Expt-6: Experiment with Compound Pendulum and determination of value of 'g'

Objectives:

1. To determine the g, the acceleration due to gravity.

2. To determine k, the radius of the gyration of the pendulum.

Apparatus:

1. Compound pendulum
2. Meter rule
3. Stop watch

Theory:

A physical pendulum or compound pendulum is a rigid object, which is free to rotate about a fixed horizontal axis. In this experiment, we use a special type of compound pendulum which is symmetric about its center of mass. This compound pendulum is nothing but a metal bar, containing a number of holes with equal intervals. The pendulum can be suspended by the help of knife edge passing through different holes. The point of suspension is known as pivot point. If we swing the bar from different holes then the moment of inertia of the pendulum and the time period will change.

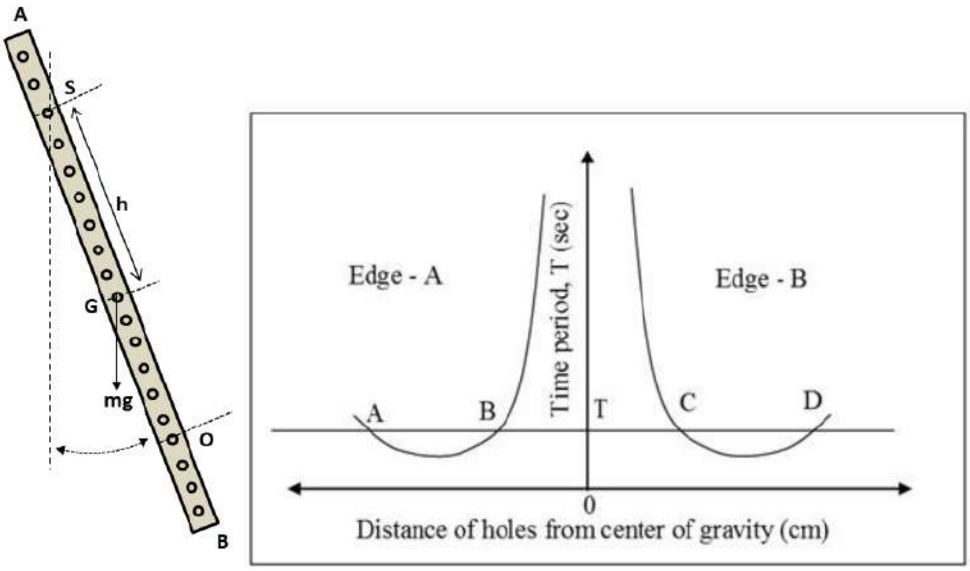

Figure 1: Demonstration of compound pendulum oscillations and corresponding time period vs distance graph.

Allowing the bar to swing it will approximately follow a simple harmonic motion. According to Newton's 2nd law of motion for rotation the torque (τ):

$$\tau = I\,\alpha \qquad (1)$$

where, I is the moment of inertia of the pendulum about the axis of rotation, and α the angular acceleration.

Torque is given by $\tau = -mgl\, sin\phi$, here l is the distance of the pivot from the center of the pendulum.

For very small angle of rotation $sin\phi$ can be approximated by $\emptyset$, then from equation (1)

$$I\frac{d^2\emptyset}{dt^2} = -mgl \tag{2}$$

By rearranging the equation, $$\frac{d^2\emptyset}{dt^2} + \frac{mgl}{I}\emptyset = 0 \tag{3}$$

This 2nd order differential equation describes the simple harmonic motion with the angular frequency, $\omega = \frac{2\pi}{T}$ and the time period from (3): $T = 2\pi\sqrt{\frac{I}{mgl}}$ (4)

Using "Parallel Axis Theorem: moment of inertia I, of an object about an axis parallel to the axis that passes through the center of mass,

$$I = I_G + ml^2 \tag{5}$$

where I_G the moment of inertia of the object about the axis through the center of mass, m is the total mass of the object, and l is the distance between the axes. We would also show that,

$$I_G = mK^2 \tag{6}$$

where K is the radius of gyration about the axis passing through the G.

Substituting Eq. 5 and Eq.6 in Eq.4 we get,

$$T = 2\pi\sqrt{\frac{\frac{K^2}{l}+l}{g}} \tag{7}$$

Comparing the time period relation for simple pendulum of length L, $T = 2\pi\sqrt{\frac{L}{g}}$, we can deduce,

$$L = l + \frac{k^2}{l} \tag{8}$$

From the above equation we can obtain a quadratic equation of l, which has 2 roots l_1 and l_2 such that,

$$L = l_1 + l_2 \tag{9}$$

$$K^2 = l_1 l_2 \tag{10}$$

The value of K and g can be determined from,

$$g = 4\pi^2\frac{L}{T^2} \tag{11}$$

$$K = \sqrt{l_1 l_2} \tag{12}$$

Since the "effective length L is composed of two roots l_1 and l_2, so there are infinite ways to combine $l_1$and l_2 to make the same L.

In this experiment, we will determine the length L and corresponding time period T graphically [see Figure 1]. If we plot a graph using table (1), two curves symmetric about the position of COM should appear. Horizontal lines in the lower portion will intersect the curves in four points. l_1 and l_2 can be determined by measuring the distances from the COM position. Using eqns. (11), acceleration due to gravity g and radius of gyration K would be calculated.

Procedure:

1. With the help of the knife edge suspend the metal bar by passing through the hook to the hole closer to the Edge A.
2. Measure the distance d from the center of gravity (middle hole) to the edge of the hole.
3. Oscillate the metal bar with an angle for a small angle.
4. Record the time for 10 oscillations using a stopwatch. Repeat it for two times and obtain the average time period T for that distance.
5. Repeat the procedure 1-4 for more holes of the bar, except the center of mass.
6. After procedure 5, again repeat procedures 1-4 by inverting the metal bar (Edge B) for all the holes.
7. Draw a graph $T\ vs\ d$ for Edge A and Edge B observations as shown in the Figure 1.
8. Draw a suitable horizontal line that intersects both curves. Mark A, B, C and D to the four points of intersection with the graph. Measure the length AC and BD, then find the length $L = \frac{AC+BD}{2}$ and corresponding T for the line and then find the value of$g$.
9. Repeat procedure 8 by drawing another horizontal line and find the value of$g$. Calculate the mean of$g$.
10. Calculate the value of K, by using the formula $K = \sqrt{l_1 \times l_2} = K = \sqrt{OA\ \times\ OC}\ = K = \sqrt{OD\ \times\ OB}$. Repeat the procedure for another line and then find the average value of K.