UNIVERSITY OF ABERDEEN

Independent samples t-test

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- Comparing groups of continuous data or categorical data?
- Number of groups for comparison
- Paired or independent groups?
- Parametric vs non-parametric tests

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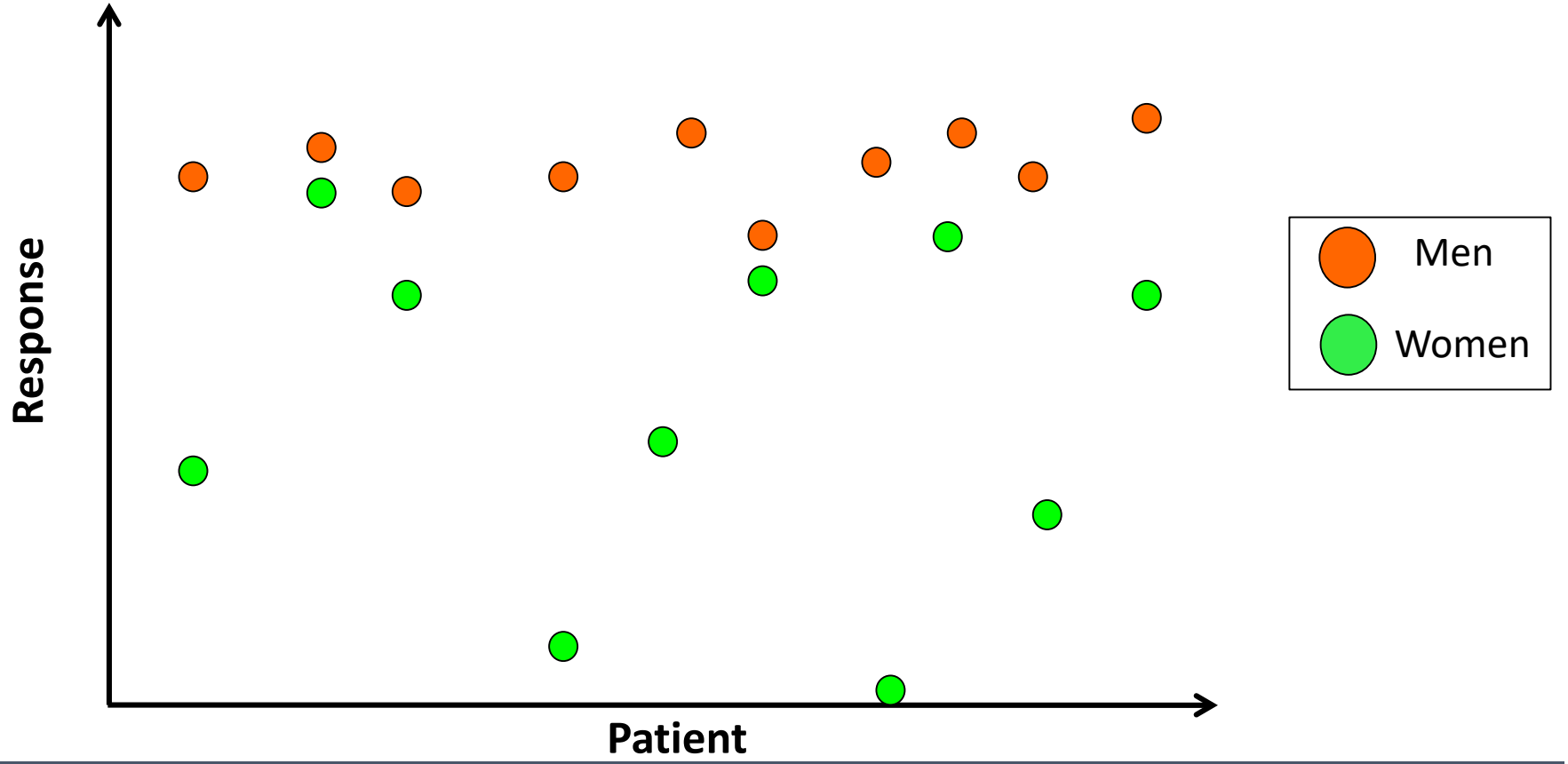
3. Variances are assumed to be equal

Unequal variances



Assumptions of the independent groups t-test

1. Distribution: Outcome variable is normally distributed in both groups

   CHECK BY PLOTTING A HISTOGRAM FOR EACH GROUP

2. Comparison is between two independent groups

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USE LEVENE'S TEST:

- H_o: There is no difference between the variance of group 1 and the variance of group 2 in the population

  (i.e. $H_o: \sigma_1^2 = \sigma_2^2$)

Example of independent samples t-test

- Is there a difference between the mean total cholesterol of males and females?

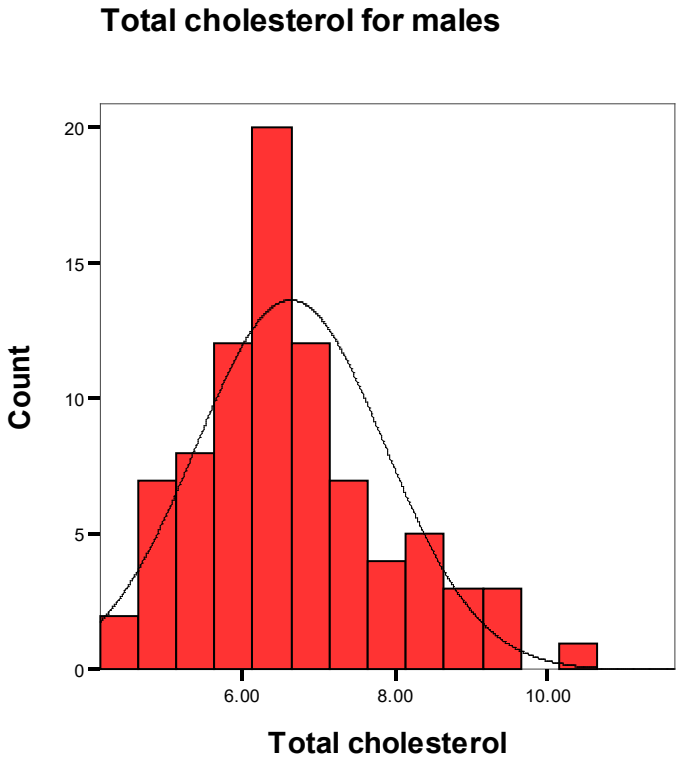
- The null hypothesis is that the group means are the same. That is:

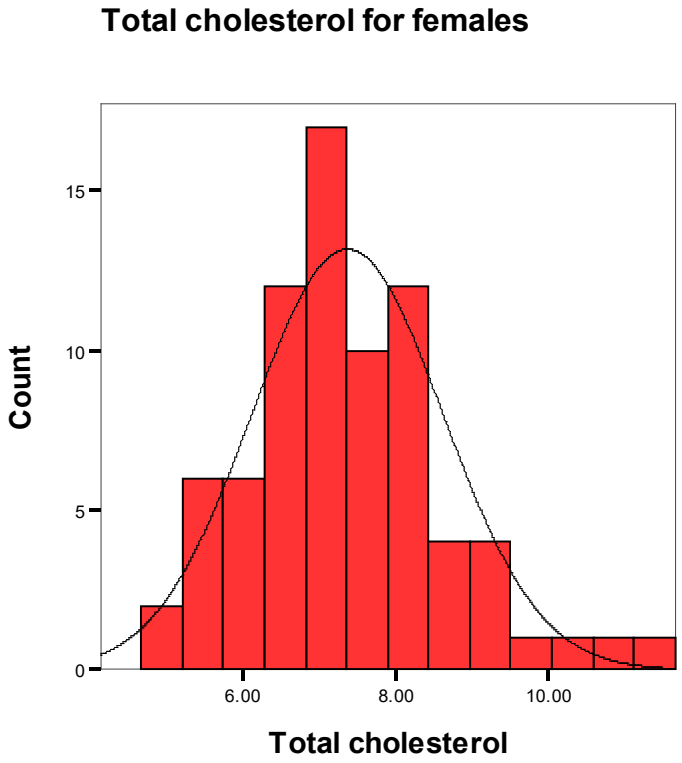
$$H_0: \mu_{Males} = \mu_{Females}$$

- The alternative hypothesis:

$$H_1: \mu_{Males} \neq \mu_{Females}$$

Normality assumption for example





Example of independent samples t-test

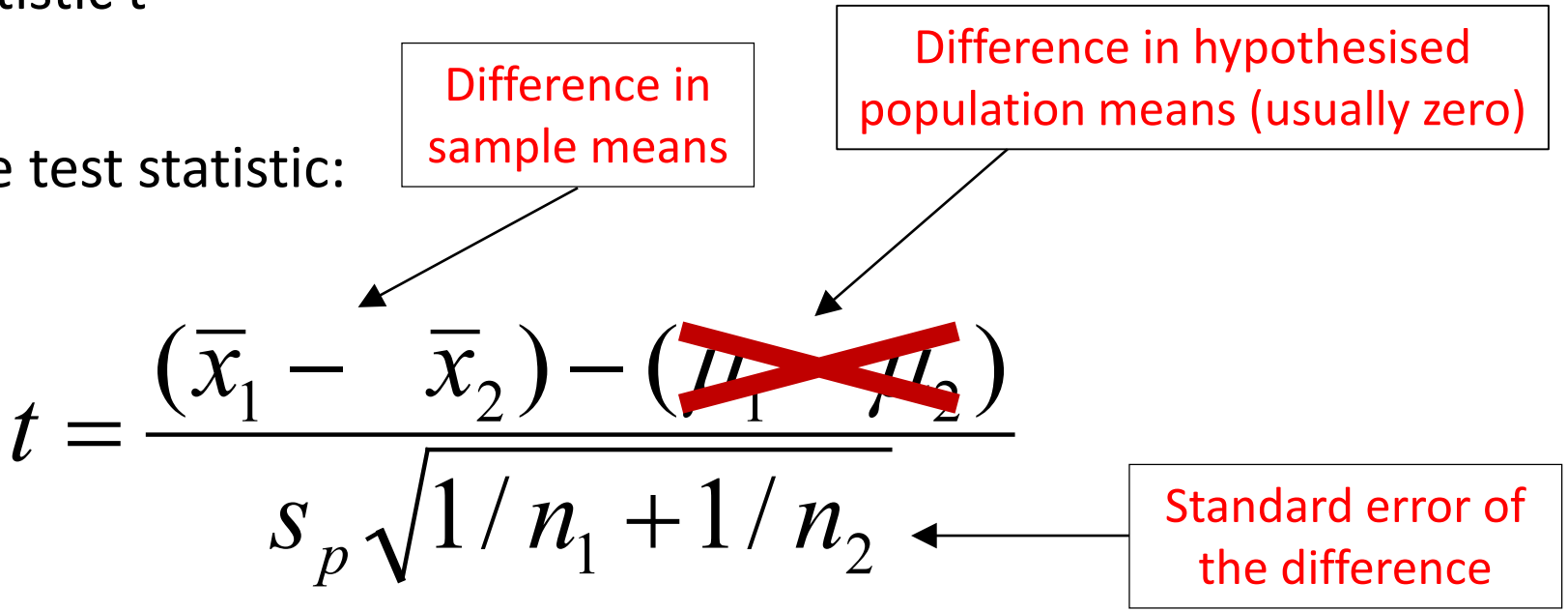
Group Statistics

	Sex	N	Mean	Std. Deviation	Std. Error Mean
Total cholesterol (mmol/l)	Female	77	7.3622	1.25750	.14331
	Male	84	6.6262	1.23788	.13506

Test statistic

- In order to perform the test, we must again construct a test statistic t

- The test statistic:



Test statistic

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p \sqrt{1/n_1 + 1/n_2}}$$

Group Statistics

	Sex	N	Mean	Std. Deviation	Std. Error Mean
Total cholesterol (mmol/l)	Female	77	7.3622	1.25750	.14331
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Test statistic

$$t = \frac{(7.36 - 6.63) - 0}{s_p \sqrt{1/77 + 1/84}}$$

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s_p is the pooled SD i.e. the average of the 2 sample variances

$$s_p = \sqrt{\frac{(df_1)s^2{}_1 + (df_2)s^2{}_2}{df_1 + df_2}}$$

Test statistic

$$t = \frac{(7.36 - 6.63) - 0}{s_p \sqrt{1/77 + 1/84}}$$

Group Statistics

	Sex	N	Mean	Std. Deviation	Std. Error Mean
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s_p is the pooled SD i.e. the average of the 2 sample variances

$$s_p = \sqrt{\frac{(76)1.2575^2 + (83)1.2379^2}{76 + 83}} = 1.247$$

Test statistic

$$t = \frac{(7.362 - 6.626) - 0}{1.247\sqrt{1/77 + 1/84}}$$

Therefore,

$$t = 3.74$$

with 77+84-2=159 df

Group Statistics

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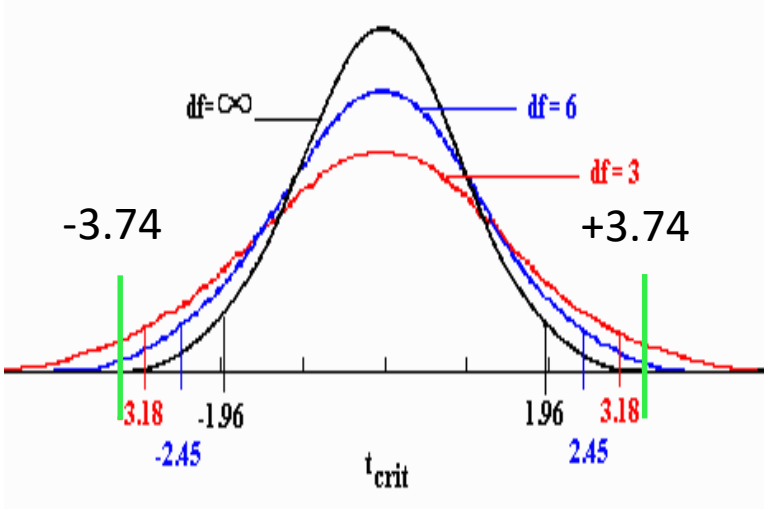

http://www.psychstat.missouristate.edu/introbook/sbk24m.htm

Output from example

Group Statistics

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Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Total cholesterol (mmol/l)	Equal variances assumed	.011	.918	3.740	159	.000	.73602	.19679	.34736	1.12467
	Equal variances not assumed			3.738	157.322	.000	.73602	.19692	.34706	1.12497

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Group Statistics

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Therefore assumption of equal variances in each group holds

Test statistic, t

df=no. of males + no. of females - 2

P<0.05

Mean difference = mean for males – mean for females

Conclusions

- Female total cholesterol levels were on average 0.736mmol/l higher than male total cholesterol.
- The value of t=3.74 is highly significant at the 5% level.
- Therefore, we reject the null hypothesis and conclude that female total cholesterol levels are significantly higher than male total cholesterol levels in the population.
- In the wider population there is a 95% chance that the actual mean difference is between 0.347 and 1.125mmol/l.

Summary

- Independent samples t-test is a parametric test that is used when we want to compare the mean of a continuous measurement in two independent groups.
- The assumptions:
  - The continuous or outcome measure must be approximately normally distributed in each group
    - Tested by drawing a histogram
  - The variance of the outcome measure is assumed equal in both groups
    - Tested using Levene's test.

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