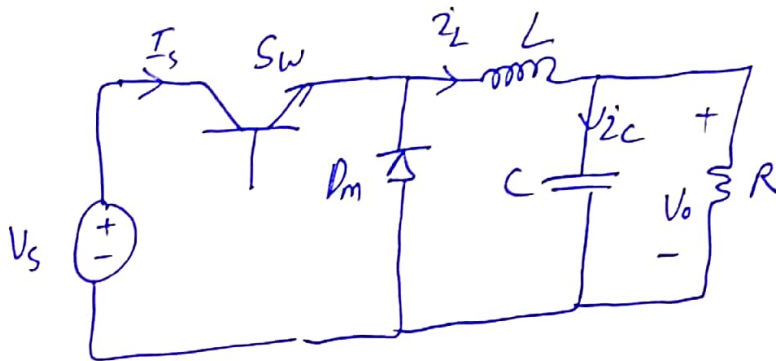


Isolated DC-DC Converters

Why we need isolation?



$$V_s = 200V$$

$$\text{Required } V_o = 2V$$

$$f_s = 200\text{kHz}$$

$$D = \frac{V_o}{V_s} = \frac{2}{200} = 0.01$$

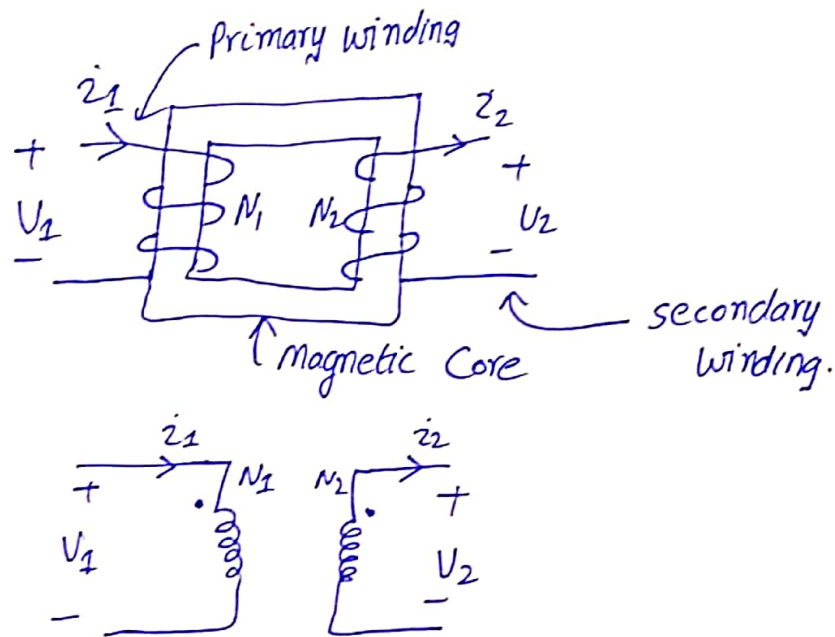
$$D = \frac{T_{on}}{T_s} \Rightarrow T_{on} = DT_s$$

$$T_{on} = (0.01)(5 \times 10^{-6}) = 50\text{ns}$$

- The on time of the switch is very small, the switch needs to be closed before it turns-on completely
- For such high conversion ratio, isolated dc-dc converters are preferred
- Many applications such as medical devices require isolation for safety reasons.
- Isolation is also required by regulatory agencies.

Transformer Model :-

- Transformer are used to provide electrical isolation between the input and output.
- To step-up or stepdown time Varying voltages and currents.

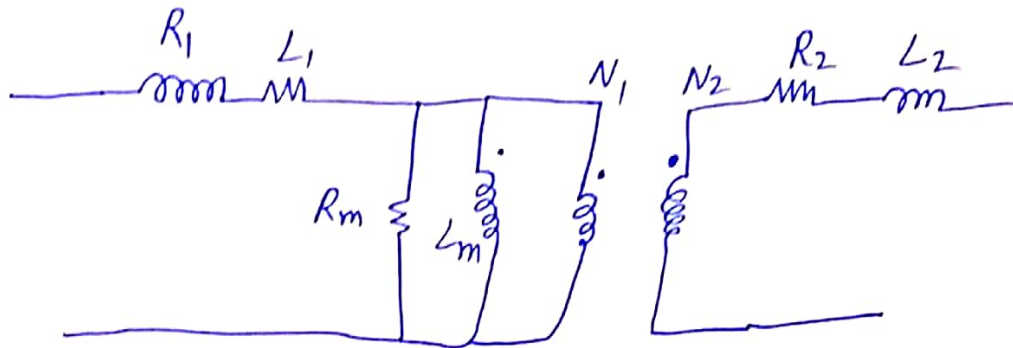


Dot Convention :-

- Dot convention is used to indicate relative Polarity between the two windings.
- When the Voltage at the dotted terminal of one winding is positive, then the Voltage at the dotted terminal of the other winding is also positive
- when current enters the dotted terminals on one winding then current leaves the dotted terminal on the other winding.

For an ideal transformer

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}$$



Actual transformer model.

$R_1 \Rightarrow$ Primary winding Resistance

$R_2 \Rightarrow$ Secondary winding Resistance

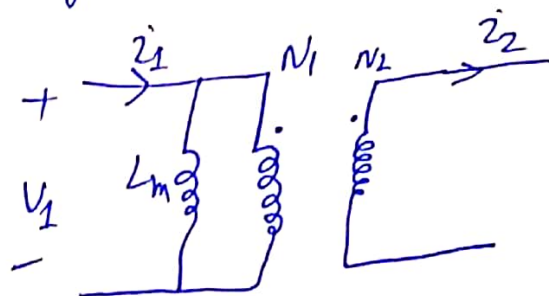
$R_m \Rightarrow$ Core loss

$L_1 \Rightarrow$ Primary winding Leakage Inductance

$L_2 \Rightarrow$ Secondary winding Leakage Inductance

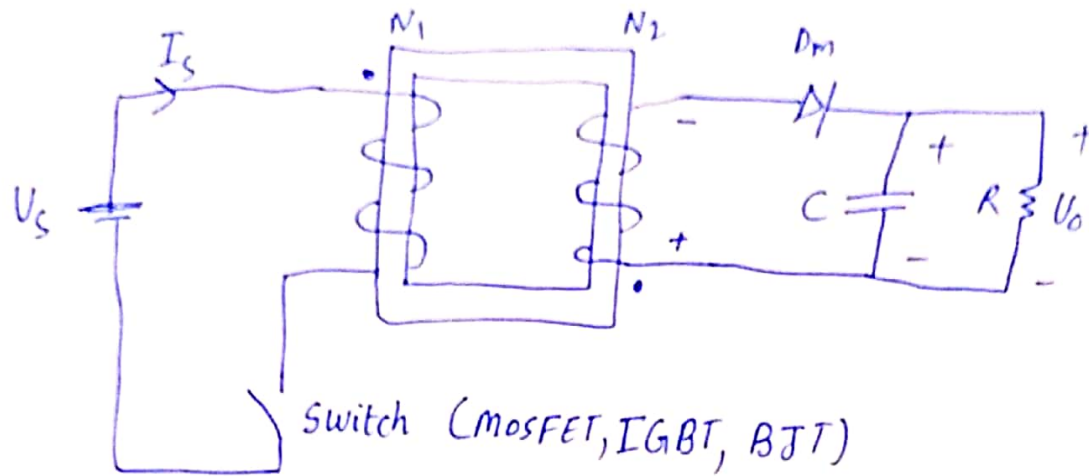
$L_m \Rightarrow$ Magnetizing Inductance

\rightarrow Generally R_1, R_2, R_m, L_1 and L_2 has no effect on the operating principle of the converter.

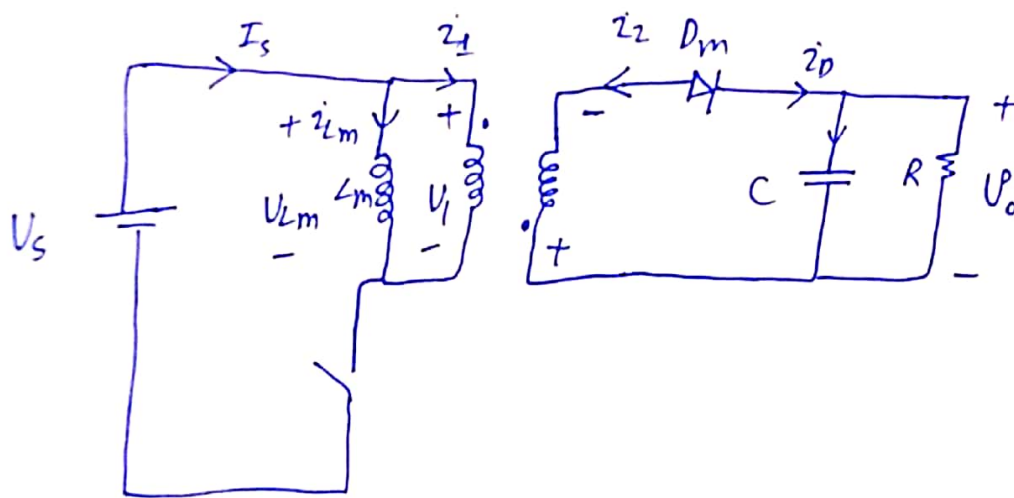


Simplified model.

Flyback Converter:-



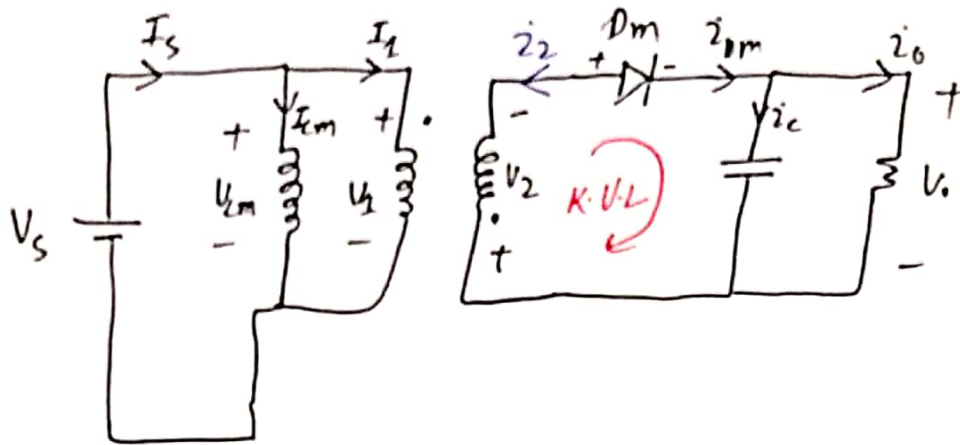
→ Replace the transformer with its simplified equivalent circuit model.



Assumption for analysis

- The Converter is operating in steady-state.
- The leakage inductance current is continuous.
- The output voltage ripple is small (small ripple approximation)
- The switch is closed for some time and open for some time during each switching cycle.

Analysis for switch Closed:-



→ The current I_1 in the primary winding is entering the dotted terminal, therefore the current i_2 should be leaving the dotted terminal. But current can not flow from cathode to anode, therefore diode D_m should be reversed bias.

Alternatively

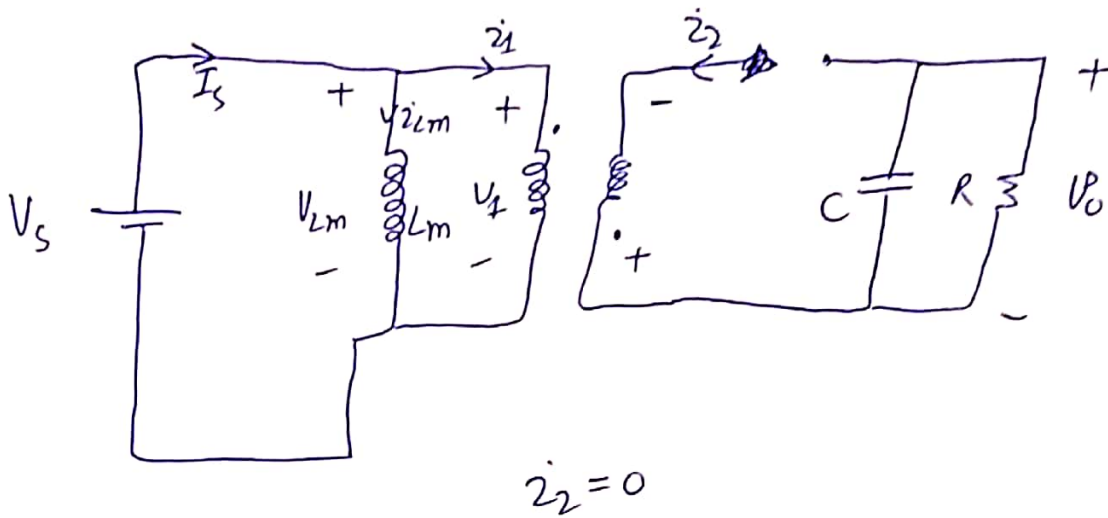
Apply K.V.L

$$V_2 + V_{Dm} + V_o = 0$$

$$V_{Dm} = \underbrace{-V_2 - V_o}$$

The voltage across the diode is negative therefore diode will be reversed bias.

Thus the equivalent circuit will become:



$$\frac{i_1}{i_2} = \frac{N_2}{N_1}$$

$$i_1 = \frac{N_2}{N_1} i_2$$

$$i_1 = 0$$

$$I_s = i_{Lm}$$

$$V_{Lm} = L_m \frac{di_{Lm}}{dt} = V_s \rightarrow \text{Eq (1)}$$

$$\int_{i_{Lmin}}^{i_{Lm}(t)} di_{Lm} = \int_0^t \frac{V_s}{L_m} dt$$

$$i_{Lm}(t) - i_{Lmin} = \frac{V_s}{L_m} t$$

$$i_{Lm}(t) = \frac{V_s}{L_m} t + i_{Lmin} \rightarrow \text{Eq (2)}$$

$$y = mx + c$$

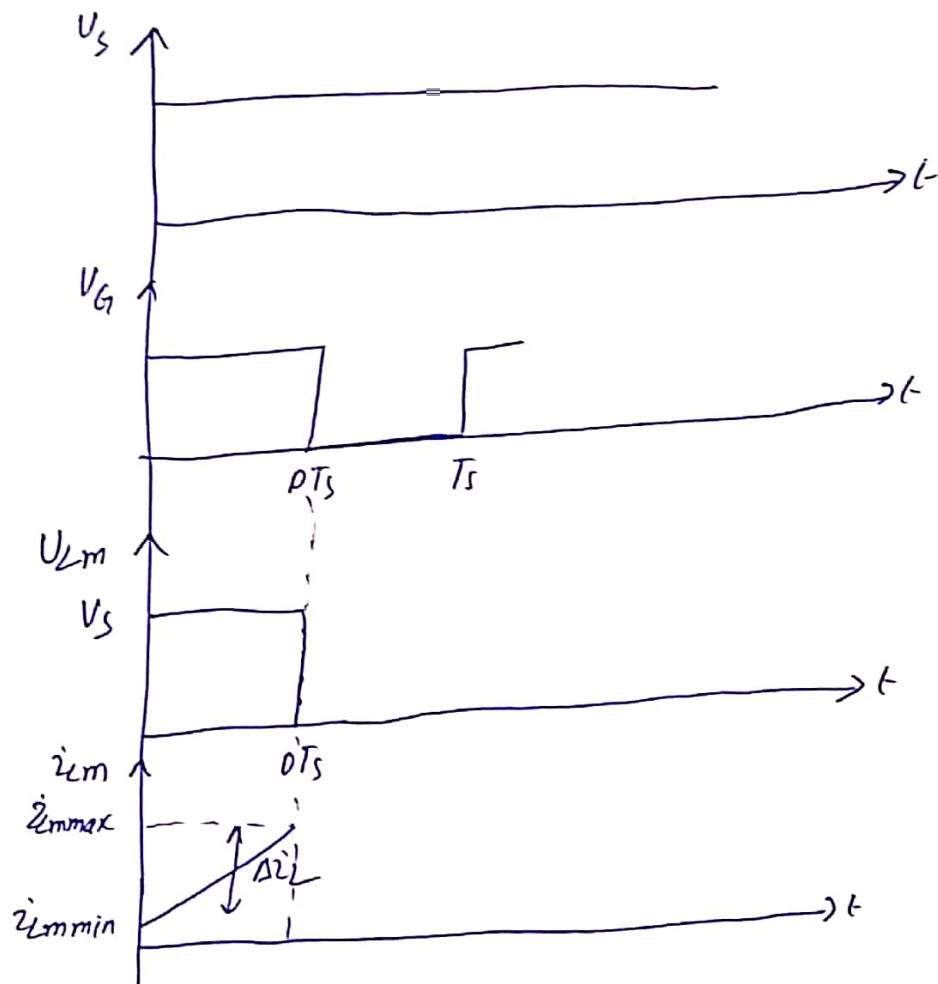
At $t = DT_s$ $i_{Lm}(t) = i_{Lmmax}$

From Eq (3)

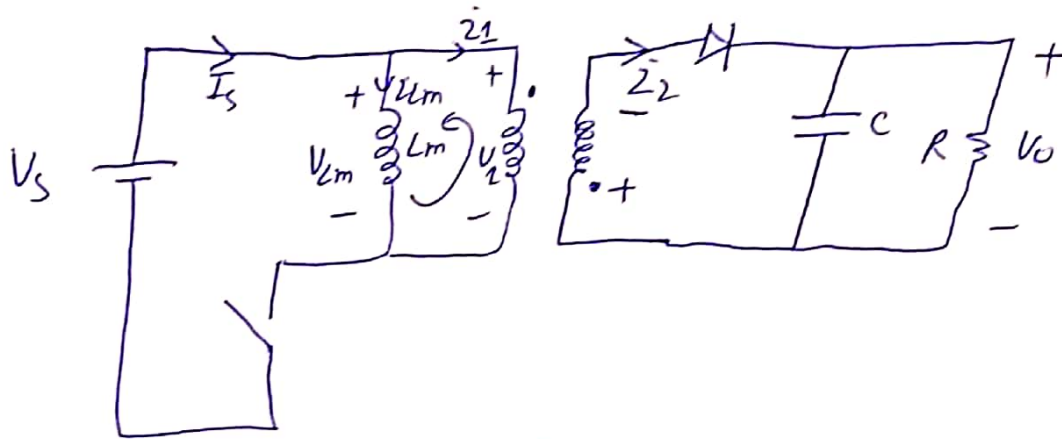
$$i_{Lmmax} = \frac{V_{in} V_s}{L_m} DT_s + i_{Lmin} \rightarrow (3)$$

$$\Delta i_{Lm} = i_{Lmmax} - i_{Lmin}$$

$$= \frac{V_{in} V_s}{L_m} DT_s \rightarrow \text{Eq (4)}$$

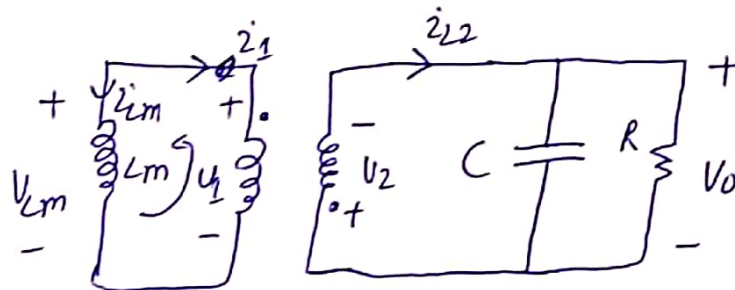


Analysis for Switch open $DT_s < t < T_s$



$$i_{Lm} = i_1$$

→ The current i_1 is leaving the dotted terminal. Therefore according to dot Convention the current i_2 should be entering the dotted terminal. Thus the diode will be forward bias. The equivalent circuit will become.



$$i_1 = -i_{Lm}$$

$$V_2 = -V_o$$

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

$$V_1 = \frac{N_1}{N_2} V_2$$

$$V_1 = -\frac{N_1}{N_2} V_o$$

$$V_{Lm} = V_1 = L_m \frac{di_{Lm}}{dt} = -\frac{N_1}{N_2} V_o \rightarrow \text{Eq } 5$$

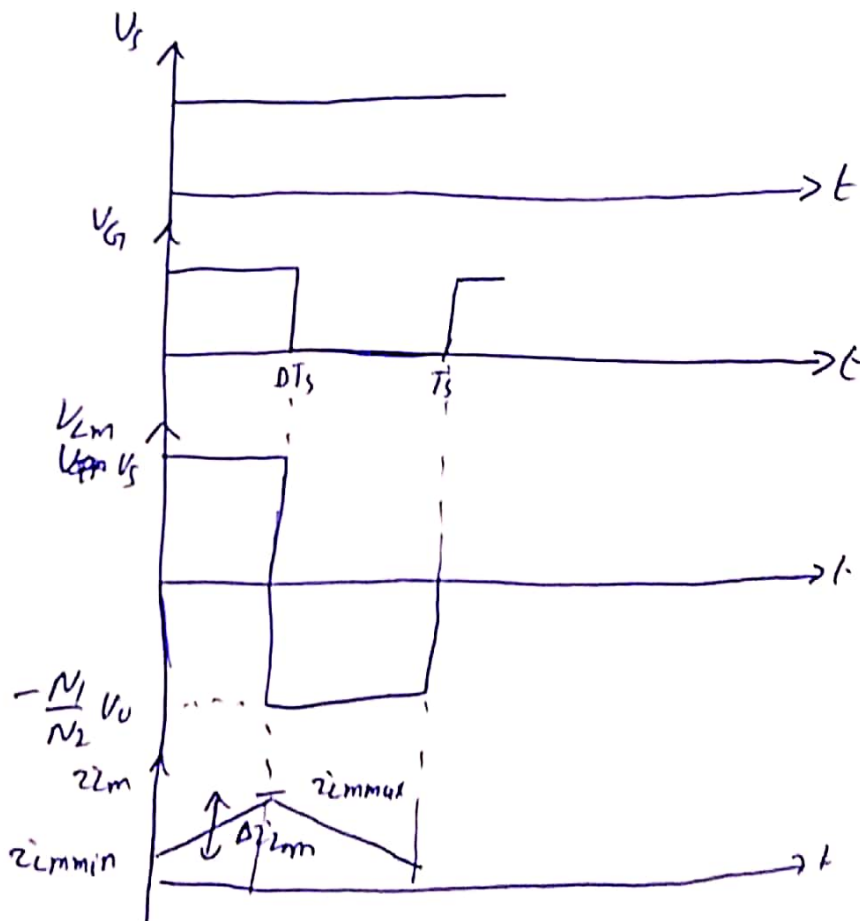
$$\int_{i_{Lm\max}}^{i_{Lm}(t)} di_{Lm} = \int_{DT_s}^t -\frac{N_1}{N_2} \frac{V_o}{L_m} dt$$

$$i_{Lm}(t) - i_{Lm\max} = -\frac{N_1}{N_2} \frac{V_o}{L_m} (t - DT_s)$$

$$i_{Lm}(t) = -\frac{N_1}{N_2} \frac{V_o}{L_m} (t - DT_s) + i_{Lm\max} \rightarrow \text{Eq } 6$$

$$\text{at } t = T_s \quad i_{Lm}(t) = i_{Lm\min}$$

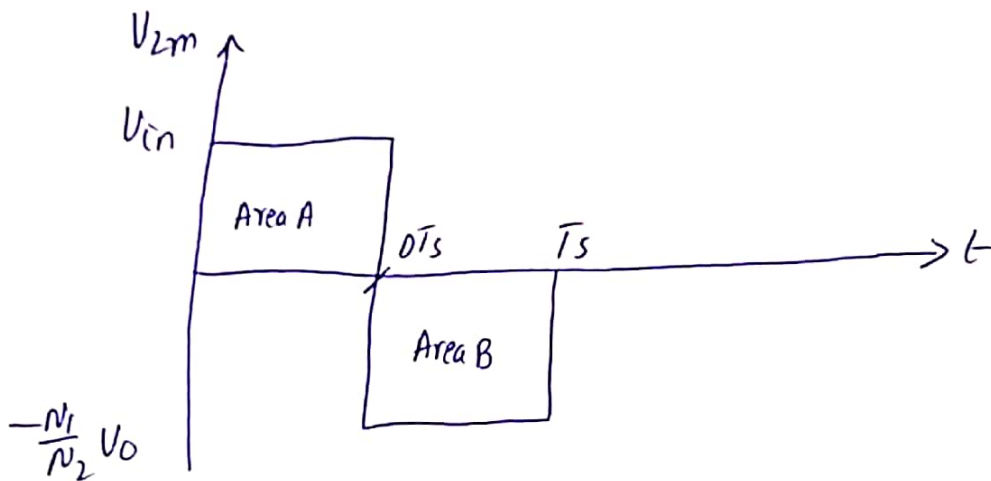
$$i_{Lm\min} = -\frac{N_1}{N_2} \frac{V_o}{L_m} (1-D)T_s + i_{Lm\max} \rightarrow \text{Eq } 7$$



$$\Delta z_m = z_{mmax} - z_{mmin}$$

$$= \frac{N_1 V_0}{N_2 L_m} (1-D) T_s \rightarrow \text{Eq. (8)}$$

Voltage Conversion Ratio:-



From Volt-Second balance

$$D T_s V_s + \left(-\frac{N_1}{N_2} V_o\right) (1-D) T_s = 0$$

solving for V_o

$$V_o = \left(\frac{D}{1-D}\right) \left(\frac{N_2}{N_1}\right) V_s$$

Alternatively

$$(\Delta i_m)_{open} = (\Delta i_m)_{closed}$$

$$V_o = \underbrace{\left(\frac{D}{1-D}\right)}_{\text{similar to buck-boost Converter but}} \left(\frac{N_2}{N_1}\right) V_s$$

With additional term the turns ratio of the transformer $\left(\frac{N_2}{N_1}\right)$

→ Flyback Converter is actually the isolated version of the buck-boost converter.

Average, Minimum and Maximum Value of the Magnetizing.

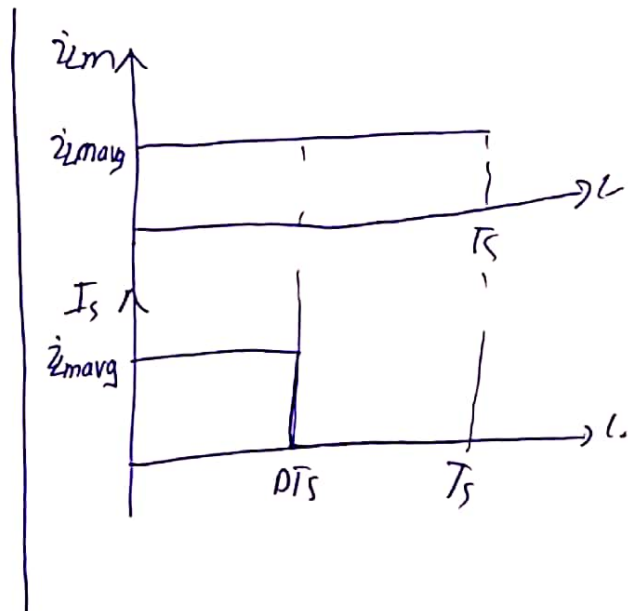
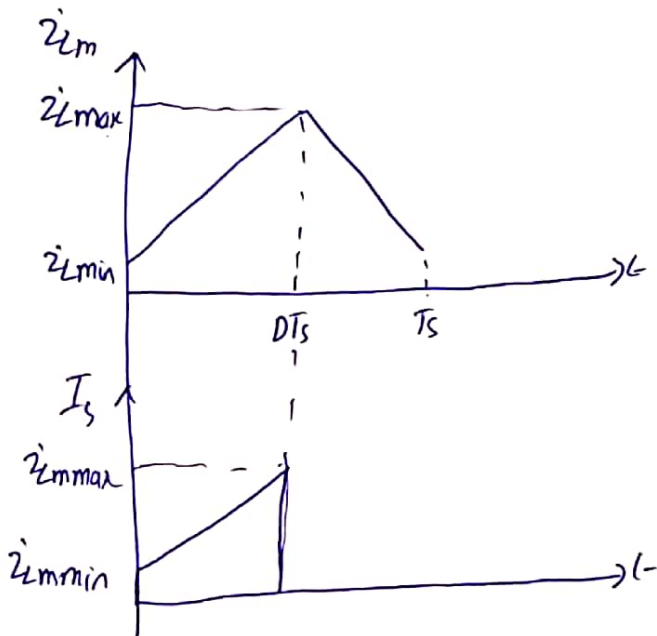
Inductance Current:-

$$P_o = V_o I_o = \frac{V_o^2}{R}$$

$$P_{in} = V_s \langle I_s \rangle$$

$$V_s \langle I_s \rangle = \frac{V_o^2}{R}$$

$$\langle I_s \rangle = \frac{V_o^2}{V_{in} R}$$



$$\langle i_s \rangle = \frac{1}{T_s} \int_0^{DT_s} i_{Lavg} dt = D i_{Lavg}$$

$$D i_{Lavg} = \frac{V_o^2}{V_{in} R}$$

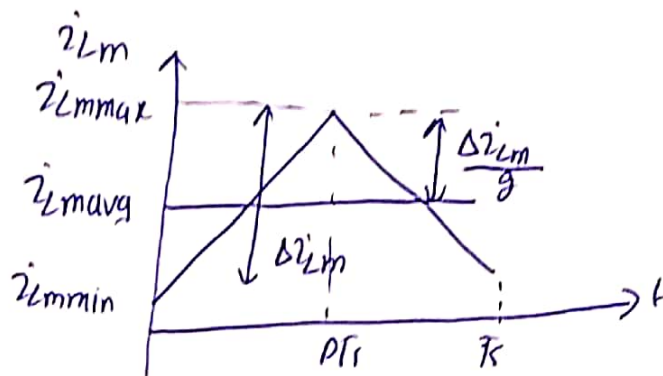
$$i_{Lavg} = \frac{V_o^2}{D V_s R}$$

$$i_{Lavg} = \frac{V_o^2}{D V_s R}$$

$$V_o = \frac{D}{(1-D)} \left(\frac{N_2}{N_1} \right) V_{inS}$$

$$i_{Lavg} = \frac{\frac{D^2}{(1-D)^2} \left(\frac{N_2}{N_1} \right)^2 V_s^2}{D V_s R}$$

$$= \frac{D V_s}{(1-D)^2 R} \left(\frac{N_2}{N_1} \right)^2$$



$$i_{Lmmax} = i_{Lmavg} + \frac{\Delta i_{Lm}}{2}$$

$$= \frac{D V_s}{(1-D)^2 R} \left(\frac{N_2}{N_1} \right)^2 + \frac{D V_s T_s}{2 L_m}$$

$$i_{Lmin} = i_{Lmavg} - \frac{\Delta i_{Lm}}{2}$$

$$= \frac{D V_s}{(1-D)^2 R} \left(\frac{N_2}{N_1} \right)^2 - \frac{D V_s T_s}{2 L_m}$$

Boundary Condition for Continuous Conduction Mode :-

$$Z_{\text{min}} = \frac{DV_s}{(1-D)^2 R} \left(\frac{N_2}{N_1} \right)^2 - \frac{DV_s T_s}{2L_m}$$

At boundary condition $Z_{\text{min}} = 0$

$$0 = \frac{DV_s}{(1-D)^2 R} \left(\frac{N_2}{N_1} \right)^2 - \frac{DV_s T_s}{2L_m}$$

Solving for L_m

$$L_{\text{min}} = \frac{(1-D)^2 R}{2 f_s} \left(\frac{N_1}{N_2} \right)^2$$

$$L > L_{\text{min}}$$

Output Voltage Ripple :-



Similar to Buck-Boost Converter output voltage ripple can be derived in the same way as derived for the buck-boost converter

$$C = \frac{DV_o}{\Delta V_o R f_s}$$