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RESEARCH PAPER

Many objective visual analytics: rethinking the design of complex engineered systems

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Abstract Many cognitive and computational challenges accompany the design of complex engineered systems. This study proposes the many-objective visual analytics (MOVA) framework as a new approach to the design of complex engineered systems. MOVA emphasizes learning through problem reformulation, enabled by visual analytics and many-objective search. This study demonstrates insights gained by evolving the formulation of a General Aviation Aircraft (GAA) product family design problem. This problem's considerable complexity and difficulty, along with a history encompassing several formulations, make it well-suited to demonstrate the MOVA framework. The MOVA framework results compare a single objective, a two objective, and a ten objective formulation for optimizing the GAA product family. Highly interactive visual analytics are exploited to demonstrate how decision biases can arise for lower dimensional, highly aggregated problem formulations.

Keywords Multi-objective optimization · Multidimensional data visualization · Product family design

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1 Introduction

Complex engineered systems display emergent behaviors arising from interactions among their component subsystems that cannot be easily predicted based on knowledge of the subsystems alone (Bloebaum and McGowan 2010; Simpson and Martins 2010). Such systems pose severe challenges to engineering design optimization. The initial challenge is to formulate a computational model of such a system (Simpson and Martins 2010). Supposing that this has been done, using it to discover a design for implementation is fraught with difficulty. The design space may be complicated by constraints, interactions, discontinuities, and nonlinearities. Discovering a design for implementation in these complex spaces requires a new generation of highly robust optimization technologies. Moreover, even under the assumption of a perfect optimization technology, defining an appropriate problem formulation for optimization represents a critical and potentially severely limiting challenge (Tsoukiàs 2008; Kasprzyk et al. 2012).

Traditionally, the process of formulation is often depicted as being linear where systems design experts have a clear understanding of objectives, constraints, goals, and performance dependencies. However, complex engineered systems, by virtue of their complexity, invalidate these assumptions (Bloebaum and McGowan 2010). Furthermore, conceiving the design of complex engineered systems as a linear "one time and done" process exposes the design process to cognitive biases (or decision errors) first identified by Hogarth (1981). Hogarth articulates how decisions can be degraded when a complex decision-making task is treated as an event rather than as a feedback-informed process of learning.

How one creates an optimization formulation depends on what optimization algorithm is selected, and vice-versa. Tsoukiàs (2008) has pointed out the interrelatedness of formulation, optimization, evaluation and negotiation. He calls for constructive decision-aiding methods that integrate these activities into a process that produces a decision as the result of learning how best to state the problem. Figure 1 illustrates our proposed Many-Objective Visual Analytics (MOVA) framework for constructive decision aiding building on the recommendations of Tsoukiàs (2008).

Our proposed MOVA framework in Fig. 1 unifies recent advances related to multi-objective evolutionary algorithms (MOEAs) with innovations in visual analytics to enable learning through continual reformulation. The capabilities of modern MOEAs and visual analytic tools make it possible to explore higher dimensional formulations than before and attain insights about design problems that can be hidden in lower dimensional formulations. Put more simply, the MOVA framework abstracts design as a continual learning process wherein decision-makers come to understand a problem while seeking its solution (Roy 1971; Tsoukiàs 2008).

Figure 2 provides an example of how adding additional problem objectives can fundamentally change the interpretation of the result. Figure 2 shows a surface, $\{(A, B, C)|\ A + B + C = 1,\ A \geq 0,\ B \geq 0,\ C \geq 0\}$, on which the nondominated solutions to a three-objective (A, B, and C) problem lie. This example illustrates a case when tradeoffs are only apparent with the addition of a third objective. This is a geometrical abstraction drawn from insights attained in the environmental sensing work of Kollat et al. (2011). In the environmental sensing application, Kollat et al. (2011) found that if the costs of sensing were ignored, other performance objectives such as network coverage and mapping uncertainties did not conflict. Alternatively, if sensing costs are considered jointly with network coverage and mapping uncertainties, then very strong objective conflicts emerge. That is, spending less on sensing causes conflicts in space because some locations are more important for coverage and others more important for uncertainty. Analogously, Fig. 2 illustrates a surface on which the problem's nondominated surface is only apparent if all three objectives are considered simultaneously. Figure 2a shows the three objective solution set projected on a single axis corresponding to Objective A. In this view, a solution that minimizes A would appear to be the sole optimum in the absence of any conflict. Figure 2b adds Objective B, revealing that, of the whole set of points that minimize Objective A, one of them ($A = 0$, $B = 0$) also minimizes Objective B. If only Objective A and Objective B were considered in the formulation, a decision-maker would assume that no conflict exists and select the solution that minimizes both objectives.

Adding Objective C as a third objective disrupts this view of the problem. Figure 2c includes Objective C, revealing the emergence of a significant tradeoff surface. The triangular surface in Fig. 2c is Pareto-optimal, revealing the effect of the constraint $A + B + C = 1$. For any fixed value of C, A and B are in conflict. The idea that A and B are in agreement represents a misconception and a source of decision error that arises from using formulations with fewer objectives. The solution that the decision-maker had thought ideal ($A = 0$, $B = 0$) is actually an extreme point ($A = 0$, $B = 0, C = 1$), and a decision-maker seeking to compromise between objectives would want to look elsewhere. Likewise, Kollat et al. (2011) found that transitioning to higher dimensional "many-objective" formulations for their environmental sensing application revealed that lower dimensional results represented extreme corners of the objective space that were of little interest to decision-makers. There have been a range of examples in the

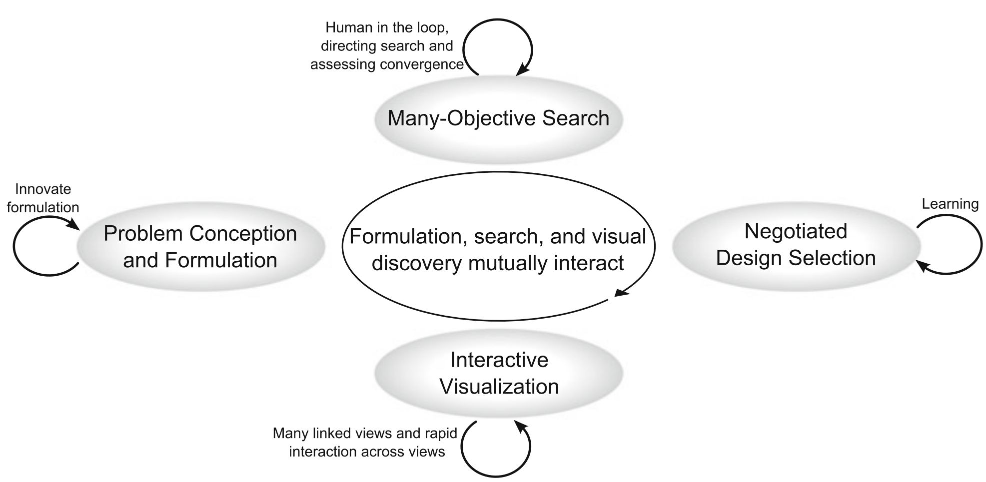

Fig. 1 The Many-Objective Visual Analytics (MOVA) framework emphasizes learning and feedbacks between the formulation, search, visual exploration, and negotiated design selection for complex engineered systems

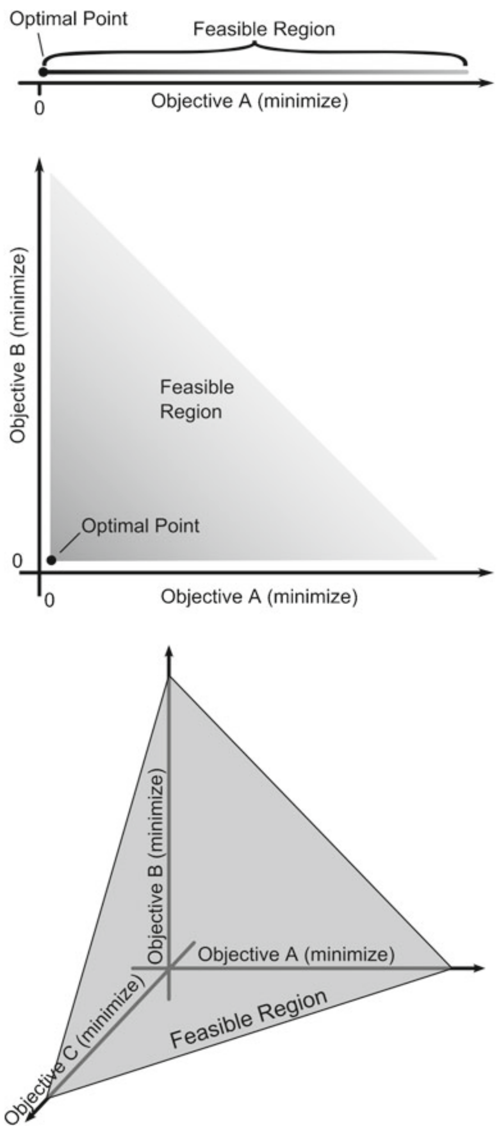

Fig. 2 Increasing the number of objectives under consideration changes not only preferences but problem conception. The solutions in (a), (b), and (c) are the same set. In (a) and (b), the set of solutions does not appear to have any tradeoffs. Minimizing Objective A yields an optimal solution that remains optimal with the addition of Objective B. In (c) it becomes clear that there is a mutual conflict among Objectives A, B, and C. The optimal point is no longer uniquely optimal, and it occupies an extreme of the nondominated set

literature illustrating how preferences shift with the addition of new objectives (Brill et al. 1990; Balling et al. 1999; Kasprzyk et al. 2009, 2012).

Tsoukiàs (2008) clearly discusses strong linkages between how problems are formulated and the optimization technologies that analysts are comfortable using. He calls for constructive decision-aiding methods that more strongly focus on producing a decision that results from learning how best to formulate the problem. With respect to problem formulation, Zeleny (1986) has argued for a de Novo approach. The de Novo approach to formulation seeks to "design an optimal system with respect to multiple objectives" and "maintain its optimality through time", that is, as objectives, goals, and constraints evolve. In the de Novo approach, objectives and constraints are not fixed but subject to continual reformulation, which helps to overcome preconception biases inherent in any fixed formulation. This learning process is represented in the feedbacks illustrated in the MOVA framework in Fig. 1. As Tsoukiàs (2008) observed, in real decision-making contexts the divisions between objectives, constraints, and decisions in classical systems analysis are false constructs. In reality, decision-makers see these components as being interchangeable parts of a formulation that should evolve with their system knowledge. Tsoukiàs highlights that the true challenge in complex engineered systems design is defining and evaluating the problem itself without making strongly limiting a priori assumptions or choices of optimization technologies. This view reflects a large body of work in constructive decision aiding (Roy 1971; Climaco 2004; Tsoukiàs 2008; Hitch 1960).

Kasprzyk et al. (2012) have recently expanded this line of thinking by generalizing the de Novo approach of Zeleny to a "many-objective" context (i.e., optimizing four or more objectives simultaneously). Their proposed many-objective de Novo planning framework seeks to allow stakeholders' insights, competing hypotheses, or preferences to inform the exploration of competing problem formulations where the number and types of decisions, constraints, and objectives can vary. This work marks a departure from classical systems analysis by ultimately seeking the "non-dominated problem" versus simply finding "non-dominated" solutions. A non-dominated solution is one that, when compared to any other solution, is superior with respect to at least one objective. This definition, however, implies the context of a mathematical problem formulation with well characterized and fixed objectives. The concept of a "non-dominated problem" refers to a problem formulation with component solutions that perform robustly across a decision-maker's evolving objectives, which may be explicit or discovered during the decision-aiding process (Kasprzyk et al. 2012).

Recent years have seen major technological innovations for many-objective optimization (Coello Coello et al. 2002; Hadka and Reed 2012a, b). In particular, recent breakthroughs for MOEAs now permit them to provide high quality approximations of the optimal tradeoffs for highly challenging problems from a broad range of domains (Coello Coello et al. 2002; Hadka et al. 2012; Ferringer et al. 2009; Reed et al. 2013; Fleming et al. 2005). MOEAs are a posteriori decision aiding tools that exploit population-based search to develop explicit representations of design relevant tradeoffs (i.e., Pareto approximate solutions). Informally, Pareto optimal solutions are those that cannot be improved in a given objective without degrading their performance in another objective. Graphing these solutions yields the Pareto front. In the a posteriori decision aiding context, the Pareto front is presented to decision-makers to inform their preferences when selecting a particular alternative.

Exploration and comprehension of the high-dimensional solution sets generated by MOEAs requires advanced tools for visualization. In the last decade, we have seen the emergence of visual analytics as a field that combines information visualization techniques with advances in computational discovery to augment human cognition (Keim et al. 2010). Visual analytic tools are designed to allow human beings to find meaning in large and complex data sets like Pareto approximation sets through automated analysis and rapid interaction across many heterogeneous linked visualization strategies. These tools make it possible to discover unanticipated solutions and to reconceive a problem in light of the structure of a solution set. Selecting a Pareto-approximate design implies an acceptable formulation for optimality, an effective method for approximating the Pareto set, and a means of understanding it. Brill et al. (1990) and Balling (1999) highlight that decision-makers' preferences are contextual. Our proposed MOVA framework seeks to aid in the discovery and understanding of what is possible as represented in actual alternatives for complex engineered systems so that decision-makers can express well-informed design preferences.

This paper demonstrates the MOVA framework using the General Aviation Aircraft (GAA) product family design problem (Simpson 1995; Simpson et al. 1996). This problem's considerable complexity and difficulty, along with a history encompassing several formulations, make it well-suited to demonstrate the MOVA framework. Section 2 describes three alternative formulations for optimizing the GAA product family, as well as the MOEA used to find solutions, and the visualization techniques used to understand them. Section 3 discusses the parameterization and execution of the MOEA for each formulation. Section 4 then uses visualization to compare the Pareto-approximate solutions obtained from each formulation and discusses how each formulation affects search and decision-making.

2 Methods

In 1950, Arrow proved his famous Impossibility Theorem (Arrow 1950): that no single objective function can simultaneously optimize utility for more than two individuals in social choice problems. For many decades, the implications of Arrow's theorem have not been directly linked to traditional aggregation-based solutions strategies for multi-criteria problems. Consequently, engineers have largely proceeded as if it does not (Tsoukiàs 2008). Franssen (2005) has recently proved that social choice problems and multi-criteria engineering design problems are formally equivalent. This implies that, in the design of complex engineered systems, any single aggregate objective function will necessarily impose preferences among the criteria. The resulting designs then are biased towards a subset of performance objectives in ways that cannot be known a priori by decision-makers, yielding the potential for severe opportunity costs.

The GAA product family design problem described in Section 2.1 illustrates how aggregated objective formulations (Section 2.2) affect decision-making. Product family design has been extensively studied and found to present numerous difficulties to formulation, optimization, and design selection (Shah et al. 2011). Product family design entails a fundamental conflict between performance and commonality. Make all members of the product family too similar, and they perform poorly. Differentiate products too much, and they become costly to manufacture. This conflict in design preferences embodies Franssen's observation that Arrow's Impossibility Theorem applies to engineering design problems. Discovering the tradeoffs between complex engineered systems' objectives can be severely challenging.

This study uses the Borg MOEA as the many-objective optimization component of the MOVA framework illustrated in Fig. 1. Hadka and Reed (2012b) have recently demonstrated that the Borg MOEA is highly robust across a broad sampling of challenging multiobjective problem classes where it met or exceeded the performance attained by eight other top performing multiobjective heuristic optimizers on a majority of the problems analyzed. Its auto-adaptive search and ease of use make it ideal for addressing evolving problem formulations within the MOVA framework. Section 2.3.1 describes the Borg MOEA in detail, and Section 2.3.2 describes the visual analytic tools that provide the decision-maker with an interactive, perceptual means to discover new solutions.

2.1 The GAA product family design problem

General aviation refers to any aircraft that is not used for military purposes or scheduled commercial airline travel. The number of people taking flights on general aviation aircraft is much lower than the number of people flying on commercial airliners. However, the number of flights is much greater. Approximately ten times as many airfields serve general aviation as serve commercial airlines. Air taxi services, flying instructors, businesspeople, and private owners use general aviation aircraft. These aircraft are generally small and propeller-driven, purchased as flexible transportation options that are unconstrained by the limited number of airports and schedules available on commercial airline flights.

In the 1990s, the National Aeronautics and Space Administration (NASA), seeking to energize domestic GAA production, sponsored a series of challenges intended to generate innovative aircraft designs (NASA and FAA 1994). Most participants produced novel designs for individual aircraft (Nolan et al. 1995). In 1995, Chen et al. (1995) proposed a new approach to the design process: design a family of three aircraft, distinguished by having two, four, or six seats, to address demand in different market segments, taking advantage of a common platform to simplify the manufacturing process. These aircraft would not themselves incorporate any novel design elements, but would instead improve the competitiveness of the manufacturer by streamlining the supply chain, reducing manufacturing costs, and increasing flexibility.

Simpson et al. (1996) addressed the three aircraft product family design problem using the popular Beechcraft Bonanza B36TC as a baseline aircraft. This choice fixed many design decisions, such as wing position (low), landing gear (retractable), and engines (one). To assess the performance of their aircraft designs, they employed NASA's General Aviation Synthesis Program, or GASP (NASA 1978), which had been developed as a means of connecting high-level aircraft design parameters to characteristics of the finished aircraft. The inputs to GASP not fixed by the choice of the B36TC as a baseline design are illustrated in Fig. 3. They describe the geometry of the propeller, wing, and fuselage, as well as parameters of the aircraft's mission.

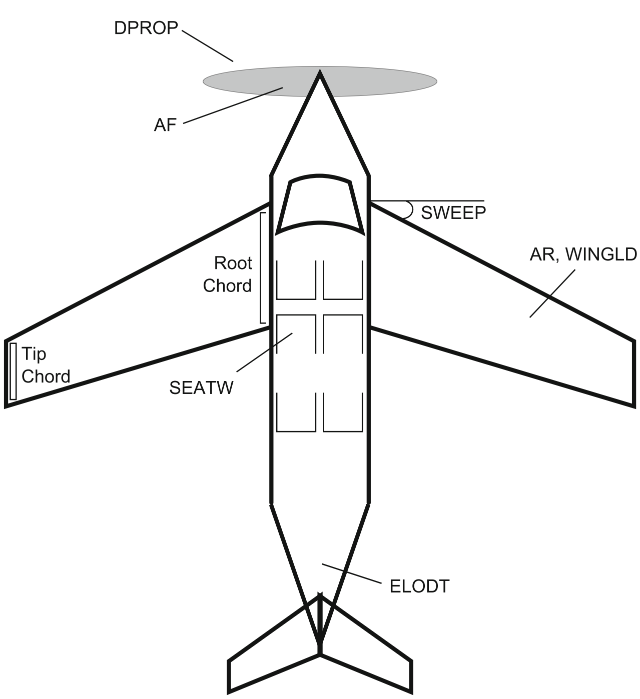

Fig. 3 The aircraft model used in the GAA product family design problem, illustrating geometric decision variables.

Table 1 Decision variables used in both formulations of the GAA product family design problem and their acceptable ranges as determined by Simpson et al. (1996)

Name	Abbreviation	Unit	Minimum	Maximum
Activity factor	AF	ratio	85	110
Aspect ratio	AR	ratio	7	11
Nominal cruising speed	CSPD	Mach	0.24	0.48
Propeller diameter	DPROP	feet	5.5	5.968
Tail elongation	ELODT	ratio	3	3.75
Seat width	SEATW	inches	14	20
Wing sweep	SWEEP	degrees	0	6
Wing taper	TAPER	ratio	0.46	1
Wing loading	WINGLD	lb/ft^2	19	25

The propeller parameters are its diameter (DPROP) and activity factor (AF). The wing parameters are aspect ratio (AR), leading edge wing sweep angle (SWEEP), and root-to tip wing taper (TAPER). The fuselage parameters are seat width (SEATW) and tail cone elongation (ELODT). The mission parameters are wing loading (WINGLD) and nominal cruising speed (CSPD). Thus there are nine adjustable design variables for each of the three aircraft in the product family. Table 1 summarizes these decision variables and their allowable ranges. The GASP technical report (NASA 1978) describes how GASP uses these parameters, and Raymer (1999) explains how they relate to aircraft performance. From among the GASP outputs, nine performance criteria were considered in comparing designs: takeoff noise (NOISE), empty weight (WEMP), direct operating cost (DOC), ride roughness coefficient (ROUGH), fuel weight (WFUEL), purchase price (PURCH), flight range (RANGE), lift-drag ratio (LDMAX), and maximum cruising velocity (VCMAX). Table 2 summarizes these outputs and their roles in the two formulations considered in this paper. To reduce the amount of time required by GASP-based evaluations of the GAA objective function(s), Simpson et al. (1996) developed a second-order response surface model relating only the design variables of interest to the outputs of interest. This response surface model eliminates design variables already fixed by the choice of the Beechcraft Bonanza B36TC as the reference platform for the product family.

2.2 Problem formulations

Twenty-seven criteria (9 criteria $\times$ 3 aircraft) exceed the capabilities of Pareto ranking based evolutionary algorithms (Teytaud 2006). In addition, human cognitive capacity is finite. Miller (1956) concluded that human beings were able to hold "seven plus or minus two" pieces of information in immediate memory. Whatever the true limit on

Table 2 Performance criteria from NASA's General Aviation Synthesis Program (GASP) and their roles in both problem formulations: the ten-objective formulation and the two-objective formulation

Name	Abbreviation	Unit	Used as a constraint	Included in GP performance	Included in ten objective performance	Preference
Takeoff noise	NOISE	dB	x		x	Minimize
Empty weight	WEMP	lb	x	x	x	Minimize
Direct operating cost	DOC	1970$/hr	x	x	x	Minimize
Ride roughness	ROUGH	Ratio	x		x	Minimize
Fuel weight	WFUEL	lb	x	x	x	Minimize
Purchase price	PURCH	1970$		x	x	Minimize
Range	RANGE	nmi	x	x	x	Maximize
Lift-drag ratio	LDMAX	Ratio		x	x	Maximize
Cruising speed	VCMAX	Knots		x	x	Maximize

immediate memory, the more than three hundred two-criterion tradeoffs incorporated in a twenty-seven criterion problem (Teytaud 2006) exceed it. Therefore the criteria must be consolidated to a reasonable extent as part of the design and decision-making process. The earliest work on the GAA problem used single-objective formulations that aggregated performance using a goal-programming approach, addressing commonality without measuring it numerically by using methods such as the Robust Concept Exploration Method (Chen et al. 1995) and the Platform Concept Exploration Method (Simpson et al. 2001). Simpson and D'Souza (2004) added a two-objective formulation to trade performance explicitly against Messac's Product Family Penalty Function (PFPF) commonality metric (Messac et al. 2002). The PFPF metric seeks to provide a measurement of the difference among the individual designs belonging to a family, by totaling the variation of each decision variable across designs. The two objective PFPF formulation gave a sense of the tradeoff between commonality and performance in an aggregated sense, but Simpson and D'Souza (2004) had to relax the constraints shown in Table 3 to obtain a set of feasible solutions with the NSGA-II optimization algorithm.

Subsequent to Simpson and D'Souza's study, Shah et al. (2011) demonstrated that the three aircraft GAA problem is challenging by showing that of 50 million designs drawn from a Latin Hypercube sample of the problem's design space only 4 solutions were feasible. Shah et al were the first study to identify feasible, Pareto-approximate solutions for the GAA problem without relaxing the original constraints. They reformulated the problem as a "many-objective" search problem, retaining PFPF as an objective, while expanding the single performance objective of Simpson and D'Souza (2004) to nine distinct objectives. Each of the nine performance objectives used a min-max or max-min (Tchebychev) formulation to optimize the worst performing aircraft while ensuring the other two aircraft in the product family were better for each criterion than the worst performer. Transitioning to this ten-objective formulation has the potential to greatly elucidate key tradeoffs

Table 3 Goals and constraints for the GAA product family design problem

Abbreviation	2-seat		4-seat		6-seat	
	Constr.	Goal	Constr.	Goal	Constr.	Goal
NOISE	75		75		75	
WEMP	2200	1900	2200	1950	2200	2000
DOC	80	60	80	60	80	60
ROUGH	2		2		2	
WFUEL	450	450	475	400	500	350
PURCH		41000		42000		43000
RANGE	2000	2500	2000	2500	2000	2500
LDMAX		17		17		17
VCMAX		200		200		200

For all constraints except RANGE, constraints are maximum allowable values. For RANGE, 2000 nautical miles is the minimum allowed for the GAA product family, while also representing a significant search challenge. Algorithms capable of finding Pareto-approximate solutions in an objective space having so many dimensions are a recent innovation (Fleming et al. 2005) and algorithms that do so in a manner insensitive to their parameterization are more recent still (Hadka and Reed 2012a, b; Hadka et al. 2012).

This study contributes a comparative analysis of three formulations for the GAA problem: the goal programming compromise formulation from Simpson's original work (Simpson 1995), the two-objective formulation of Simpson and D'Souza (2004) and the ten-objective formulation of Shah et al. (2011). It demonstrates the value of the MOVA framework in advancing designers' ability to understand the tradeoffs in their systems and to discover diverse high-quality design strategies. Additionally, we have used state of the art multiobjective search (Hadka and Reed 2012b; Hadka et al. 2012) to find new solutions for all three problem formulations. These new solutions improve on those found in previous studies. Of the three previous studies cited (Simpson 1995; Simpson and D'Souza 2004; Shah et al. 2011), only Shah et al were able to find feasible solutions without allowing a three percent violation of the problem constraints. In this study, we contrast the consequences of using objective functions that aggregate incommensurable quantities versus using a many objective formulation.

As Venkataraman and Haftka (2004) observed, increases in computing power over time generally enable greater model fidelity, but there are other uses to which that computing power can be applied, including optimization. Alternative strategies could be employed to exploit the increased capabilities for computational optimization. One strategy for many-objective problems could be to make several optimization runs with various criterion weightings to increase the diversity of the solutions generated. This strategy presents interpretation challenges, as there would now be as many tradeoffs as weighting combinations, within each of which all of the other weightings would appear inferior. Alternatively, the many-objective formulation strategy employed in this study shifts away from highly-aggregated objectives. Compared with a weighting approach, a linear combination of objectives in a many-objective formulation addresses an exponentially increasing number of subproblems that are solved simultaneously (Di Pierro et al. 2007). For example, solving a ten-objective problem implicitly includes solutions to 1,023 subproblems, including 45 two-objective subproblems, 120 of three objectives, 210 of four objectives, 252 of five objectives, 210 of six objectives, 120 of seven objectives, 45 of eight objectives, 10 of nine objectives, and one of ten objectives. Given an appropriately powerful optimization algorithm, the ability to address an exponentially increasing number of tradeoffs is a significant benefit of using many-objective formulations.

The goal programming (GP) formulation considered in this study is one of several originally proposed (Simpson et al. 1996). It assigns inequality constraints to six of the performance criteria and goals to seven of them. Designs failing to meet the constraints are considered infeasible, and the objective function (z) is designed to minimize the deviation from goals. The aggregate performance objective z is formulated mathematically as:

$$z = \sum_{i \in \{2,4,6\}} \sum_{j=1}^{7} \frac{d_{ij}}{7} \tag{1}$$

where i is the number of seats and d_{ij} is a goal deviation function for the jth objective. The goal deviation function d_{ij} measures the distance for the aircraft with i seats from the goal for the jth objective. It is defined as:

$$d_{ij} = \max\left\{\frac{\text{goal}_{ij} - \text{attained}_{ij}}{\text{goal}_{ij}}, 0\right\} \quad \text{to maximize} \tag{2}$$
$$d_{ij} = \max\left\{\frac{\text{attained}_{ij} - \text{goal}_{ij}}{\text{attained}_{ij}}, 0\right\} \quad \text{to minimize}$$

The two-objective formulation of D'Souza and Simpson adds as its second objective an explicit measurement of commonality: the Product Family Penalty Function (PFPF). PFPF is a measure of the total distance of the three individual aircraft designs from the mean design in normalized decision space, across all nine decision variables. The formulation for PFPF used in this study is:

$$\text{PFPF} = \sum_{j=1}^{9}\left(\frac{\sqrt{\sum_{i \in \{2,4,6\}} \left(x_{ij} - \bar{x_j}\right)^2}}{\text{UB}_j - \text{LB}_j}\right) \tag{3}$$

where UB_j and LB_j are respectively the upper and lower bounds on the jth decision variable. This computation differs slightly from Messac's original (Messac et al. 2002). Because bounded ranges are available for the decision variables, each decision variable is normalized by its range rather than its mean. This improves the stability of the computation for SWEEP, which can have an average angle of zero degrees, by keeping very small values out of the denominator. Just like the aggregate performance objective z (1), PFPF attempts to transform nine separate and incommensurable quantities, in this case the L_2 (Euclidean) distance from the actual design to the mean design in decision variable space, into a single aggregate objective. Because the two-objective formulation incorporates the single-objective formulation as one objective, an approximately optimal solution for the single-objective formulation can be selected from the Pareto-approximate solution set for the two-objective formulation by choosing the solution that minimizes z.

The ten-objective formulation of Shah et al. (2011) replaces the highly-aggregated performance objective z with nine separate performance objectives. Rather than trying to reconcile incommensurable performance criteria through normalization, it recognizes that, within the twenty-seven performance criteria output by the response surface model, there are nine groups of three commensurable performance criteria. That is, for each aircraft in the family, there are nine performance criteria, and each of those is commensurable with the same criterion for the other two aircraft. The resulting formulation reflects the problem's structure; it aggregates three commensurable performance criteria into one objective by applying a minimum or maximum function. These min-max/max-min objectives are expressed mathematically as:

$$\begin{aligned} z_j &= \min\{z_{2j}, z_{4j}, z_{6j}\} \quad \text{to maximize} \\ z_j &= \max\{z_{2j}, z_{4j}, z_{6j}\} \quad \text{to minimize} \end{aligned} \tag{4}$$

where z_j is the aggregate objective, and z_{ij} is the jth performance criterion for the aircraft with i seats. The consequence of using such a formulation is that improving a min-max/max-min objective always improves the worst performing member of the family. While this formulation makes it difficult to discover extremely high-performing individual aircraft designs, it guarantees product family designs with good performance for all three aircraft. This problem formulation reflects an implicit design goal: equivalently strong performance for all members of the product family.

The decision to separate the performance criteria into logically and numerically coherent objectives, while keeping the commonality criteria in a single aggregate objective, reflects the priorities of the ten-objective formulation. The formulation emphasizes the discovery of a high-performing product family design, while maintaining a degree of commonality between the aircraft in the family. A dual ten-objective formulation emphasizing commonality could also be conceived, with nine objectives for decision variable commonality and a single aggregate performance objective, as could an eighteen-objective formulation with nine objectives each for performance and commonality. Choosing the ten-objective formulation we used instead of these alternatives expresses a preference for performance over commonality, since nine objectives represent performance and only one represents commonality.

From a cognitive perspective, the aggregate objectives z and PFPF are difficult to interpret. They are designed so that minimizing them optimizes performance and commonality with respect to a particular set of preferences, but they are not designed in a way that supports understanding the solution. For example, we would be at a loss to explain how a design with $z = 0.5$ is worse than one with $z = 0.4$. Is it slower? More expensive? It might even be faster, but with a greater loss in range. Likewise for PFPF, a lower value might indicate that the three aircraft can use a common propeller, or it might mean that they can use a common tail cone, or it might mean neither: that the three aircraft are closer to having common parts, but they still have small differences that actually make it impossible to take advantage of commonality. Therefore, choosing a significance level for these objectives cannot be done in advance. When a many-objective search using ε-nondominance sorting (Section 2.3.1) is used to find Pareto-approximate solutions, the ε values for z and PFPF are chosen not on the basis of significant difference, but so that the search produces a reasonable number of solutions.

2.3 Many-objective visual analytics

The MOVA framework as illustrated in Fig. 1 applies the more general work in visual analytics (Thomas et al. 2005; Keim et al. 2010) to the discovery of solutions to many-objective design problems. Many-objective design problems are those, like the GAA product family problem, having four or more objectives. Visual analytics, according to Keim, "combines automated analysis techniques with interactive visualizations for an effective understanding, reasoning, and decision-making on the basis of very large and complex datasets" (Keim et al. 2010). Keim's definition arises from a broad study of practices in many disciplines. Visual analytics identifies the emerging common requirements of analysts and decision-makers in diverse fields, from astrophysics to medicine. In the MOVA framework proposed in this study, automated analysis takes the form of an optimization algorithm capable of approximating the Pareto-efficient solution set of a many-objective design problem. Sections 2.3.1 and 2.3.2 describe the two primary software components used to demonstrate a specific instance of the MOVA framework. Section 2.3.1 describes the Borg MOEA metaheuristic algorithm used in this work to generate solutions to the GAA product family design problem. Section 2.3.2 introduces the interactive visualization tools used to visualize the complex solution set of the GAA problem.

2.3.1 Evolutionary many-objective search

The search component of the MOVA framework exploits the Borg MOEA as recently introduced and benchmarked by Hadka and Reed (2012a, b). These studies have shown the Borg MOEA to be one of the most powerful MOEAs currently available. Broadly, MOEAs emulate the process of evolution by natural selection to approximate a problem's Pareto optimal solution set. Pareto optimal solutions are those which are not inferior in every respect to all of the other feasible solutions (also termed nondominated solutions). MOEAs have proven effective in addressing problems with difficult structures, including nonlinearity, nonconvexity, and discontinuity (Coello Coello et al. 2002).

The Borg MOEA assimilates many recent advances in the field of many-objective evolutionary algorithms: a highly efficient steady-state algorithm structure (Deb et al. 2005), ε-box dominance archiving (Laumanns et al. 2002; Kollat and Reed 2007b), adaptive population sizing (Kollat and Reed 2005), time continuation (Goldberg 2002), and adaptive use of multiple recombination operators (Vrugt and Robinson 2007). In addition, the Borg MOEA introduces a new measure to detect search stagnation termed ε-progress.

The Borg MOEA exploits an auto-adaptive multi-operator search, in which the search algorithm uses recombination operators in proportion to their effectiveness at producing ε-box dominance archive members. Borg's use of multiple search operators that compete with each other is motivated by the No Free Lunch Theorem of Wolpert and Macready (1997), which implies that no one search operator is the most well-suited to every problem. The competing operators include simulated binary crossover (Deb and Agrawal 1994), parent centric crossover (Deb et al. 2002), uniform normally distributed crossover (Kita et al. 1999), simplex crossover (Tsutsui et al. 1999), differential evolution crossover (Storn and Price 1997), polynomial mutation (Deb and Agrawal 1994), and uniform mutation. The Borg MOEA overcomes the difficulty in selecting a recombination operator appropriate to the landscape of a given search problem. Instead, the most effective operator is allowed to produce the most solutions, and if another recombination operator becomes more effective, then it is used instead. This operator adaptation makes the Borg MOEA a more general optimization framework, where specific search operator instances are tailored to the application being solved. Finally, the Borg MOEA introduces ε-progress. This measurement requires that the algorithm improve solutions' performance by numerically meaningful distances as defined by user-specified ε values (i.e., the significant precision of objectives.) The ε-progress measurement provides a means to detect stagnation; when the Borg MOEA detects inadequate ε-progress, it auto-adaptively increases the diversity of the search while initiating a new algorithm run with a population of archive solutions and random solutions generated using uniform mutation. Hadka and Reed (2012a) provide a detailed discussion of the features of the Borg MOEA.

2.3.2 Visual analytics

Many visualization software packages with visual analytic capabilities exist. These tools facilitate dynamic and highly interactive exploration of many-objective solution sets using brushing, dynamic filtering, linked heterogeneous plots, and statistical tools like principal component analysis and K-means clustering. They are available to users of packages like R (R project 2012), JMP (SAS Institute 2012), and Spotfire (TIBCO 2012). Software packages both commercial and academic provide visual analytic capabilities specifically in support of engineering design decision-making: among others, ATSV (Stump et al. 2003), VIDEO (Kollat and Reed 2007a), and modeFrontier (ESTECO SpA 2012). The multidisciplinary optimization community has also provided a number of visualization approaches, including Cloud Visualization (Eddy and Lewis 2002), Graph Morphing (Winer and Bloebaum 2002), BrickViz (Kanukolanu et al. 2006), the Visual Dependency Structure Matrix (English and Bloebaum 2008), and Hyper-Radial Visualization (Naim et al. 2008).

This software fills a need identified by Keim et al. (2010): handling the problem of information overload. Many-objective optimization produces a set of solutions approximating the Pareto frontier. This set of solutions may be quite large, and it is by nature complex, representing the tradeoffs among many objectives. The number of possible tradeoffs among objectives grows rapidly with the number of objectives. Visualization makes these tradeoffs available to perception, reducing a large number of many-dimensional data points, an overload of information, to a single visual object. Dynamic filtering with linked plots makes it possible to isolate interesting solutions and evaluate them in the most relevant contexts; likewise, brushing highlights those solutions and permits comparison with the rest. Applied to the approximate Pareto set generated by a many-objective optimization algorithm, brushing and dynamic filtering across linked plots is an essential element of the MOVA framework. It combines the analytical power of many-objective optimization with the perceptual power of the human vision system to produce actionable discoveries from a computer model of an engineering design problem (Keim et al. 2010; Ware 2004).

To visualize the Pareto approximation sets produced in this study, we employed 3D scatter plots linked with parallel-coordinate plots (Inselberg 2009; Kipouros et al. 2008). Within the visual analytics literature these types of linked heterogeneous plots have been found to improve their users' ability to comprehend complex data sets (Keim et al. 2010; Seo and Shneiderman 2005). More specifically, linking 3D scatter plots to parallel-coordinate plots has been employed successfully in a range of many-objective applications (Fleming et al. 2005; Kasprzyk et al. 2012; Ferringer et al. 2009; Hadka et al. 2012) to communicate the nature of high-dimensional Pareto approximate sets and evaluate commonality tradeoffs among different products in a family (Slingerland et al. 2010).

The final selection of solutions requires narrowing the set down to a small number of diverse alternatives (Brill et al.

1990). Since the two-objective Pareto approximate front has a one-dimensional topological geometry, selecting diverse solutions is as simple as finding points on a curve with the greatest mutual distance. These solutions have little diversity, but the selection procedure is very straightforward. The ten-objective solution set, in contrast, lies on a hypersurface; its true dimension has not been determined, but it must be between two and nine dimensions (Teytaud 2006, 2007; Brockhoff et al. 2007). Assuming for the moment that all ten objectives are in conflict to some degree, there are ten solutions that are optimal for each objective, plus 45 solutions that occupy the midpoint in a compromise between two objectives, plus 120 solutions that occupy the midpoint in a compromise between three objectives, and so on.

Even supposing that not all ten objectives are in conflict, adequately representing the true diversity of preferences accommodated by the ten-objective solution set would take more than a few solutions. This means that obtaining a small set of diverse solutions from a many-objective set requires some input from a decision-maker. In particular, the decision-maker must express some preferences about the performance criteria. This is not necessary for the two-objective formulation because its aggregate performance objective enforces a preference structure a priori on the performance criteria, but for the ten-objective problem formulation, a posteriori preference articulation enables the discovery of designs suitable to the decision-maker's needs.

One of the distinguishing features of visual analyitcs is that it enables the discovery of features in a data set through brushing and dynamic filtering across multiple linked heterogeneous views (Keim et al. 2010). The search for features of interest corresponds to a progressive articulation of preference Fonseca and Fleming (1998), where the choice to focus on a particular feature is an expression of preference informed by the visualization process. This contrasts to explicit preference weighting schemes such as the Hypothetical Equivalents and Inequivalents Method (See et al. 2003) or Analytic Hierarchy Process (Saaty 1990) that require an a priori specification of preferences that are assumed to remain static (i.e., no learning feedbacks). Our proposed MOVA framework emphasizes learning via progressive articulation of preference through visual analysis where preferences are implicit and possibly dynamic with new insights.

3 Computational experiments

The Borg MOEA's auto-adaptive search makes it easy to use and is insensitive to its parameterization as demonstrated by Hadka and Reed (2012b). A key contribution of these studies is a shift from traditional MOEAs, which are severely sensitive to their parameterization, towards auto-adaptive search that has a very high probability of success as long as the Borg MOEA is allowed enough function evaluations to adapt its choice of search operators to the problem. A detailed study of the effectiveness of the Borg MOEA's auto-adaptive search capabilities on different formulations of the GAA problem can be found in (Woodruff et al. 2012). Table 4 provides the default Borg MOEA parameters used in this study. It should be noted that because the algorithm has been proven to be highly robust across its full parameterization space, use of the default parameterization in Table 4 is justified (Hadka et al. 2012).

A key component of the Borg MOEA is its use of ε-dominance archiving, which requires users to specify the ε values representing significant precision that should be used when evaluating each objective, as shown in Table 5. For the performance objectives in the ten-objective problem, ε values were chosen corresponding to the minimum numerically meaningful difference that should be distinguished for each objective. ε-dominance is critical for the Borg MOEA's theoretical proofs for converging to diverse approximations of global many-objective Pareto fronts (Hadka and Reed 2012a).

Table 4 Parameters used for the Borg MOEA

Parameter	Value
Initial population size	100
Tournament selection size	2
SBX rate	1.0
SBX distribution index	15.0
DE crossover rate	1.0
DE step size	0.5
PCX parents	3
PCX offspring	2
PCX eta	0.1
PCX zeta	0.1
SPX parents	3
SPX offspring	2
SPX epsilon	2
UNDX parents	3
UNDX offspring	2
UNDX eta	0.5
UNDX zeta	0.5
UM rate	$\frac{1}{27} = \frac{1}{\text{number of DV}}$
PM rate	$\frac{1}{27} = \frac{1}{\text{number of DV}}$
PM distribution index	20

The following operators are included: simulated binary crossover (SBX), differential evolution (DE), parent centric mating (PCX), simplex crossover (SPX), uniform normally distributed crossover (UNDX), uniform mutation (UM), and polynomial mutation (PM)

Table 5 ε resolutions used with the Borg MOEA for the ten- and two-objective formulations of the GAA problem

Formulation	Variable	ε
Ten objective	NOISE	0.1
	WEMP	20.0
	DOC	2.0
	ROUGH	0.025
	WFUEL	15.0
	PURCH	1000.0
	RANGE	50.0
	LDMAX	0.1
	VCMAX	2.0
	PFPF	0.1
Two objective	Performance (z)	0.001
	PFPF	0.001

The two-objective formulation uses much finer ε resolution for PFPF than the ten-objective formulation

For PFPF, which has no direct physical interpretation, an ε value was chosen such that the final solution set would distinguish 20 levels of commonality. Excessively precise evaluations of PFPF below this value are of questionable value and only serve to add difficulty to the search problem. An initial attempt to use the same ε values for PFPF in both the ten-objective formulation and the two-objective formulation failed. Unfortunately the aggregated two-objective formulation does not cover a diverse range of PFPF. If we use the ten-objective ε values this formulation yields only two solutions that are considered numerically different.

To provide a set of two-objective solutions that capture some level of commonality-performance tradeoff as shown in the ten-objective formulation, it was necessary to make the ε resolution two orders of magnitude finer. As highly-aggregated objectives, both PFPF and z severely limit the diversity of solutions discovered during search. Consequently, the ε resolution for PFPF was made more precise by two orders of magnitude for the two-objective formulation to increase the number of solutions that could be used for comparison with the ten-objective formulation. The two-objective formulation uses $\varepsilon = 0.001$ for PFPF, while the ten-objective formulation uses $\varepsilon = 0.1$ for PFPF. Although the two-objective ε values are excessively precise, we wanted to maximize the diversity of solutions that could be discovered (even if they represent small variations around one basic GAA design strategy in reality). The ε value used for the aggregate performance objective (z) in the two-objective formulation was also 0.001; since the z objective also has no direct interpretation, the value 0.001 was chosen to provide a similar amount of resolution in both commonality and performance objectives.

4 Results

This section demonstrates the MOVA framework by comparing the decision-making process under the three alternative GAA formulations presented in Section 2. This study demonstrates how the problem formulation affects not only the algorithmic search for solutions, but also the decision-making process carried out on those solutions, in direct analogy to Fig. 2.

4.1 Evaluating the approximation sets

Figure 4a shows a single-objective projection of the Pareto-approximate solution sets for each of the three formulations, as evaluated in terms of the goal programming aggregate performance objective z (1). Each point in this plot represents a fully specified GAA product family for the 2, 4, and 6-seat planes. The solution from the single-objective formulation is colored yellow, those from the two-objective formulation are colored blue, and those from the ten-objective formulation, red. The solution from the single-objective formulation dominates all of the rest in Fig. 4a, although the solutions from the two-objective formulation gather close to it. The solutions from the ten-objective formulation, however, are separated from the others by a gap, and spread out along the axis.

Figure 4b provides a two-objective projection of the same solutions. The two objectives are those from the two-objective formulation: z and the commonality measurement PFPF (3). Here, the single-objective solution no longer dominates the two-objective solutions, but instead is one of the two-objective formulation's nondominated solutions. The ten-objective solutions remain dominated in this view. The single-objective projection in Fig. 4a would lead the decision-maker to assume that the single-objective solution is a clear winner, with the two-objective solutions close by, and the ten-objective solutions badly dominated. The two-objective projection in Fig. 4b would do little to change this opinion. The yellow single-objective solution provides better performance but worse commonality (i.e., PFPF) than any two-objective solution; so, it remains nondominated with respect to the two-objective formulation. The ten-objective solutions in Fig. 4b appear to be scattered haphazardly, and appear largely dominated with respect to both z and PFPF.

Franssen's demonstration that engineering design is subject to Arrow's theorem (Franssen 2005) implies that optimizing these aggregate measures cannot and does not optimize the individual performance criteria themselves. Arrow's theorem says that the aggregation of preference across multiple individuals will always favor some individuals over others. Franssen's extension of Arrow's theorem to engineering design says that aggregating preference across

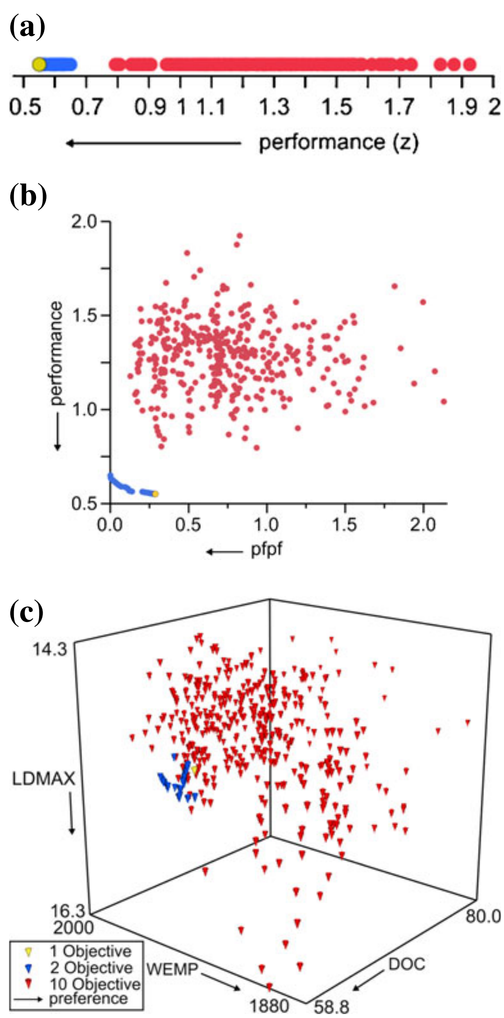

Fig. 4 All three plots show the solutions from all three formulations. Each cone represents performance objectives computed across all three aircraft. The direction of preference is toward minimization, indicated by *arrows* on the three axes. **a** Shows the solutions in terms of the goal-programming objective z. A single solution dominates all of the others. **b** Shows the solutions in terms of z and commonality (PFPF). The one- and two-objective solutions completely dominate the ten-objective solutions in this view. **c** Shows the solutions in terms of three of the nine min-max objectives from the ten-objective formulation. Here, the one- and two-objective solutions perform well on only one of the three visible objectives

multiple criteria will always favor some criteria over others in a manner that will be difficult to ascertain a priori. The aggregate performance objective z is no exception. It must by its nature favor some of the criteria it aggregates over others. Which criteria this formulation favors cannot be determined until after a search has already occurred, because it depends on the range of values that are feasible for each of the aggregated criteria.

The one- and two-objective formulations use goal programming to aggregate the criteria. They reward the search for satisfying constraints and for making progress toward goals for each criterion. These goals are specified a priori, that is, in advance of search or optimization, in the absence of a more global understanding of what is possible for the GAA problem. Performance aggregation in the one- and two-objective formulations using a priori goals rewards and ignores the underlying system's performance objectives in complex ways that a decision-maker would not be able to predict. Indeed, even after the search has been run, understanding the effect of a particular set of a priori goals requires comparing the solution set to solutions resulting from other higher objective count formulations. One approach to doing this is sampling the space of possible goals. However with a problem like this one, having seven goals, the space of possible goals is quite large. Even sampling only two levels for each goal would require searching 128 combinations. This consideration leads to the alternative ten-objective formulation described in Section 2.2 which, by replacing goal attainment with objective seeking, removes the enforced weighting of criteria and allows the search to seek optimal values for each criterion simultaneously.

Figure 4c presents an alternative view of the three sets of solutions. This three-dimensional plot shows the three sets of solutions as measured against the DOC, WEMP, and LDMAX objectives, which are three of the ten min-max objectives in the ten-objective formulation. DOC, WEMP, and LDMAX were chosen because they exhibited the strongest mutual tradeoff of any three out of the ten objectives. An arrow along each axis indicates the direction of preference. The ideal point for these three objectives is thus the near corner of the volume as annotated on Fig. 4c. Given the limitations of geometrical projection and human perception, this plot necessarily collapses a large hypervolume into a three dimensional volume. Despite the highly reduced three-dimensional projected volume, comparing the ten-objective solutions with the one- and two-objective solutions shows the deceptive nature of Fig. 4a and b. The one- and two-objective solutions occupy a tiny portion of the three-dimensional projected volume. In this view, the relative merit of many ten-objective solutions compared to the one- and two-objective solutions becomes apparent.

While the one- and two-objective solutions attain relatively attractive values for DOC, they come nowhere near the ten objective solutions for WEMP and LDMAX. Seeing the ten-objective solutions in this view explains their apparent disorganization and failure to obtain strong performance in Fig. 4a and b. Rather than seeking to optimize solutions to meet a single particular set of preferences, the ten-objective search better covers the space of possible design preferences. None of the ten-objective solutions meet the performance of the one- and two-objective solutions on the aggregate performance objective z used in the one- and two-objective formulation. This means that the particular preference expressed by the goals used in that objective was not exactly duplicated by any of the solutions from the ten-objective formulation, although there are solutions that are very similar.

Figure 4 provides a real application example of the hypothetical illustrations in Brill et al. (1990) and in Fig. 2, which demonstrate that an increase in the number of objectives under consideration in a given problem formulation can cause a decision-maker's preferences among solutions to change dramatically. In our example, we make the additional point that an increase in objectives can change not only preferences between solutions, but the entire conception of a problem. Figure 4 provides a direct analogy with the didactic example of Fig. 2: *the solutions that appear so dominant in one- and two-objective views of the problem are seen to lie on the extreme edge of the set of nondominated solutions within the full context of the problem.*

Figure 5 presents a complementary view of the three solutions sets. While Figure 4 shows geometrical projections of the objective space for each of the formulations, Fig. 5 shows all of the performance criteria, together with PFPF, in three parallel-coordinate plots. As the inventor of the parallel-coordinate plot observed (Inselberg 1997), two of the chief benefits of using parallel-coordinate plots is that they scale to an unlimited number of dimensions and they present the different coordinates in a uniform manner. This makes it possible to consider the problem in full, with all of the criteria independently, and without being tied to the objective formulations which are needed to make the problem computationally tractable.

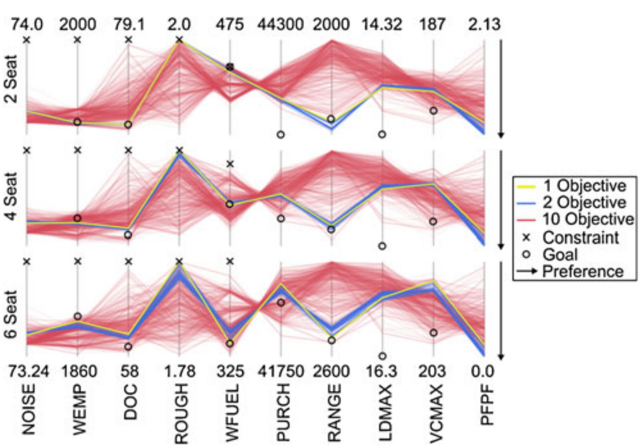

Fig. 5 Parallel-coordinate plots showing all nine performance criteria for each aircraft, along with PFPF. PFPF is duplicated across the three plots. The direction of preference is the same for each axis, so *crossing lines* indicate conflicting criteria. The single-objective solution is shown in *yellow*, the two-objective solutions in *blue*, and the ten-objective solutions in *red*. The two-objective solutions exhibit much less diversity than the ten-objective solutions. They manifest the implicit preference structure they share with the single-objective formulation, imposed by aggregating all of the performance criteria into a single objective

Figure 5 shows a set of three parallel-coordinate plots: one for each of the three aircraft in the GAA product family. Each of these parallel axis plots shows the nine aircraft performance criteria. Rather than the aggregate performance objective z or the min-max performance objectives used in the ten-objective formulation, Fig. 5 shows the individual performance measurements as predicted by the response surface model described in Section 2.2. Each parallel-coordinate plot in Fig. 5 represents a solution as a set of eight connected line segments crossing the nine axes. Solutions cross each axis at the values of their corresponding performance in a given criterion. Because a solution represents a set of designs for the whole product family, each solution appears on all three plots, with different values for the performance criteria corresponding to each member of the GAA product family.

Figure 5 displays a comprehensive view of all solutions from the three alternative formulations of the GAA problem, indicating not only the values attained for each criterion by each solution, but also the constraint levels, indicated by an X, and the goal values, indicated by an O. For ease of comparison, each axis has been oriented so that the direction of preference for a given performance criterion is always downward. This makes it easy to see when criteria on adjacent axes trade off against one another. If many of the line segments representing solutions cross each other between two adjacent axes, then there is a tradeoff between those two criteria. If no tradeoff, or a weak tradeoff exists, then the solutions take nearly parallel paths between adjacent axes. The ordering of axes does strongly influence the identifiability of tradeoffs and users should carefully consider an array of alternative orderings as part of their visual analytic exploration of Pareto approximate set data. For more guidance, we refer the reader to Seo and Shneiderman (2005). For example, DOC and ROUGH trade off against each other, but there is little conflict between NOISE and WEMP.

The parallel-coordinate plots in Fig. 5 reinforce the observation that compared to the ten-objective solutions, the two-objective solutions offer minimal diversity. The single-objective formulation makes no attempt to provide diverse solutions. Comparing the two-objective solutions to the goals and constraints that define the two-objective formulation yields some insight into how the two-objective formulation drives the search for solutions. NOISE has no effect, because all feasible solutions meet the constraint. WEMP presents an achievable goal for all three aircraft, so the two-objective solutions, consistent with their formulation, just barely achieve it.

The ten-objective solutions demonstrate that considerable further improvement in WEMP is possible, but because the aggregate performance objective in the two-objective formulation does not reward it, no two-objective solutions cover that part of the space. The one- and two-objective solutions fail to attain the goals for VCMAX, not because the goals are unattainable, but because the aggregate performance objective z rewards improvement in VCMAX less than it penalizes the accompanying losses on other criteria. This illustrates Franssen's argument that engineering design is subject to Arrow's Impossibility Theorem. Whether or not the decision-maker actually values VCMAX less than the other criteria, forcing all the criteria into a single objective function imposes preferences among them, and in this case VCMAX loses out. Preemptive goal programming formulations, such as the alternative formulations from Simpson et al. (1996), take the surprise out of this imposition of preference by making it explicit. The reason for choosing the non-preemptive formulation for z used in this study is that it is ostensibly neutral with respect to the various goals. Figure 5 demonstrates that this neutrality consists of allowing preferences to be determined arbitrarily by the problem space rather than specifying them in advance. The single objective formulation of the GAA product family design problem in Fig. 5 clearly illustrates the unintended consequences of aggregation.

Figure 6 provides a visualization of the decision space for the aircraft as a set of nine parallel-coordinate plots. Each of the plots in Fig. 6 corresponds to one of the decision variables for each aircraft. This layout follows that introduced by (Slingerland et al. 2010) for visualizing product family designs. A solution appears as a set of two connected line segments in each plot. Where those line segments cross each of the axes represents the value of the corresponding decision variable. Each solution appears on each of the nine plots. Where Fig. 5 displayed the 9 explicitly modeled performance criteria for each aircraft, Fig. 6 makes it possible to consider what those aircraft would actually look like if manufactured. Since no mathematical formulation can adequately capture all of a decision-maker's concerns, Fig. 6 invites the consideration of unmodeled objectives (Brill et al. 1990). Some of the solutions, for instance, include a 6 seat aircraft with seats 15 inches wide—narrower than most full-grown adults. A hypothetical decision-maker might be unwilling to commit to a design offering so little elbow room. In addition, Fig. 6 presents an alternate view of commonality. Where PFPF aggregates commonality into a single number, Fig. 6 displays commonality visually: where the two line segments go straight across the three axes, all three aircraft use the same value for that decision variable. Diagonal line segments, in contrast, show a lack of commonality. This display allows the decision-maker to evaluate commonality in terms of its effect on the design of the three aircraft in the product family, unlike the PFPF aggregate, which allows the decision-maker to consider commonality only in the abstract.

Comparing the three formulations in Fig. 6, it becomes clear that the relative lack of diversity for the two-objective solutions in the objective space corresponds to a lack of diversity in the designs as expressed in decision space. All of the two-objective solutions are approximately the same as the single-objective solution. Adding commonality in the form of PFPF to trade off against aggregate performance as represented by z exposes a very narrow tradeoff among designs that are scarcely different from one another. The

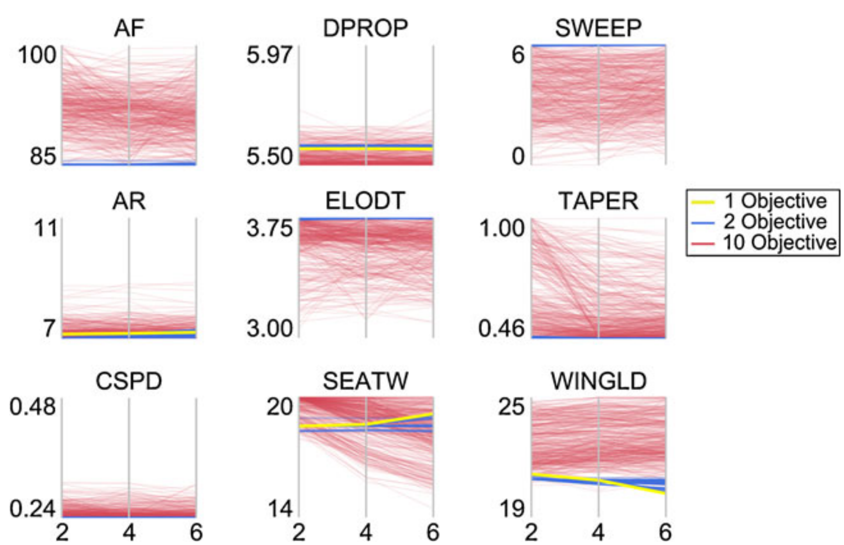

Fig. 6 Parallel-coordinate plots showing decision variables. Each of the nine plots shows the values of a decision variable across all three aircraft in a product family. *Lines going straight* across a plot show high commonality for that decision variable, while *diagonal lines* indicate variation between aircraft. Constraints are binding, or nearly so, on the one objective solution and on all of the two objective solutions for six of the nine decision variables. The ten-objective solutions, in contrast, cover the feasible design space. Where the single-objective solution can be distinguished from the two-objective solutions, it is highlighted in *yellow*. Otherwise, it is the same as the two-objective solutions

single-objective solution, and all of the two-objective solutions are bound by the constraints for AF, SWEEP, ELODT, TAPER, and CSPD, and confined to a comparatively narrow band for DPROP, AR, SEATW, and WINGLD. None of these solutions exhibits a large amount of variation between the three aircraft.

The ten-objective solutions, in contrast, explore a wide variety of configurations for all twenty-seven decision variables, consistent with their much greater diversity in objective space. A diverse set of alternatives like that provided by the ten-objective formulation is desirable because it permits the decision-maker to explore the whole nondominated design space and evaluate its solutions, forming hypotheses about the relationships among objectives and decision variables, and testing them against the selected solutions. This is the procedure illustrated by Inselberg (1997) and recommended by Brill et al. (1990). For the GAA problem, a diverse set of alternatives allows a decision-maker to explore different design and marketing scenarios and decide which among them represents the best manufacturing and business strategy.

4.2 Choosing a design

Figures 5 and 6 show the diversity of solutions offered by the ten-objective formulation, and this presents both an opportunity and a challenge to the decision-maker. The opportunity is that, within such a diverse set of solutions, the decision-maker will discover a solution that opens up a new market segment or dominates an old one. The problem is that there are so many solutions to the ten-objective problem, each occupying a different point on the many-objective Pareto-approximate front. Brill's recommendation (Brill et al. 1990) in this regard is to present a small number of maximally diverse solutions. This section describes how a decision-maker equipped with appropriate visual analytic software can reduce a large set of solutions as they discover the tradeoffs for attainable performance.

Figure 7 illustrates the process of selecting solutions. It shows the same three projections as Fig. 4, highlighting selected solutions. Figure 7a highlights the single optimal solution (Solution A) to the single-objective formulation. Solution A is uniquely optimal with respect to z, although as was observed in the discussion of Fig. 5, it represents a particular set of preferences with regard to the full set of performance criteria. Figure 7b puts PFPF on the X axis and z on the Y axis. Three highlighted solutions in Fig. 7b represent the greatest possible diversity among the two-objective solutions. One of these is Solution A, which minimizes z. Its extreme position reflects the change in perspective brought by adding a second objective. Now that PFPF is considered as well, Solution A seems extreme, revealing the myopia (Brill et al. 1990; Hogarth 1981) of the single-objective formulation. Its opposite extreme is also highlighted in Fig. 7b, as well as the solution (Solution B) closest to the ideal point for PFPF and z. It is from among these three solutions that a decision-maker using the two-objective formulation would choose a solution. Supposing that the decision-maker's priority reflects the two-objective formulation's emphasis on balancing performance with commonality, then the solution selected for implementation would be

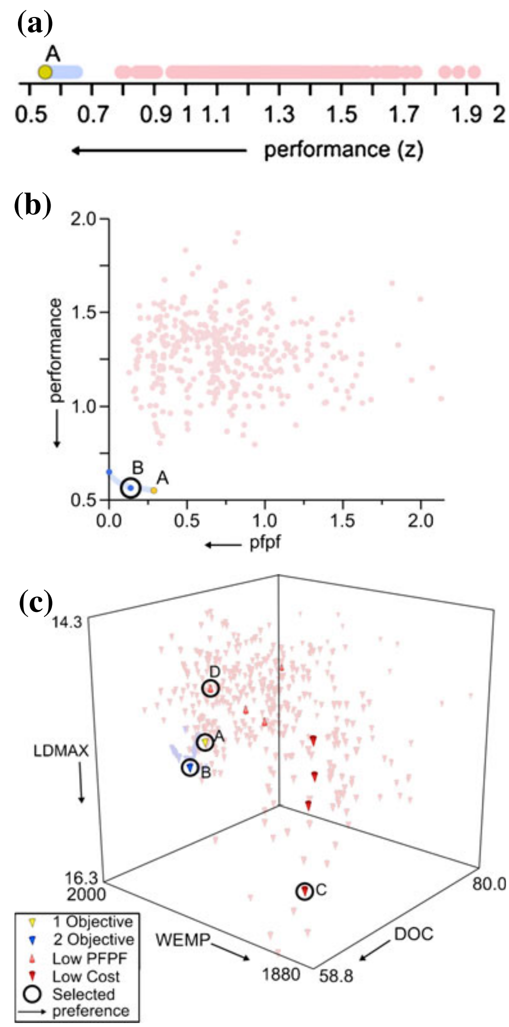

Fig. 7 The one-, two-, and three- objective projections shown in (**a**), (**b**), and (**c**) are the same as those in Fig. 4. The single-objective solution is marked in *yellow*, the two-objective solutions in *blue*, and the ten-objective solutions in *red*. The solutions for (a) and (b) have been brushed according to the preference structures of the one- and two-objective formulations. The solution marked in (a) minimizes the performance deviation function z, while the solutions marked in (b) represent the two-objective tradeoff between z and commonality (PFPF). Of the many possible preference structures supported by the ten-objective formulation, two are indicated in (c): low PFPF and low cost (DOC and PURCH). One solution from each preference structure is highlighted as representative. Solution A is the optimal single-objective solution, Solution B is the best compromise point between z and PFPF, Solution C represents the low cost solutions, and Solution D represents the low PFPF solutions

Table 6 Decision variables for each of the selected designs

Solution	Aircraft	CSPD	AR	SWEEP	DPROP	WINGLD	AF	SEATW	ELODT	TAPER
One	2 seat	0.240	7.128	6.000	5.573	21.16	85.20	18.55	3.738	0.468
objective	4 seat	0.240	7.150	5.966	5.564	20.85	85.21	18.64	3.750	0.460
(A)	6 seat	0.240	7.186	5.999	5.563	20.19	85.45	19.17	3.750	0.460
Two	2 seat	0.240	7.000	5.989	5.575	20.90	85.13	18.55	3.749	0.462
objective	4 seat	0.240	7.028	5.997	5.575	20.65	85.22	18.55	3.749	0.4600
(B)	6 seat	0.240	7.172	6.000	5.577	20.40	85.10	18.66	3.750	0.460
Low	2 seat	0.240	7.678	4.611	5.619	22.76	88.43	18.30	3.630	0.514
cost	4 seat	0.240	7.770	4.848	5.597	23.84	88.34	16.02	3.620	0.587
(C)	6 seat	0.241	7.650	4.420	5.592	23.06	89.19	15.44	3.623	0.508
Low	2 seat	0.268	7.094	1.468	5.509	22.62	89.25	19.23	3.657	0.566
PFPF	4 seat	0.268	7.181	1.289	5.502	22.48	88.47	18.97	3.635	0.547
(D)	6 seat	0.275	7.075	1.482	5.510	22.48	89.03	19.39	3.644	0.563

the compromise solution, namely, Solution B. Since there is little variation between solutions in the two-objective formulation, we henceforth take Solution B to represent the two-objective formulation.

Figure 7c shows solutions from all three formulations, plotted against the min-max objectives DOC, WEMP, and LDMAX. This figure highlights solutions corresponding to two of the many possible preferences that could be imposed on the ten-objective solution set. Dark red glyphs, with tips pointing down, stand for solutions with low cost: they represent solutions with purchase price below $42,500 and direct operating cost below $60 per nautical mile. Light red glyphs, with tips pointing up, represent solutions with high commonality: that is, with PFPF below 0.17. The final selection of a solution for implementation is then a refinement on the preferences already articulated. It does not necessarily represent the decision-maker's original intent, but may instead be a design with desirable characteristics that the decision-maker had not conceived of before the set of solutions became known (i.e., a discovery that arises during the process of interactive visualization). Among the low-cost solutions, there was a solution with high LDMAX and VCMAX that presented an opportunity to produce a family of cheap, fast maneuverable aircraft. This is

Table 7 Performance criteria and objectives for each of the selected designs

Solution	Aircraft	NOISE	WEMP	DOC	ROUGH	WFUEL	PURCH	RANGE	LDMAX	VCMAX	PFPF
One	2 seat	73.5	1900	60	2.000	435	42000	2500	15.6	196	
objective	4 seat	73.5	1940	62	2.000	405	43000	2450	15.4	192	
(A)	6 seat	73.5	1980	62	2.000	345	44000	2500	15.3	188	
	Objective	73.5	1980	62	2.000	435	44000	2450	15.3	188	0.3
Two	2 seat	73.5	1900	60	2.000	435	42000	2550	15.6	196	
objective	4 seat	73.5	1940	62	2.000	405	43000	2500	15.3	192	
(B)	6 seat	73.5	1960	62	2.000	360	44000	2450	15.3	190	
	Objective	73.5	1960	62	2.000	435	44000	2450	15.3	190	0.1
Low	2 seat	73.7	1880	58	2.000	450	42000	2200	15.9	200	
cost	4 seat	73.6	1880	58	1.975	465	42000	2050	15.8	202	
(C)	6 seat	73.6	1880	60	2.000	450	42000	2250	15.8	200	
	Objective	73.7	1880	60	2.000	465	42000	2050	15.8	200	0.8
Low	2 seat	73.3	1900	70	1.925	435	42000	2250	15.3	196	
PFPF	4 seat	73.3	1940	70	1.925	405	43000	2200	15.1	194	
(D)	6 seat	73.3	1960	72	1.875	360	44000	2150	14.9	192	
	Objective	73.3	1960	72	1.925	435	44000	2150	14.9	192	0.2

Values are rounded to the nearest ε

Solution C, shown in Fig. 7c. Alternatively, one of the low-PFPF solutions had very low takeoff noise and low ride roughness, Solution D, also shown in Fig. 7. This presented an opportunity to market a family of aircraft with a smooth, quiet ride, while taking advantage of parts commonality to simplify the manufacturing process. The ten-objective formulation makes it possible to accommodate both sets of preferences, and any set of preferences among the decision criteria, without knowing anything about those preferences in advance. Indeed, preferences are likely to develop and evolve opportunistically in response to available solutions with desirable characteristics, like Solutions C and D. So while the brushing itself is done on a parallel-coordinate plot, its effects can be viewed on the geometrical projection of Fig. 7c.

Tables 6 and 7 provide detailed descriptions of Solutions A, B, C, and D. Table 6 displays the values of their decision variables, and Table 7 contains their performance criteria and objective function values. From Table 6, it can be observed that Solution B achieves its low PFPF by holding CSPD, SWEEP, DPROP, AF, SEATW, ELODT, and TAPER nearly constant, and allowing WINGLD and AR to vary across the three aircraft. Comparing the four solutions shown in Table 6, big differences among the three product family designs appear in the wing sweep angle (SWEEP), wing taper ratio (TAPER), and wing loading (WINGLD). This points to the configuration of the wing as a critical differentiator between product family designs. Another notable difference among the three product family designs is the seat width. Solution C squeezes relatively small seats into the four- and six-seat aircraft, Solution B holds the line across all three aircraft with wider seats. Solution D provides the most generous seats of all, which along with its low takeoff noise and low roughness makes it a solution that has not only high commonality but a very comfortable ride.

5 Conclusion

This study demonstrates the benefits of adding objectives and reducing aggregation, using the conceptual design of a GAA product family. We trace the evolution of problem conception and design selection from an early single-objective formulation first published in 1995, through a two-objective formulation from 2004, to a ten-objective formulation introduced in 2010. Moving from the early, highly aggregated formulations to the ten-objective formulation reveals the problem structure and leads to the discovery of solutions that could not be found using the lower-dimensional formulations.

The example of the GAA problem parallels the theoretical recommendations posed by Brill et al. (1990), that increasing the number of objectives considered in complex, ill-defined planning problems can avoid myopic decision biases. Our single-objective and two-objective formulations for the GAA product family rely heavily on an aggregate performance objective. This objective imposes unintended preferences among the performance criteria as a result of Arrow's impossibility theorem (Franssen 2005). Solutions found using the ten-objective formulation reveal that the aggregate performance objective drives optimization to an extreme corner of the problem space. Brushing and dynamic filtering across linked plots enabled a rapid and flexible exploration of the high-dimensional objective space. Using these visual analytic tools, we found two families of solutions having distinctly different performance advantages aligned with two of the many possible preference structures that could be imposed a posteriori on this solution set. The existence of these solutions and the consequences of aggregation would not be known in advance; so, finding these solutions using an aggregate performance objective would be nearly impossible.

This work suggests several areas for further exploration. Considering problem formulation broadly includes not only the objective formulation, but also the underlying model. For the GAA problem considered in this study, we used a response surface metamodel. Given that the solutions obtained using a metamodel may perform differently when evaluated against the original model, and the original model may disagree with a physical design realization, it would be instructive to consider changing the model or metamodel as part of the constructive learning process embodied in our proposed MOVA framework. In addition, this study considered only performance, not robustness. Adding robustness objectives to a problem represents another aspect of the learning process: not only asking what solutions are good, but what solutions are reliably good as our understanding of the problem changes? Furthermore, the ideas embodied by the MOVA framework are applicable beyond just the GAA problem, and beyond product family design problems, to the whole domain of complex engineered systems. Finally, the conceptual basis of the MOVA framework has connections to several large bodies of literature, from economics to cognitive psychology to evolutionary optimization. Exploring these links may yield further insights and refinements to many-objective visual analytics to support putting the human "back-in-the-loop" during the design of complex engineered systems (Simpson and Martins 2011).

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