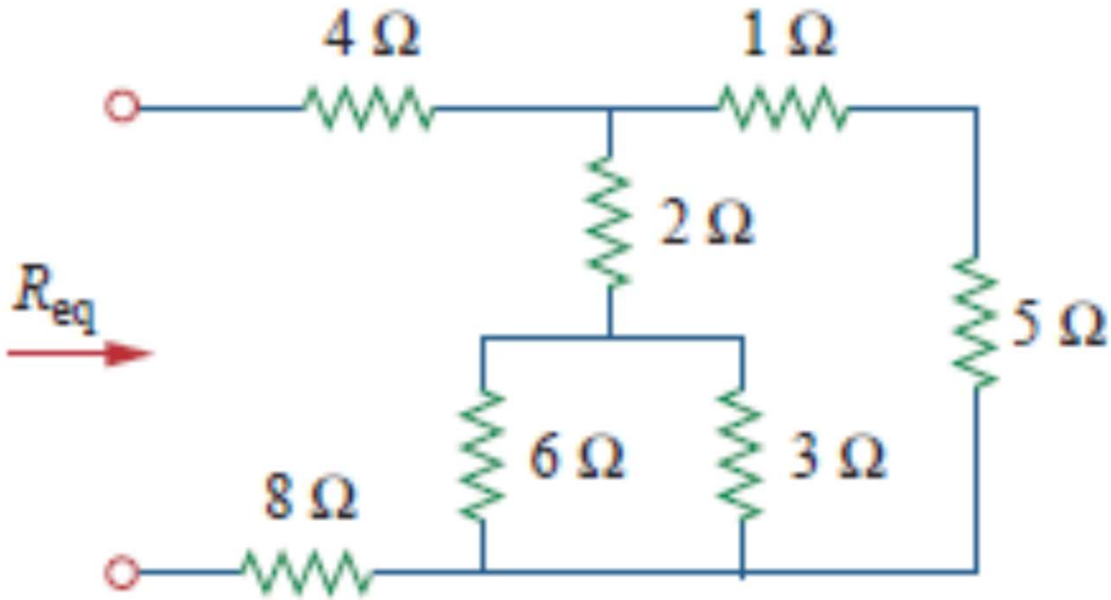
The equivalent resistance of a circuit with N resistors in parallel is:

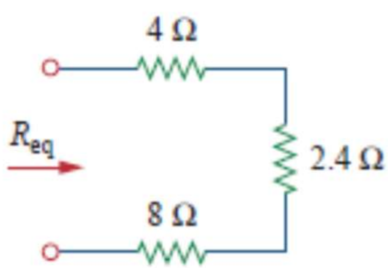
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

The total current i is shared by the resistors in inverse proportion to their resistances. The current divider can be expressed as:

$$i_n = \frac{v}{R_n} = \frac{iR_{eq}}{R_n}$$

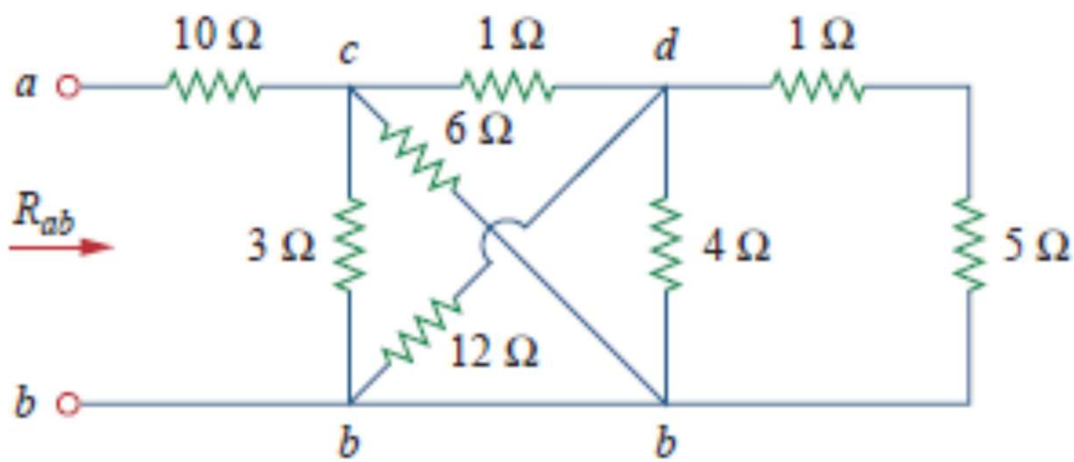
EXAMPLE





$$R_{eq} = 4\ \Omega + 2.4\ \Omega + 8\ \Omega = 14.4\ \Omega$$

EXAMPLE



$$2\ \Omega \parallel 3\ \Omega = \frac{2 \times 3}{2 + 3} = 1.2\ \Omega$$

This 1.2-Ω resistor is in series with the 10-Ω resistor, so that

$$R_{ab} = 10 + 1.2 = 11.2\ \Omega$$

EXAMPLE

FIG. 6.13
Example 6.7.

Solution: The network is redrawn in Fig. 6.14:

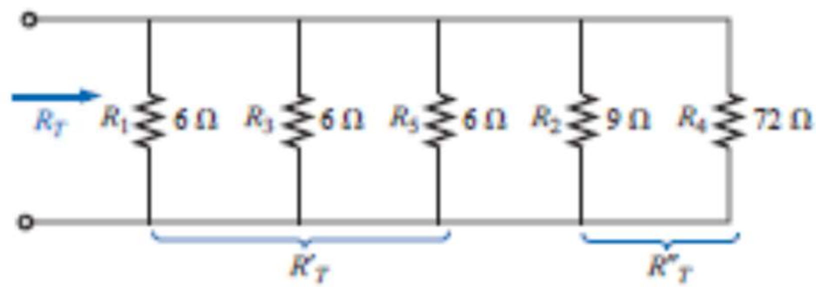


FIG. 6.14
Network of Fig. 6.13 redrawn.

$$R'_T = \frac{R}{N} = \frac{6\,\Omega}{3} = 2\,\Omega$$

$$R''_T = \frac{R_2 R_4}{R_2 + R_4} = \frac{(9\,\Omega)(72\,\Omega)}{9\,\Omega + 72\,\Omega} = \frac{648\,\Omega}{81} = 8\,\Omega$$

and $$R_T = R'_T \parallel R''_T$$

In parallel with

$$= \frac{R'_T R''_T}{R'_T + R''_T} = \frac{(2\,\Omega)(8\,\Omega)}{2\,\Omega + 8\,\Omega} = \frac{16\,\Omega}{10} = \mathbf{1.6\,\Omega}$$

EXAMPLE

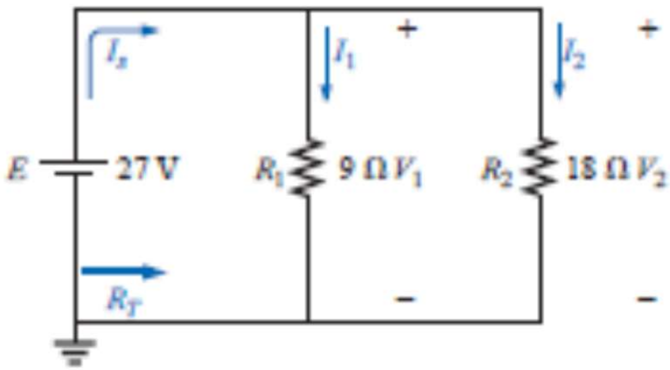


FIG. 6.22
Example 6.11.

EXAMPLE 6.11 For the parallel network of Fig. 6.22:

a. Calculate R_T.
b. Determine I_s.
c. Calculate I_1 and I_2, and demonstrate that $I_s = I_1 + I_2$.
d. Determine the power to each resistive load.
e. Determine the power delivered by the source, and compare it to the total power dissipated by the resistive elements.

Solutions:

a. $R_T = \dfrac{R_1 R_2}{R_1 + R_2} = \dfrac{(9\ \Omega)(18\ \Omega)}{9\ \Omega + 18\ \Omega} = \dfrac{162\ \Omega}{27} = \mathbf{6\ \Omega}$

EXAMPLE

EXAMPLE 6.12 Given the information provided in Fig. 6.23:

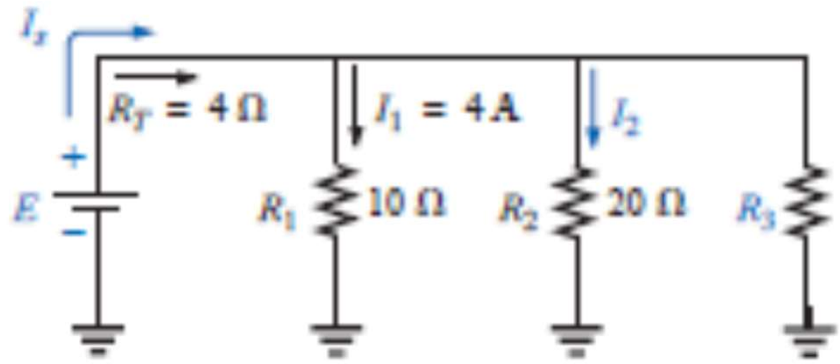


FIG. 6.23
Example 6.12.

a. Determine R_3.
b. Calculate E.
c. Find I_s.
d. Find I_2.
e. Determine P_2.

EXAMPLE

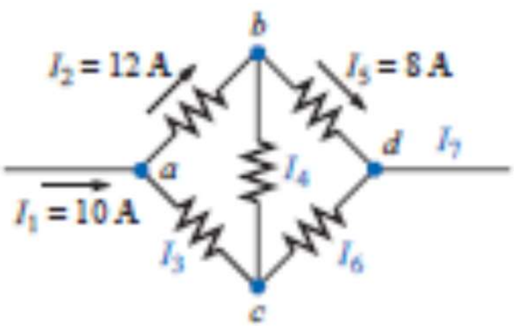


FIG. 6.30
Example 6.16.

EXAMPLE 6.16 Find the magnitude and direction of the currents I_3, I_4, I_6, and I_7 for the network of Fig. 6.30. Even though the elements are not in series or parallel, Kirchhoff's current law can be applied to determine all the unknown currents.

Solution: Considering the overall system, we know that the current entering must equal that leaving. Therefore,

$$I_7 = I_1 = \mathbf{10\ A}$$

Since 10 A are entering node *a* and 12 A are leaving, I_3 must be supplying current to the node.

CURRENT DIVIDER RULE

For the particular case of *two parallel resistors,* as shown in Fig. 6.34,

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

and

$$I_1 = \frac{R_T}{R_1} I = \frac{\frac{R_1 R_2}{R_1 + R_2}}{R_1} I$$

Note difference in subscripts.

and

$$\boxed{I_1 = \frac{R_2 I}{R_1 + R_2}} \qquad (6.10)$$

Similarly for I_2,

$$\boxed{I_2 = \frac{R_1 I}{R_1 + R_2}} \qquad (6.11)$$

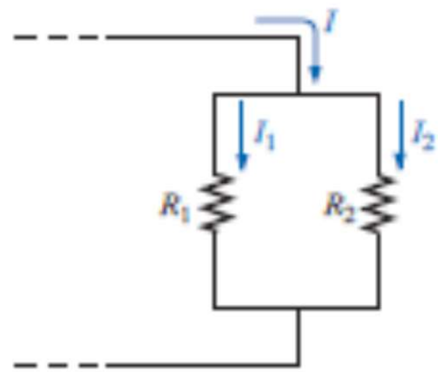


FIG. 6.34
Developing an equation for current division between two parallel resistors.

EXAMPLE

EXAMPLE 6.18 Find the current I_1 for the network of Fig. 6.36.

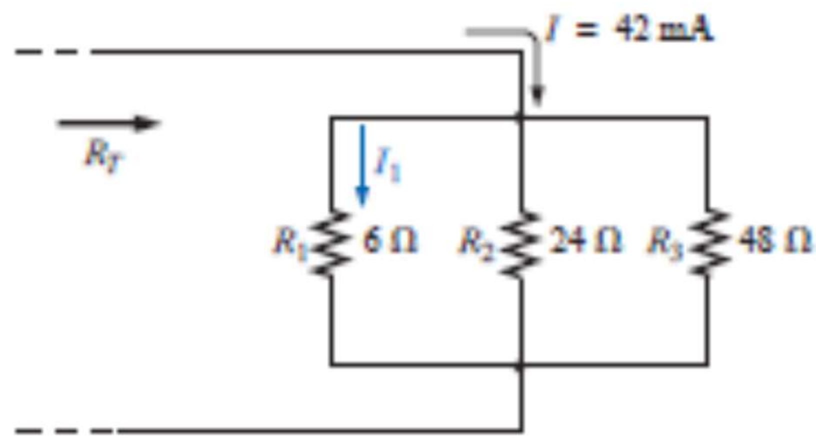
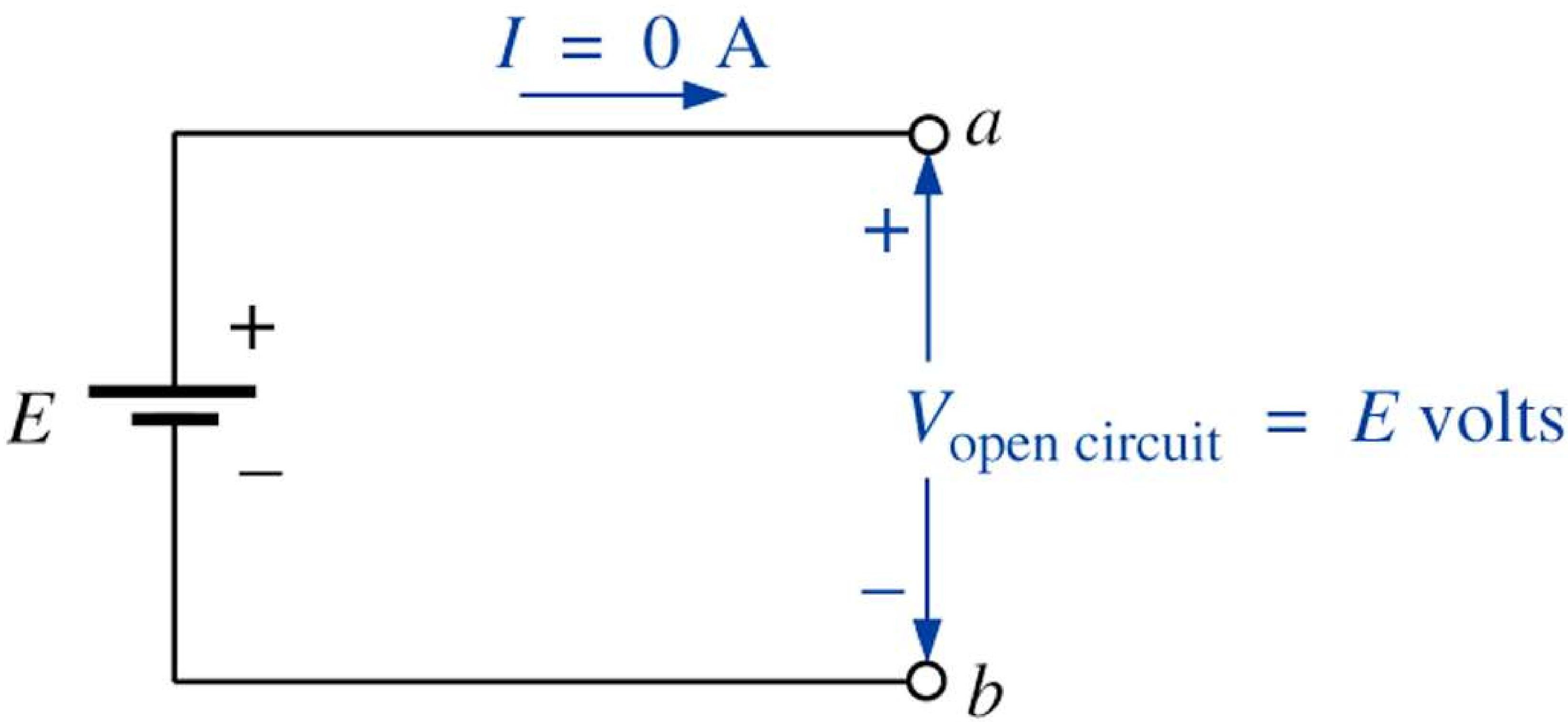


FIG. 6.36
Example 6.18

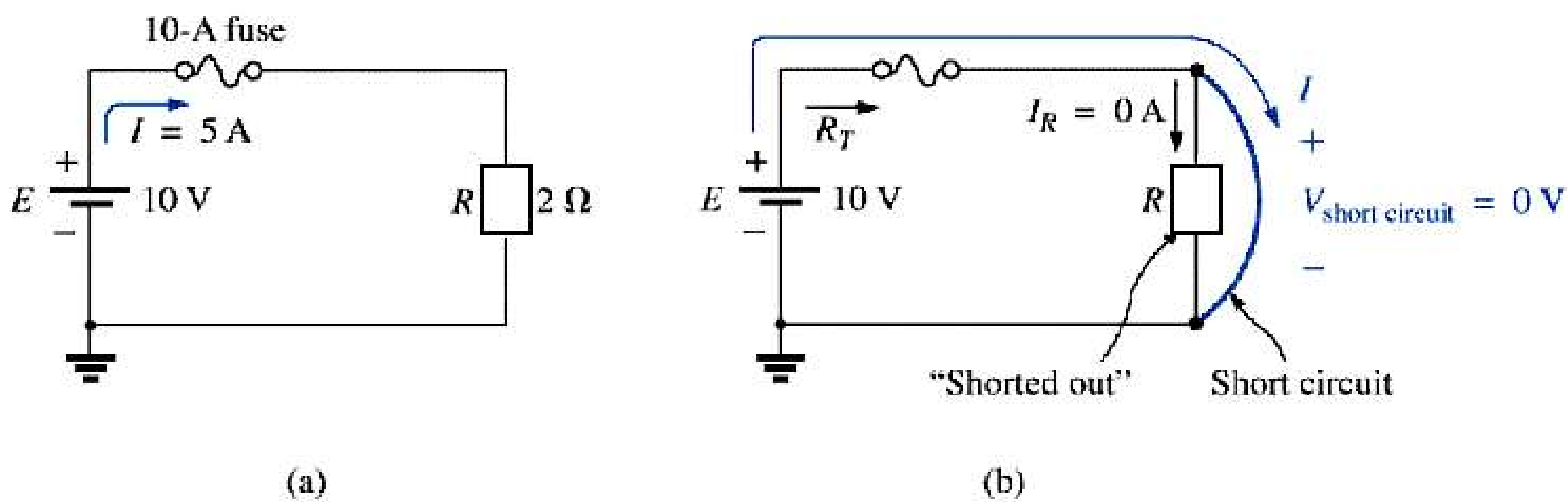
Solution: There are two options for solving this problem. The first is to use Eq. (6.9) as follows:

$$\frac{1}{R_T} = \frac{1}{6\ \Omega} + \frac{1}{24\ \Omega} + \frac{1}{48\ \Omega} = 0.1667\text{ S} + 0.0417\text{ S} + 0.0208\text{ S}$$
$$= 0.2292\text{ S}$$

OPEN CIRCUIT



SHORT CIRCUIT



EXAMPLE

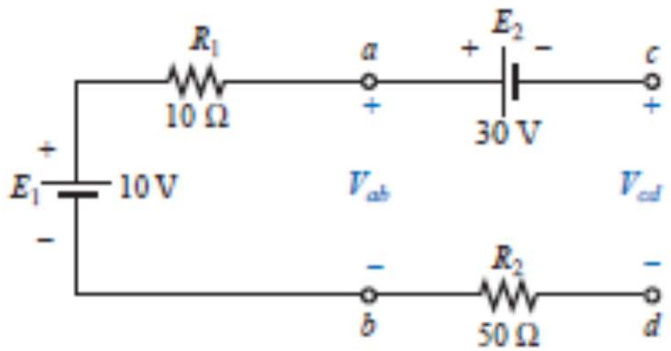


FIG. 6.46
Example 6.22.

EXAMPLE 6.22 Determine the voltages V_{ab} and V_{cd} for the network of Fig. 6.46.

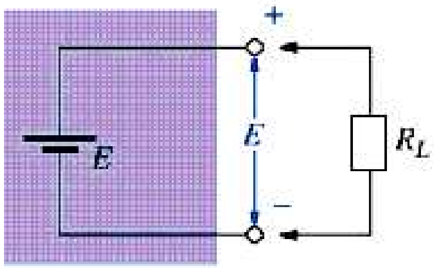
Solution: The current through the system is zero amperes due to the open circuit, resulting in a 0-V drop across each resistor. Both resistors can therefore be replaced by short circuits, as shown in Fig. 6.47. The voltage V_{ab} is then directly across the 10-V battery, and

$$V_{ab} = E_1 = \mathbf{10\ V}$$

The voltage V_{cd} requires an application of Kirchhoff's voltage law:

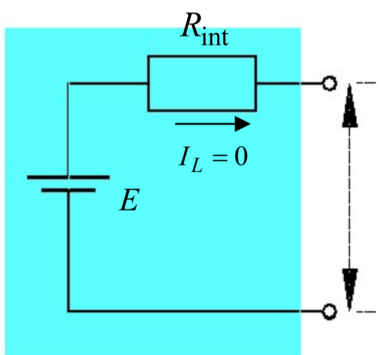
$$+E_1 - E_2 - V_{cd} = 0$$

INTERNAL RESISTANCE OF A VOLTAGE SOURCE



(a)

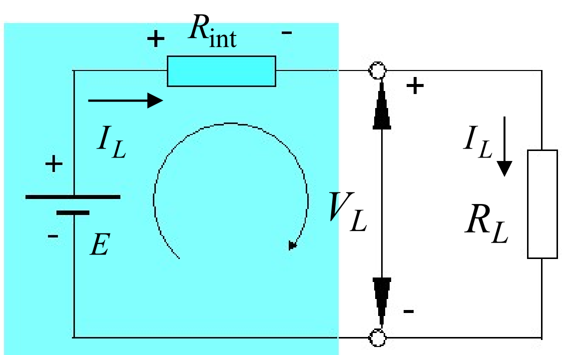
Ideal voltage source – no internal resistance



R_{int} = internal resistance of source

No Load Voltage

$$V_{NL} = E$$

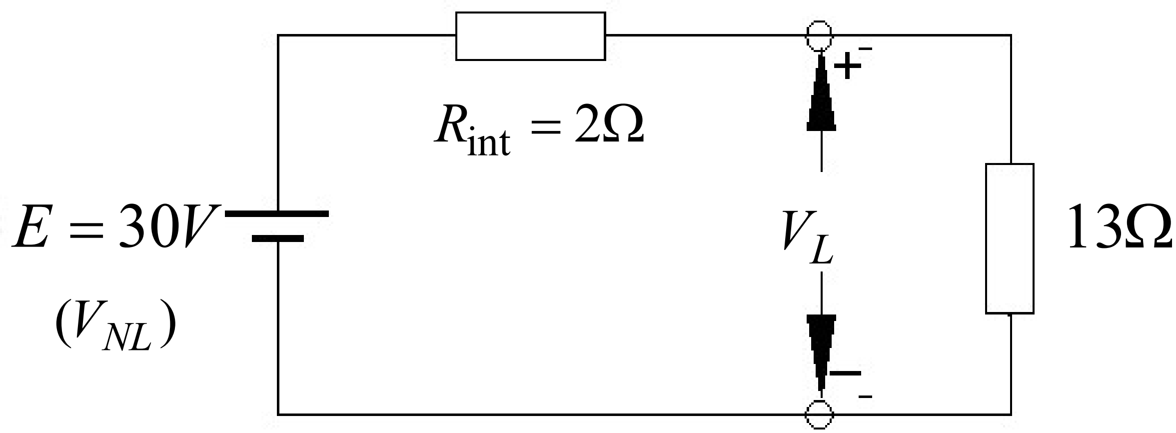


Load Voltage

$$V_L = \frac{R_L}{R_L + R_{int}} . E$$

EXAMPLE

Find V_L and power loss in R_{int}



$$I_L = \frac{30V}{2\Omega + 13\Omega} = \frac{30V}{15\Omega} = 2A$$

$$V_L = V_{NL} - I_L \times R_{int} = 30V - (2A)(2\Omega) = 26V$$

$$P_{lost} = I^2{}_L R_{int} = (2A)^2 (2\Omega) = (4)(2) = 8W$$

VOLTAGE REGULATION.

$$\% regulation = \frac{V_{NL} - V_{FL}}{V_{NL}} \times 100$$