Walkthrough on Channel Equalization

Disclaimer

This walkthrough is solely based on the author's self-visualization of the assignment. Readers are asked to study further(Reference book's 9.1-9.5 or wiki article on Viterbi algorithm) to understand the workflow clearly on their own.

1 What to do:in short

Apply Viterbi algorithm to estimate the bit sequence received from a channel, by building a **Hidden Markov Model** and observing the continuous valued responses.

1.1 Training phase : in short

You have to build a hidden markov model based on randomly generated data, i.e: bit sequences.

1.2 Test phase:in short

You have to apply Viterbi algorithm based on the built hidden markov model and randomly generated data to estimate the bit sequence and compare the accuracy.

2 Explanation

2.1 Hidden Markov Model

A HMM consists of some parameters explained below. I am going to brief it through a temparature check-up and health check up models

- **States**: Domain of the states the sequence can turn into time to time:$\{healthy, sick\}$

- **States' prior probabilities**: Initial probability that the sequence will start from a particular state : $\{healthy : 0.7, sick : 0.3\}$

- **State to state transition probabilities**: Probabilities that a sequence will make transition from a particular state to another state. If the transition is not possible,the probability will be 0: $\{(healthy, healthy) : 0.8, (healthy, sick) : 0.2, (sick, healthy) : 0.4, (sick, sick) : 0.6\}$

- **Observation Space**: Domain of the observations. It can be discrete: $\{working, cold, hot, cannotmove\}$ or can be continious ,i.e body temparature while being in any of the states.

- **Observation set**: A set of observations for different times in a sequence; i.e. a patient's body temparature readings for 4 days: $\{98.0, 102.0, 101.8, 99.0\}$

2.2 Viterbi Algorithm

Viterbi algorithm will take all of the above mentioned spaces/sets as parameters and will return the most likely path of the states the sequence went through; i.e: for the stated example, the states of the patient for those 4 days might be $\{healthy, sick, sick, healthy\}$

3 What we have to do in this assignment

Let I be a sequence of bits, i.e. $I = 0101011110$.
Let there be a channel equalizer with impulse response $f(.)$ and noise associated with it is normally distributed with mean 0 and variance σ^2. The channel will send a sequence of bits and in other side a real valued response x will be recieved. It is assumed that $f(.)$ is a linear combination of previous n sent bits and noise, i.e., for the kth bit's response x_k

$$x_k = \sum_{i=0}^{n-1} h_i I_{k-i} + \eta_k$$

n, the number of last bits the current response will depend, $H = \{h_0, h_1, ..., h_{n-1}\}$, i.e. $\{1.2, 2.3, 0.1\}$ for $n = 3$ and variance σ^2 of noise will have to be taken from a file.
The noise η_k will be generated randomly under normal distribution with

mean 0 and variance σ^2.
From the sequence of real valued observations $\{x_1, x_2, x_3, ..., x_N\}$, i.e. {2.3,1.2,....,4.6}, we will have to determine the most likely sequence of N bits, i.e. $\hat{I_1}\hat{I_2}\hat{I_3}...\hat{I_N} =$ 011.....1 using Viterbi algorithm and HMM.
Firstly,we need to build the HMM. Let's assume that $n = 3$.
States will be the all possible sequences of 3 bits $I_{k-2}I_{k-1}I_k$: 000,001,010,.....111.
There will be 8 states.
Each state will have same prior probability $\frac{1}{8}$.
There are 8×8 transition possiblities, theoritically. However, you can only jump from any state to only two other states by excluding I_{k-2} and including I_{k+1}. So,there can be $8 \times 2 = 16$ transitions. For each state, those two transitions will have 0.5 probabilities each, while the other 6 transitions will have 0 probability.
Observation space will be real number for each state, distributed normally.
Determining observation probabilities for each state is the thing **WHERE THE TRAINING IS DONE**. Observation probabilities will be under Normal distribution. You need to derive their means and variances.
Generate random sequences of bits, pass them through the channel, gather response sequences for each bit sequence. From each bit sequences, look for the response value received for each state present there. By this way, for each state, accumulate all the real valued responses ranging over the generated sequences and take their average. By this, you will get the mean of normal distribution of the observation probability of each state, as the average of samples determine the population mean of the estiamed distribution. As noted in the specification, variance is not needed for this . You can assume it as 1. Or if you want to be meticulous, take the sample variance of the responses for each state (don't forget, for sample variance, you have to divide by $n - 1$, not by n).
TESTING!!!. Generate samples of sequence of bits,transmit them through the channel and gather the response sequence. Apply viterbi algorithm to get the most possible state path. From the state path, i.e. 010–101–011–111, you will get the estimated bit sequence, i.e. 010111.
Compare the accuracy now, by hamming or Euclidean or if possible, Mahalonbis distance(!!).

4 Conclusion

DONT FORGET!!. Don't put 1 or 0 as value in the equation for the bits. Put +1 for 1 (you don't say!) and −1 for 0.

Remember, you have to implement a class for the channel with H and σ^2 as class variables and a transmit function which will generate the response x_k based on given previous bit sequence (as per specification).
Also, be careful about implementing efficiently.